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ORIGINAL ARTICLE

Ion acoustic solitons and double layers in electron–positron–ion plasmas with dust particulates

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Abstract Propagation of small but finite amplitude ion acoustic solitons and double layers are investigated in electron–positron–ion plasmas in presence of highly negatively charged impurities or dust. The presence of negatively charged dust particulates can result in existence of two critical concentrations of ion–electron density ratio α. One of them α_D decides the existence of double layers, whereas the other one α_R decides the nature of the solitons and double layers. The system supports both compressive and rarefactive solitons as well as double layers. The parameter regimes of transitions from compressive to rarefactive solitons and double layers are also specified.

Keywords Electron–positron–ion–dust multi-component plasma · Ion acoustic solitary waves · Ion acoustic double layers

1 Introduction

Electron–positron–ion plasma is usually characterized as a fully ionized gas consisting of electrons and positrons, the masses of which are equal (Tandberg-Hansen and Emsile 1988). The presence of ions leads to the existence of several low frequency waves which otherwise do not propagate in electron–positron plasmas. Electron–positron plasmas are found in astrophysical plasmas such as in magnetosphere of pulsars, in active galactive nuclei, in early universe, and in the regions of the accretion disks surrounding the central black holes (Golderich and Julian 1969; Sturrock 1971; Misner et al. 1973; Michel 1982; Rees 1983; Miller and Witta 1987). In the last few years, there has been considerable interest in different types of coherent linear and nonlinear wave structures such as solitons, double layers, vortices, etc., in electron–positron plasmas (Shukla et al. 1986; Yu et al. 1986; Tandberg-Hansen and Emsile 1988; Tajima and Taniuti 1990; Bharuthram 1992; Shukla et al. 1993; Shukla and Stenflo 1993; Shukla 1993) as well as in multi-component electron–positron–ion plasmas (Rizzato 1988; Berezhiani et al. 1994; Popel et al. 1995). These nonlinear wave structures arise due to the balance between nonlinearity and dispersion of the wave.

However, most of the astrophysical plasmas usually contain highly charged (negative/positive) impurities or dust particulates in addition to the electrons, positrons and ions. Clearly, the properties of wave motions in electron–positron–ion–dust plasma should be different from those in three component electron–positron–ion plasmas (Rizzato 1988; Berezhiani et al. 1994; Popel et al. 1995). For example, the presence of highly charged dust particulates modifies the dispersion relation for ion acoustic wave by increasing its phase velocity in electron–ion plasma (D'Angelo 1990; Shukla and Silin 1992; Barkan et al. 1996; Nakamura et al. 1999). Also it does modify the nonlinear properties of ion acoustic waves (Popel et al. 1996; Luo et al. 1999; Nakamura and Sharma 2001; Shukla 2000; Ghosh et al. 2000b; 2000a; Ghosh 2006). Thus it is instructive to examine the collective behaviours of ion acoustic waves in electron–positron–ion–dust plasmas.

In this paper the coherent nonlinear wave structures such as solitons and double layers of small but finite ampli-

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tude ion acoustic wave in electron–positron–ion–dust plasmas are investigated which were not considered in the earlier investigations (Rizzato 1988; Berezhiani et al. 1994; Popel et al. 1995). The paper is organized in the following manner. Formulation of the problem and the basic equations are given in Sect. 2. The nonlinear modified KdV equation is derived in Sect. 3. The nature of the steady state solution of the modified KdV equation and its different physical consequences are discussed in Sect. 4. Numerical results are presented in Sect. 5. Section 6 deals with the conclusions.

2 Formulation and basic equations

A four-component plasma consisting of electrons (e) with number density n_e, positrons (p) with number density n_p, singly charged cold positive ions (i) with number density n_i and negatively fixed charged immobile dust grains (d) with number density n_{d0} is considered. At $x=-\infty$, the plasma is assumed to be in its equilibrium state defined by $\phi=0$; $n_e=n_{e0}$; $n_p=n_{p0}$; $n_i=n_{i0}$ so that the plasma is quasi-neutral

$$n_{e0}+z_d n_{d0}=n_{i0}+n_{p0}\Rightarrow \alpha=n_{i0}/n_{e0}=1+\delta_d-\delta_p \quad (1)$$

where $\delta_d=z_d n_{d0}/n_{e0}$, $\delta_p=n_{p0}/n_{e0}$ and z_d is the number of negatively elementary charges on the dust grains.

