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ORIGINAL ARTICLE

Soliton energy of the Kadomtsev–Petviashvili equation in warm dusty plasma with variable dust charge, two-temperature ions, and nonthermal electrons

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Abstract The propagation of nonlinear waves in warm dusty plasmas with variable dust charge, two-temperature ions, and nonthermal electrons is studied. By using the reductive perturbation theory, the Kadomtsev–Petviashivili (KP) equation is derived. The energy of the soliton has been calculated. By using standard normal modes analysis a linear dispersion relation has been obtained. The effects of variable dust charge on the energy of the soliton and the angular frequency of the linear wave are also discussed. It is shown that the amplitude of solitary waves of the KP equation diverges at the critical values of plasma parameters. We derive solitons of a modified KP equation with finite amplitude in this situation.

Keywords Dust · Soliton · Nonthermal

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1 Introduction

Dusty plasma physics studies the properties of heavier charged dust in the presence of electrons and ions (Rao et al. 1990). These media have been observed in the planetary rings, the earth's magnetosphere, comet tails, and so on (Shukla and Mamun 2002; Havens et al. 1992). Moreover, the studies of the dusty plasma media are very attractive because of their theoretical features and their applications (D'Angelo 1995; Mamun et al. 1996). Tagare (1997) extended the model of Mamun et al. (1996) to study plasma consisting of cold dust particles and two-temperature isothermal ions. They also examined a necessary condition that must be satisfied to achieve the validity of the two-temperature ions assumption. As is well known, in most practical dusty plasma experiments, a gas flow which is usually introduced can charge quickly, while maintaining a relatively low temperature. In most investigations, dust charge variation has been neglected. However, from the physical point of view, dust grains have variable charge due to fragmentation, coalescence, and other phenomena (Bharuthram and Shukla 1992; Nejoh 1997). Xie et al. (1999) investigated small and large amplitude dust acoustic solitary waves in dusty plasma with variable dust charge and two-temperature ions. In most of the theoretical studies on dusty plasma, the reductive perturbation method has been used for deriving the Korteweg–de Vries (KdV), Zakharov–Kuznetsov (ZK), and Kadomtsev–Petviashvili (KP) equations (Duan 2002; El-Labany et al. 2004, 2006). The effects of variable dust charge, dust temperature, and trapped electrons on small amplitude dust acoustic waves are investigated in (El-Labany and El-Taibany 2003). Gill et al. (2006) have derived a KP equation for dusty plasma with variable dust charge and two-temperature ions. Space plasma observations indicate the presence of ion and electron populations which are not in thermodynamic equilibrium (Lundin et al. 1989; Futaana et al. 2003). Recently, motivated by the latter class of events, Cairns et al. (1995) used a nonthermal distribution of electrons to study the ion acoustic solitary structures observed by the FREJA satellite. Sagdeev's pseudo-potential technique is used to investigate the existence of double layers in dusty plasma with nonthermal electrons and two-temperature ions in (Das and Chatterjee 2009). Tribeche and

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Boumezoued (2008) investigated the effect of nonthermal electrons with an excess of fast energetic electrons on large amplitude electrostatic solitary waves in a charge-varying dusty plasma. In the present paper, we consider the motion of dust particles with variable charge in the presence of nonthermal electrons and two-temperature ions. In Sect. 2, the basic set of equations is introduced and in Sect. 3, by using the reductive perturbation method (RPM), the KP equation has been derived. Section 4 contains a discussion on the energy of the soliton. The linear dispersion relation and the effect of variable dust charge on this relation are also discussed in this section. We discuss the critical parameters of solitonic solutions and derive a modified KP equation in Sect. 5. Conclusions are given in Sect. 6.

2 Basic equations

We consider the propagation of dust acoustic waves in collisionless, unmagnetized warm dusty plasma consisting of nonthermal electrons, two-temperature ions, and highly negatively charged dust grains. Total charge neutrality at equilibrium requires that

$$n_{0\mathrm{e}} + n_{0\mathrm{d}} Z_{0\mathrm{d}} = n_{0\mathrm{il}} + n_{0\mathrm{ih}} \tag{1}$$

where $n_{0\mathrm{e}}, n_{0\mathrm{d}}, n_{0\mathrm{il}}$ and $n_{0\mathrm{ih}}$ are the equilibrium values of electron, dust, lower temperature ion, and higher temperature ion number densities, respectively. $Z_{0\mathrm{d}}$ is the unperturbed number of charges on the dust particles. The following set of normalized two-dimensional equations of continuity, motion for the adiabatic dust, and Poisson describe the dynamics of a dust acoustic wave in such plasma:

