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ORIGINAL ARTICLE

Electron-acoustic solitons in plasma with nonthermal electrons

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Abstract Nonlinear electron-acoustic solitary waves (EASWs) are studied using Sagdeev's pseudo-potential technique in a collisionless unmagnetized plasma consisting of a cold electron fluid, nonthermal hot electrons and stationary ions. It is shown that the presence of fast nonthermal electrons may modify the parametric region where electron-acoustic solitons may exist. Our investigation is of wide relevance to astronomers and space scientists working on interstellar space plasmas.

Keywords Electron-acoustic waves · Solitary waves · Sagdeev potential · Mach number · Nonthermal electrons

1 Introduction

An electron-acoustic wave (EAW) can exist in a two-temperature (cold and hot) electron plasma (Watanabe and Taniuti 1977). The phase speed of the electron-acoustic wave is much larger than the thermal speed of cold electrons and ions, but is usually smaller than the thermal speed of the hot electron component. Watanabe and Taniuti (1977) used a linear electrostatic Vlasov dispersion equation to show that electron-acoustic waves can be destabilized in such a plasma. Later on, Yu and Shukla (1983) and Gary and Tokar (1985) obtained a dispersion relation for EAWs in a two (electron–ion) and three component plasma (two electrons and ions), respectively. The electron-acoustic solitary wave (EASW) is a localized nonlinear wave phenomena which arises due to a delicate balance between nonlinearity and dispersion. EASWs have been studied theoretically (Schamel 2000) as well as numerically (Valentini et al. 2006). They have been observed in experiments with pure electron plasmas (Kabantsev et al. 2006) and in laser-produced plasmas (Sircombe et al. 2006) and related numerical simulations (Ghizzo et al. 2006). Nonlinear propagation of EASWs has been investigated by several authors (Yu and Shukla 1983; Gary and Tokar 1985; Mace and Hellberg 1983; Dubouloz et al. 1991; Mace et al. 1991; Berthomier et al. 2002; Mamun and Shukla 2002; Mamun et al. 2002; Chatterjee and Roychoudhury 1995; Berthomier et al. 2000; Clarmann et al. 2002; Mace and Helberg 2001; Berthomier et al. 2003; Shukla et al. 2004). This propagation has been studied in unmagnetized two-temperature electron plasmas (Dubouloz et al. 1991; Chatterjee and Roychoudhury 1995; Berthomier et al. 2000; Clarmann et al. 2002) as well as magnetized plasmas (Mamun and Shukla 2002; Mace and Helberg 2001; Berthomier et al. 2003; Shukla et al. 2004). Mamun et al. (2002) derived a modified Korteweg–de Vries (mKdV) equation in a collisionless plasma having cold fluid electrons, hot non-isothermal electrons obeying a vortex-like distribution, and stationary ions. They investigated the effect of electron trapping on small but finite amplitude acoustic solitary waves. On another side, space plasma observations indicate clearly the presence of ion and electron populations which are far away from their thermodynamic equilibrium (Shukla et al. 1986; Ghosh and Bharuthram 2008;

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Pakzad 2009a, 2009b). Fast energetic electron distributions are observed in different regions of the magnetosphere. In a very interesting and influential paper (Cairns et al. 1995), Cairns et al. showed that the presence of a nonthermal distribution of electrons may change the nature of ion sound solitary structures and allow for the existence of rarefactive ion-acoustic solitary structures very like those observed by the Freja and Viking satellites. A possible scenario is that lower hybrid turbulence produces, through modulational instability, cavities which collapse until the lower hybrid wave amplitude is sufficient to trap and accelerate a substantial number of electrons. Singh and Lakhina (2004) derived the Sagdeev pseudopotential for localized electron-acoustic waves in an unmagnetized three component plasma with nonthermal electrons. They showed that the inclusion of nonthermal electrons leads to the existence of negative potential structures.

It is well known that the inclusion of nonthermal electrons may change the properties as well as the regime of existence of solitons (Singh and Lakhina 2004). The aim of the present paper is therefore to study the effect of electron nonthermality on the existence and possible realization of electron-acoustic solitary waves within the theoretical framework of Mamun and Shukla model (Mamun and Shukla 2002). Our investigation may be of wide relevance to astronomers and space scientists working on interstellar and space plasmas. The manuscript is organized as follows: In the next section, we present the basic equations of our theoretical model and derive the pseudo-potential associated to localized electron-acoustic solitary waves. Our results are presented and discussed in Sect. 3. A summary of our results and conclusions is given in Sect. 4.