The nonlinear propagation of low phase velocity (in comparison with the electron and positron thermal velocities) ion acoustic waves is governed by the following normalized equations

$$\frac{\partial N_i}{\partial T}+\frac{\partial (N_i V_i)}{\partial X}=0 \quad (2)$$

$$\frac{\partial V_i}{\partial T}+V_i\frac{\partial V_i}{\partial X}=-\frac{1}{\alpha_p}\frac{\partial \Phi}{\partial X} \quad (3)$$

$$\sigma\frac{\partial^2\Phi}{\partial X^2}=N_e-\sigma\alpha_p N_i-\delta_p N_p+\delta_d \quad (4)$$

where $\alpha_p=\alpha/\sigma=n_{i0}/\sigma n_{e0}$, $\sigma=(1+\delta_p/\sigma_p)$, $\sigma_p=T_p/T_e$, T_p and T_e are the temperatures of positrons and electrons respectively. The electrons and positrons are in thermodynamical equilibrium so that their densities obey the following Boltzmann distribution

$$N_e=\exp(\Phi); \qquad N_p=\exp\left(-\frac{\Phi}{\sigma_p}\right). \quad (5)$$

The following normalizations are used in the above equations (2)–(5):

$$X=\frac{x}{\lambda_D};\quad T=\omega_i t;\quad \Phi=\frac{e\phi}{T_e};\quad V_i=\frac{v_i}{c_{ia}};$$
$$N_i=\frac{n_i}{n_{i0}};\quad N_e=\frac{n_e}{n_{e0}};\quad N_p=\frac{n_p}{n_{p0}} \quad (6)$$

where $\lambda_D(=1/\sqrt{\lambda_{De}^{-2}+\lambda_{Dp}^{-2}}=\lambda_{De}/\sqrt{\sigma})$ is plasma Debye Length, $\lambda_{De}(=\sqrt{\varepsilon_0 T_e/n_{e0}e^2})$, $\lambda_{Dp}(=\sqrt{\varepsilon_0 T_p/n_{p0}e^2})$ are the electron and positron Debye Length, $\omega_i(=\sqrt{n_{i0}e^2/\varepsilon_0 m_i})$ is ion plasma frequency and $c_{ia}(=\sqrt{T_e\alpha_p/m_i})$ is ion acoustic speed.

3 Dynamics of solitons

To study the dynamics of small but finite amplitude solitons, the independent variables are stretched as

$$\xi=\sqrt{\varepsilon}\left(X-V_{ph}T\right);\qquad \tau=\varepsilon^{3/2}T \quad (7)$$

where V_{ph} is the normalized phase velocity of the linear ion acoustic wave and ε is a small parameter characterizing the strength of the nonlinearity. The dynamical variables are expanded in powers of ε as follows

$$\begin{aligned}N_i&=1+\varepsilon N_i^{(1)}+\varepsilon^2 N_i^{(2)}+\cdots,\\ V_i&=\varepsilon V_i^{(1)}+\varepsilon^2 V_i^{(2)}+\cdots,\\ \Phi&=\varepsilon\Phi^{(1)}+\varepsilon^2\Phi^{(2)}+\cdots.\end{aligned} \quad (8)$$

Now to the lowest order in ε, (2)–(5) yield,

$$V_i^{(1)}=V_{ph}N_i^{(1)},\quad \Phi^{(1)}=\alpha_p V_{ph}^2 N_i^{(1)},\quad \Phi^{(1)}=\alpha_p N_i^{(1)}. \quad (9)$$

The above set of (9) self-consistently yields the linear wave phase velocity

$$V_{ph}=1\Rightarrow\frac{\omega}{k}=\sqrt{\frac{T_e\alpha_p}{m_i}}=\sqrt{\frac{T_e\alpha}{m_i\sigma}}=\sqrt{\frac{T_e(1+\delta_d-\delta_p)}{m_i(1+\delta_p/\sigma_p)}}. \quad (10)$$

This is the normalized phase velocity of long wavelength ion acoustic waves in electron–positron–ion–dust plasmas. It is interesting to note that in absence of dust particulates i.e. for $\delta_d=0$, the phase velocity of ion acoustic waves ($\omega/k=\sqrt{T_e(1-\delta_p)/m_i(1+\delta_p/\sigma_p)}$) in electron–positron–ion plasmas can be recovered (Popel et al. 1995). On the other hand, in absence of positrons i.e. for $\delta_p=0$, the phase velocity of ion acoustic waves ($\omega/k=\sqrt{T_e(1+\delta_d)/m_i}=\sqrt{T_e n_{i0}/m_i n_{e0}}$) in electron–ion–dust plasmas i.e. phase velocity of dust ion acoustic waves can be recovered (Shukla and Silin 1992).

