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ORIGINAL ARTICLE

New agegraphic dark energy in $f(R)$ gravity

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Abstract In this paper, we study a cosmological application of the new agegraphic dark energy density in the $f(R)$ gravity framework. We employ the new agegraphic model of dark energy to obtain the equation of state for the new agegraphic energy density in a spatially flat universe. Our calculations show, taking $n < 0$, that it is possible to have w_Λ crossing -1. This implies that one can generate a phantom-like equation of state from a new agegraphic dark energy model in a flat universe in the modified gravity cosmology framework. Also, we develop a reconstruction scheme for the modified gravity with $f(R)$ action.

Keywords New agegraphic · $f(R)$ Gravity · Dark energy · Phantom

1 Introduction

Many cosmological observations, such as Type 1a supernovae (SNe Ia) (Riess et al. 1998, 2004; Knop et al. 2003; Perlmutter et al. 1999), and findings from the Wilkinson Microwave Anisotropy Probe (WMAP) (Bennett et al. 2003; Spergel et al. 2003), Sloan Digital Sky Survey (SDSS) (Tegmark et al. 2004a, 2004b; Seljak et al. 2005; Adelman-McCarthy et al. 2005; Abazajian et al. 2003, 2004a, 2004b), and Chandra X-ray observatory (Allen et al. 2004) etc., have shown that our universe is undergoing an accelerated expansion. They also suggest that our universe is spatially flat, and consists of about 70% dark energy with negative pressure, 30% dust matter (cold dark matter plus baryons), and negligible radiation. In order to explain why the cosmic acceleration happens, many theories have been proposed. It is the most accepted idea that a mysterious dominant component, dark energy, with negative pressure, leads to this cosmic acceleration, though its nature and cosmological origin still remain enigmatic at present. An alternative proposal for dark energy is the dynamical dark energy scenario. The cosmological constant puzzles may be better interpreted by assuming that the vacuum energy is canceled to exactly zero by some unknown mechanism and by introducing a dark energy component with a dynamically variable equation of state. The dynamical dark energy proposal is often realized by some scalar field mechanism which suggests that the energy form with negative pressure is provided by a scalar field evolving down a proper potential.

In recent years, many string theorists have devoted themselves to understanding the cosmological constant or dark energy within the string framework. The famous Kachru-Kallosh-Linde-Trivedi (KKLT) model (Kachru et al. 2003) is a typical example, which tries to construct metastable de Sitter vacua in the light of type IIB string theory. Furthermore, a string landscape idea (Susskind 2003) has been proposed for shedding light on the cosmological constant problem based upon the anthropic principle and multiverse speculation. Although we lack a quantum gravity theory today, we still can make some attempts to probe the nature of dark energy according to some principles of quantum gravity. An interesting attempt in this direction is called the "holographic dark energy" proposal

(Cohen et al. 1999; Horava and Minic 2000; Thomas 2002; Hsu 2004; Li 2004; Pavon and Zimdahl 2005; Enqvist and Sloth 2004; Ke and Li 2005; Huang et al. 2005; Elizalde et al. 2005; Wang et al. 2005; Kim et al. 2006; Hu and Ling 2006; Li et al. 2006, 2008; Setare 2006a, 2006b, 2007a, 2007b, 2007c; Setare et al. 2007; Zhao 2007).

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Such a paradigm has been constructed in the light of the holographic principle of quantum gravity ('t Hooft 1993; Susskind 1995), and thus it presents some interesting features of an underlying theory of dark energy. More recently, a new dark energy model, dubbed agegraphic dark energy, has been proposed (Cai 2007) (see also Neupane 2009; Kim et al. 2008; Zhang et al. 2008), which takes into account the Heisenberg uncertainty relation of quantum mechanics together with the gravitational effect in general relativity.