2 Basic equation

Let us consider a collisionless unmagnetized plasma consisting of a cold electron fluid, hot electrons obeying a nonthermal distribution and stationary ions. The nonlinear dynamics of the electron-acoustic solitary waves is governed by the following set of normalized basic equations

$$\begin{aligned} &\frac{\partial n_c}{\partial t} + \frac{\partial (n_c u_c)}{\partial x} = 0, \\ &\frac{\partial u_c}{\partial t} + u_c \frac{\partial u_c}{\partial x} - \alpha \frac{\partial \phi}{\partial x} = 0, \\ &\frac{\partial^2 \phi}{\partial x^2} - \frac{1}{\alpha} n_c - n_h + \left(1 + \frac{1}{\alpha}\right) = 0, \end{aligned} \tag{1}$$

n_h is the nonthermal hot electron density and it is given by Cairns et al. (1995)

$$\begin{aligned} &n_h = \left(1 - \beta\phi + \beta\phi^2\right) \exp(\phi), \\ &\beta = \frac{4\delta}{1 + 3\delta}. \end{aligned} \tag{2}$$

Here δ is a parameter determining the number of nonthermal electrons present in our plasma model.

In the above equations, n_c (n_h) is the cold (hot) electron number density normalized by its equilibrium value n_{c0} (n_{h0}), u_c is the cold electron fluid velocity normalized by $C_e = (k_B T_h/\alpha m_e)^{1/2}$, ϕ is the electrostatic wave potential normalized by $k_B T_h/e$, k_B is the Boltzmann's constant, e the electron charge, m_e its mass, and $\alpha = n_{h0}/n_{c0}$. The time and space variables are in units of the cold electron plasma period ω_{pc}^{-1} and the hot electron Debye radius λ_{Dh}, respectively.

Now, making use of the transformation $\chi = x - Mt$ (where M is the Mach number, solitary wave speed/C_e) along with the appropriate boundary conditions for localized perturbations ($\phi \to 0$, $u_c \to 0$ and $n_c \to 1$ as $\chi \to \infty$), one obtains the following quadrature

$$\frac{1}{2}\left[\frac{d\phi}{d\chi}\right]^2 + V(\phi) = 0 \tag{3}$$

where $V(\phi)$ is the Sagdeev potential and reads

$$\begin{aligned} V(\phi) = &\frac{M^2}{\alpha^2}\left(1 - \sqrt{1 + 2\alpha\phi/M^2}\right) \\ &- \left[1 + 3\beta - 3\beta\phi + \beta\phi^2\right] \exp(\phi) \\ &+ (1 + 1/\alpha)\phi + (1 + 3\beta). \end{aligned} \tag{4}$$

Equation (4) can be regarded as an "energy integral" of an oscillating particle of unit mass, with a velocity $d\phi/dx$ and position ϕ in a potential well $V(\phi)$. Prior to any numerical integration, it is instructive to discuss the conditions under which equation (4) leads to soliton solutions by analysing the Sagdeev potential $V(\phi)$ (Verheest et al. 2005). Solitonic solutions of (4) exist under the following conditions:

(i) $V(\phi) = dV(\phi)/d\phi = 0$ and $d^2V/d\phi^2 < 0$ at $\phi = 0$.

(ii) There exists a nonzero ϕ_m for which $V(\phi_m) \geq 0$, which means that a quasi-particle of zero total energy will be reflected at the position $\phi = \phi_m$.

(iii) $V(\phi) < 0$ when $0 < \phi < \phi_m$ for positive solitary waves or $\phi_m < \phi < 0$ for negative solitary waves, where ϕ_m is a maximum or a minimum value of ϕ. This means that $V(\phi)$ has to be a potential trough in which quasi-particles are trapped and experience oscillations. It may me instructive to note that the cold electron number density is obtained from the first two equations in (1) as

$$n_c = \frac{M}{\sqrt{M^2 + 2\alpha\phi}}. \tag{5}$$

3 Results and discussion

Making use of the above mentioned existence conditions, one can obtain the domain of allowable Mach numbers.

Condition $\mathrm{d}^2V/\mathrm{d}\phi^2 < 0$ for the existence of localized electron-acoustic solitary waves requires the Mach number to satisfy

$$M^2 > \frac{1}{1-\beta}. \tag{6}$$

It follows that the lower limit

$$M_{\min} = \sqrt{\frac{1}{1-\beta}} \tag{7}$$

is greater than its Boltzmannian counterpart ($M_{\min} = 1$, for $\beta = 0$) ruling out the possibility of the existence of subsonic electron-acoustic solitons. The upper limit of M, $M_{\max}$, can be found by the condition $V(\phi_c) \geq 0$, where $\phi_c = -\frac{M_{\max}^2}{2\alpha}$ is the extremum value of ϕ for which the cold electron density n_c is real. Thus, we have

$$\begin{aligned} V(\phi_c) = {} & \frac{M_{\max}^2}{\alpha^2} - \left[1 + 3\beta + \frac{3\beta}{2\alpha}M_{\max}^2 + \beta\left(\frac{M_{\max}^2}{2\alpha}\right)^2\right] \\ & \times \exp\left(-\frac{M_{\max}^2}{2\alpha}\right) - \frac{1+1/\alpha}{2\alpha}M_{\max}^2 + (1+3\beta) \\ & \geq 0. \end{aligned} \tag{8}$$

