

# Statefinder diagnostic for dilaton dark energy

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**Abstract** Statefinder diagnostic is a useful method which can differ one dark energy model from the others. The Statefinder pair  $\{r, s\}$  is algebraically related to the equation of state of dark energy and its first time derivative. We apply in this paper this method to the dilaton dark energy model based on Weyl-Scaled induced gravitational theory. We investigate the effect of the coupling between matter and dilaton when the potential of dilaton field is taken as the Mexican hat form. We find that the evolving trajectory of our model in the  $r - s$  diagram is quite different from those of other dark energy models.

**Keywords** Dark energy · Statefinder · Dilaton · Mexican hat potential · Attractor

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Since the first observational data from SNe Ia (Riess 1998) is issued in 1998, exploring the nature of dark energy has been one of the most challengeable problems in theoretical physics and modern astrophysics. All data from SNe Ia (Riess et al. 2004; Perlmutter et al. 1999; Bahcall et al. 1999) together with data from WMAP5 (Hinshaw et al. 2008; Nolte et al. 2008) and SDSS (Tegmark et al. 2004) strongly

show us that, the Universe is spatially flat with about one third of the critical energy density being in non-relativistic matter and about two thirds of the critical energy density being in a smooth component with large negative pressure (dark energy), and is undergoing an accelerated expansion phase. Of course, with the recent data on the galaxy power spectrum from 2dF Galaxy Survey combined with CMB data (Spergel et al. 2003), the existence of dark energy (DE) can be proved without using the supernova data at all (Percival et al. 2002).

So far, many models have been proposed to fit the observations including cosmological constant  $\Lambda$  (Weinberg 1989; Sahni and Starobinsky 2000; Carroll 2001; Peebles and Ratra 2003; Padmanabhan 2003), quintessence (Amendola et al. 2006a; Elizalde et al. 2004; Nojiri et al. 2004; Nojiri and Odintsov 2006; Boisseau et al. 2000; Esposito-Farese and Polarski 2001; Zhang 2005b; Setare 2006; Faraoni and Jensen 2006; Wetterich 1988; Copeland et al. 2006; Ferreira and Joyce 1998; Frieman et al. 1995; Brax and Martin 2000; Barreiro et al. 2000; Zlatev et al. 1999; Padmanabhan and Choudhury 2002; Sen 2002b; Armendariz-Picon et al. 1999; Feinstein 2002; Fairbairn and Tytgat 2002; Frolov et al. 2002; Kofman and Linde 2004; Acatrinei and Sochichiu 2003; Alexander 2002; Padmanabhan 2002; Mazumadar et al. 2001; Sarangi and Tye 2002; Huang and Lu 2006; Huang et al. 2007a; Huang et al. 2007b; Fang et al. 2007), phantom (Amendola et al. 2006b; Amendola 2004; Gannouji et al. 2006; Lu 2005; Lu et al. 2005; Fang et al. 2006; Li and Hao 2004; Chiba et al. 2000; Singh et al. 2003; Carroll et al. 2003), holographic dark energy (Li 2004; Huang and Li 2004; Ito 2005; Ke and Li 2005; Huang and Li 2005; Gong et al. 2005; Zhang 2005a), Quintom (Hao et al. 2005; Guo et al. 2005; Feng 2006), tachyon (Sen 2002a; Garousi 2000; Garousi 2003; Bergshoeff et al. 2000; Kluson 2000; Gibbons 2002; Sami et al. 2002; Sami 2003; Piao et al. 2002;

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Kofman and Linde 2002) and Chaplygin gas (Mak and Harko 2005; Lu et al. 2005) so on. The essential characteristics of these dark energy models are contained in the parameter of its equation of state,  $p = \omega\rho$ , where  $p$  and  $\rho$  denote the pressure and energy density of dark energy, respectively, and  $\omega$  is a state parameter. Among these models, cosmological constant  $\Lambda$  model may be the simplest candidate. This constant term in Einstein field equation can be regarded as an fluid with the equation of state parameter  $\omega = -1$ . However, there are two serious problems with the cosmological constant, namely the fine-tuning and the cosmic coincidence. Firstly, in the framework of quantum field theory, the vacuum expectation value is 123 order of magnitude larger than the observed value of  $10^{-47}$  GeV<sup>4</sup>. The absence of a fundamental mechanism which sets the cosmological constant zero or very small value is the cosmological constant “fine-tuning” problem. Secondly, to explain in this way a constant vacuum energy density of  $10^{-47}$  GeV<sup>4</sup>, which is not only small but is also just the right value that it is just beginning to dominate the energy density of the Universe now, would require an unbelievable coincidence.