To order ε^2, using relations (9) and (10), the following equations are obtained

$$\frac{2}{\alpha_p}\frac{\partial\Phi^{(1)}}{\partial\tau}+\frac{3}{\alpha_p^2}\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi}=\frac{\partial N_i^{(2)}}{\partial\xi}-\frac{1}{\alpha_p}\frac{\partial\Phi^{(2)}}{\partial\xi} \quad (11)$$

$$\sigma \frac{\partial^2 \Phi^{(1)}}{\partial \xi^2} = \sigma \Phi^{(2)} - \sigma \alpha_p N_i^{(2)} + \left(1 - \frac{\delta_p}{\sigma_p^2}\right) \frac{\Phi^{(1)^2}}{2}. \quad (12)$$

Finally, eliminating all the second order quantities from (11), (12) and using equations in (14), the following KdV equation is obtained

$$\frac{\partial \Phi^{(1)}}{\partial \tau} + \alpha_2 \Phi^{(1)} \frac{\partial \Phi^{(1)}}{\partial \xi} + \beta \frac{\partial^3 \Phi^{(1)}}{\partial \xi^3} = 0. \quad (13)$$

The coefficients of nonlinearity α_2 and dispersion β are as follows

$$\alpha_2 = \beta \left[3 - \frac{\alpha_p}{\sigma}\left(1 - \frac{\delta_p}{\sigma_p^2}\right)\right] = \beta \left[3 - \frac{\alpha}{\sigma^2}\left(1 - \frac{\delta_p}{\sigma_p^2}\right)\right]; \quad \beta = \frac{1}{2}. \quad (14)$$

It follows that in the limit $\delta_p = 0, \delta_d = 0 \Rightarrow \alpha = 1$, $\sigma = 1$, the well-known results $\alpha_2 = 1$ and $\beta = \frac{1}{2}$ is recovered (Davidson 1972).

Transforming the KdV equation (13) with $\Phi^{(1)} = \psi$ to the wave frame $\eta = \xi - U\tau$ and integrating the transformed equation with respect to η with the help of the boundary conditions $\psi, d_\eta \psi$ and $d_\eta^2 \psi \to 0$ as $|\eta| \to \infty$, the following equation is obtained

$$\frac{1}{2}\left(\frac{d\psi}{d\eta}\right)^2 + V(\psi, U) = 0; \quad V(\psi, U) = \frac{\psi^2}{2\beta}\left(\frac{\alpha_2}{3}\psi - U\right) \quad (15)$$

where $V(\psi, U)$ is the Sagdeev potential.

For the existence of solitary waves, one requires that the Sagdeev potential $V(\psi, U)$ has to satisfy the following conditions (Sagdeev 1966)

$$\begin{aligned} &\text{(i)} \quad V(\psi, U)\big|_{\psi=0} = 0 = \partial_\psi V(\psi, U)\big|_{\psi=0}, \\ &\text{(ii)} \quad V(\psi, U)\big|_{\psi=\psi_S} = 0, \\ &\text{(iii)} \quad V(\psi, U) < 0; \quad 0 < |\psi| < |\psi_S|, \end{aligned} \quad (16)$$

where ψ_S is the amplitude of the solitary wave. These conditions yield the following single soliton solution of (13)

$$\psi = \Phi^{(1)} = \psi_S \operatorname{Sech}^2\left(\frac{\eta}{\Delta_S}\right); \quad \psi_S = \frac{3U}{\alpha_2}; \quad \Delta_S = \sqrt{\frac{4\beta}{U}} \quad (17)$$

where Δ_S is the spatial width of the soliton. The nature of the solitons (compressive or rarefactive) depends on the sign of the coefficient of quadratic nonlinearity α_2 (see (14)). Compressive solitons exist for $\alpha_2 > 0$ and rarefactive solitons exist for $\alpha_2 < 0$ which corresponds to the following condition

$$\alpha_2 > (<)\, 0 \Rightarrow \alpha < (>)\, \alpha_R = \frac{3\sigma^2}{(1 - \delta_p/\sigma_p^2)} = \frac{3(1 + \delta_p/\sigma_p)^2}{(1 - \delta_p/\sigma_p^2)}$$

provided $\delta_p < \sigma_p^2$. (18)

It is evident from (18) that $\alpha_R > 1$. In absence of negatively charged dust particulates i.e. for $\delta_d = 0$, the charge neutrality condition (1) reveals that always $\alpha < 1$ and thereby $\alpha < \alpha_R \Rightarrow \alpha_2 > 0$ i.e. ion acoustic waves possess only compressive solitons (Popel et al. 1995). But in presence of negatively charged dust particulates i.e. $\delta_d \neq 0$, α can easily be greater than unity if $\delta_d > \delta_p \Rightarrow z_d n_{d0}/n_{p0} > 1$ (see (1)) i.e. the charge density ratio of dust–positron is greater than unity.