$$\begin{aligned}
&\frac{\partial n_\mathrm{d}}{\partial t} + \frac{\partial}{\partial x}(n_\mathrm{d} u_\mathrm{d}) + \frac{\partial}{\partial y}(n_\mathrm{d} v_\mathrm{d}) = 0\\
&\frac{\partial u_\mathrm{d}}{\partial t} + u_\mathrm{d}\frac{\partial u_\mathrm{d}}{\partial x} + v_\mathrm{d}\frac{\partial u_\mathrm{d}}{\partial y} + \frac{\sigma_\mathrm{d}}{n_\mathrm{d}}\frac{\partial P_\mathrm{d}}{\partial x} = Z_\mathrm{d}\frac{\partial \phi}{\partial x}\\
&\frac{\partial v_\mathrm{d}}{\partial t} + u_\mathrm{d}\frac{\partial v_\mathrm{d}}{\partial x} + v_\mathrm{d}\frac{\partial v_\mathrm{d}}{\partial y} + \frac{\sigma_\mathrm{d}}{n_\mathrm{d}}\frac{\partial P_\mathrm{d}}{\partial y} = Z_\mathrm{d}\frac{\partial \phi}{\partial y}\\
&\frac{\partial p_\mathrm{d}}{\partial t} + u_\mathrm{d}\frac{\partial p_\mathrm{d}}{\partial x} + v_\mathrm{d}\frac{\partial P_\mathrm{d}}{\partial y} + 3P_\mathrm{d}\left(\frac{\partial u_\mathrm{d}}{\partial x} + \frac{\partial v_\mathrm{d}}{\partial y}\right) = 0\\
&\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = Z_\mathrm{d} n_\mathrm{d} + n_\mathrm{e} - n_\mathrm{il} - n_\mathrm{ih}
\end{aligned} \tag{2}$$

where u_d and v_d are the velocity components of the dust particles in the x- and y-directions, and they are normalized by the effective dust acoustic speed $C_\mathrm{d} = \sqrt{Z_{0\mathrm{d}} T_\mathrm{eff}/m_\mathrm{d}} \cdot P_\mathrm{d}$ and ϕ are the pressure of the dust particles and electrostatic potential, and they are normalized by $Z_\mathrm{d} n_{\mathrm{d}0} T_\mathrm{d}$ and T_eff/e, respectively, in which

$$T_\mathrm{eff} = \left[\frac{1}{Z_{0\mathrm{d}} n_{0\mathrm{d}}}\left(\frac{n_{0\mathrm{e}}}{T_\mathrm{e}} + \frac{n_{0\mathrm{il}}}{T_\mathrm{il}} + \frac{n_{0\mathrm{ih}}}{T_\mathrm{ih}}\right)\right]^{-1}$$

is the effective temperature. n_d and Z_d are the dust number density and the variable charge number of dust grains, and they are normalized by $n_{0\mathrm{d}}$ and $Z_{0\mathrm{d}}$, respectively. The time and space variables are normalized by the dust plasma period $\omega_\mathrm{pd}^{-1} = \sqrt{m_\mathrm{d}/4\pi n_{0\mathrm{d}} Z_{0\mathrm{d}}^2 e^2}$ and the Debye length $\lambda_\mathrm{d} = \sqrt{T_\mathrm{eff}/4\pi Z_{0\mathrm{d}} n_{0\mathrm{d}} e^2}$, respectively. Electrons and ions are assumed to be distributed with nonthermal and Maxwell–Boltzmann distribution functions, respectively. So the related dimensionless number densities for electrons (n_e), low temperature ions (n_il), and high temperature ions (n_ih) are

$$\begin{aligned}
n_\mathrm{e} &= \frac{1}{\delta_1 + \delta_2 - 1}\left[1 - \frac{4\alpha}{1+3\alpha}\beta_1 s\phi\right.\\
&\quad \left. + \frac{4\alpha}{1+3\alpha}(\beta_1 s\phi)^2\right]\exp(\beta_1 s\phi)\\
n_\mathrm{il} &= \frac{\delta_1}{\delta_1 + \delta_2 - 1}\exp(-s\phi)\\
n_\mathrm{ih} &= \frac{\delta_2}{\delta_1 + \delta_2 - 1}\exp(-\beta s\phi)
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
&\beta_1 = \frac{T_\mathrm{il}}{T_\mathrm{e}}, \quad \beta_2 = \frac{T_\mathrm{ih}}{T_\mathrm{e}}, \quad \beta = \frac{\beta_1}{\beta_2} = \frac{T_\mathrm{il}}{T_\mathrm{ih}},\\
&s = \frac{T_\mathrm{eff}}{T_\mathrm{il}} = \frac{\delta_1 + \delta_2 - 1}{\delta_1 + \delta_2\beta + \beta_1}, \quad \delta_1 = \frac{n_{0\mathrm{il}}}{n_{0\mathrm{e}}},\\
&\delta_2 = \frac{n_{0\mathrm{ih}}}{n_{0\mathrm{e}}}, \quad \sigma = \frac{T_\mathrm{d}}{T_\mathrm{eff}}
\end{aligned} \tag{4}$$

where $T_\mathrm{d}, T_\mathrm{e}, T_\mathrm{il}, T_\mathrm{ih}$ are the temperature of the dust and electrons and the low temperature and high temperature of the ions. $\alpha$is a nonthermal parameter which determines the number of fast (nonthermal) electrons. From (1) it follows that

$$\delta_1 + \delta_2 - 1 \geq 0 \tag{5}$$

The dust charge variable Q_d is obtained from the charge-current balance equation (Melandsø et al. 1993)

$$\left(\frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla}\right) Q_\mathrm{d} = I_\mathrm{e} + I_\mathrm{il} + I_\mathrm{ih} \tag{6}$$