Quintessence model has been widely studied, and its state parameter  $\omega$  which is time-dependent, is greater than  $-1$ . In this paper, we regard dilaton in Weyl-scaled induced gravitational theory as a quintessence coupled with matter. It is well known that scalar-tensor theories are the most natural extensions of general relativity, in particular they contain local Lorentz invariance, constancy of nongravitational constants and respect the weak equivalence principle (Riazuelo and Uzan 2002; Will 1993; Bartolo and Pietroni 2000; Damour and Nordtvedt 1993; Chen and Kamionkowsky 1999; Baccigaluppi et al. 2000; Amendola 2000; Amendola 2001; Chen et al. 2001). In our previous papers (Huang et al. 2006), we have constructed a dilatonic dark energy model which belongs to nonminimal quintessence (Uzan 1999; Amendola 2000; Perrotta et al. 2000; Esposito-Farese and Polarski 2001; Amendola 1999; de Ritis et al. 2000; Bertolami and Martins 2000), based on Weyl-scaled induced gravitational theory. We found that when the dilaton field was not gravitational clustered at small scales, the effect of dilaton can not change the evolutionary law of baryon density perturbation, and the density perturbation can grow from  $z \sim 10^3$  to  $z \sim 5$ , which guarantees the structure formation. We have also investigated the property of the attractor solutions and concluded that the coupling between dilaton and matter affects the evolutive process of the Universe, but not the fate of the Universe.

With the remarkable increase in the accuracy of cosmological observational data during the last few years and the appearance of more general models of dark energy than a cosmological constant, advancing beyond quantities Hubble parameter  $H(t) \equiv \frac{\dot{a}}{a}$  and deceleration parameter  $q_0$  becomes a necessity. For this reason, Sahni et al. (2003), Alam et al.

(2003) propose a new geometrical diagnostic pair  $\{r, s\}$  for dark energy, which is called statefinder and can be expressed as follows.

$$r \equiv \frac{\ddot{a}}{aH^3}, \quad s \equiv \frac{r-1}{3(q-\frac{1}{2})} \quad (1)$$

where  $r$  is a natural next step beyond  $H$  and  $q$ . We can easily see that this diagnostic is constructed from the  $a(t)$  and its derivatives up to the third order. So, the statefinder probes the expansion dynamics of the universe through higher derivatives of the expansion factor. By far, many models (Chang et al. 2007; Gorini et al. 2003; Zhang 2005b; Zhang 2005a; Wu and Yu 2005; Zhang et al. 2006) have been differentiated by this geometrical diagnostic method. Its important property is that  $\{r, s\} = \{1, 0\}$  is a fixed point for the flat  $\Lambda$ CDM FRW cosmological model. Departure of a given DE model from this fixed point is a good way of establishing the “distance” of this model from flat  $\Lambda$ CDM. In this paper, we will investigate the evolutive trajectory of our model in the  $r-s$  diagram when the potential of dilaton field is taken as the Mexican hat potential, and show the difference between our model and the others, special  $\Lambda$ CDM.