4 Dynamics of weak double layers

In the previous section it is seen that as $\alpha \to \alpha_R$, the coefficient of quadratic nonlinearity $\alpha_2 \to 0$ and in this case soliton solution is not possible. Hence to study the nonlinear wave propagation characteristics at $\alpha \approx \alpha_R$, higher order nonlinear effects must be included that are also important in the study of double layers. To study the dynamics of weak double layers, the independent variables are stretched as

$$\xi = \varepsilon\left(X - V_{ph}T\right); \quad \tau = \varepsilon^3 T. \quad (19)$$

On substituting the stretching (19) into (2)–(5), one can easily obtain the earlier results. Next order terms i.e. $O(\varepsilon^3)$ of (2)–(3) and using first order solutions (9) and (10), the following second order solutions are obtained

$$N_i^{(2)} = \frac{1}{\alpha_p}\Phi^{(2)} + \frac{3}{2\alpha_p^2}\Phi^{(1)^2}; \quad V_i^{(2)} = \frac{1}{\alpha_p}\Phi^{(2)} + \frac{1}{2\alpha_p^2}\Phi^{(1)^2}. \quad (20)$$

Poisson equation (4) with the help of (5) and (6) at $O(\varepsilon^2)$ yields

$$-\gamma \Phi^{(1)^2} = 0; \quad \gamma = \frac{\alpha_2}{\alpha_p}. \quad (21)$$

Since $\Phi^{(1)} \neq 0$, it follows that at least $|\gamma| \sim O(\varepsilon)$ which is justified because at $\alpha \approx \alpha_R$, $|\alpha_2| \ll 1$ implying $|\gamma| \ll 1$ and thus $\gamma \Phi^{(1)^2} \sim O(\varepsilon^3)$. Hence it should be included in the next higher order i.e. $O(\varepsilon^3)$ of Poisson equation.

Finally, eliminating all the third order variables i.e. the terms $O(\varepsilon^4)$ using the first and second order solutions (see (9), (10) and (20)) and neglecting a higher order term

$\gamma\partial_\xi(\Phi^{(2)}\Phi^{(1)}) \sim O(\varepsilon^5)$, the following modified form of KdV equation is derived

$$\frac{\partial\Phi^{(1)}}{\partial\tau} + \alpha_2\Phi^{(1)}\frac{\partial\Phi^{(1)}}{\partial\xi} + \alpha_3{\Phi^{(1)}}^2\frac{\partial\Phi^{(1)}}{\partial\xi} + \beta\frac{\partial^3\Phi^{(1)}}{\partial\xi^3} = 0. \tag{22}$$

The coefficient of cubic nonlinearityα_3 is as follows

$$\begin{aligned}\alpha_3 &= \frac{1}{4\alpha_p^2}\left[15 - \frac{\alpha_p^2}{\sigma}\left(1 + \frac{\delta_p}{\sigma_p^3}\right)\right] \\ &= \frac{\sigma^2}{4\alpha^2}\left[15 - \frac{\alpha^2}{\sigma^3}\left(1 + \frac{\delta_p}{\sigma_p^3}\right)\right].\end{aligned} \tag{23}$$

Following the same procedure as done in Sect. 3, the Sagdeev potential corresponding to the modified KdV equation (23) is given by

$$V(\psi, U) = \frac{\psi^2}{2\beta}\left(\frac{\alpha_3}{6}\psi^2 + \frac{\alpha_2}{3}\psi - U\right). \tag{24}$$