where $\vec{V} = (u_\mathrm{d}, v_\mathrm{d})$ and I_e, I_il and I_ih are the electron and ion (low and high temperature) currents. Notice that the characteristic time for dust motion is around 10^{-3} s (Barkan et al. 1995), while the dust charging time is typically about 10^{-9} s (Winske and Jones 1995). So the dust charge reaches its equilibrium position quickly. Thus $\frac{\mathrm{d}Q_\mathrm{d}}{\mathrm{d}t} \ll I_\mathrm{e}, I_\mathrm{il}, I_\mathrm{ih}$ and the charge-current balance equation (6) reads (Melandso 1996)

$$I_\mathrm{e} + I_\mathrm{il} + I_\mathrm{ih} \approx 0 \tag{7}$$

The electron and ion currents are (Nejoh 1997)

$$I_e = -e\pi r^2\left(\frac{8T_e}{\pi m_e}\right)^{\frac{1}{2}}\left(\frac{1}{\delta_1+\delta_2-1}\right)\left(\frac{1}{1+3\alpha}\right)\exp(\beta_1 s\psi) \times\left\{\left[1+\frac{24\alpha}{5}-\frac{16\alpha}{3}\beta_1 s\phi+4\alpha\beta_1^2 s^2\phi^2\right] + \beta_1 s\psi\left(1+\frac{8\alpha}{5}-\frac{8\alpha}{3}\beta_1 s\phi+4\alpha\beta_1^2 s^2\phi^2\right)\right\}$$

$$I_{il} = e\pi r^2\left(\frac{8T_{il}}{\pi m_i}\right)^{\frac{1}{2}}\left(\frac{\delta_1}{\delta_1+\delta_2-1}\right)\exp(-s\phi)\left(1-\frac{e\Phi}{T_{il}}\right) \tag{8}$$

$$I_{ih} = e\pi r^2\left(\frac{8T_{ih}}{\pi m_i}\right)^{\frac{1}{2}}\left(\frac{\delta_2}{\delta_1+\delta_2-1}\right)\exp(-\beta s\phi)\left(1-\frac{e\Phi}{T_{ih}}\right)$$

where Φ denotes the dust grain surface potential relative to the plasma potential ϕ. If the thermal velocities of electrons and ions are larger than their streaming velocities, then from (6) we have

$$\sqrt{\frac{\beta_1}{\mu_i}}\delta_1\exp(-s\phi)(1-s\psi)+\sqrt{\frac{\beta_2}{\mu_i}}\delta_2\exp(-\beta s\phi)(1-\beta s\psi) - \left(\frac{1}{1+3\alpha}\right)\exp(\beta_1 s\psi) \times\left\{\left[1+\frac{24\alpha}{5}-\frac{16\alpha}{3}\beta_1 s\phi+4\alpha\beta_1^2 s^2\phi^2\right] + \beta_1 s\psi\left(1+\frac{8\alpha}{5}-\frac{8\alpha}{3}\beta_1 s\phi+4\alpha\beta_1^2 s^2\phi^2\right)\right\} \tag{9}$$

where $\psi = e\Phi/T_{eff}$ and $\mu_i = m_i/m_e \cong 1840$. The dust charge $Q_d = C\Phi$ is calculated by using (7) in which C is the capacitance of the dust grains. Z_d is defined as

$$Z_d = \psi/\psi_0 \tag{10}$$

where $\psi_0 = \psi(\phi = 0)$ is the dust surface floating potential with respect to the unperturbed plasma potential at an infinite region. By substituting $\phi = 0$ into (9), we have

$$b_1\delta_1(1-s\psi_0)+b_2\delta_2(1-\beta s\psi_0) = \left(\frac{1}{1+3\alpha}\right)\exp(\beta_1 s\psi) \times\left\{1+\frac{24\alpha}{5}+\beta_1 s\psi_0\left(1+\frac{8\alpha}{5}\right)\right\} \tag{11}$$

where $b_1 = \sqrt{\beta_1/\mu_i}$ and $b_2 = \sqrt{\beta_2/\mu_i}\cdot Z_d$ can be expanded with respect to ϕ as follows:

$$Z_d = 1+\gamma_1\phi+\gamma_2\phi^2+\gamma_3\phi^3+\cdots \tag{12}$$

where $\gamma_1 \equiv \frac{\psi_0'}{\psi_0} = \frac{r_b}{r_a}$, $\gamma_2 \equiv \frac{\psi_0''}{2\psi_0} = \frac{r_c}{r_a}$ and $\gamma_3 \equiv \frac{\psi_0'''}{6\psi_0} = \frac{r_d}{r_a}$ come from expanding ψ near ψ_0, so we can obtain

$$r_a = \delta_1 b_1+\delta_2 b_2\beta+\beta_1\left[\delta_1 b_1(1-s\psi_0) + \delta_2 b_2(1-\beta s\psi_0)\right] + \left(\frac{\beta_1}{1+3\alpha}\right)\exp(\beta_1 s\psi_0)\left(1+\frac{8\alpha}{5}\right)$$

$$r_b = \delta_1 b_1(1-s\psi_0)+\delta_2 b_2(1-\beta s\psi_0) + \left(\frac{\exp(\beta_1 s\psi_0)}{1+3\alpha}\right)\left(\frac{16\alpha}{3}\beta_1+\frac{8\alpha}{5}\beta_1^2 s\psi_0\right)$$

