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ORIGINAL ARTICLE

Effect of oblateness and radiation pressure on angular frequencies at collinear points

N.I. Vishnu Namboodiri · D. Sudheer Reddy · Ram Krishan Sharma

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Abstract In the three-dimensional restricted three-body problem, by considering the more massive primary as an oblate spheroid with its equatorial plane coincident with the plane of motion as well as source of radiation, it is found that the collinear point L_1 comes nearer to the primaries with the increase in oblateness and radiation pressure, while L_2 and L_3 move away from the more massive primary with the increase in oblateness and come nearer to it with the increase in radiation pressure. It is noted that the angular frequency s_1 at L_1 increases with oblateness as well as with radiation pressure. s_2 increases with oblateness and decreases with radiation pressure and s_3 decreases with oblateness and increases with radiation pressure. A study on the norms of the characteristic roots λ and s at L_1, L_2 and L_3 is carried out.

It is established that for certain oblateness and radiation pressure parameters there is a one-to-one commensurability at the collinear points L_2, L_3 between the planar angular frequencies ($s_{2,3}$) and the corresponding angular frequency (s_z) in the z-direction, and that at L_1 no such commensurability exists. At L_2 and L_3, the value of oblateness parameter providing the commensurability decreases with the increase in the radiation pressure. However, the commensurable angular frequencies and eccentricity of the periodic orbits decrease at L_2 and increase at L_3, with the increase in the radiation pressure.

Keywords Restricted three-body problem · Conditional periodic orbits · Commensurability · Collinear points · Angular frequencies · Oblateness and radiation pressure

N.I. Vishnu Namboodiri · D. Sudheer Reddy · R.K. Sharma (✉)
Applied Mathematics Division, Vikram Sarabhai Space Centre, Thiruvananthapuram 695022, India
e-mail: ksvariables@yahoo.co.in

1 Introduction

The restricted three-body problem possesses five stationary solutions called Lagrangian points, three of which called collinear equilibria lie on the line joining the primaries and the other two called equilateral equilibria make equilateral triangles with primaries. In general, the collinear equilibria are unstable while equilateral points are stable, in the Lyapunov sense, only in a certain region for the mass parameter. However, under certain initial conditions periodic solutions to the first variational equations that represent the infinitesimal (linearized) periodic orbits around the collinear points can be established. It is known that in the three-dimensional case also there are only five stationary points $L_i\ (i = 1, 2, \dots, 5)$ for the problem located at exactly the same places as in the corresponding planar case—three of which ($L_{1,2,3}$) are collinear with the primaries.

Various authors have made studies on Lagrangian points in the restricted three-body problem by considering the more massive primary or both primaries as source of radiation. Some of the important contributions are by Radzievsky (1950, 1953), Chernikov (1970), Perezhogin (1976), Kunitsyn and Perezhogin (1978), Bhatnagar and Chawla (1979), Schuerman (1980), Simmons et al. (1985), Kunitsyn and Tureshbaev (1985), Lukyanov (1988), Todoran (1994), Ragos and Zagouras (1988a, 1988b), Xue-tang et al. (1994), Kalantonis et al. (2006) and Papadakis (2006). Some of the significant studies carried out related to the Lagrangian points by considering the oblateness of one or both the primaries with their equatorial planes coincident with the plane of motion, are by Vidyakin (1974), Sharma (1975), Sharma and Subba Rao (1975, 1976, 1978). Subba Rao and Sharma (1975, 1997), Markellos et al. (1996), and Douskos and Markellos (2006). By including the effect of oblateness of the more massive primary, Sharma and Subba Rao (1978) established that oblateness of the more massive primary induces a one-to-one commensurability between the planar angular frequencies ($s_{2,3}$) and the corresponding angular frequency (s_z) in the z-direction at $L_{2,3}$ for $0 \le \mu \le 0.5$ and that no such commensurability existed for L_1. However, the values of the oblateness coefficient (A_1) involved at L_2 were found to be high, while those at L_3 being small for small values of μ could be useful in generating periodic orbits of third kind.

Sharma (1982) studied the linear stability of the triangular points in the planar case by considering the more massive primary as an oblate spheroid as well as a source of radiation and found that the critical mass value decreases with the increase in the oblateness and radiation force and becomes zero for $q = ((2+3A_1)(1.5A_1)^{(3/2)})/4$, where A_1 is the oblateness coefficient of the more massive primary and $q = 1 - F_p/F_g$ is the mass reduction factor constant for a given particle, F_p being the force due to radiation pressure and F_g is the force due to gravitational field. Sharma (1987) and Sharma and Ishwar (1995) studied the linear stability of the triangular points in the planar case by considering the more massive primary as source of radiation and smaller primary as oblate spheroid and found that the critical mass value decreases with the increase in oblateness and radiation pressure. Recently Abdul Raheem and Singh (2006, 2008) studied the combined effects of perturbations due to coriolis forces, centrifugal forces, radiation pressure and oblateness on the linear stability of the triangular libration points.