For the existence of double layers, one requires that the Sagdeev potential $V(\psi, U)$ has to satisfy the following conditions (Sagdeev 1966; Bharuthram and Shukla 1986)

$$\begin{aligned}&\text{(i)} \quad V(\psi, U)\big|_{\psi=0} = 0 = V(\psi, U)\big|_{\psi=\psi_D}, \\ &\text{(ii)} \quad d_\psi V(\psi, U)\big|_{\psi=0} = 0 = d_\psi V(\psi, U)\big|_{\psi=\psi_D}, \\ &\text{(iii)} \quad d_\psi^2 V(\psi, U)\big|_{\psi=0,\psi_D} < 0\end{aligned} \tag{25}$$

where $\psi = 0$, ψ_D are two extreme points of the Sagdeev potential $V(\psi, U)$. The condition (i) of (25) yields

$$\begin{aligned}&U = -\frac{\alpha_3}{6}\psi_D^2; \\ &\psi_D = -\frac{\alpha_2}{\alpha_3} \Rightarrow V(\psi, U) = \frac{\alpha_3\psi^2}{12\beta}(\psi_D - \psi)^2.\end{aligned} \tag{26}$$

The double layer solution of (22) is given by

$$\begin{aligned}&\psi = \Phi^{(1)} = \frac{\psi_D}{2}\left[1 - \tanh\left(\frac{2\eta}{\Delta_D}\right)\right]; \\ &\Delta_D = 4\sqrt{\frac{6\beta}{-\alpha_3}}\Big/|\psi_D|\end{aligned} \tag{27}$$

where $\Delta_D$is the thickness of the double layer.

It should be noted from (26) and (27) that for the existence of double layer the following condition must be satisfied

$$-\frac{\alpha_3}{\beta} > 0 \Rightarrow \alpha_3 < 0 \quad \text{as } \beta\left(=\frac{1}{2}\right) > 0. \tag{28}$$

It is observed from the expression for the coefficient of cubic nonlinearity (23) that condition (28) is satisfied if and only if

$$\alpha_3 < 0 \Rightarrow \alpha > \alpha_D = \sqrt{\frac{15\sigma^3}{1 + \delta_p/\sigma_p^3}}. \tag{29}$$

The nature of the double layers i.e. whether the system will support a compressive or rarefactive double layer, depends upon the sign of coefficient of quadratic nonlinear term α_2. It follows from (26) that a compressive or rarefactive double layer exists according as

$$\alpha_2 > (<)\, 0 \Rightarrow \alpha < (>)\, \alpha_R. \tag{30}$$

5 Numerical analysis and discussions

All the equations obtained in the previous section are analyzed numerically. The variations of coefficient of quadratic nonlinearity $\alpha_2(\alpha, \delta_p, \sigma_p)$ (see (14)) are drawn in Fig. 1. This figure shows that α_2 increases with the increase of α $(= n_{i0}/n_{e0})$, whereas it decreases with the increase of δ_p $(= n_{p0}/n_{e0})$ at low σ_p $(= T_p/T_e)$ [Fig. 1: $\sigma_p = 0.1$] for $\delta_p > \sigma_p^2$. On the other hand at high σ_p for $\delta_p < \sigma_p^2$ [Fig. 1: $\sigma_p = 5$], α_2 acts conversely. From the comparison between the solid and dotted lines in Fig. 1, it is important to note that $\alpha_2 > 0$ for all values of σ_p and $\alpha \leq 1 \Rightarrow \delta_d < \delta_p$, whereas $\alpha_2 < 0$ for all values of $\alpha > 1 \Rightarrow \delta_d > \delta_p$ and $\delta_p < \sigma_p^2$ [dotted line in Fig. 1: $\delta_p = 0.2$, $\sigma_p = 5$]. Thus the ion acoustic waves possess negative ($\alpha_2 < 0$) potentials only if $\delta_p < \sigma_p^2$ and $\delta_d > \delta_p \Rightarrow z_d n_{d0}/n_{p0} > 1$, i.e.

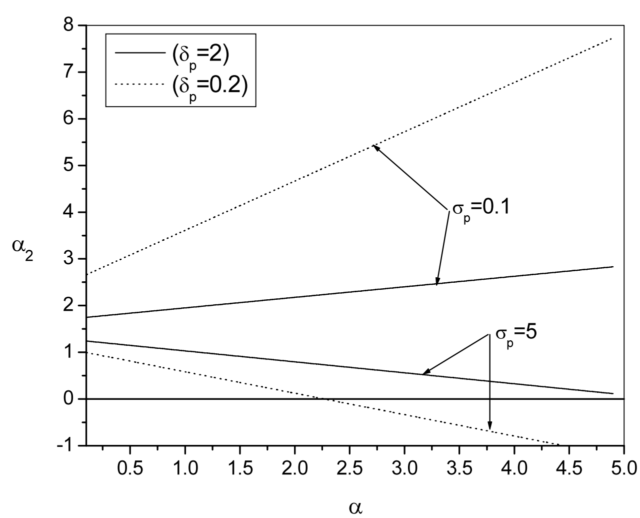

Fig. 1 Variations of coefficient of quadratic nonlinearity α_2 (see (14)) with ion–electron density ratio $\alpha = n_{i0}/n_{e0}$ for different positron–electron density ratio δ_p $(= n_{p0}/n_{e0})$ and positron–electron temperature ratio σ_p $(= T_p/T_e)$