The action of the Weyl-scaled induced gravitational theory is as follows:

$$S = \int d^4X \sqrt{-g} \times \left[ \frac{1}{2}R(g_{\mu\nu}) - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma) + L_{fluid}(\psi) \right] \quad (2)$$

where  $L_{fluid}(\psi) = \frac{1}{2}g^{\mu\nu}e^{-\alpha\sigma}\partial_\mu\psi\partial_\nu\psi - e^{-2\alpha\sigma}V(\psi)$ ,  $\alpha = \sqrt{\frac{3M_p^2}{2\varpi+3}}$  with  $\varpi > 3500$  (Will 2001) being an important parameter in Weyl-scaled induced gravitational theory,  $\sigma$  is dilaton field,  $g_{\mu\nu}$  is the Pauli metric which can really represent the massless spin-two graviton and should be considered to be physical metric (Cho 1992), and  $V(\sigma)$  is the potential of dilaton field. The conventional Einstein gravity limit occurs as  $\sigma \rightarrow 0$  for an arbitrary  $\varpi$  or  $\varpi \rightarrow \infty$  with an arbitrary  $\sigma$ . When  $V(\sigma) = 0$ , it will result in the Einstein-Brans-Dicke theory.

By varying action (2) and working in FRW universe, we obtain the field equations of Weyl-scaled induced gravitational theory:

$$H^2 = \frac{1}{3M_p^2}(\rho_m + \rho_\sigma), \quad (3)$$

$$\dot{H} = -\frac{1}{2M_p^2}(\rho_m + \rho_\sigma + p_\sigma), \quad (4)$$

$$\dot{\rho}_m + 3H\rho_m = \frac{1}{2}\alpha\dot{\sigma}\rho_m, \quad (5)$$

$$\dot{\rho}_\sigma + 3H\dot{\sigma}^2 = \frac{1}{2}\alpha e^{-\alpha\sigma}\rho_m, \quad (6)$$

where  $\rho_m$  is dark matter energy density,  $\rho_\sigma$  is dilaton dark energy density and radiation is neglected. The effective energy density and pressure of dilaton dark energy can be expressed as follows

$$\rho_\sigma = \frac{1}{2}\dot{\sigma}^2 + V(\sigma), \tag{7}$$

$$p_\sigma = \frac{1}{2}\dot{\sigma}^2 - V(\sigma). \tag{8}$$

For matter  $p_m = 0$ , we get  $\rho_m \propto \frac{e^{\frac{1}{2}\alpha\sigma}}{a^3}$  from (5). Using (3) and the  $e$ -folding transformation  $N = lna$ , we have

$$H = H_i \left[ \frac{\frac{1}{2}\dot{\sigma}^2 + V(\sigma)}{\rho_{c,i}} + \Omega_{m,i} e^{-\frac{1}{2}\alpha\sigma} e^{-3N} \right]^{\frac{1}{2}} \tag{9}$$

where  $H_i^2 = \frac{\rho_{c,i}}{3M_p^2}$ ,  $\rho_{c,i}$  is the critical energy density of the universe at initial time  $t_i$ .  $H_i$ ,  $\Omega_{m,i}$ , denote the Hubble parameter, matter energy density parameter at initial time  $t_i$  respectively.

Using the definition of (1) and (3–6), one can find that

$$r = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} = 1 - \frac{3}{2}\Omega_\sigma\omega'_\sigma + \frac{9}{2}\omega_\sigma\Omega_\sigma(1 + \Omega_\sigma) - \frac{3}{4}\alpha\sigma'(1 - \Omega_\sigma) - \frac{3}{4}\frac{\alpha e^{-\alpha\sigma}}{H}(1 - \Omega_\sigma)(1 + \omega_\sigma), \tag{10}$$

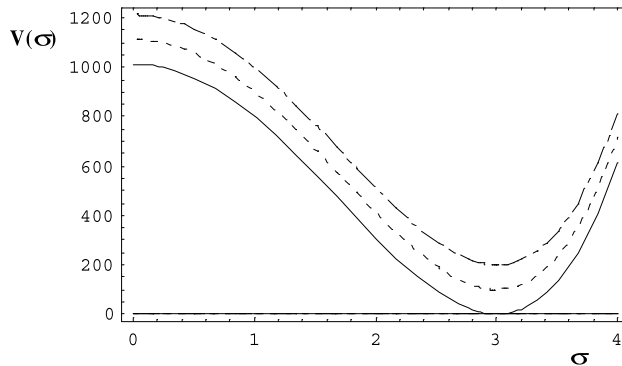
$$q = -1 - \frac{\dot{H}}{H^2} = \frac{1}{2}(1 + 3\omega_\sigma\Omega_\sigma), \tag{11}$$