In this paper we study the restricted three-body problem when the more massive primary is a source of radiation as well as an oblate spheroid with its equatorial plane coincident with the plane of motion. Expressions for locations of the collinear points are found and a numerical study on the effect of oblateness and radiation pressure is carried out. Study on the effect of oblateness and radiation pressure on the characteristic roots of the variational equations is also carried out. A study is carried out to find out one-to-one commensurability between the planar and the z-direction angular frequencies at the collinear points $L_{1,2,3}$. It is found that one-to-one commensurability exist at L_2 and L_3 between $s_{2,3}$ and s_z ($L_{2,3}$), and no such commensurability exists at L_1. At L_2, the values of A_1 providing one-to-one commensurability between s_2 and s_z decrease with the increase in the radiation pressure and the values of the angular frequency s_2 and eccentricity of the periodic orbits also decrease. However, at L_3 though the values of A_1 providing one-to-one commensurability between s_3 and s_z decrease with increase in radiation pressure, the values of s_3 and eccentricity of the periodic orbits increase with the increase in the radiation pressure. This study can be useful in becoming one of the method in generating periodic orbits of third kind at the collinear points.

2 Equations of motion

In the dimensionless synodic coordinate system (x, y), the equations of motion are (Szebehely 1967; Sharma 1982)

$$\begin{aligned} \ddot{x} - 2n\dot{y} &= \frac{\partial \Omega}{\partial x}, \\ \ddot{y} + 2n\dot{x} &= \frac{\partial \Omega}{\partial y}, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \Omega = {}& \frac{n^2}{2}[(1-\mu)r_1^2 + \mu r_2^2] + q\frac{(1-\mu)}{r_1} \\ & + \frac{\mu}{r_2} + q\frac{(1-\mu)}{2r_1^3}A_1, \end{aligned} \tag{2}$$

with $r_1^2 = (x-\mu)^2 + y^2, r_2^2 = (x+1-\mu)^2 + y^2, n^2 = 1 + \frac{3}{2}A_1$. Jacobi's integral is

$$\dot{x}^2 + \dot{y}^2 = 2\Omega - C. \tag{3}$$

The mass parameter $\mu = m_1/(m_1+m_2)$, where m_1 and m_2 are masses of primaries $m_1 > m_2$, such that $m_1 + m_2 = 1$, the oblateness coefficient $A_1 = (AE^2 - AP^2)/5R^2$, where AE and AP are the dimensional equatorial and polar radii of the more massive primary and R is the distance between the primaries.

From (3) the curves of zero velocity are given by $2\Omega(x, y) = C$, where C is the Jacobian constant. The liberation points in the xy-plane are given by

$$\frac{\partial \Omega}{\partial x} = 0, \qquad \frac{\partial \Omega}{\partial y} = 0,$$

i.e.,

$$\begin{aligned} & n^2 x - \frac{q(1-\mu)(x-\mu)}{r_1^3} - \frac{\mu(x+1-\mu)}{r_2^3} \\ & \quad - \frac{3A_1 q(1-\mu)(x-\mu)}{2r_1^5} = 0, \end{aligned} \tag{4}$$

$$y\left[n^2 - \frac{q(1-\mu)}{r_1^3} - \frac{\mu}{r_2^3} - \frac{3A_1(1-\mu)q}{2r_1^5}\right] = 0. \tag{5}$$

3 Location of collinear equilibrium points

When $y = 0$, (4) determines the locations of the collinear points $L_1(x_1, 0)$, $L_2(x_2, 0)$ and $L_3(x_3, 0)$, where

$$\begin{aligned} & x_1 = \mu - 1 - \xi_1, \qquad x_2 = \mu - 1 + \xi_2, \\ & x_3 = \mu + \xi_3. \end{aligned} \tag{6}$$

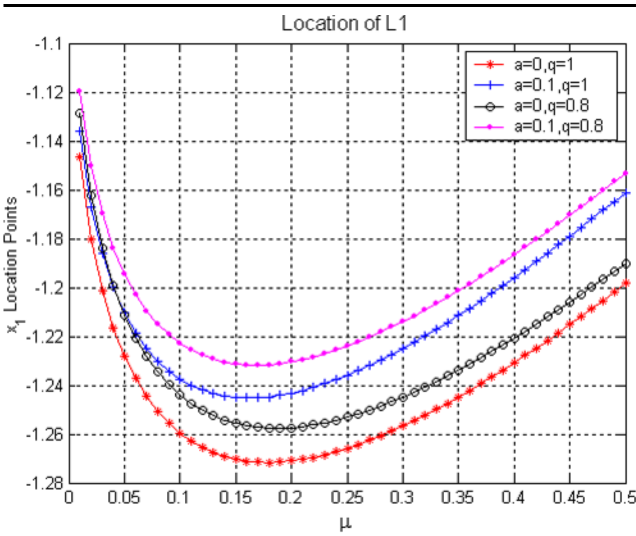

Fig. 1 Location of L_1 versus μ

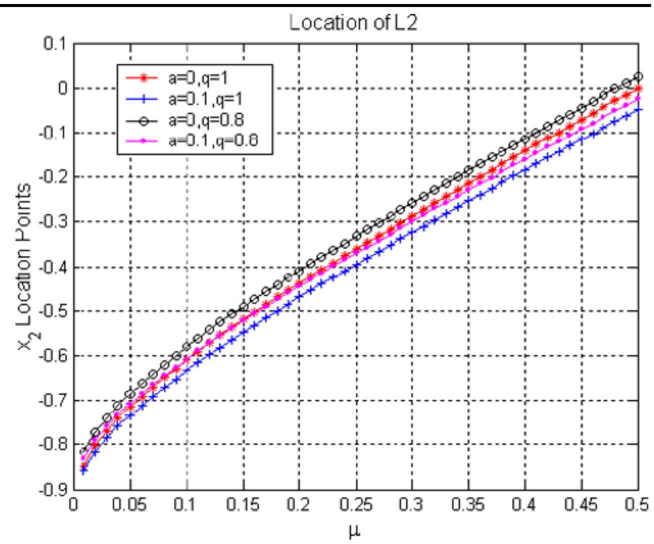

Fig. 2 Location of L_2 versus μ

ξ_1, ξ_2, ξ_3 satisfying the seventh degree polynomials:

$$\begin{aligned}
&(3A+2)\xi_1^7 + (A_1(15-3\mu) - 2\mu + 10)\xi_1^6 \\
&\quad + (-8\mu + A_1(30-12\mu) + 20)\xi_1^5 \\
&\quad + (\mu(2q-12) - 2q + A_1(30-18\mu) + 20)\xi_1^4 \\
&\quad + (\mu(4q-8) - 4q + A_1(15-12\mu) + 10)\xi_1^3 \\
&\quad + (A_1(\mu(3q-3) - 3q + 3) \\
&\quad + \mu(2q-4) - 2q + 2)\xi_1^2 - 8\mu\xi_1 - 2\mu = 0, \qquad (7)
\end{aligned}$$

$$\begin{aligned}
&(3A_1+2)\xi_2^7 + (A_1(3\mu-15) + 2\mu - 10)\xi_2^6 \\
&\quad + (-8\mu + A_1(30-12\mu) + 20)\xi_2^5 \\
&\quad + (\mu(2q+10) - 2q + A_1(18\mu-30) - 20)\xi_2^4 \\
&\quad + (-4\mu q + 4q + A_1(15-12\mu) + 10)\xi_2^3 \\
&\quad + (A_1(\mu(3q-3) - 3q - 3) \\
&\quad + \mu(2q-10) - 2q - 2)\xi_2^2 + 8\mu\xi_2 - 2\mu = 0, \qquad (8)
\end{aligned}$$

$$\begin{aligned}
&(3A_1+2)\xi_3^7 + (A_1(3\mu+6) + 2\mu + 4)\xi_3^6 \\
&\quad + (A_1(6\mu+3) + 4\mu + 2)\xi_3^5 \\
&\quad + (2\mu q + 2q + 3A_1\mu)\xi_3^4 + (4\mu q - 4q)\xi_3^3 \\
&\quad + (A_1(3\mu q - 3q) + 2\mu q - 2q)\xi_3^2 \\
&\quad + A_1(6\mu q - 6q)\xi_3 + A_1(3\mu q - 3q) = 0. \qquad (9)
\end{aligned}$$

The locations of the triangular points ($y \neq 0$) in xy-plane are given by Sharma (1982)

$$(2+3A_1)r_1^5 - 2qr_1^2 - 3A_1 q = 0, \qquad r_2^3 = \frac{2}{2+3A_1}.$$

Figures 1, 2 and 3 provide the locations of the collinear points for μ up to 0.5 for different values of A_1 and q. It is noted that L_1 comes nearer to the primaries with the increase in oblateness and radiation pressure. L_2 and L_3 move away from the more massive primary with the increase in oblateness and come nearer to it with the increase in radiation pressure.

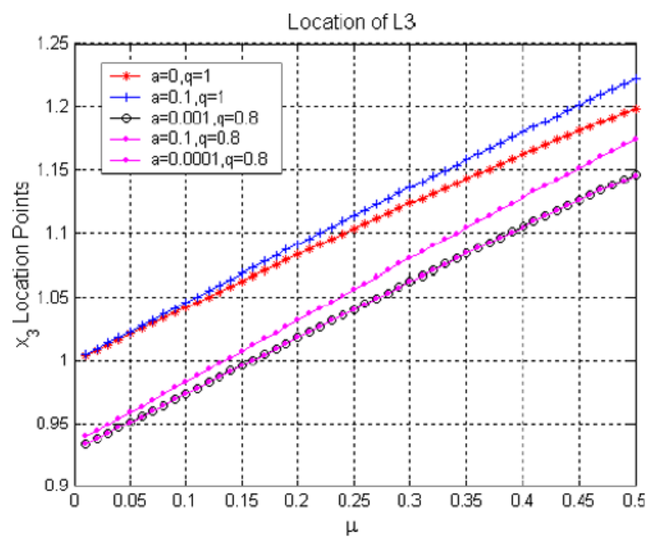

Fig. 3 Location of L_3 versus μ

4 Variational equations and characteristic exponents

The variational equations in the linear analysis become

$$\begin{aligned}
\ddot{\xi} - 2n\dot{\eta} &= \Omega_{xx}^0 \xi + \Omega_{xy}^0 \eta, \\
\ddot{\eta} + 2n\dot{\xi} &= \Omega_{xy}^0 \xi + \Omega_{yy}^0 \eta,
\end{aligned} \qquad (10)$$

where $x = a + \xi$, $y = b + \eta$, and the superscript '0' indicates that the second derivatives are to be evaluated at the points