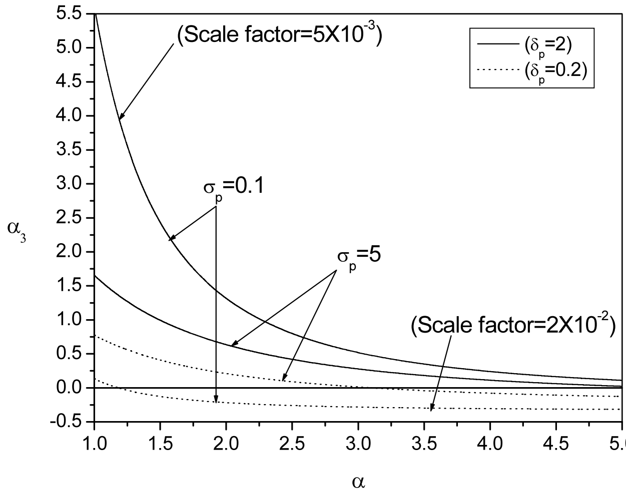

Fig. 2 Variations of coefficient of cubic nonlinearity α_3 (see (23)) with ion–electron density ratio $\alpha = n_{i0}/n_{e0}$ for different positron–electron density ratio δ_p $(= n_{p0}/n_{e0})$ and positron–electron temperature ratio σ_p $(= T_p/T_e)$

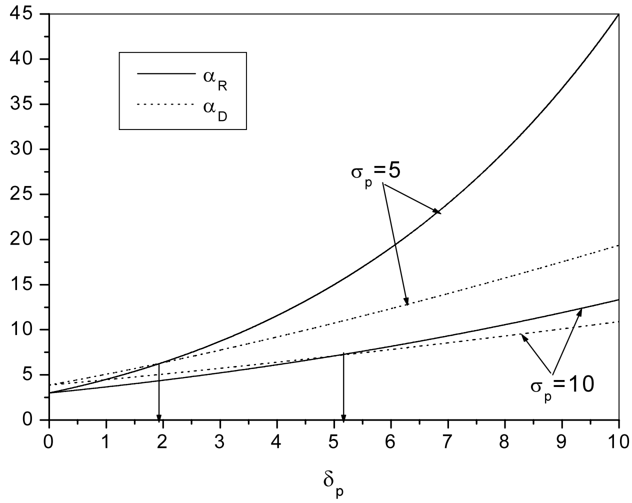

Fig. 3 Variations of critical ion–electron density ratios α_R (see (19)) and α_D (see (29)) with positron–electron density ratio δ_p $(= n_{p0}/n_{e0})$ for different positron–electron temperature ratio σ_p $(= T_p/T_e)$

dust charge density is greater than the positron charge density. The solitary wave amplitude, which is proportional to α_2^{-1} (see (17)), decreases with α, but it increases (decreases) with increase of δ_p for $\delta_p > \sigma_p^2$ $(\delta_p < \sigma_p^2)$. Also note that $\alpha_2 > 0 \Rightarrow \Phi^{(1)} > 0 \Rightarrow N_i^{(1)} > 0$ and $\alpha_2 < 0 \Rightarrow \Phi^{(1)} < 0 \Rightarrow N_i^{(1)} < 0$ [by virtue of (9) and (17)]. Thus solitary wave with positive potential implies compressive soliton with ion density enhancement and solitary wave with negative potential implies rarefactive soliton with ion density depletion.

The variations of coefficient of cubic nonlinearity $\alpha_3(\alpha, \delta_p, \sigma_p)$ (see (23)) are drawn in Fig. 2. It is observed from this figure that α_3 decreases with the increase of α (> 1) and finally $\alpha_3 < 0$, for higher values of α $(\gg 1)$, which is the condition for the formation of double layer (see (28)). This happens only because of the presence of negatively charged dust grains $(\delta_d \neq 0)$ as in absence dust grains $(\delta_d = 0)$, α is always less than unity and increase of α implies increase of δ_d (see (1)). Thus as the negatively charged dust density increases they are able to build up the charge separation effect, which is required to maintain and support the required electric field for double layer potential structure.