$$\begin{aligned} r_c &= \delta_1 b_1 s^2(1-s\psi_0) + \left(2\delta_1 b_1 s+\delta_2 b_2\beta^2 s^2\right)\psi_0' + 2\delta_2 b_2\beta^2 s^2-\delta_2 b_2\beta^3 s^3\psi_0 - \left(\frac{\beta_1 s}{1+3\alpha}\right)\exp(\beta_1 s\psi_0) \\ &\times\left\{\left(\beta_1 s\psi_0'^2+\psi_0''\right)\left[1+\frac{24\alpha}{5}+\beta_1 s\psi_0\left(1+\frac{8\alpha}{5}\right)\right]\right\} - \left(\frac{2\beta_1 s\psi_0'}{1+3\alpha}\right)\exp(\beta_1 s\psi_0)\left[-\frac{16\alpha}{3}\beta_1 s + \beta_1 s\psi_0'\left(1+\frac{8\alpha}{5}\right)-\frac{8\alpha}{3}\beta_1^2 s^2\psi_0\right] \end{aligned} \tag{13}$$

and

$$\begin{aligned} r_d &= -\left(\delta_1 b_1 s^3+\delta_2 b_2\beta^3 s^3\right)\psi_0' + \left(2\delta_1 b_1 s+\delta_2 b_2\beta^2 s^2\right)\psi_0''+2\delta_2 b_2\beta^2 s^2 - \left(\frac{\beta_1 s}{1+3\alpha}\right)\exp(\beta_1 s\psi_0) \\ &\times\left\{\left(\beta_1 s\psi_0'^2+\psi_0''\right)\left[1+\frac{24\alpha}{5}+\beta_1 s\psi_0\left(1+\frac{8\alpha}{5}\right)\right]\right\} \times\left\{\beta_1 s\psi_0'\left(2+\frac{8\alpha}{5}\right)\left(\beta_1 s\psi_0'^2+\psi_0''\right) + \left(2\beta_1 s\psi_0'\psi_0''+\psi_0'''\right)\left[1+\frac{24\alpha}{5}+\beta_1 s\psi_0\left(1+\frac{8\alpha}{5}\right)\right]\right\} \\ &- \left(\frac{2\beta_1 s\psi_0''+\beta_1 s\psi_0'}{1+3\alpha}\right)\exp(\beta_1 s\psi_0)\left[-\frac{16\alpha}{3}\beta_1 s + \beta_1 s\psi_0'\left(1+\frac{8\alpha}{5}\right)-\frac{8\alpha}{3}\beta_1^2 s^2\psi_0\right] - \left(\frac{2\beta_1 s\psi_0'}{1+3\alpha}\right)\exp(\beta_1 s\psi_0)\left[\beta_1 s\psi_0''\left(1+\frac{8\alpha}{5}\right) - \frac{8\alpha}{3}\beta_1^2 s^2\psi_0'\right] \end{aligned} \tag{14}$$

3 The derivation of the KP equation

According to the general method of reductive perturbation theory, we choose the independent variables as

$$\xi = \varepsilon(x - \lambda t), \qquad \tau = \varepsilon^3 t, \qquad \eta = \varepsilon^2 y \tag{15}$$

where ε is a small dimensionless expansion parameter which characterizes the strength of nonlinearity in the system, and λ is the phase velocity of the wave along the x direction. We can expand the physical quantities which have appeared in (2) in terms of the expansion parameter ε as

$$\begin{aligned}
n_{\rm d} &= 1 + \varepsilon^2 n_{1\rm d} + \varepsilon^4 n_{2\rm d} + \cdots \\
u_{\rm d} &= \varepsilon^2 u_{1\rm d} + \varepsilon^4 u_{2\rm d} + \cdots \\
v_{\rm d} &= \varepsilon^3 v_{1\rm d} + \varepsilon^5 v_{2\rm d} + \cdots \\
\phi &= \varepsilon^2 \phi_1 + \varepsilon^4 \phi_2 + \cdots \\
P_{\rm d} &= 1 + \varepsilon^2 P_{1\rm d} + \varepsilon^4 P_{2\rm d} + \cdots \\
Z_{\rm d} &= 1 + \varepsilon^2 Z_{1\rm d} + \varepsilon^4 Z_{2\rm d} + \cdots
\end{aligned} \tag{16}$$

Substituting (15) and (16) into (2) and collecting terms with the same powers of ε, from the coefficients of the lowest order we have

$$\begin{aligned}
& n_{1\rm d} = \frac{\phi_1}{3\sigma - \lambda^2}, \quad n_{1\rm d} = \frac{u_{1\rm d}}{\lambda}, \\
& \lambda^2 = 3\sigma + \left[1 + \gamma_1 - \frac{4\alpha\beta_1}{(1+3\alpha)(\delta_1 + \delta_2\beta + \beta_1)}\right]^{-1}, \\
& P_{1\rm d} = \frac{3}{\lambda} u_{1\rm d}, \quad \frac{\partial v_{1\rm d}}{\partial \xi} = -\frac{1}{v_0}\frac{\partial \phi_1}{\partial \eta}
\end{aligned} \tag{17}$$