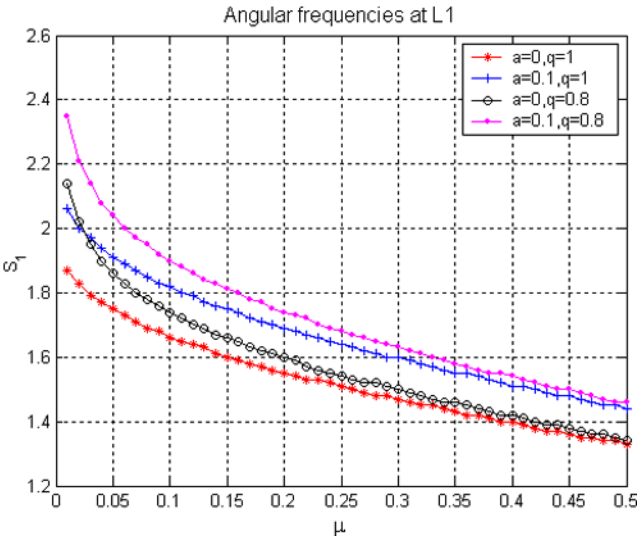

Fig. 4 Angular frequencies at L_1 versus μ

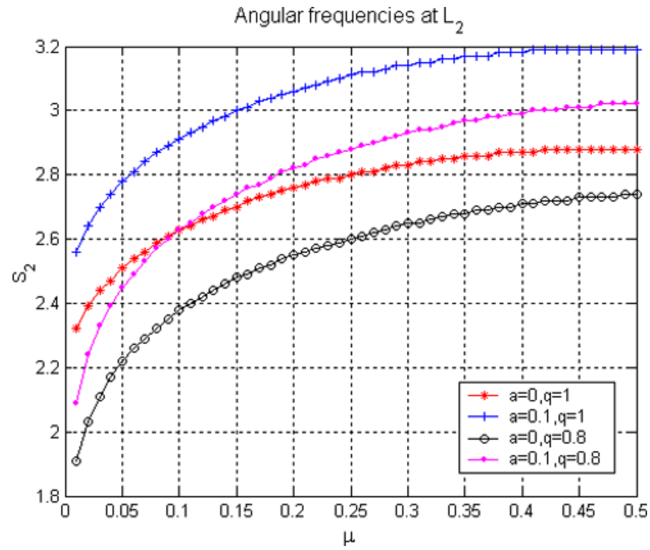

Fig. 5 Angular frequencies at L_2 versus μ

$L_i(a, b)$, $i = 1, 2, 3$.

The characteristic equation of (10) is given by

$$\lambda^4 + [4n^2 - \Omega_{xx}(a, b) - \Omega_{yy}(a, b)]\lambda^2 + [\Omega_{xx}(a, b)\Omega_{yy}(a, b) - \Omega^2_{xy}(a, b)] = 0. \tag{11}$$

At the collinear points, we have

$$\Omega_{xx} = n^2 + \frac{2q(1-\mu)}{r_1^3} + \frac{2\mu}{r_2^3} + \frac{6q(1-\mu)A_1}{r_1^5} > 0,$$

$$\Omega_{xy} = 0,$$

$$\Omega_{yy} = n^2 - \frac{q(1-\mu)}{r_1^3} - \frac{\mu}{r_2^3} - \frac{3q(1-\mu)A_1}{r_1^5} < 0.$$

Consequently, $\Omega_{xx}\Omega_{yy} - (\Omega_{xy})^2 < 0$.

The roots λ_i $(i = 1, 2, 3, 4)$ of (10) are

$$\lambda_{1,2} = \pm[-\beta_1 + (\beta_1^2 + \beta_2^2)^{1/2}]^{1/2} = \pm\lambda, \tag{12}$$

$$\lambda_{3,4} = \pm[-\beta_1 - (\beta_1^2 + \beta_2^2)^{1/2}]^{1/2} = \pm is, \tag{13}$$

where

$$\beta_1 = 2n^2 - (\Omega_{xx} + \Omega_{yy})/2,$$

$$\beta_2^2 = -(\Omega_{xx}.\Omega_{yy}) > 0.$$

The eccentricity of the periodic orbit is given by $e = \sqrt{(1 - 1/\beta_3^2)}$ and synodic period of the orbit is $2\pi/s$, where $\beta_3 = (s^2 + \Omega_{xx})/2ns$.

Figures 4, 5 and 6 provide the angular frequencies s_1, s_2 and s_3 at L_1, L_2 and L_3 for the mass parameter μ up to 0.5 for different values of A_1 and q. It may be noted from Fig. 4 that s_1 increases with oblateness as well as radiation pressure. The increase is more with radiation pressure for smaller values of μ. It may be noted from Fig. 5 that s_2 increases with oblateness and decreases with radiation pressure. However, it may be seen from Fig. 6 that s_3 decreases with oblateness and increases with radiation pressure. Figures 7 and 8 provide the norms of λ and s at the collinear points L_1, L_2, and L_3. It may be noted that the norms of λ and s at L_2 and L_3 are seen to be strictly increasing with mass ratio μ, while both norms at L_1 are strictly decreasing with μ. It may be seen that for the unperturbed case ($A_1 = 0$, $q = 1$) as in Deprit (1965), the value of λ and s at L_1 and L_3 coincide at $\mu = 0.5$ and at L_1 and L_2, these values coincide at $\mu = 0$. In the perturbed case, we observe that the oblateness coefficient increases λ and s values at L_1, L_2, and L_3. The increase in the radiation pressure increases λ and s L_1 and L_3 and decreases them at L_2. The values of λ and s

