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ORIGINAL ARTICLE

Bulk viscous string cosmological models with electromagnetic field

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Abstract LRS Bianchi type-I string cosmological models are studied in the frame work of general relativity when the source for the energy momentum tensor is a bulk viscous stiff fluid containing one dimensional strings embedded in electromagnetic field. The bulk viscosity is assumed to be inversely proportional to the scalar expansion. The physical and kinematical properties of the models are discussed. The effects of Viscosity and electromagnetic field on the physical and kinematical properties are also investigated.

Keywords String · Viscosity · Electromagnetic field

1 Introduction

String cosmological models are widely studied in recent times because of their prime role in the description of the evolution of the early phase of universe. One dimensional strings are believed to occur as a topological stable defect during the initial phase of the universe. The physical basis of the strings lies in the fact that the density perturbation arising out of the cosmic strings leads to the formation of large scale structures like galaxies (Vilenkin 1981). Since the universe is filled with lot of extended objects like galaxies, string cosmology can greatly help in the understanding of the early phase of evolution of the universe. Moreover, Grand Unified Theories (GUT) predicts the presence of strings in the initial epoch.

A good many authors have investigated about different aspects of the string cosmological models either in the frame work of Einstein Relativity (ER) or in modified theories of gravity using various space times. Besides the pioneer works of Letelier (1983) and Stachel (1980), Bali and Pradhan (2007), Bali et al. (2007), Yadav et al. (2007a, 2007b), Banerjee et al. (1990), Rahaman et al. (2003), Ram and Singh (1995), Singh and Singh (1999), Tikekar and Patel (1992), Reddy (2003a, 2003b), Pradhan et al. (2007), Katore and Rane (2006), Bali and Anjali (2006), Tripathy et al. (2008a, 2008b, 2009) have studied string cloud cosmology. String cosmologies with bulk viscosity have been studied by Wang (2004, 2005, 2006), Bali et al. (2007, 2008) and Tripathy et al. (2008b).

Magnetic field plays an important role in the description of the energy distribution in the universe as it contains highly ionized matter. Strong magnetic field may be created due to adiabatic compression in cluster of galaxies. Cosmic anisotropies may also be attributed to the large scale magnetic fields. Katore and Rane (2006), Banerjee et al. (1990), Pradhan et al. (2007), Saha and Visinescu (2008) have studied the behaviour of string cosmological models in presence of electromagnetic field. The necessity of electromagnetic field along with the topological defects in the form of cosmic string has already been established in our earlier work (Tripathy et al. 2008a). In a recent work (Tripathy et al. 2008b),

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we have justified the need of viscosity in describing the evolution of the properties of universe. In that work (Tripathy et al. 2008b), we have assumed a general linear relationship between the string tension density and the energy density and the viscous fluid is taken to be barotropic i.e. the proper pressure is considered to be directly proportional to the energy density.

The present work is a sequel to our earlier work (Tripathy et al. 2008b). For the sake of simplicity, many authors consider the string tension density λ to be proportional to energy density ρ, the proportionality constant being a constant for the whole range of cosmic time. In fact, both λ and ρ should evolve with time and in turn the proportionality relation should be disturbed in course of the growth of the cosmic time. In the present work, from an idealized stiff cosmic fluid we have tried to gauge the evolving relationship between λ and ρ instead of taking any specific relation between λ and ρ. Also we have studied the effects of bulk viscosity and magnetic field in the evolution of the properties of the universe.

The organization of the paper is as follows: In Sect. 2, the basic field equations for an LRS Bianchi type-I bulk viscous string fluid model have been derived. In Sect. 3, the solutions of the field equations are discussed in the absence of bulk viscosity. In Sect. 4, two different time dependent viscous magnetized string fluid models have been discussed in detail along with their physical and kinematical properties. Conclusions of the work are presented at the end in Sect. 5.

2 The field equations and their consequences

The LRS Bianchi type-I metric is considered in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2(dy^2 + dz^2) \tag{1}$$

where A and B are the metric potentials considered as functions of cosmic time only.