Because the holographic energy density belongs to a dynamical cosmological constant, we need a dynamical frame to accommodate it instead of general relativity. Einstein's theory of gravity may not describe gravity at very high energy. The simplest alternative to general relativity is the Brans-Dicke scalar-tensor theory (Brans and Dicke 1961). Modified gravity provides a natural gravitational alternative for dark energy (Nojiri et al. 2006; Nojiri and Odintsov 2007a, 2007b; Cognola et al. 2007). Moreover, modified gravity presents natural unification of the early-time inflation and late-time acceleration because of the different role of gravitational terms relevant at small and at large curvature. Also, modified gravity may naturally describe the transition from a non-phantom phase to a phantom one without the necessity of introducing the exotic matter. But among the most popular modified gravities which may successfully describe the cosmic speed-up is $F(R)$ gravity. Very simple versions of this theory like $1/R$ (Capozziello 2002; Capozziello et al. 2003; Carroll et al. 2004) and $1/R + R^2$ (Nojiri and Odintsov 2003) may lead to the effective quintessence/phantom late-time universe (to see solar system constraints on modified dark energy models refer to Nojiri and Odintsov 2007a, 2007b; Cognola et al. 2007). Another theory proposed as gravitational dark energy is scalar Gauss-Bonnet gravity (Nojiri et al. 2005, 2006; Carter and Neupane 2006a, 2006b; Moffat and Toth 2007), which is closely related with low-energy string effective action.

In this paper, using the new agegraphic model of dark energy in a spatially flat universe, we obtain the equation of state for agegraphic dark energy density in the framework of modified gravity. We show the phantomic description of the new agegraphic dark energy in a flat universe with $n < 0$. Also, we develop a reconstruction scheme for the modified gravity with $f(R)$ action, and we use the known new agegraphic energy density for this reconstruction.

2 Modified gravity and new agegraphic dark energy

The action of modified gravity is given by

$$S = \int \sqrt{-g}\, \mathrm{d}^4x [f(R) + L_m], \tag{1}$$

where L_m is the matter Lagrangian density. The equivalent form of the above action is (Nojiri et al. 2006, 2007b; Nojiri and Odintsov 2007a; Cognola et al. 2007)

$$S = \int \mathrm{d}^4x \sqrt{-g}[P(\phi)R + Q(\phi) + L_m], \tag{2}$$

where P and Q are proper functions of the scalar field ϕ. By the variation of the action (2) with respect to ϕ, we obtain

$$P'(\phi)R + Q'(\phi) = 0, \tag{3}$$

which may be solved with respect to ϕ:

$$\phi = \phi(R). \tag{4}$$

By the variation of the action (2) with respect to the metric $g_{\mu\nu}$, one can obtain

$$\frac{-1}{2} g_{\mu\nu}[P(\phi)R + Q(\phi)] - R_{\mu\nu}P(\phi) + \nabla_\mu \nabla_\nu P(\phi) - g_{\mu\nu}\nabla^2 P(\phi) + \frac{1}{2}T_{\mu\nu} = 0, \tag{5}$$

where $T_{\mu\nu}$ is the energy-momentum tensor. The equations corresponding to a standard spatially flat Friedmann-Robertson-Walker (FRW) universe are

$$\rho = 6H^2 P(\phi) + Q(\phi) + 6H\frac{\mathrm{d}P(\phi)}{\mathrm{d}t} \tag{6}$$

$$p = -\left(4\dot{H} + 6H^2\right)P(\phi) - Q(\phi) - 2\frac{\mathrm{d}^2P(\phi)}{\mathrm{d}t^2} - 4H\frac{\mathrm{d}P(\phi)}{\mathrm{d}t} \tag{7}$$

where p and ρ are the pressure and energy density due to the scalar field in the modified gravity framework. By combining (6) and (7) and deleting $Q(\phi)$, we find

$$p + \rho = -2\frac{\mathrm{d}^2P(\phi)}{\mathrm{d}t^2} + 2H\frac{\mathrm{d}P(\phi)}{\mathrm{d}t} - 4\dot{H}P(\phi). \tag{8}$$

Now we suggest a correspondence between the new agegraphic dark energy scenario and the above modified dark energy model. According to the new agegraphic dark energy, we have the following relation for energy density (Wei and Cai 2008; Kim et al. 2008; Wu et al. 2008; Cui et al. 2009):

$$\rho_\Lambda = 3n^2 M_p^2 \eta^{-2}, \tag{9}$$

where the numerical factor $3n^2$ is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, and η is conformal time, given by

$$\eta = \int \frac{\mathrm{d}t}{a} = \int \frac{\mathrm{d}a}{a^2 H}. \tag{10}$$

The critical energy density, $\rho_{\rm cr}$, is given by the following relation:

$$\rho_{\rm cr} = 3H^2. \tag{11}$$

Using the definitions $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{\rm cr}}$ and $\rho_{\rm cr} = 3M_p^2 H^2$, we get

$$H\eta = \frac{n}{\sqrt{\Omega_\Lambda}} \tag{12}$$

and we obtain the equation of state for the agegraphic energy density. Let us consider the dark energy dominated universe. In this case, the dark energy evolves according to its conservation law,

