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ORIGINAL ARTICLE

An exact Bianchi type-V cosmological model in Saez-Ballester theory of gravitation

V.U.M. Rao · T. Vinutha · M. Vijaya Shanthi

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Abstract An exact Bianchi type-V cosmological model is obtained in a scalar-tensor theory of gravitation proposed by Saez and Ballester (Phys. Lett. A 113:467, 1986) in case of perfect fluid distribution. Some physical properties of the model are also discussed.

Keywords Bianchi type-V · Saez-Ballester theory · Perfect fluid distribution

1 Introduction

Einstein's general theory of relativity (1916) has provided a sophisticated theory of gravitation. It has been very successful in describing gravitational phenomena. It has also served as a basis for models of the universe. The homogeneous isotropic expanding model based on general relativity appears to provide a grand approximation to the observed large scale properties of the universe. However, since Einstein first published his theory of gravitation a no. of modifications have been proposed from time to time which seek to incorporate into the theory certain desirable features lacking in the original theory. For example, Einstein himself pointed out that general relativity does not account satisfactorily for the inertial properties of matter, i.e. Mach's principle is not substantiated by general relativity. So, in recent years, there have been some interesting attempts to generalize the general theory of relativity by incorporating Mach's principle and other desired features which are lacking in the original theory.

Brans and Dicke (1961) introduced a scalar-tensor theory of gravitation involving a scalar function in addition to the familiar general relativistic metric tensor. In this theory the scalar field has the dimension of inverse of the gravitational constant and its role is confined to its effects on gravitational field equations. Subsequently Saez and Ballester (1986) developed a scalar-tensor theory in which the metric is coupled with a dimension less scalar field. In spite of the dimension less character of the scalar field an antigravity regime appears. This theory suggests a positive way to solve the missing matter problem in non-flat FRW cosmologies.

Saez and Ballester field equations for combined scalar and tensor field are

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = -8\pi T_{ij} \tag{1.1}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n\phi^{;i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{1.2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is an Einstein tensor, T_{ij} is the stress energy tensor of the matter, ω is the dimensionless constant, n is also a constant.

The equation of motion

$$T^{ij}_{,j} = 0 \tag{1.3}$$

is a consequence of the field (1.1) and (1.2).

Singh and Agarwal (1991), Shri and Tiwari (1998), Singh and Shri (2003), Reddy and Venkateswara Rao (2001) and

V.U.M. Rao (✉) · M. Vijaya Shanthi
Department of Applied Mathematics, Andhra University, Visakhapatnam, AP, India
e-mail: umrao57@hotmail.com

T. Vinutha
Department of Basic Sciences and Humanities (Mathematics), College of Engineering, GITAM University, Visakhapatnam, AP, India

Reddy (2003), Reddy et al. (2006) are some of the authors who have investigated several aspects of this theory. Singh and Chaubey (2006) have studied Bianchi type-V model with a perfect fluid and with the term lambda in general relativity. Saha (2006) has studied the evaluation of Bianchi type-I space-time in the presence of a perfect fluid. Several people worked on Bianchi type-V model with a perfect fluid in general relativity.

In this paper we discuss Bianchi type-V cosmological model in a scalar-tensor theory proposed by Saez and Ballester in the presence of perfect fluid distribution. Spatially homogeneous Bianchi models in Saez and Ballester theory in the presence of perfect fluid with or without radiation are quite important to discuss the early stages of evolution of the universe.

Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. The simplicity of the field equations and relative ease of solutions made Bianchi space times useful in constructing models of spatially homogeneous and anisotropic cosmologies. A complete list of all solutions of Einstein's equations for Bianchi type's I–IX with perfect fluid is given by Krammer et al. (1980).

2 Metric and field equations

We consider the Bianchi type-V cosmological model in the form

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 e^{-2mx} dy^2 - a_3^2 e^{-2mx} dz^2. \tag{2.1}$$

With the metric functions a_1, a_2, a_3 are functions of t only and m is a constant.

The energy momentum tensor for the perfect fluid is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \tag{2.2}$$

where ρ is the density and p is the pressure.

Also

$$g_{ij} u^i u^j = 1. \tag{2.3}$$

In the co moving coordinate system, we have from (2.2) and (2.3)

$$T_1^1 = T_2^2 = T_3^3 = -p, \qquad T_4^4 = \rho \quad \text{and}$$
$$T_j^i = 0 \quad \text{for } i \neq j. \tag{2.4}$$

The quantities ρ, p and the scalar field ϕ in the theory depend on t only.

The field (1.1–1.3) for the metric (2.1) with the help of (2.2–2.4) can be written as

$$\frac{m^2}{a_1^2} - \frac{\ddot{a}_2}{a_2} - \frac{\ddot{a}_3}{a_3} - \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{1}{2}\omega\phi^n\dot{\phi}^2 = 8\pi p, \tag{2.5}$$

$$\frac{m^2}{a_1^2} - \frac{\ddot{a}_1}{a_1} - \frac{\ddot{a}_3}{a_3} - \frac{\dot{a}_1\dot{a}_3}{a_1a_3} + \frac{1}{2}\omega\phi^n\dot{\phi}^2 = 8\pi p, \tag{2.6}$$

$$\frac{m^2}{a_1^2} - \frac{\ddot{a}_1}{a_1} - \frac{\ddot{a}_2}{a_2} - \frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{1}{2}\omega\phi^n\dot{\phi}^2 = 8\pi p, \tag{2.7}$$