$$s \equiv \frac{r - 1}{3(q - \frac{1}{2})} = 1 + \omega_\sigma - \frac{1}{3}\frac{\omega'_\sigma}{\omega_\sigma} - \frac{\alpha\sigma' (1 - \Omega_\sigma)}{6\omega_\sigma \Omega_\sigma} - \frac{1}{6}\frac{\alpha e^{-\alpha\sigma}}{H} \frac{1 - \Omega_\sigma}{\Omega_\sigma} \frac{1 + \omega_\sigma}{\omega_\sigma} \tag{12}$$

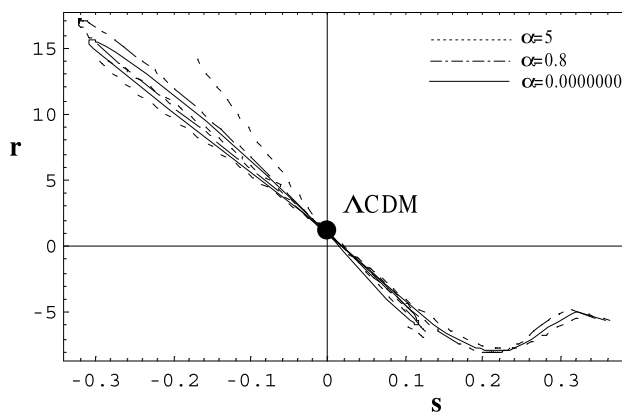
where a prime denotes the derivative with respect to the  $e$ -folding time  $N = lna$  and  $\Omega_i \equiv \frac{\rho_i}{3M_p^2 H^2}$  for  $i = m$  and  $\sigma$ .

Now, let us consider the Mexican hat potential  $V(\sigma) = \frac{\mu}{4}(\sigma^2 - \varepsilon^2)^2 + V_0$  where  $\mu, \varepsilon$  and  $V_0$  are all constant. For this type of Mexican hat potential, it has two extremum points in the range  $\sigma \geq 0$ : a minimum at  $\sigma = \varepsilon$  and a maximum at  $\sigma = 0$ . The non-conventional parameter  $V_0$  in this potential moves the potential up and down, which is equivalent to adding a cosmological constant to the usual Mexican hat potential. We show the features of the Mexican hat potential mathematically in Fig. 1.

In Fig. 2, the  $\{r, s\}$  phase portrait is shown numerically. When the coupling parameter  $\alpha$  is set 5 (dot line), 0.8 (dot-dashed line) and 0.00000001 (real line), the evolutive trajectories of  $r(s)$  are very similar. This means that the intensity of coupling between dilaton and matter changes the evolutive trajectory of  $r(s)$  weakly. This result is consistent with a conclusion obtained from our previous paper (Nojiri et al. 2004): the coupling between dilaton and matter



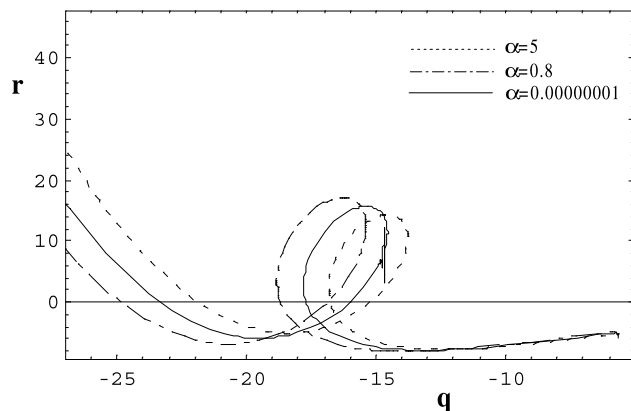
**Fig. 1** The Mexican hat potential  $V(\sigma) = \frac{\mu}{4}(\sigma^2 - \varepsilon^2)^2 + V_0$ . We set  $V_0 = 0$  (real line), 100 (dot line), 200 (dot-dashed line)