and for the higher orders of ε

$$\begin{aligned}
& -\lambda\frac{\partial n_{2\rm d}}{\partial \xi} + \frac{\partial n_{1\rm d}}{\partial \tau} + \frac{\partial(u_{2\rm d} + n_{1\rm d}u_{1\rm d})}{\partial \xi} + \frac{\partial v_{1\rm d}}{\partial \eta} = 0 \\
& -\lambda\frac{\partial u_{2\rm d}}{\partial \xi} + \frac{\partial u_{1\rm d}}{\partial \tau} + u_{1\rm d}\frac{\partial u_{1\rm d}}{\partial \xi} + \sigma\frac{\partial P_{2\rm d}}{\partial \xi} - \sigma n_{1\rm d}\frac{\partial P_{1\rm d}}{\partial \xi} \\
& \quad = \gamma_1\phi_1\frac{\partial \phi_1}{\partial \xi} + \frac{\partial \phi_2}{\partial \xi} \\
& -\lambda\frac{\partial v_{1\rm d}}{\partial \xi} + \sigma\frac{\partial P_{1\rm d}}{\partial \eta} = \frac{\partial \phi_1}{\partial \eta} \\
& -\lambda\frac{\partial P_{2\rm d}}{\partial \xi} + u_{1\rm d}\frac{\partial P_{1\rm d}}{\partial \xi} + 3P_1\frac{\partial u_{1\rm d}}{\partial \xi} + 3\frac{\partial v_{1\rm d}}{\partial \eta} + 3\frac{\partial u_{2\rm d}}{\partial \xi} \\
& \quad + \frac{\partial P_{1\rm d}}{\partial \tau} = 0 \\
& \frac{\partial^2 \phi_1}{\partial \xi^2} \\
& \quad = \gamma_1 n_{1\rm d}\phi_1 + n_{2\rm d} \\
& \quad + \left[1 + \gamma_1 - \frac{4\alpha\beta_1}{(1+3\alpha)(\delta_1 + \delta_2\beta + \beta_1)}\right]\phi_2 \\
& \quad + \left[\gamma_2 - \frac{1}{2}\frac{(\delta_1 + \delta_2 - 1)^2(\delta_1 + \delta_2\beta^2 - \beta_1^2)}{(\delta_1 + \delta_2\beta + \beta_1)^2}\right]\phi_1^2
\end{aligned} \tag{18}$$

The KP equation is derived from the above equations as

$$\frac{\partial}{\partial \xi}\left[\frac{\partial \phi_1}{\partial \tau} + A\phi_1\frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3}\right] + C\frac{\partial^2 \phi_1}{\partial \eta^2} = 0 \tag{19}$$

where

$$\begin{aligned}
A &= \frac{1}{2\lambda}\Bigg\{-2 + (\lambda^2 - 3\sigma)^2\Bigg[(\delta_1 + \delta_2\beta^2 - \beta_1^2) \\
&\quad \times \frac{(\delta_1 + \delta_2 - 1)}{(\delta_1 + \delta_2\beta + \beta_1)^2} - 2\gamma_2\Bigg] \\
&\quad + 3\gamma_1(\lambda^2 - 3\sigma) - \frac{\lambda^2 + 9\sigma}{\lambda^2 - 3\sigma}\Bigg\} \\
B &= \frac{1}{2\lambda}(\lambda^2 - 3\sigma)^2, \quad C = \frac{\lambda}{2}
\end{aligned} \tag{20}$$

If the charge of the dust particles is constant ($\gamma_1 = \gamma_2 = 0$), "A" becomes

$$\begin{aligned}
A &= \frac{1}{2\lambda}\Bigg\{\frac{(\delta_1 + \delta_2\beta^2 - \beta_1^2)(\lambda^2 - 3\sigma)^2(\delta_1 + \delta_2 - 1)}{(\delta_1 + \delta_2\beta + \beta_1)^2} \\
&\quad - 3\left(\frac{\lambda^2 + \sigma}{\lambda^2 - 3\sigma}\right)\Bigg\}
\end{aligned} \tag{21}$$

where

$$\lambda^2 = 3\sigma + \left[1 - \frac{4\alpha\beta_1}{(1+3\alpha)(\delta_1 + \delta_2\beta + \beta_1)}\right]^{-1}$$

The KP equation is widely used in plasma physics and theoretical physics. Duan et al. (2004) have studied the resonance of the KP equation, theoretically. The stationary solution of (19) can be written as (Gill et al. 2006)

$$\begin{aligned}
\phi_1 &= \phi_0 \sec h^2\left(\frac{\xi + \eta - U\tau}{W}\right), \\
\phi_0 &= \frac{3(U - C)}{A}, \quad W = 2\sqrt{\frac{B}{U - C}}
\end{aligned} \tag{22}$$

where ϕ_0 and W are the amplitude and width of the soliton, respectively.