Conditions (7) and (8) restrict the region of allowable Mach numbers M, for which the existence of electron-acoustic solitons is possible. This region depends sensitively on the electron nonthermal effects. For the sake of comparison, Fig. 1 depicts the profile of the Sagdeev potential $V(\phi)$ for different values of the electron nonthermal parameter $\beta = 0.3, 0.4, 0.5, 0.6$. The remaining parameters are kept constant, viz., $\alpha = 0.5$ and $M = 2$. As is evident from the well structure of $V(\phi)$, the EAW associated nonlinear potential exhibits a rarefactive spatially localized (soliton-like) structure. Each pick value of ϕ corresponds to a left zero of $V(\phi)$ in Fig. 1. The results reveal that the spatial patterns of EASW are significantly modified by the electron nonthermal effects. An increase of β (i.e., the nonthermal character of the plasma becomes more important) causes the pulse amplitude to decrease leading ultimately (beyond a certain critical value β_c) to a complete disappearance of the EASWs.

Let us now investigate the effect of the plasma parameters on the domain of allowable Mach numbers ($M_{\min} < M < M_{\max}$). Interestingly, we found that for $\beta = 0.1$ and $\alpha = 2, 4, 10$ and 50, electron-acoustic solitons may exist for $1.054 < M < 2.799$, $1.054 < M < 2.373$, $1.054 < M < 2.201$ and $1.054 < M < 2.125$, respectively. Having in mind that $\alpha = \frac{n_{h0}}{n_{c0}}$, it turns out that the domain of admissible Mach numbers shrinks as the relative number of hot electrons increases. Next, keeping $\alpha = 2$, the effects of the electron nonthermality on the admissible Mach numbers are investigated. Our numerical results show that for $\beta = 0.1, 0.9$,

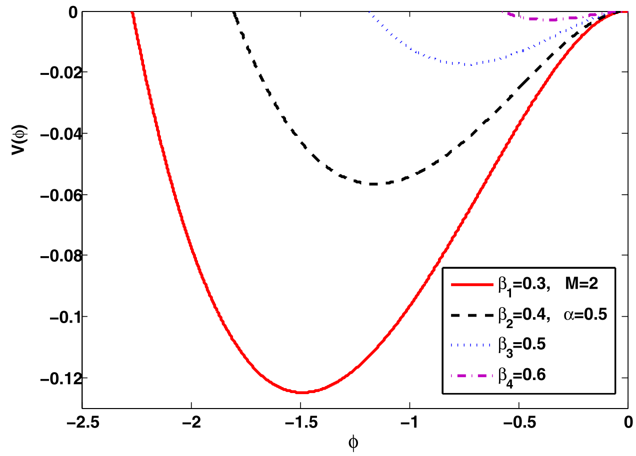

Fig. 1 Plot of the Sagdeev potential $V(\phi)$ for different values of β, with $M = 2$, $\mu = 0.1$, and $\alpha = 0.5$

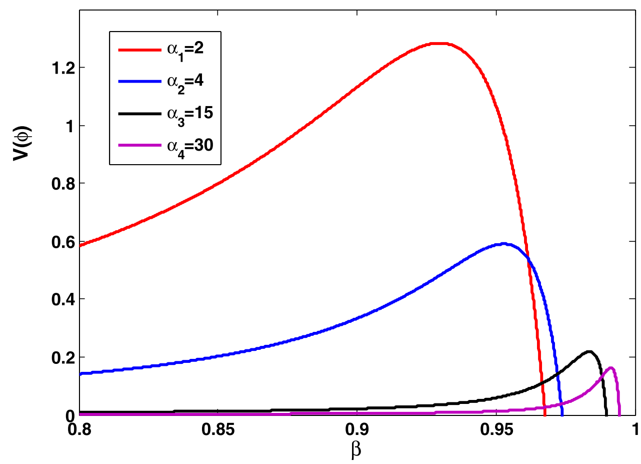

Fig. 2 Variation of $V(\beta)$ with β for different values of α, when $M_{\min} = M_{\max}$

and 0.95, EASWs exist for $1.054 < M < 2.799$, $3.16 < M < 5.405$, and $4.47 < M < 5.52$, respectively. Thus the Mach number regime in which solitons may exist gets modified depending on the number of fast nonthermal electrons present in our plasma model. In fact, in the presence of a hot nonthermal electron population, larger soliton speeds are required, involving higher Mach numbers. It may be useful to note that for $\alpha = 2$ and $\beta = 0.99$, one obtains $M_{\min} = 10$ and $M_{\max} = 5.611$, i.e., $M_{\min} > M_{\max}$. This confirms that critical nonthermal parameters β_c exist beyond which no EASWs may appear as is evident from Fig. 2 from which we can deduce that for $\alpha = 2, 4, 15$ and 30, $\beta_c = 0.967, 0.973, 0.989$, and 0.994, respectively. It turns out that the effect of increasing α is to shift the upper limit β_c toward higher values, enlarging therefore the domain of possible relevant values of the electron nonthermal parameter.