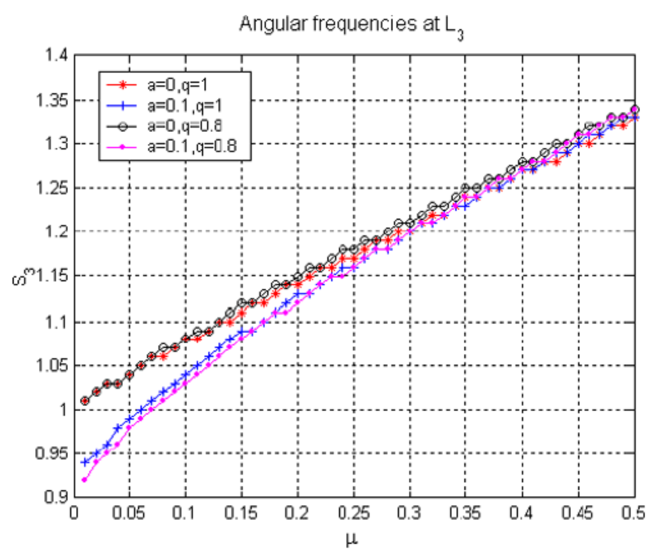

Fig. 6 Angular frequencies at L_3 versus μ

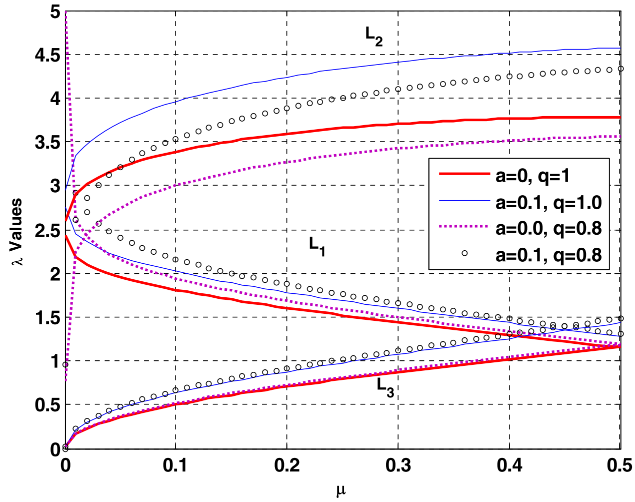

Fig. 7 λ values at L_1, L_2, L_3 vs. mass parameter for different combinations of oblateness and radiation pressure

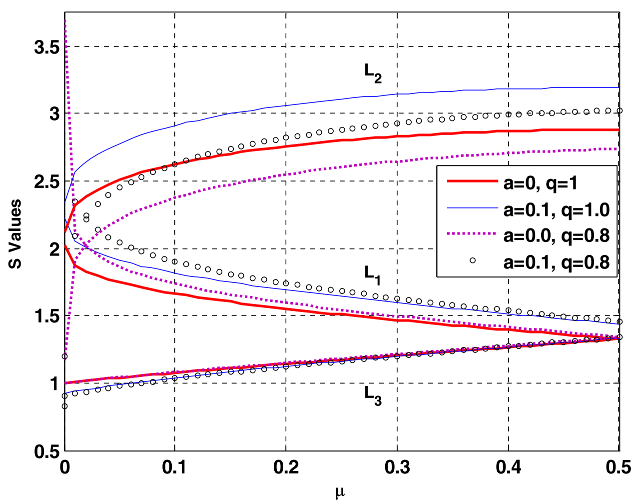

Fig. 8 s values at L_1, L_2, L_3 versus mass parameter different combinations of oblateness and radiation pressure

at L_1 and L_3 become equal for $\mu < 0.5$ and at L_1 and L_2, these values become equal for $\mu > 0$. It is interesting to note that with increase in oblateness, λ and s values at L_1 and L_2 coincide at $\mu = 0$ and with increase in radiation pressure, λ and s values at L_1 and L_3 coincide at $\mu = 0.5$, as in the unperturbed case. However, their values are higher.

5 Three-dimensional case

The equations of motion are (Szebehely 1967)

$$\ddot{x} - 2n\dot{y} = \frac{\partial \Omega}{\partial x},$$
$$\ddot{y} + 2n\dot{x} = \frac{\partial \Omega}{\partial y}, \tag{14}$$
$$\ddot{z} = \frac{\partial \Omega}{\partial z},$$

where

$$\Omega = \frac{n^2}{2}[(1-\mu)r_1^2 + \mu r_2^2] + q\frac{(1-\mu)}{r_1} + \frac{\mu}{r_2} + q\frac{(1-\mu)}{2r_1^3}A_1 - \frac{3q(1-\mu)A_1 z^2}{2r_1^5}, \tag{15}$$

with $r_1^2 = (x-\mu)^2 + y^2 + z^2$, $r_2^2 = (x+1-\mu)^2 + y^2 + z^2$.