Solitonic solutions of the KP equation can be compared with other results. The above results are reduced to those of (Xie et al. 1999; Gill et al. 2006) for Maxwell distributed electron ($\alpha = 0$) and cold plasma ($\sigma = 0$). Also, the above solitonic solutions are comparable with the results of Tagare (1997) for dusty plasmas containing cold dust particles and two-temperature ions. The above-mentioned equations for warm plasma with one ion and without fast electrons ($\alpha = 0$), when $\gamma_1 = \gamma_2 = 0$, agree with those of Duan

(2002). He showed that, in this medium, the nonlinear term is always negative. For cold dusty plasmas without nonthermal electrons and dust particles with constant charge (Gill et al. 2006), that is, $\alpha = \sigma = 0$ and $\gamma_1 = \gamma_2 = 0$ we have

$$A = \frac{1}{2}\left\{\frac{(\delta_1 + \delta_2\beta^2 - \beta_1^2)(\delta_1 + \delta_2 - 1)}{(\delta_1 + \delta_2\beta + \beta_1)^2} - 3\right\}$$

Obviously, $(\delta_1 + \delta_2\beta^2 - \beta_1^2)$ is always less than $(\delta_1 + \delta_2\beta + \beta_1)$, but for the term $\frac{(\delta_1+\delta_2-1)}{(\delta_1+\delta_2\beta+\beta_1)}$ we have

$$\frac{(\delta_1 + \delta_2 - 1)}{(\delta_1 + \delta_2\beta + \beta_1)} = \frac{(\delta_1 + \delta_2 - 1)}{(\delta_1 + \delta_2 - 1) + 1 + \beta_1 - (1 - \beta)\delta_2}$$

It is clear that the above term is less than 1 if $\delta_2 < \frac{1+\beta_1}{1-\beta}$, and in this case "$A$" is always negative and rarefactive solitons always exist. Also, the above-mentioned term is more than 1 if $\delta_2 > \frac{1+\beta_1}{1-\beta}$, and in this case "$A$" can have positive or negative values. In these cases, both compressive and rarefactive solitary waves can be propagated. It can also be easily shown that in cold dusty plasma ($\sigma = 0$) with fixed dust charge ($\gamma_1 = \gamma_2 = 0$), compressive solitons can exist if $(\delta_1 + \delta_2\beta^2 - \beta_1^2)(\delta_1 + \delta_2 - 1) > 3(\delta_1 + \delta_2\beta + \beta_1 - G\beta_1)^2$. The coefficients of the dispersive terms "B" and "C" and also the nonlinear term "A" are functions of relative densities, relative temperatures, γ_1 and γ_2. It is possible that the competition between the nonlinear term and dispersion terms leads to the formation of a soliton.

4 Energy of soliton and linear dispersion relation

The study of the amplitude and width of solitons is a common way to further recognize waves in plasmas. The other way is the study of the soliton's energy. El-Shewy (2007) showed that increasing the population of nonthermal electrons decreases the energy of electron acoustic solitary waves. Also, the effect of electron inertia on the energy of a soliton in relativistic plasma has been investigated by Malik (1999).

The energy of a soliton can be obtained from the following equation (Singh and Honzawa 1993):

$$E = \int_{-\infty}^{+\infty} u_{1d}^2 \, d\xi \tag{23}$$

After the integration, we obtain (Singh and Honzawa 1993)

$$E = \frac{4}{3}u_m^2 W = \frac{24\lambda^2}{(3\sigma - \lambda^2)^2}\frac{(U - C)^2}{A^2}\sqrt{\frac{B}{U - C}} \tag{24}$$

Figure 1a shows the variation of soliton energy with respect to γ_1. The following set of parameters have been chosen for this figure (Gill et al. 2004; Das and Chatterjee 2009):

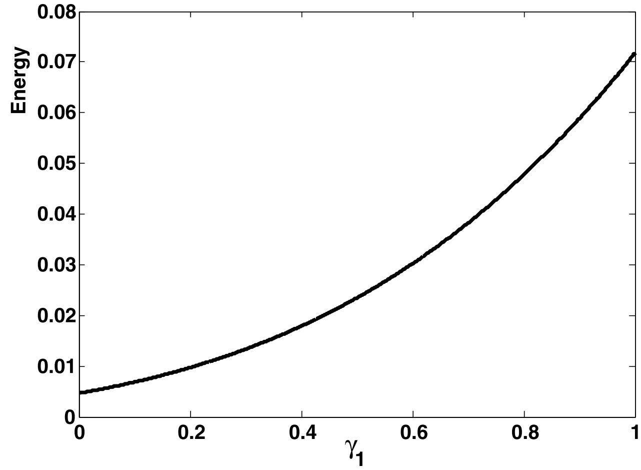

Fig. 1a Energy of the soliton as a function of γ_1 for $\delta_1 = 0.1$, $\delta_2 = 10$, $\sigma = 0.002$, $\beta = 0.001$, $\beta_1 = 0.0001$, $\alpha = 0.25$, $\gamma_2 = 0$, $U = 1$