Finally, it may be useful to note that the nature of small-amplitude, $\phi \ll 1$, electron-acoustic localized structures may be obtained explicitly. Expanding in $\phi \ll 1$ and retaining leading-order terms, we find that (3) reduces to

$$V(\phi) = A_2\phi^2 + A_3\phi^3 + 0(\phi^4) \tag{9}$$

where

$$A_2 = \frac{1}{2M^2} + \frac{\beta}{2} - \frac{1}{2}, \quad A_3 = -\frac{\alpha}{2M^4} - \frac{1}{6}. \tag{10}$$

Performing the last step in deriving soliton solutions, namely, solving the energy equation (9), one gets

$$\phi(\chi) = \phi_m \,\mathrm{Sech}^2\left(\frac{\chi - \chi_0}{\Delta}\right) \tag{11}$$

where

$$\phi_m = -A_2/A_3, \quad \text{and} \quad \Delta = (-2/A_2)^{1/2}. \tag{12}$$

Solution (11) represents a small-amplitude stationary electron-acoustic solitary wave provided $A_2 < 0$ or $M^2 > 1/(1-\beta)$. Now retaining terms up to ϕ^4, (3) may be expanded as

$$V(\phi) = A_2\phi^2 + A_3\phi^3 + A_4\phi^4 + 0(\phi^5) \tag{13}$$

where

$$A_4 = \frac{5\alpha^2}{8M^6} - \frac{\beta}{8} - \frac{1}{24}. \tag{14}$$

Double layers solutions of (13) exist if (i) $V(\phi) = V(\phi_m) = 0$, (ii) $(\frac{dV}{d\phi})_{\phi=0} = (\frac{dV}{d\phi})_{\phi=\phi_m} = 0$, and $(\frac{d^2V}{d\phi^2})_{\phi=0,\phi_m} < 0$, where 0 and ϕ_m are two extreme points of the Sagdeev potential $V(\phi)$. Applying the first two boundary conditions, we obtain

$$2\phi_m = -A_3/A_4 \tag{15}$$

and $V(\phi)$ can be rewritten as

$$V(\phi) = A_4\phi^2(\phi_m - \phi)^2. \tag{16}$$

Performing the last step in deriving double layer solution, one gets

$$\phi(\chi) = \frac{\phi_m}{2}\left[1 - \tanh\left(\frac{2\chi}{\Delta}\right)\right] \tag{17}$$

where $\Delta = (-\frac{8}{A_4})^{1/2}/|\phi_m|$ represents the width of the small electron-acoustic DL provided

$$A_4 < 0 \quad \text{or} \quad M^6 > \frac{5\alpha^2}{\beta + 1/3}. \tag{18}$$

It is obvious that the domain of allowable Mach numbers M depends drastically on the plasma parameters and, in particular, on the electron nonthermal effects. In view of (10), (12) and (18), it becomes evident that our plasma model can only admit small-amplitude electron- acoustic double layers with negative potential.

4 Conclusion

To conclude, we have addressed the problem of electron-acoustic oscillations in an unmagnetized collisionless plasma comprising cold fluid electrons, nonthermal hot electrons and stationary ions. The pseudo-potential approach has been used. Our results show that in such a plasma spatially localized electron-acoustic structures, the height and nature of which depend sensitively on the plasma parameters, can exist. The EAW associated nonlinear potential exhibits a rarefactive spatially localized (soliton-like) structure. The spatial patterns of the EASW are significantly modified by the electron nonthermal effects. An increase of β causes the pulse amplitude to decrease leading ultimately to a complete disappearance of the EASWs. In addition, the domain of allowable Mach numbers shrinks as the relative number of hot electrons increases. The Mach number regime in which solitons may exist gets modified depending on the number of fast nonthermal electrons present in our plasma model. Critical nonthermal parameters β_c exist beyond which no EASWs may appear. These critical nonthermal parameters are shifted toward higher values as α is increased, enlarging therefore the domain of possible relevant values of the electron nonthermal parameter. Considering the wide relevance of nonlinear oscillations, we stress that the results of the present investigation should be useful in understanding the nonlinear features of localized electrostatic-acoustic structures in different regions of the magnetosphere (Singh and Lakhina 2004) as well as other physical phenomena like condensation of double layers (Moslem 2006; Verheest et al. 2005).

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