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ORIGINAL ARTICLE

Bianchi type-III universe with perfect fluid

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Abstract We study Bianchi type-III cosmological model filled with perfect fluid in the presence of cosmological constant $\Lambda(t)$. The Hubble law utilised yields a constant value of deceleration parameter. Physical and Kinematical properties of the model have also studied.

Keywords Bianchi type-III model · Cosmological constant · Perfect fluid

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1 Introduction

The Einstein field equations are a coupled system of highly non-linear differential equations and we seek physical solutions to the field equations for applications in cosmology and astrophysics. In order to solve the field equations, we normally assume a form for the matter content or support that spacetime admits Killing vector symmetries (Kramer et al. 1980). Solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter which was proposed by Berman (1983). It is interesting to observe that this law yields a constant value for deceleration parameter. Forms for deceleration parameter which are variable have been investigated recently by Beesham (1993a, 1993b). In the simplest case the Hubble law yields a constant deceleration parameter have been studied by Berman (1983), Berman and Gomide (1988), Johri and Desikan (1994), Maharaj and Naidoo (1993) and others.

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Our intention in this paper is to extend the results obtained by Berman (1983), Berman and Gomide (1988), Pradhan and Vishwakarma (2002), Singh and Kumar (2006), Wang (2003, 2004, 2005, 2006). By obtaining solution to the Einstein field equations with variable cosmological constants in the Bianchi type-III space time, we have derived some solutions in perfect fluid and we reached similar solution to that of Beesham (1993a, 1993b), Kalligas et al. (1992) and Arbab (1998).

2 Field equations

We consider the homogenous and isotropic space-time described by Bianchi type-III metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2\alpha x}dy^2 + C^2dz^2 \tag{2.1}$$

where A, B and C being the functions of t only, and α is a constant.

In Gaussian normal coordinate of this metric, the unit flow vector $\mathbf{v}^i$ is the normalised time like eigen vector of $\mathbf{T}_{ij}$. The energy-momentum tensor of the perfect fluid given by

$$\mathbf{T}_i{}^j = (\epsilon + p)\mathbf{v}_i\mathbf{v}^j + p\delta_i{}^j \tag{2.2}$$

where $\mathbf{v}^i$ is the unit flow vector satisfying

$$g_{ij}\mathbf{v}^i\mathbf{v}^j = 1 \tag{2.3}$$

Here ϵ is total energy density of a perfect fluid while p is the corresponding pressure. p and ϵ are related by our equation of state which will be studied below in detail. In a co-moving system of coordinates from (2.3) one finds

$$T^1{}_1 = T^2{}_2 = T^3{}_3 = p, \quad T_4{}^4 = \epsilon \tag{2.4}$$

In the context of (2.1) and (2.2), the Einstein field equations with time dependent cosmological constant Λ and $8\pi G = c = 1$, i.e.

$$R_i{}^j - \frac{1}{2}\delta_i{}^j R = \mathbf{T}_i{}^j - \delta_i{}^j \Lambda \tag{2.5}$$

yield independent equations

$$\frac{B_4 C_4}{BC} + \frac{B_{44}}{B} + \frac{C_{44}}{C} = p - \Lambda, \tag{2.5a}$$

$$\frac{C_4 A_4}{CA} + \frac{A_{44}}{A} + \frac{C_{44}}{C} = p - \Lambda, \tag{2.5b}$$

$$\frac{A_4 B_4}{AB} + \frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{\alpha^2}{A^2} = p - \Lambda, \tag{2.5c}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{\alpha^2}{A^2} = -\epsilon - \Lambda, \tag{2.5d}$$

$$\alpha\left(\frac{A_4}{A} - \frac{B_4}{B}\right) = 0 \tag{2.5e}$$

where suffix 4 denote ordinary differentiation with respect to cosmic time ‘t’.

As (2.5a)–(2.5e) having only five independent equation in the six unknown A, B, C, ϵ, p and Λ. An extra equation is needed to solve the system completely, which we shall obtain in the following by using property of constant deceleration parameters.

The law of variation of Hubble parameter was first proposed by Berman (1983) in FRW models and that yields a constant value of deceleration parameter. Recently Singh et al. (2008) have presented Bianchi type-I models in self-creation cosmology with constant deceleration parameter.

The deceleration parameter

$$q = \frac{\frac{-S_{44}}{S}}{\left(\frac{S_4}{S}\right)^2} \tag{2.6}$$

and S is function of t defined to be

$$S^3 = ABC \tag{2.7}$$

We assume that the variation of the Hubble parameter is given by the equation

$$H = \frac{D}{3}(AB^2)^{-n/3} \quad (\text{for } B = C) \tag{2.8}$$

here obeying condition

$$AB^2 = \left(\frac{n}{3}Dt + c_1\right)^{3/n} \tag{2.9}$$

where D and n are constants.