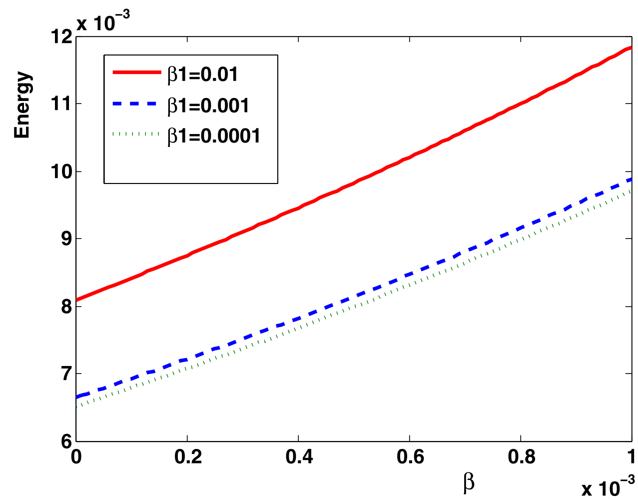

Fig. 1b Energy of the soliton as a function of β for $\delta_1 = 0.1$, $\delta_2 = 10$, $\sigma = 0.002$, $\alpha = 0.2$, $\gamma_1 = 0.2$, $\gamma_2 = 0$, $U = 1$ and different values of β_1

$\beta = 0.001, \beta_1 = 0.0001, \delta_1 = 0.1, \delta_2 = 10, \sigma = 0.002, \alpha = 0.25, \gamma_2 = 0, u = 1$

We can also choose $0 \le \gamma_1 \le 1$ based on the values of dust charge in (Tribeche and Boumezoued 2008; Xie et al. 1999).

Figure 1a indicates that an increase in the parameter γ_1 increases the energy of solitary waves. Figure 1b shows the variation of soliton energy with respect to β for different values of β_1. In this figure we see that an increase in β increases the soliton energy. On the other hand, the energy of the soliton is increased when the temperature of cold ions (β_1) increases. More investigation shows that the energy is increased slightly by increasing α. Therefore, α has a very weak effect on the energy.

Now we derive the linear dispersion relation. The linear dispersion relation shows the amount of decay rate in the linear approximation for a wave packet. Therefore, we

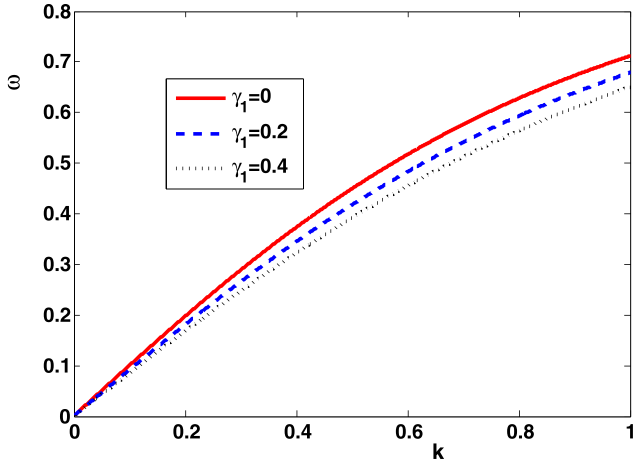

Fig. 2 The angular frequency with respect to k for $\alpha=0.28$, $\delta_1=0.1$, $\delta_2=10$, $\sigma=0.002$, $\beta=0.001$, $\beta_1=0.0001$, $\gamma_1=0, 0.2, 0.4$, $\gamma_2=0$

can calculate the distance that a wave packet can travel before decaying. This is the essential difference between linear and nonlinear media. The characteristic frequency range of a dust acoustic wave in isotropic, collisionless, weakly coupled plasma was theoretically predicted by Rao et al. (1990). According to standard normal mode analysis, by linearization of dependent variables n_{d}, ϕ and Z_{d} in terms of their equilibrium and perturbed parts (Verheest and Lakhina 1996; Samanta et al. 2007), we have

$$n_{\mathrm{d}}=1+n_{1\mathrm{d}}, \quad \phi=\phi_1, \quad u_{\mathrm{d}}=u_{1\mathrm{d}}, \\ Z_{\mathrm{d}}=1+Z_{1\mathrm{d}}=1+\gamma_1\phi_1 \tag{25}$$

Then, we may assume that all the perturbed quantities are proportional to $\mathrm{e}^{\mathrm{i}(kx-\omega t)}$ with k being the wave propagation constant in the direction of the x-axis, and so we have $\frac{\partial}{\partial t}=-\mathrm{i}\omega$, $\frac{\partial}{\partial x}=\mathrm{i}k$. Substituting (25) into (2) and using their linear terms, one can obtain the linear dispersion relation as

$$\omega^2=\frac{k^2}{k^2+H}+3k^2\sigma \tag{26}$$

where

$$H=1+\gamma_1-\frac{G\beta_1}{\delta_1+\delta_2\beta+\beta_1} \tag{27}$$

Figure 2 shows the angular frequency (ω) as a function of k for $\gamma_1=0, 0.2, 0.4$ and $\gamma_2=0$.

Figure 2 indicates that increasing $k(\gamma_1)$ leads to increasing (decreasing) values for ω. For real values of ω, all perturbation variables oscillate harmonically, and if any or all of the ω's have positive imaginary parts, then the system is unstable since those normal modes will grow in time (Samanta et al. 2007; Krall and Trivelpiece 1986).

5 Modified KP equation

The strength of the nonlinear term in the KP equation depends on the value of parameter "A", which is a function of $\beta_1, \beta, \delta_1, \delta_2, \sigma_{\mathrm{i}}, \gamma_1$ and γ_2. Note that, for some parameters (which are called critical values), there exists a singularity in the soliton amplitude, which is proportional to $1/A$, at $A=0$. For example, in (21) if

$$\left(\delta_1+\delta_2\beta^2-\beta_1^2\right)\frac{(\delta_1+\delta_2-1)}{(\delta_1+\delta_2\beta+\beta_1)^2}=\frac{3(\lambda^2+\sigma)}{(\lambda^2-3\sigma)^3} \tag{28}$$

we will have $A=0$.