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + P_\Lambda) = 0. \tag{13}$$

By considering the definition of agegraphic energy density ρ_Λ, one can find that

$$\dot{\rho_\Lambda} = \frac{-2}{a\eta}\rho_\Lambda. \tag{14}$$

Substituting this relation into (13), we obtain

$$w_\Lambda = \frac{2\sqrt{\Omega_\Lambda}}{3an} - 1, \tag{15}$$

and then we see that w_Λ can cross the phantom divide if $n < 0$.

As one can redefine the scalar field ϕ properly, we may choose

$$\phi = t. \tag{16}$$

Now using (9), (15), one can rewrite (8) as

$$2\frac{\mathrm{d}^2 P(t)}{\mathrm{d}t^2} - 2H\frac{\mathrm{d}P(t)}{\mathrm{d}t} + 4\dot{H}P(t) - \frac{2\Omega_\Lambda^{3/2}H^2}{an} = 0. \tag{17}$$

In principle, by solving (17) we find the form of $P(\phi)$. Using (6), (9), we also find the form of $Q(\phi)$ as

$$Q(\phi) = 3\Omega_\Lambda H^2 - 6H^2 P(\phi) - 6H\frac{\mathrm{d}P(\phi)}{\mathrm{d}t}. \tag{18}$$

3 Modified gravity and its reconstruction from the new agegraphic dark energy

In this section, we consider another approach (Capozziello et al. 2006; Nojiri and Odintsov 2006) to realistic cosmology in the new agegraphic modified gravity. We start with general $f(R)$ gravity action (1) but without the matter term. For the spatially flat FRW universe we have

$$\rho = f(R) - 6\left(\dot{H} + H^2 - H\frac{\mathrm{d}}{\mathrm{d}t}\right)f'(R), \tag{19}$$

$$p = f(R) - 2\left(-\dot{H} - 3H^2 + \frac{\mathrm{d}^2}{\mathrm{d}t^2} + 2H\frac{\mathrm{d}}{\mathrm{d}t}\right)f'(R), \tag{20}$$

where

$$R = 6\dot{H} + 12H^2. \tag{21}$$

Again we use the new agegraphic dark energy density and substitute (9) into (19):

$$3\Omega_\Lambda H^2 = f(R) - 6\left(\dot{H} + H^2 - H\frac{\mathrm{d}}{\mathrm{d}t}\right)f'(R), \tag{22}$$

thus

$$f(R) = 3\Omega_\Lambda H^2 + 6\left(\dot{H} + H^2 - H\frac{\mathrm{d}}{\mathrm{d}t}\right)f'(R). \tag{23}$$

Using (9), (15), and substituting $f(R)$ into (20), one can obtain

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}f'(R) - H\frac{\mathrm{d}}{\mathrm{d}t}f'(R) + 2\dot{H}f'(R) + f(R) + \frac{\Omega_\Lambda^{3/2}H^2}{an} = 0. \tag{24}$$

We shall consider the following simple solution:

$$a = a_0(t_s - t)^{h_0}, \tag{25}$$

where a_0, h_0, and t_s are constant. Substituting (25) into (21) gives us the following relation for scalar curvature:

$$R = \frac{12h_0^2 - 6h_0}{(t_s - t)^2}. \tag{26}$$

Using (10), (25) we can write

$$\eta = \int_t^{t_s} \frac{\mathrm{d}t}{a_0(t_s - t)^{h_0}} = \frac{1}{a_0(1 - h_0)(t_s - t)^{h_0 - 1}}. \tag{27}$$