From the comparison between the solid [Fig. 2: $\delta_p = 2$] and dotted [Fig. 2: $\delta_p = 0.2$] curves, it is observed that for a fixed σ_p, the criterion for formation of double layer $\alpha_3 < 0$ (see (28)) is satisfied at low δ_p but not at high δ_p. This is attributed to the fact that the positrons are able to penetrate and mix better with the electrons through their thermal motion $(T_p \neq 0)$. Thus as the positron density increases they are able to destroy the charge separation effect required for the double layer potential structure.

Also from the comparison between the dotted curves [Fig. 2: $\delta_p = 0.2$], it is observed that for a fixed δ_p, the criterion for formation of double layer $(\alpha_3 < 0)$ (see (28)) is satisfied for low value of α at low σ_p $(= 0.1)$ but for high value of α at high σ_p $(= 5)$. This means that the system needs more negatively charged dust grains to form double layer at higher value of σ_p than at lower values of σ_p. This happens because of the fact that increase (decrease) in σ_p implies increase (decrease) of thermal velocity of positron and thereby enhancing (reducing) their ability to mix with the electrons and decrease (increase) the charge separation effect supporting double layer potential structure. The dotted curves also show that the double layer exists for both low and high values of σ_p.

The variations of critical points $\alpha_R(\delta_p, \sigma_p)$ (see (18)) and $\alpha_D(\delta_p, \sigma_p)$ (see (29)) with δ_p are drawn in Fig. 3. This figure reveals that both the critical points increase with the increase of δ_p and decrease with the increase of σ_p [Fig. 3: solid and dotted curves]. It is observed that there exists a critical value of δ_p^{cr} [~ 1.98 for $\sigma_p = 5$ and ~ 5.015 for $\sigma_p = 10$], below which $\alpha_R < \alpha_D$ and above which $\alpha_R > \alpha_D$. Thus the system supports rarefactive double layer for $\delta_p < \delta_p^{cr}$ and $\alpha > \mathrm{Max}(\alpha_R, \alpha_D)$, whereas it supports compressive double layer for $\delta_p > \delta_p^{cr}$ and $\alpha_D < \alpha < \alpha_R$. But no double layer exists (only rarefactive soliton exists) for $\delta_p < \delta_p^{cr}$ and $\alpha < \alpha_R < \alpha_D$.

The profiles of solitary potentials ψ $(= \Phi^{(1)})$ (see (17)) as a function of η are drawn in Fig. 4 for different δ_p and σ_p satisfying the condition $\delta_p < \sigma_p^2$. This figure shows that for positive potentials the amplitude of the solitary wave decreases with the increase of δ_p [Fig. 4: solid curve vs dotted curve], whereas it increases with the increase of σ_p [Fig. 4: solid curve vs dashed curve]. But in case of negative potentials the amplitude varies conversely.

The profiles of stationary double layers ψ $(= \Phi^{(1)})$ (see (27)) as a function of η are depicted in Fig. 5 for dif-

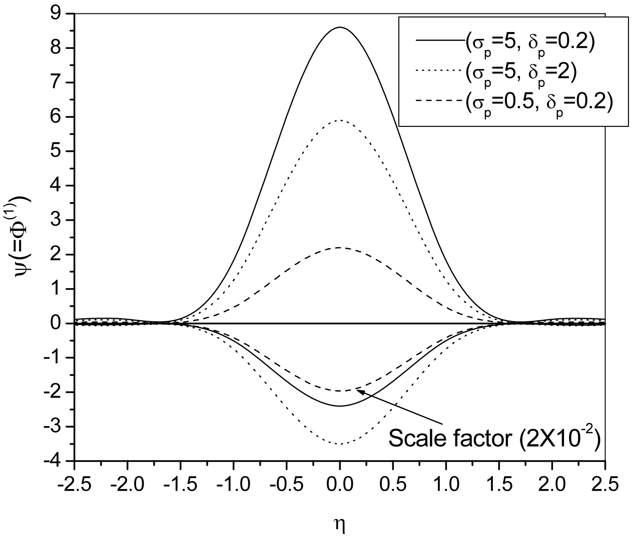

Fig. 4 Profiles of solitary potentials ψ $(=\Phi^{(1)})$ (see (17)) with η for different positron–electron density ratio δ_p $(=n_{p0}/n_{e0})$ and positron–electron temperature ratio σ_p $(=T_p/T_e)$