This implies that the stretching coordinates mentioned above are not valid for these critical parameters and tone must use new stretching coordinates to obtain the modified KP equation containing a higher-order nonlinear term. The method is similar to what has appeared in (Tagare 1997).

$$\begin{aligned}
n_{\mathrm{d}}&=1+\varepsilon n_{1\mathrm{d}}+\varepsilon^2 n_{2\mathrm{d}}+\varepsilon^3 n_{3\mathrm{d}}+\cdots\\
u_{\mathrm{d}}&=\varepsilon u_{1\mathrm{d}}+\varepsilon^2 u_{2\mathrm{d}}+\varepsilon^3 u_{3\mathrm{d}}+\cdots\\
v_{\mathrm{d}}&=\varepsilon^2 v_{1\mathrm{d}}+\varepsilon^3 v_{2\mathrm{d}}+\varepsilon^4 v_{3\mathrm{d}}+\cdots\\
\phi&=\varepsilon\phi_1+\varepsilon^2\phi_2+\varepsilon^3\phi_3+\cdots\\
Z_{\mathrm{d}}&=1+\varepsilon Z_{1\mathrm{d}}+\varepsilon^2 Z_{2\mathrm{d}}+\varepsilon^3 Z_{3\mathrm{d}}+\cdots\\
&=1+\varepsilon\gamma_1\phi_1+\varepsilon^2(\gamma_1\phi_2+\gamma_2\phi_1)\\
&\quad+\varepsilon^3\left(\gamma_1\phi_3+2\gamma_2\phi_1\phi_2+\gamma_3\phi_1^3\right)+\cdots
\end{aligned} \tag{29}$$

Substituting the above expansions into (1) and collecting different orders of ε, we can derive the following equation:

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau}+D\phi_1^2\frac{\partial\phi_1}{\partial\xi}+E\frac{\partial}{\partial\xi}(\phi_1\phi_2)+B\frac{\partial^3\phi_1}{\partial\xi^3}\right] \\ +C\frac{\partial^2\phi_1}{\partial\eta^2}=0 \tag{30}$$

where

$$\begin{aligned}
D&=\frac{3}{4\lambda^2}\left\{\gamma_3\left(\lambda^2-3\sigma\right)\left[\left(\delta_1+\delta_2-\beta_1^2\right)^2\right.\right.\\
&\quad\left.\left.\times\frac{G(\delta_1+\delta_2-1)}{(\delta_1+\delta_2\beta+\beta_1)^2}-\gamma_1\right]+\frac{\lambda^2+9\sigma}{\lambda^2-3\sigma}\right\}\\
&\quad+\frac{1}{2\lambda}\left\{\left[3+\left(\delta_1+\delta_2\beta^2-\beta_1^2\right)\right.\right.\\
&\quad\left.\times\frac{(\delta_1+\delta_2-1)}{(\delta_1+\delta_2\beta+\beta_1)^2}+\gamma_2\right]\\
&\quad\left.\times\left(\lambda^2-3\sigma\right)\left[2+3\gamma_1\left(\lambda^2-3\sigma\right)\right]\right\}
\end{aligned} \tag{31}$$

At $A=0$, (30) reduces to the modified KP equation

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau}+D\phi_1^2\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}\right]+C\frac{\partial^2\phi_1}{\partial\eta^2}=0 \tag{32}$$

This equation has solitonic solutions. One soliton solution for this equation is (Wang et al. 2005)

$$\phi_1 = \pm\phi_{\mathrm{m}}\mathrm{sech}[(\xi + \eta - u\tau)/W] \quad (33)$$

where u is velocity. $\phi_{\mathrm{m}} = \sqrt{6(u-C)/D}$ and $W = \sqrt{B/(u-C)}$ are the amplitude and width of solitons, respectively. Solitons exist when $D > 0$.

6 Conclusion

We investigated dust acoustic solitary waves in unmagnetized dusty plasmas with variable dust charge, two-temperature ions, and nonthermal electrons. The KP equation was obtained, and then a solitonic solution of this equation was studied. Our investigations are summarized as follows.

(i) Both rarefactive and compressive solitary waves can be propagated.
(ii) The energy of solitons is increased when the dust charges are increased, but it is almost independent of the nonthermal parameter. Also, an increase in the parameters β and β_1 increases the energy of solitary waves
(iii) The linear dispersion relation is derived. We have shown that increasing the charge of dust grains leads to decreasing values of ω.
(iv) Since the nonlinear coefficient of the KP equation, "A", can be positive or negative, it can also be zero. But a solitonic solution cannot be established when "A" is zero; therefore, "A" has critical values. In this situation, we derived a modified KP equation, and the solitonic solutions of this equation are finite.
(v) Our investigations can be useful in understanding the behavior of a dust acoustic wave in space and astrophysical plasma environments and also in understanding physical phenomena like condensation of dust grains and double layers (Das and Chatterjee 2009; Djebli and Marif 2009) that clearly indicate the presence of nonthermal particle populations.

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