The energy momentum tensor for a bulk viscous fluid containing one dimensional strings embedded in an electromagnetic field is taken as

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij} - \lambda x_i x_j + E_{ij} - \xi\theta(u_i u_j + g_{ij}) \tag{2}$$

where ρ is the rest energy density of the system, λ is the string tension density and $\xi(t)$ is the time dependent coefficient of bulk viscosity, p is the pressure of the cosmic fluid and θ is the expansion scalar. $u^i = (0, 0, 0, 1)$ is the four velocity vector and x^i, a space like vector represents the anisotropic direction of the string. u^i and x^i satisfy the equations

$$g_{ij}u^i u^j = -1 \tag{3}$$

$$g_{ij}x^i x^j = 1 \tag{4}$$

and

$$u^i x_i = 0 \tag{5}$$

Choosing x^i parallel to $\partial/\partial x$, we have

$$x^i = (A^{-1}, 0, 0, 0) \tag{6}$$

E_{ij} is the part of the energy momentum tensor corresponding to the electromagnetic field and is defined as

$$E_{ij} = \frac{1}{4\pi}\left(g^{sp}F_{is}F_{jp} - \frac{1}{4}g_{ij}F_{sp}F^{sp}\right) \tag{7}$$

where F_{sp} is the electromagnetic field tensor. Quantizing the axis of the magnetic field along the x-direction we can have the only non-vanishing component of F_{sp} as F_{23}. Assuming an infinite electrical conductivity,

$$F_{14} = F_{24} = F_{34} = 0$$

From Maxwell's equation

$$F_{23} = -F_{32} = H = \text{Constant} \tag{8}$$

For LRS Bianchi type-I metric considered in (1), the components of electromagnetic field can be expressed as

$$E_{11} = \frac{-H^2 A^2}{8\pi B^4} = -\eta A^2 \tag{9a}$$

$$E_{22} = E_{33} = \frac{H^2}{8\pi B^2} = \eta B^2 \tag{9b}$$

$$E_{44} = \frac{H^2}{8\pi B^4} = \eta \tag{9c}$$

where η is defined by (9c).

Einstein field equations are expressed as

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} \tag{10}$$

where R_{ij} is the Ricci tensor and R is the Ricci scalar. The conservation law for the energy momentum tensor T_{ij} is given by

$$T^{ij}_{;j} = 0 \tag{11}$$

Here semicolon (;) denotes covariant differentiation. The units taken are such that $8\pi G = c = 1$, where G is the Newtonian gravitational constant and c is the speed of light in free space.

The field equations (10) with the help of (2)–(9) for the metric (1) take the explicit forms:

$$2\dot{\beta} + 3\beta^2 = -p + \lambda + \eta + \xi\theta \tag{12}$$

$$\dot{\alpha}+\dot{\beta}+\alpha^2+\beta^2+\alpha\beta=-p-\eta+\xi\theta \tag{13}$$

$$\beta^2+2\alpha\beta=\rho+\eta \tag{14}$$

After some algebraic simplification, we get, from these field equations

$$\dot{\alpha}+\dot{\beta}+\alpha^2+2\beta^2+3\alpha\beta=(\rho-p)+\xi\theta \tag{15}$$

where the overhead dots represent ordinary time derivatives. In the above equations the Hubble parameters are defined as

$$\alpha=\frac{\dot{A}}{A} \quad \text{and} \quad \beta=\frac{\dot{B}}{B}$$

Equation (15) and other field equations are highly non linear in nature and compel us to assume certain plausible physical conditions to obtain determinate solutions. We assume that the coefficient of viscosity should vary with the expansion scalar in such a manner that

$$\xi\theta=\xi_0=\text{Constant} \tag{16}$$

The expansion scalar θ and the shear scalar σ for the metric (1) are defined as

$$\theta=u^l_{;l}=\alpha+2\beta \tag{17}$$

$$\sigma^2=\alpha^2+3\beta^2+2\alpha\beta+2\theta^2 \tag{18}$$

In accordance with the fact that the shear scalar σ be proportional to the scalar expansion θ (Saha and Visinescu 2008), we can assume a linear relationship between α and β i.e $\alpha=k\beta$ (k being an arbitrary constant). Under this assumption, the shear scalar takes the form,

$$\sigma^2=\left(\frac{3k^2+10k+11}{(k+2)^2}\right)\theta^2 \tag{19}$$

For a stiff cosmological fluid ($p=\rho$) containing one dimensional strings, (15) can be reduced to

$$\frac{\dot{\beta}}{k+2}+(\beta^2-m)=0 \tag{20}$$

where

$$m=\frac{\xi_0}{k^2+3k+2} \tag{21}$$

which can be defined only for $k\neq-1$, and $k\neq-2$.