From (2.6) and (2.8), we have the deceleration parameter is constant:

$$q = n - 1 \tag{2.10}$$

After solving the Einstein field equations (2.5a)–(2.5e), we get

$$A = \left(\frac{n}{3}Dt + c_1\right)^{\frac{1}{n}} (k_3)^{-2/3} \times \exp\left[\frac{2k_1}{D(n-3)\left(\frac{n}{3}Dt + c_1\right)^{\frac{n-3}{n}}}\right] \tag{2.11}$$

$$B = \left(\frac{n}{3}Dt + c_1\right)^{\frac{1}{n}} (k_3)^{-1/3} \times \exp\left[\frac{-k_1}{D(n-3)\left(\frac{n}{3}Dt + c_1\right)^{\frac{n-3}{n}}}\right] \tag{2.12}$$

where k_1, k_3 being integration constants and field equations satisfying always for $k_1 = 0$.

Taking into account that the pressure and energy density obey a equation of state of type $p = \epsilon$ (Zeldovich Universe), we conclude that ϵ and p, from (2.5c) and (2.5d) i.e.

$$\Lambda = \left(\frac{n}{3}Dt + c_1\right)^{-2}\left[\frac{-D^2(6-2n)}{18} + \frac{\alpha^2(k_3)^{1/3}}{\left(\frac{n}{3}Dt + c_1\right)^{1/n}} \times \exp\left\{\frac{4k_1}{D(n-3)\left(\frac{n}{3}Dt + c_1\right)^{\frac{3-n}{n}}}\right\}\right] \tag{2.13}$$

From (2.13) and (2.5a), we find

$$p = \left(\frac{n}{3}Dt + c_1\right)^{-2}\left[2nD^2 + 18k_1^2\left(\frac{n}{3}Dt + c_1\right)^{-5/n} + 18\alpha^2(k_3)^{1/3}\left(\frac{n}{3}Dt + c_1\right)^{-1/n} \times \exp\left[\frac{4k_1}{D(n-3)\left(\frac{n}{3}Dt + c_1\right)^{\frac{3-n}{n}}}\right]\right] \tag{2.14}$$

The ratio between Λ and ϵ is given by

$$\Omega = \frac{\Lambda}{\epsilon} = \frac{-D^2(6-2n) + \left(\frac{n}{3}Dt + c_1\right)^{-1/n} 18\alpha^2(k_3)^{1/3} \exp\left[\frac{4k_1}{D(n-3)\left(\frac{n}{3}Dt+c_1\right)^{\frac{3-n}{n}}}\right]}{2nD^2 + 18k_1^2\left(\frac{n}{3}Dt + c_1\right)^{-5/n} + 18\alpha^2(k_3)^{1/3}\left(\frac{n}{3}Dt + c_1\right)^{-2/n} \exp\left[\frac{4k_1}{D(n-3)\left(\frac{n}{3}Dt+c_1\right)^{\frac{3-n}{n}}}\right]} \tag{2.15}$$

Here shear σ and expansion scalar θ for the model are

$$\sigma = \frac{1}{\sqrt{3}} k_1 \left(\frac{n}{3} Dt + c_1\right)^{\frac{-3}{n}}, \tag{2.16}$$

$$\theta = D\left(\frac{n}{3} Dt + c_1\right)^{-1} \tag{2.17}$$

From (2.16) and (2.17), we find relationship between shear σ and θ expansion scalar, we have

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}} \frac{k_1}{D} \left(\frac{n}{3} Dt + c_1\right)^{\frac{-3}{n}+1} \tag{2.18}$$

There is a relationship between energy density and expansion scalar θ, we find

$$\frac{\varepsilon}{\theta^2} = \frac{1}{\sqrt{3}} \frac{k_1}{D} \left(\frac{n}{3} Dt + c_1\right)^{\frac{-3}{n}+2} \tag{2.19}$$

In the model we observe that the spatial volume V is zero at $t = -3c_1/nD = t_0$ (say) and expansion scalar is infinite which shows that the universe start evolving with zero volume at $t = t_0$ with an infinite rate of expansion. The scale factors also vanish at $t = 0$ and hence the model has a point type singularity. The pressure p, energy density ε and shear σ diverses at the initial singularity. As t increases the spatial volume increases but the expansion decreases. Thus the rate of expansion slows down with the increase in time. Also ε, p decreases as the time increases. As $t \to \infty$ scalar factors and volume become infinite whereas ε, p, σ and θ tend to zero. Therefore the model would essentially give an empty universe as large time t. The ratio $\frac{\varepsilon}{\theta^2}$ tend to zero as $t \to \infty$ provided n less than $3/2$, so the model approaches isotropy for large values of t. Thus the model representing shearing-non rotating and expanding model with a big-bang start.

Cosmological constant $\Lambda(t)$ is constant initially and then decreases, it becomes zero asymptotically which is similar solution to Beesham. We see that the quantity $\Lambda \sim t^{-2}$ for $\alpha = 0$ this also follows from the model of Kalligas et al. (1992), Lopez and Nanopoulos (1996).

3 Conclusion

In this paper we take time dependent Λ-term in Einstein theory of general relativity and deceleration parameter constant. We have obtained solution of the Bianchi type-III universe.The model has point type singularity at initial epoch which is similar solution to Beesham. We see that the quantity $\Lambda \sim t^{-2}$ for $\alpha = 0$ this also follows from the model of Kalligas et al. (1992) and Lopez and Nanopoulos (1996).

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