Jacobi's integral is

$$\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = 2\Omega - C.$$

The singularities of the manifold of the state of motion are obtained from the equations $\dot{x} = 0$, $\dot{y} = 0$, $\dot{z} = 0$ and $\Omega_x = 0$, $\Omega_y = 0$, $\Omega_z = 0$.

The variational equations in the linear analysis become

$$\ddot{\xi} - 2n\dot{\eta} = \Omega_{xx}^0 \xi + \Omega_{xy}^0 \eta,$$
$$\ddot{\eta} + 2n\dot{\xi} = \Omega_{xy}^0 \xi + \Omega_{yy}^0 \eta, \tag{16}$$
$$\ddot{\zeta} = \Omega_{zz}^0 \zeta,$$

where $x = a + \xi$, $y = b + \eta$, $z = c + \zeta$ and the superscript '0' indicates that the second derivatives are to be evaluated at the points L_i (a, b, c), $i = 1, 2, 3$. It may be noted from (16) that the motion in xy plane does not influence the motion in the z-direction. At the collinear points, since we have

$$\Omega_{xx} + \Omega_{yy} + \Omega_{zz} = 2n^2,$$

the mean motion in the z-direction is

$$s_z = (-\Omega_{zz})^{1/2} = (\Omega_{xx} + \Omega_{yy} - 2n^2)^{1/2}. \tag{17}$$

Figures 9, 10 and 11 provide the angular frequencies s_z at L_1, L_2 and L_3 for the mass parameter μ up to 0.5 for different values of A_1 and q. It may be noted from Fig. 9 that s_z increases at L_1 with oblateness as well as radiation pressure. The increase is more with radiation pressure with smaller values of μ. It may be noted from Fig. 10 that at L_2, s_z increases with oblateness and decreases with radiation pressure. However, it may be seen from Fig. 11 that s_z decreases with oblateness and increases with radiation pressure.

6 One to one commensurability

To find out one-to-commensurability between the planar angular frequencies $s_{1,2,3}$ and the three-dimensional angular frequency s_z at the collinear points, (13) and (17) are equated and the values of the oblateness parameter A_1 are found by fixing the values of the mass parameter μ and the

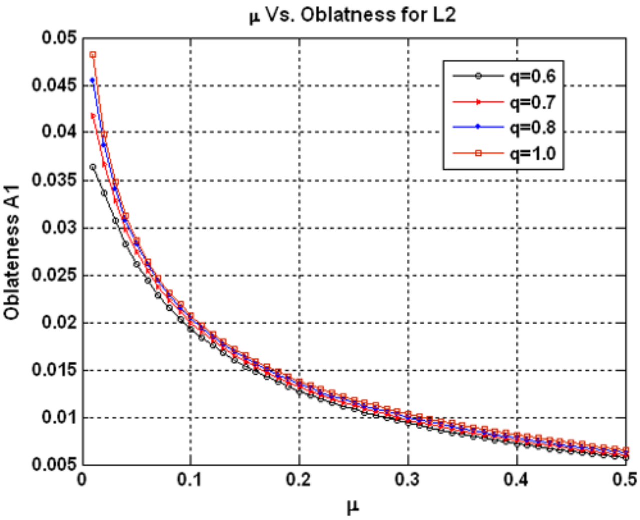

Fig. 9 Oblateness coefficient (A_1) versus mass parameter (μ) for $q = 0.6, 0.7, 0.8, 1.0$ when angular frequency $s_2 = s_z$ at L_2

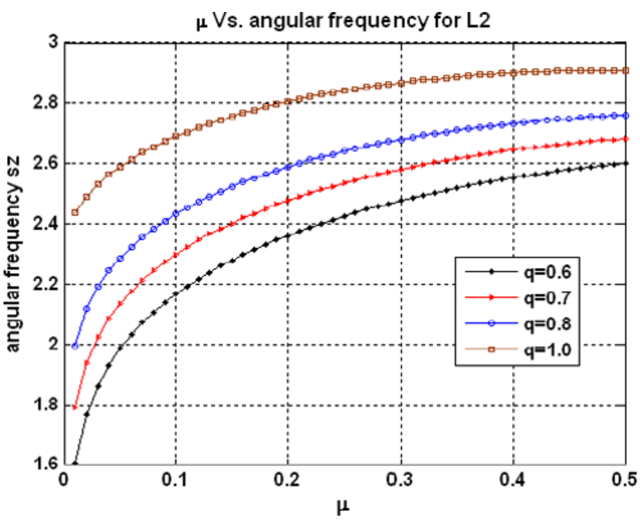

Fig. 10 Mass parameter (μ) versus commensurable angular frequency at $s_2 = s_z$ for $q = 0.6, 0.7, 0.8, 1.0$

radiation parameter q. It is found that the collinear points L_2 and L_3 have one-to-one commensurability, and that at L_1 no such commensurability exists. Figure 9 provides the values of A_1 at L_2 for the mass parameter μ up to 0.5 for $q = 0.6$, 0.7, 0.8, 1.0. It is noted that A_1 decreases with the increase in the radiation pressure effect. Figures 10 and 11 provide the value of the angular frequency ($s_2 = s_z$) and the eccentricity of the conditional periodic orbits, for μ up to 0.5 for $q = 0.6$, 0.7, 0.8, 1.0. It is noticed that both the parameters s_2 and e decrease with the increase in the radiation force. Similar results are provided for L_3 in the Figs. 12, 13 and 14. It may be noted that in the case of L_3, though A_1 decreases with the increase in the radiation pressure effect for obtaining one-to-one commensurability between s_3 and s_z, however, the parameters $s_3 = s_z$ and eccentricity of the conditional periodic