Form the field equations (12)–(14) different properties of the cosmological model taken in the form of metric (1) can be expressed as

Sting tension density:

$$\lambda=(1-k)\dot{\beta}+(2-k-k^2)\beta^2-2\eta \tag{22}$$

Proper pressure:

$$p=\text{energy density } \rho=(2k+1)\beta^2-\eta \tag{23}$$

Particle density:

$$\rho_p=\rho-\lambda=\eta+(k^2+3k-1)\beta^2-(1-k)\dot{\beta} \tag{24}$$

Coefficient of bulk viscosity:

$$\xi=\left(\frac{k+1}{k+2}\right)\frac{\dot{\beta}}{\beta}+(k+1)\beta \tag{25}$$

3 Non-viscous magnetized string cosmological model

In the absence of bulk viscosity (20) has obvious solution,

$$\beta=\frac{1}{(k+2)t+B_0} \tag{26}$$

where B_0 is an integration constant.

From (26) we get

$$B=B_1\big[(k+2)t+B_0\big]^{\frac{1}{k+2}} \tag{27}$$

Since B_0 simply brings a shift in time, we can safely choose it to be zero. Thus,

$$B=B_1\big[(k+2)t\big]^{\frac{1}{k+2}} \tag{28}$$

and consequently

$$A=A_1\,[(k+2)t]^{\frac{k}{k+2}} \tag{29}$$

where A_1, B_1 are constants.

With suitable choice of constants, in the absence of bulk viscosity, the metric for the model can be written as

$$ds^2=-dt^2+[(k+2)t]^{\frac{2k}{k+2}}dx^2+[(k+2)t]^{\frac{2}{k+2}}[dy^2+dz^2] \tag{30}$$

In the absence of bulk viscosity, the physical properties of the model are

$$\eta=\frac{H^2}{8\pi\,[(k+2)t]^{\frac{4}{k+2}}} \tag{31}$$

$$\lambda=-\frac{4}{(k^2+4k+4)t^2}-\frac{H^2}{4\pi\,[(k+2)t]^{\frac{4}{k+2}}} \tag{32}$$

$$p=\rho=\frac{2k+1}{(k^2+4k+4)t^2}-\frac{H^2}{8\pi\,[(k+2)t]^{\frac{4}{k+2}}} \tag{33}$$

$$\rho_P=\rho-\lambda=\frac{2k+5}{(k^2+4k+4)t^2}+\frac{H^2}{8\pi\,[(k+2)t]^{\frac{4}{k+2}}} \tag{34}$$

The kinematical properties of the model (30) are

The spatial volume: $V = \sqrt{-g} = AB^2 = (k+2)t$ (35)

$$\theta = \frac{1}{t} \tag{36}$$

$$\sigma^2 = \left(\frac{3k^2+10k+11}{k^2+4k+4}\right)\frac{1}{t^2} \tag{37}$$

In the absence of bulk viscosity, all the properties of the model evolve with time. For all real values of k, the string tension density λ assumes negative values. A negative value of the string tension density implies the string phase of the universe disappears and the universe is left only with an anisotropic fluid of particles. The model is expanding in nature. As is evident from the expressions (32) and (33), the ratio $\gamma = \frac{\rho}{\lambda}$ evolves with time instead of being a constant for the whole range of time.

In the absence of viscosity as well as the electromagnetic field, the properties of the model assume the forms

$$\lambda = -\frac{4}{(k^2+4k+4)t^2} \tag{38}$$

$$p = \rho = \frac{2k+1}{(k^2+4k+4)t^2} \tag{39}$$

$$\rho_P = \rho - \lambda = \frac{2k+5}{(k^2+4k+4)t^2} \tag{40}$$

The switching off of the electromagnetic field makes the ratio $\gamma = \frac{\rho}{\lambda} = -\frac{2k+1}{4}$ a constant of time. In the other words, the inclusion of magnetic field alters the relationship between the energy density and string tension density all through the cosmic time.

4 Bulk viscous magnetized string cosmological models

For finite bulk viscosity of the cosmic fluid, (20) can be solved for the following cases:

Case-I: $\beta = \pm\sqrt{m}; \quad m > 0$ (41)

Case-II: $\dfrac{\dot\beta}{\beta^2 - m} = -(k+2)$ (42)

4.1 Model-I

$\beta = \pm\sqrt{m}$ with $m > 0$, on integration yields

$$B = B_2 e^{\pm\sqrt{m}t} \tag{43}$$

Since $\alpha = k\beta$, we have

$$A = A_2 e^{\pm kt\sqrt{m}} \tag{44}$$

where A_2, B_2 are constants.