Now using definition ρ_Λ and the above relation, we obtain the time behavior of agegraphic dark energy as

$$\rho_\Lambda = \frac{3n^2 a_0^2 (1 - h_0)^2}{(t_s - t)^{2 - 2h_0}}. \tag{28}$$

Substituting the above ρ_Λ into (19) and using (25), (26), one can obtain

$$\frac{72h_0^2(1 - 2h_0)}{(t_s - t)^4}f''(R) - \frac{6h_0(h_0 - 1)}{(t_s - t)^2}f'(R) + f(R) = \frac{3n^2 a_0^2(1 - h_0)^2}{(t_s - t)^{2 - 2h_0}}. \tag{29}$$

Again we use (26) and rewrite the above differential equation as follows:

$$f''(R) + \frac{a}{R}f'(R) + \frac{b}{R^2}f(R) = \frac{d}{R^{1+h_0}} \tag{30}$$

where

$$a = \frac{h_0 - 1}{2}, \qquad b = \frac{1 - 2h_0}{2},$$
$$d = \frac{-a_0^2 n^2 (1 - h_0)^2}{4} (6h_0(2h_0 - 1))^{h_0}. \tag{31}$$

The solution of differential equation (30) is given by

$$f(R) = C_1 R^{\frac{1}{2}(\frac{3-h_0}{2} - \sqrt{\frac{(h_0-3)^2}{4} + 4h_0 - 2})} + C_2 R^{\frac{1}{2}(\frac{3-h_0}{2} + \sqrt{\frac{(h_0-3)^2}{4} + 4h_0 - 2})} + \frac{n^2 a_0^2 (1 - h_0)^2 (6h_0(2h_0 - 1))^{h_0}}{2h_0(2 - h_0) R^{h_0 - 1}} \tag{32}$$

where C_1, C_2 are constant. Therefore, a consistent modified gravity with the new agegraphic dark energy in a flat space has the above form. To generate the accelerating expansion in the present universe, let us consider that $f(R)$ could be a small constant in the present universe, that is,

$$f(R_0) = -2R_0, \quad f'(R_0) \sim 0, \tag{33}$$

where $R_0 \sim (10^{-33}\ \text{eV})^2$ is the current curvature (Nojiri and Odintsov 2007a, 2007b; Cognola et al. 2007). By imposing the conditions (33) on the solution (32), we can obtain the constants C_1 and C_2 as follows:

$$C_1 = -\frac{(1 - h_0)^2 a_0^2 n^2 (6h_0(2h_0 - 1))^{h_0} (v + h_0 - 1)}{(v - uR_0) R_0^{h_0 + u - 1}} - \frac{2v}{(v - uR_0) R_0^{u-1}}, \tag{34}$$

$$C_2 = \frac{(1 - h_0)^2 a_0^2 n^2 (6h_0(2h_0 - 1))^{h_0} (uR_0 + h_0 - 1)}{(v - uR_0) R_0^{h_0 + v - 1}} + \frac{2u}{(v - uR_0) R_0^{v-2}}, \tag{35}$$

where

$$u = \frac{1}{2}\left(\frac{3 - h_0}{2} - \sqrt{\frac{(h_0 - 3)^2}{4} + 4h_0 - 2}\right),$$
$$v = \frac{1}{2}\left(\frac{3 - h_0}{2} + \sqrt{\frac{(h_0 - 3)^2}{4} + 4h_0 - 2}\right). \tag{36}$$

4 Conclusions

To solve cosmological problems and because of our lack of knowledge, for instance, to determine what could be the best candidate for dark energy to explain the accelerated expansion of the universe, cosmologists try to approach the best results as precisely as they can by considering all possibilities. Among the different candidates to play the role of dark energy, the new agegraphic dark energy model has emerged as a possible model with an equation of state across -1. In this paper, we have studied the cosmological application of the new agegraphic dark energy density in the $f(R)$ modified gravity framework. By considering the agegraphic energy density as a dynamical cosmological constant, we have obtained the equation of state for the agegraphic energy density in the $f(R)$ gravity framework. We have shown that if $n < 0$, the new agegraphic dark energy model also will behave like a phantom model of dark energy, the amazing feature of which is that the equation of state of dark energy component w_Λ crosses -1. Also, we have developed a reconstruction scheme for modified gravity with $f(R)$ action. We have considered the energy density in (19) in new agegraphic form; then by the assumption of a simple solution such as (25), we can obtain a differential equation for $f(R)$. The solution of this differential equation gives us a modified gravity action which is consistent with the new agegraphic dark energy scenario.

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