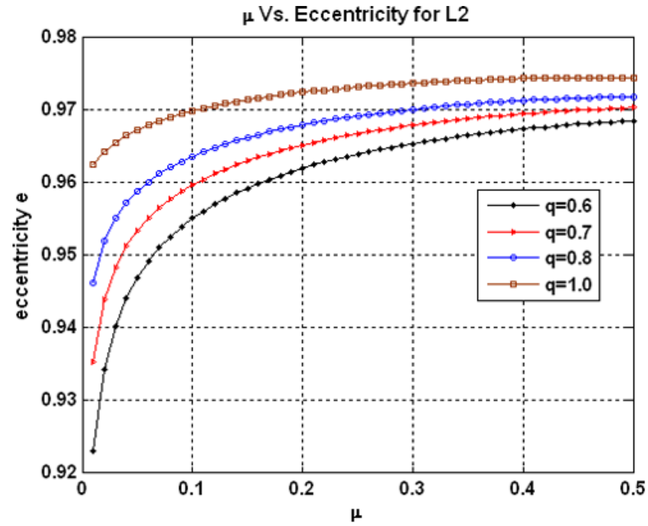

Fig. 11 Eccentricity (e) of the periodic orbits at L_2 when $s_2 = s_z$ versus mass parameter (μ) for $q = 0.6, 0.7, 0.8, 1.0$

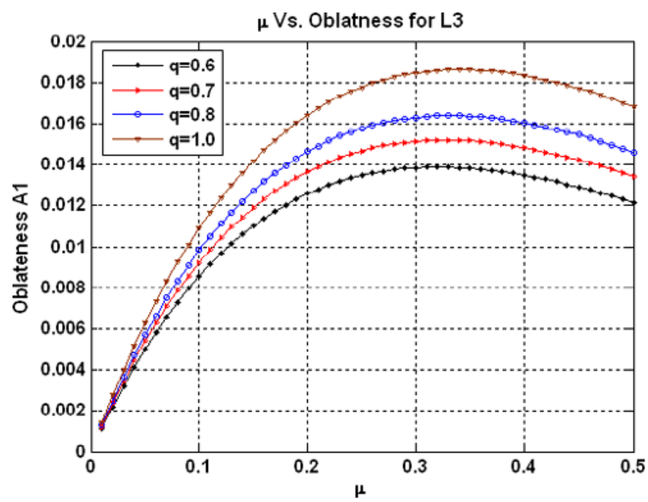

Fig. 12 Oblateness coefficient (A_1) versus mass parameter (μ) for $q = 0.6, 0.7, 0.8, 1.0$ when angular frequency $s_3 = s_z$

orbits increase with the increase in the radiation force. Advantage at L_3 is seen in terms of the decrease in the oblateness parameter in obtaining the one-to-one commensurable angular frequencies with the increase in the radiation force.

Figure 15 provides the variation of angular frequencies s_1 and s_z for the mass parameter μ up to 0.5 at L_1 ($s_1 < s_z$), for $A1 = 0.1$, $q = 0.9$, $A1 = 0.2$, $q = 0.1$. It is noted that at L_1 no commensurability exists between s_1 and s_z. The difference between s_1 and s_z increases with mass parameter. Also it is seen that as radiation pressure increases, s_1 and s_z also increase. Figure 16 shows that the increase in oblateness increases the eccentricity for fixed values of q, also as radiation pressure increases eccentricity increases. The eccentricity decreases with respect to mass parameter for a fixed value of q and A_1.

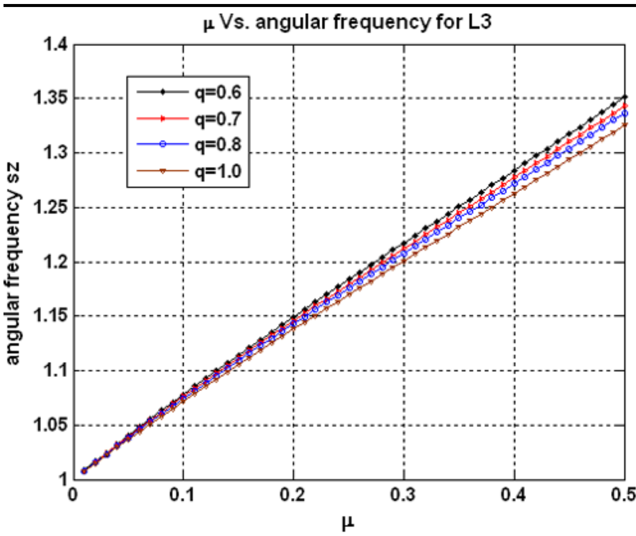

Fig. 13 Mass parameter (μ) versus commensurable angular frequency at $s_2 = s_z$ for $q = 0.6, 0.7, 0.8, 1.0$, for L_3