The metric for the model with suitable choice of constants can be written as

$$ds^2 = -dt^2 + e^{\pm 2kt\sqrt{m}}dx^2 + e^{\pm 2t\sqrt{m}}(dy^2+dz^2) \tag{45}$$

The physical properties of the model (45) defined for $k \neq -1$, and $k \neq -2$ are

$$\eta = \frac{H^2}{8\pi e^{\pm 4t\sqrt{m}}} \tag{46}$$

The string tension density:

$$\lambda = (2-k-k^2)m - \frac{H^2}{4\pi e^{\pm 4t\sqrt{m}}} \tag{47}$$

The proper energy density and the proper pressure are

$$\rho = p = (2k+1)\,m - \frac{H^2}{8\pi e^{\pm 4t\sqrt{m}}} \tag{48}$$

Particle density:

$$\rho_p = \rho - \lambda = (k^2+3k-1)m + \frac{H^2}{8\pi e^{\pm 4t\sqrt{m}}} \tag{49}$$

Coefficient of viscosity:

$$\xi = \pm(k+1)\sqrt{m} \tag{50}$$

The kinematical aspects of the model are
The spatial volume:

$$V = \sqrt{-g} = AB^2 = e^{\pm(k+2)t\sqrt{m}} \tag{51}$$

The scalar expansion:

$$\theta = \pm(k+2)\sqrt{m} \tag{52}$$

The shear scalar:

$$\sigma = (3k^2+10k+11)m \tag{53}$$

For positive values of $\sqrt{m}$, when $t \to 0, \eta$ assumes finite value but when $t \to \infty, \eta \to 0$ i.e. the effect of magnetic field gradually decreases with the growth of cosmic time and vanishes at vary large values of time.

At large t, i.e. $t \to \infty, \eta \to 0$ for $m > 0$, which implies

$$\lambda = (2-k-k^2)m = \left(\frac{1-k}{1+k}\right)\xi_0 \tag{54}$$

For $k > 1, \lambda < 0$ which implies that at large values of cosmic time the universe is filled with an anisotropic fluid of only particles. This is substantiated by the fact that at large cosmic time the particle density assumes positive finite value.

Magnetic field greatly affects the physical properties of the model. In the absence of magnetic field, for $k > 1$,

$m > 0$, the string tension density $\lambda < 0$ implying the disappearance of the string phase. If at all they exist they should co-exist with magnetic field. This kind of result reinforces the result that we have already got in our earlier work (Tripathy et al. 2008a, 2008b). However, there can be a string phase for suitable choice of the parameters k and H. For $\sqrt{m} > 0$ and $k > -2, \theta > 0$ implies that the universe is expanding in nature. Also for $\sqrt{m} < 0$ and $k < -2$, the universe is expanding in nature. For all other values of k the model provides contracting universe.

In the absence of magnetic field, the properties of the model are

$$\lambda = \left(\frac{1-k}{1+k}\right)\xi_0 \tag{55}$$

$$\rho = p = (2k+1)m \tag{56}$$

$$\rho_P = (k^2+3k-1)m \tag{57}$$

From (47) and (48) it is clear that the relation between ρ and λ i.e. $\gamma = \frac{\rho}{\lambda}$ evolves with time. But in the absence of magnetic field, $\gamma = \frac{\rho}{\lambda} = \frac{2k+1}{2-k-k^2}$ is a constant of time. In other words magnetic field has a greater role in the evolution of the string phase of the universe.

Another interesting feature of the model is that the coefficient of bulk viscosity and the expansion scalar do not evolve with time and assume constant values. This may be attributed to the nature of solution we have chosen for this particular model i.e. we have assumed for this model that $\beta^2 = m$ as a constant quantity and the expansion scalar is directly proportional to β. Viscosity affects all the properties of the model through the factor m. Since $\frac{\sigma}{\theta} \neq 0$, for all real values of k, the anisotropic behaviour of the model is maintained through the whole range of cosmic time.

4.2 Model-II

For $m > 0$, and $\beta^2 \neq m$ integration of (42) yields

$$\beta = \nu + \frac{2\nu b_1 e^{-2\nu(k+2)t}}{1 - b_1 e^{-2\nu(k+2)t}} \tag{58}$$

where $\nu^2 = m$, which on further integration generates

$$B = B_3[1 - b_1 e^{-2\nu(k+2)t}]^{\frac{1}{k+2}} e^{\nu t} \tag{59}$$

and consequently

$$A = A_3[1 - b_1 e^{-2\nu(k+2)t}]^{\frac{k}{k+2}} e^{k\nu t} \tag{60}$$

where $b_1 > 0$ and $b_1 \neq 1$, B_3, A_3 are constants. With the choice of $A_3 = B_3 = 1$, the metric for the model can be expressed as

$$ds^2 = -dt^2 + [1 - b_1 e^{-2\nu(k+2)t}]^{\frac{2k}{k+2}} e^{2k\nu t} dx^2 + [1 - b_1 e^{-2\nu(k+2)t}]^{\frac{2}{k+2}} e^{2\nu t}(dy^2 + dz^2) \tag{61}$$