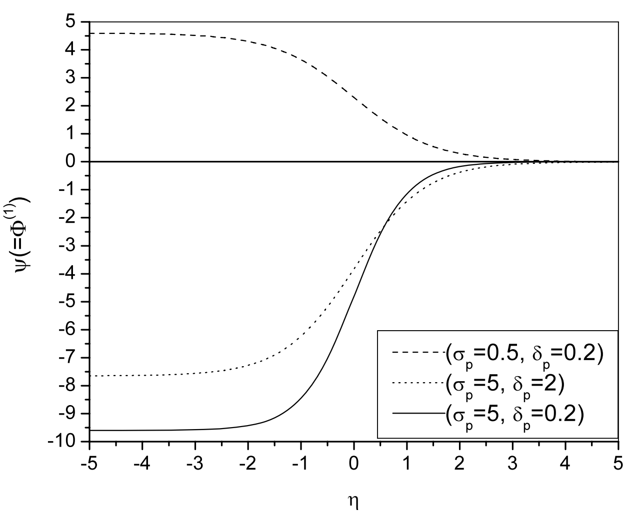

Fig. 5 Profiles of double layers ψ $(=\Phi^{(1)})$ (see (27)) with η for different positron–electron density ratio δ_p $(=n_{p0}/n_{e0})$ and positron–electron temperature ratio σ_p $(=T_p/T_e)$

ferent δ_p and σ_p satisfying the condition $\delta_p < \sigma_p^2$. This figure shows that the system support compressive double layers only at low δ_p and σ_p [Fig. 5: dashed curve]. Otherwise the system supports only rarefactive double layers [Fig. 5: solid and dotted curves].

6 Summary

The nonlinear propagation characteristics of ion acoustic waves in electron–positron–ion–dust plasmas are investigated. The results of this investigation can be summarized as follows:

(a) The presence of negatively charged dust particulates causes the generation of solitons (compressive and rarefactive) and double layers (compressive and rarefactive) in electron–positron–ion plasma, whereas, in a dust free electron–positron–ion plasma only compressive solitons are observed (Popel et al. 1995).

(b) The rarefactive soliton exists only for $\alpha > \alpha_R$, $\delta_p < \sigma_p^2$ and the amplitude of the soliton decreases (increases) with the increase of $\delta_p(\sigma_p)$. In case of $\delta_p < \sigma_p^2$, the compressive soliton exists only for $\alpha < \alpha_R$ and the amplitude of the soliton decreases (increases) with the increase of $\delta_p(\sigma_p)$. But in case of $\delta_p > \sigma_p^2$, the compressive soliton exists for all values of α and the amplitude of the soliton increases with the increase of both δ_p and σ_p.

(c) Depending upon the values of σ_p, there exists a critical value of δ_p $(=\delta_p^{cr})$, which determines the range of α for the existence of both compressive and rarefactive double layers. Compressive double layers exist only for $\delta_p > \delta_p^{cr}$ and $\alpha_D < \alpha < \alpha_R$, whereas rarefactive double layers exist for $\delta_p < \delta_p^{cr}$ and $\alpha > \mathrm{Max}(\alpha_R, \alpha_D)$.

(d) The necessary conditions for the existence of weak double layer are respectively $\gamma \sim O(\varepsilon)$ (see (21)) and $\alpha_3 < 0$ (see (28)). For the plasma parameters considered here $\gamma \sim O(10^{-1})$ and $\alpha_3 < 0$. Thus the necessary conditions are easily satisfied.

(e) The results of this investigation should be useful in understanding the properties of low phase velocity localized electrostatic perturbations that may appear in astrophysical plasmas. Also double layers are formed due to net potential differences of electrostatic fields and any charged particle traversing the double layer is directly accelerated by this potential differences. Therefore, double layers act as the possible charged particle accelerators. Thus the results may be relevant to the magnetosphere of pulsars (Sturrock 1971; Michel 1982), solar flare (Alfven and Carlqvist 1967), Jupiter's magnetosphere (Smith and Goertz 1978) and also in the auroral plasma (Temerin et al. 1982), where it is believed that the charged particle acceleration takes place due to double layers.

(f) Finally, the present investigation deals with the case $\nu_d/\omega_i \approx 0$ (ν_d is the dust charging frequency) i.e. the charge on the dust grains is fixed. But in many real life situations the charge on dust grain varies with time and the charge variation under the assumption that $\omega_i/\nu_d \neq 0$ produces anomalous dissipation, which generates shock wave in the nonlinear regime (Popel et al. 1996; Ghosh et al. 2000b). The study incorporating dust charge variations is under consideration and the results shall be reported elsewhere in future.

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