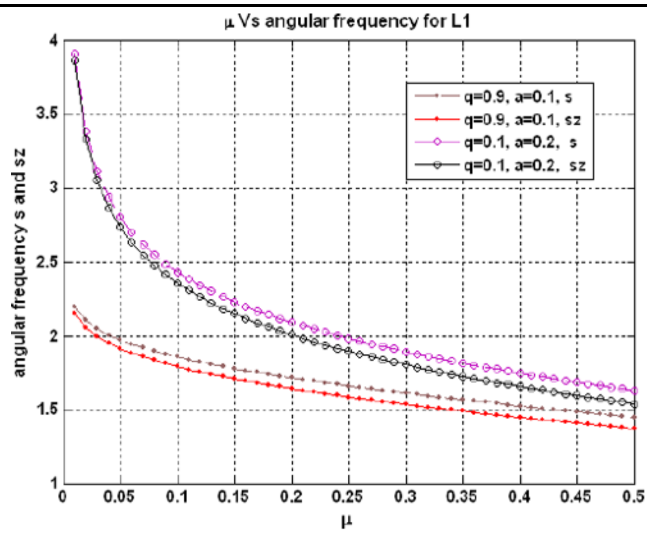

Fig. 15 Mass parameter (μ) versus angular frequencies (s and s_z) for $q = 0.9$, $A_1 = 0.1$; $q = 0.1$, $A_1 = 0.2$ at L_1

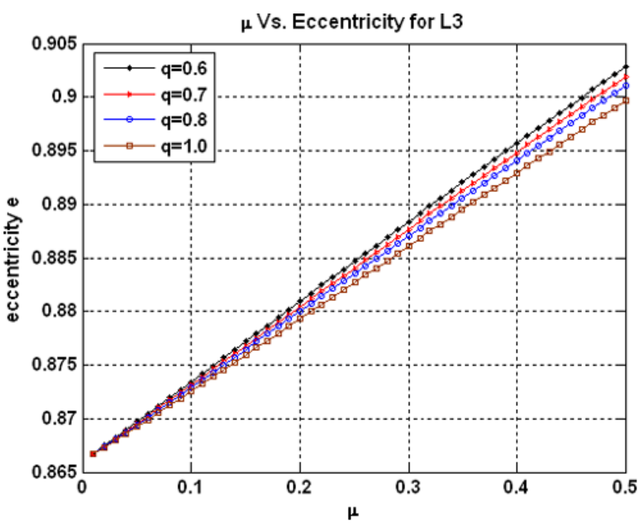

Fig. 14 Eccentricity (e) of the periodic orbits at L_3 when $s_3 = s_z$ versus mass parameter (μ) for $q = 0.6, 0.7, 0.8, 1.0$

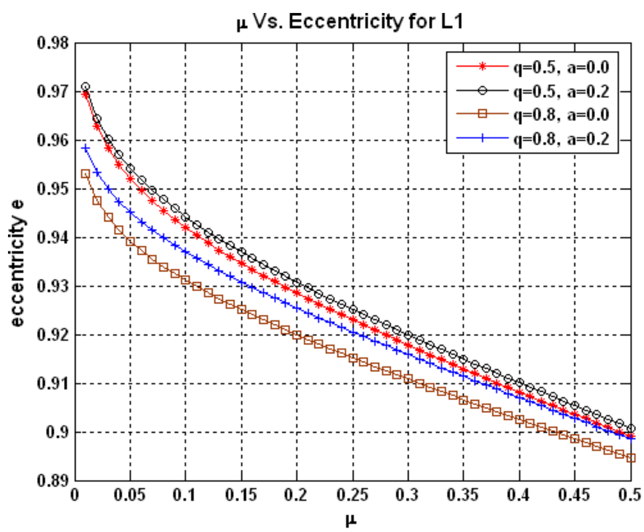

Fig. 16 Eccentricity of the periodic orbits at L_1 versus mass parameter (μ) at L_1 with $q = 0.5$, $A_1 = 0$; $q = 0.5$, $A_1 = 0.2$; $q = 0.8$, $A_1 = 0$; $q = 0.8$, $A_1 = 0.2$

7 Conclusions

A study on the effect of oblateness and radiation pressure on the location of linear collinear points, and the absolute values of the characteristic roots is carried out. We observe that, in the absence of radiation pressure, the increase in oblateness coefficient increases λ value for L_1, L_2, and L_3. Added to this effect if radiation pressure is also considered increasing, then, at L_1, L_3, λ value further increase, but at L_2 its value decreases. The effect of oblateness and radiation pressure on s values is same as that of λ values at L_1 and L_2, but at L_3 the increase in oblateness and radiation pressure decreases s values near $\mu = 0$ and increases slightly near $\mu = 0.5$ in comparison to the unperturbed case.

We have found that one-to-one commensurability exists between planar angular frequency, s, and the angular frequency in z-direction, s_z, at L_2 and L_3, and no such commensurability exists at L_1. At L_2, and L_3, the value of oblateness parameter providing the commensurability decreases with the increase in the radiation pressure. However, the commensurable angular frequencies and eccentricity of the periodic orbits decrease at L_2 and increase at L_3, with the increase in the radiation pressure. There is no one-to-one commensurability if the body is not oblate and the effect of radiation pressure decreases its eccentricity.

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