The physical properties of the model are

$$\eta = \frac{H^2}{8\pi[1 - b_1 \exp(-2\nu(k+2)t)]^{4/(k+2)} \exp(4\nu t)} \tag{62}$$

$$\lambda = (2-k-k^2)m - 2\eta \tag{63}$$

$$\rho = p = (2k+1)\beta^2 - \eta \tag{64}$$

$$\rho_p = \rho - \lambda = (k^2-k-2)m + (2k+1)\beta^2 + \eta \tag{65}$$

where β is defined by (58).

The bulk viscosity

$$\xi = \frac{\xi_0}{\theta} = \frac{\xi_0}{(k+2)\nu[1 + \frac{2b_1\exp(-2\nu(k+2)t)}{1-b_1\exp(-2\nu(k+2)t)}]} \tag{66}$$

The kinematical properties of the model are

Spatial volume:

$$V = \sqrt{-g} = AB^2 = [1 - b_1\exp(-2\nu(k+2)t)]\exp((k+2)\nu t) \tag{67}$$

The scalar expansion:

$$\theta = (k+2)\nu\left[1 + \frac{2b_1\exp(-2\nu(k+2)t)}{1-b_1\exp(-2\nu(k+2)t)}\right] \tag{68}$$

As in the previous model (45), here as $t \to 0, \eta$ assumes a finite value but when $t \to \infty, \eta \to 0$ i.e. the effect of the magnetic field on the model gradually vanishes for large values of cosmic time. When $t \to \infty, \lambda < 0$ for $k > 1$ implying particles dominate the universe. Magnetic field greatly affects the properties of the model. In the absence of magnetic field

$$\lambda = (2-k-k^2)m, \qquad \rho = p = (2k+1)\beta^2 \quad \text{and}$$

$$\rho_p = (2k+1)\beta^2 + (k^2-k-2)m$$

As in the previous bulk viscous model (45), in this model, the absence of magnetic field for $k > 1$, results in a negative string tension density and a positive particle energy density i.e. $\lambda < 0$ and $\rho_p > 0$. This result signifies that in the absence of magnetic field universe contains a fluid of only particles and strings do not exist.

Also in the model, the ratio $\gamma = \frac{\rho}{\lambda}$ varies with the cosmic time instead of being a constant quantity, as usually taken, for the whole range of time. However, in this model, γ is not a constant quantity even in the absence of magnetic field. The rate of expansion depends upon the choice of k, ν and b_1. The expansion rate decreases with the increase in time. As is assumed earlier, viscosity affects all the properties of the model through the factor ν. The anisotropic behaviour of the model is maintained all through the time since $\frac{\sigma}{\theta} \neq 0$ for all real values of k.

5 Conclusion

The field equations for LRS Bianchi type-I model are solved in the frame work of Einstein Relativity when the source of energy-momentum tensor is a stiff viscous fluid containing one dimensional strings embedded in electromagnetic field. The bulk viscosity is assumed to be related inversely with the scalar expansion. Besides a non-viscous cosmological model, two different plausible bulk viscous string fluid cosmological models along with their physical and kinematical aspects are discussed. For both the models, it is concluded that with the switching off of the magnetic field the string part of the universe disappears leaving the universe to be filled with an anisotropic fluid of particles, fostering the idea that string should co-exist with electromagnetic field. At the beginning of the universe, electromagnetic field has a dominant role in establishing a string dominated era. However, with the growth of time the particles gradually evolve. At large cosmic time, when the effect of electromagnetic field is negligible, particles dominate over the strings to fill the volume of the universe. Magnetic field plays a greater role in the evolution of the relationship of the string tension density and the energy density. In fact, magnetic field makes the ratio $\gamma = \frac{\rho}{\lambda}$ time dependent. Viscosity plays a greater role in the evolution of the properties of the model. In the absence of bulk viscosity the string phase of the universe disappears. In the model (45) viscosity comes out to be a constant quantity but in the model (61), viscosity evolves with time maintaining a reciprocal relationship with the scalar expansion.

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