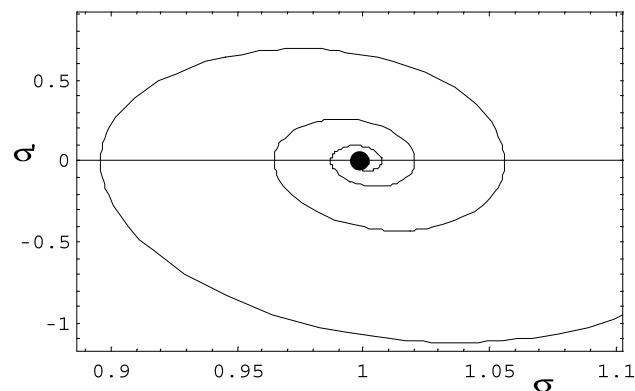
**Fig. 2** The  $r - s$  diagram of Mexican hat potential  $V(\sigma) = \frac{\mu}{4}(\sigma^2 - \varepsilon^2)^2 + V_0$ . Curves  $r(s)$  evolves in the  $e$ -fold time interval  $N \in [0, 0.878]$ . The black dot corresponds the fixed point of  $\Lambda$ CDM,  $\{r = 1, s = 0\}$ .  $\alpha$  denoting the intensity of coupling between dilaton and matter, is set for  $\alpha = 0.00000001$  (real line),  $\alpha = 0.8$  (dot-dashed line) and  $\alpha = 5$  (dot line) respectively

affects the evolutive process of the Universe, but not the fate of the Universe. We can easily see that the trajectory of  $r(s)$  will pass the fixed point  $\{r = 1, s = 0\}$  of  $\Lambda$ CDM in the future and is different from the other dark energy models. Figure 3 shows the evolutive behavior of parameter  $r$  with respect to deceleration parameter in the range of  $e$ -folding time  $N \in [0, 0.92]$ . The evolutive behavior for different coupling parameter  $\alpha$  will tend to the same one. Figure 4 shows that the shape of the evolutive trajectory of  $\sigma' - \sigma$  is a stable spiral corresponding to the equation of state  $\omega = -1$  and the dilaton dark energy density parameter  $\Omega = 1$ , which are important features for a dark energy model that can meet the current observations.

In summary, we apply the statefinder diagnostic to the dilaton dark energy model based on the Weyl-scaled induced gravitational theory with Mexican hat potential  $V(\sigma) = \frac{\mu}{4}(\sigma^2 - \varepsilon^2)^2 + V_0$ . The effect of coupling between dilaton and matter on the evolutive trajectory of  $r(s)$  with respect to the  $e$ -folding time  $N = lna$ , is investigated in this



**Fig. 3** The diagram  $r - q$  of Mexican hat potential  $V(\sigma) = \frac{\mu}{4}(\sigma^2 - \varepsilon^2)^2 + V_0$  when we set  $\alpha = 0.00000001$  (real line),  $\alpha = 0.8$  (dot-dashed line) and  $\alpha = 5$  (dot line) respectively



**Fig. 4** The phase portrait of attractor in the Mexican hat potential for  $\alpha = 0.00000001$ . The trajectory of attractor is a stable spiral

paper. First, we get the attractor solution is a stable spiral. Second, according to the numerical results, we get the coupling between dilaton and matter changes the evolutive behavior of  $r(s)$  very weakly and the trajectories of  $r(s)$  for different coupling parameter  $\alpha$  will always pass the fixed point  $\{r = 1, s = 0\}$  corresponding to  $\Lambda$ CDM model. At last, we find that the evolutive trajectory of  $r(s)$  forms a swirl before reaches attractor and is quite different from those of other dark energy models (Sahni et al. 2003; Alam et al. 2003; Chang et al. 2007; Gorini et al. 2003; Zhang 2005b; Zhang 2005a; Wu and Yu 2005; Zhang et al. 2006).

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