$$\frac{3m^2}{a_1^2} - \frac{\dot{a}_1\dot{a}_2}{a_1a_2} - \frac{\dot{a}_1\dot{a}_3}{a_1a_3} - \frac{\dot{a}_2\dot{a}_3}{a_2a_3} - \frac{1}{2}\omega\phi^n\dot{\phi}^2 = -8\pi\rho, \tag{2.8}$$

$$2m\frac{\dot{a}_1}{a_1} - m\frac{\dot{a}_2}{a_2} - m\frac{\dot{a}_3}{a_3} = 0, \tag{2.9}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) + \frac{n}{2\phi}\dot{\phi}^2 = 0, \tag{2.10}$$

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) = 0. \tag{2.11}$$

Here the over head dot denotes differentiation with respect to t.

Equation (2.9) on integration gives

$$a_1^2 = c_1 a_2 a_3.$$

Without loss of generality by taking the constant of integration $c_1 = 1$, we get

$$a_1^2 = a_2 a_3. \tag{2.12}$$

The field equations (2.5) to (2.8) by the use of (2.12) yield

$$\frac{a_1}{a_2} = d_1\left(\frac{a_2}{a_3}\right)^{k_1}, \tag{2.13a}$$

$$\frac{a_2}{a_3} = d_2\left(\frac{a_3}{a_1}\right)^{k_2}, \tag{2.13b}$$

$$\frac{a_3}{a_1} = d_3\left(\frac{a_1}{a_2}\right)^{k_3} \tag{2.13c}$$

where $k_1, k_2, k_3, d_1, d_2, d_3$ are constants of integration.

Using (2.12) in (2.13a–2.13c), we get

$$a_2 = \frac{1}{A}a_1, \tag{2.14a}$$

$$a_3 = A a_1. \tag{2.14b}$$

Since the equations are highly non-linear, to get a determinate solution, let

$$a_1 = at + b \tag{2.15}$$

where a, b are constants of integration and $a \neq 0$.

From (2.14a), (2.14b) and (2.15), we get

$$a_2 = \frac{1}{A}(at+b), \tag{2.16}$$

$$a_3 = A(at+b). \tag{2.17}$$

From (2.10) we get

$$a_1 a_2 a_3 \dot{\phi}\phi^{\frac{n}{2}} = k_4$$

where k_4 is a constant of integration.

Using (2.12) in the above equation, we get

$$a_1^3 \dot{\phi}\phi^{\frac{n}{2}} = k_4. \tag{2.18}$$

Using (2.17) in the above equation, we get

$$\phi = \left\{\frac{2}{n+2}\left[\frac{k_5}{2a(at+b)^2} + k_6\right]\right\}^{\frac{2}{n+2}} \tag{2.19}$$

where k_5 and k_6 are constants of integration.

Now substituting (2.16), (2.17) and (2.19) in (2.5), we get

$$p = \frac{(m^2-a^2)}{8\pi(at+b)^2} + \frac{\omega k_4^2}{16\pi(at+b)^6} \tag{2.20}$$

and substituting (2.16), (2.17) and (2.19) in (2.8), we get

$$\rho = \frac{-3(m^2-a^2)}{8\pi(at+b)^2} + \frac{\omega k_4^2}{16\pi(at+b)^6}. \tag{2.21}$$

Here we will get the following three cases

(i) $m > a$,
(ii) $m < a$,
(iii) $m = a$.

In cases of (i) and (ii) at the late epoch i.e., as $t \to \infty$, the second term in (2.20) and (2.21) decrease much faster, then there is a possibility for ρ and p respectively to become negative at certain stage of evolution, which is physically not possible. Hence the only possibility is case (iii) i.e., $m = a$.

In case of $m = a$,

$$\rho = p = \frac{\omega k_4^2}{16\pi(at+b)^6}. \tag{2.22}$$

Here we get a Zeldovich fluid distribution.

The corresponding metric can now be written in the form

$$ds^2 = dt^2 - (at+b)^2 dx^2 - \frac{1}{A^2}(at+b)^2 e^{-2mx} dy^2 - A^2(at+b)^2 e^{-2mx} dz^2. \tag{2.23}$$

Thus (2.23) together with (2.19) and (2.22) constitutes an exact Bianchi type-V cosmological Zeldovich universe in Saez Ballester theory of gravitation with $a = m$.

3 Physical properties

The model represented by (2.23) represents spatially homogeneous Bianchi type-V cosmological model in the scalar-tensor theory of gravitation proposed by Saez and Ballester (1986). The scalar field ϕ is given by (2.19), the pressure and energy density of the model are given by (2.22) respectively. It should be noted that the scalar field, energy density and the pressure of the model have singularity at $t = \frac{-b}{a}$, $a \neq 0$.

Spatial volume: $$V = (-g)^{\frac{1}{2}} = (at+b)^3 e^{-2mx}, \tag{3.1}$$

Expansion scalar: $$\theta = \frac{3a}{(at+b)}, \tag{3.2}$$

The shear scalar: $$\sigma^2 = \sigma^{ij}\sigma_{ij} = \frac{3a^2}{2(at+b)^2}. \tag{3.3}$$

It can be seen that for large 't' the physical and kinematical parameters given by (3.2) and (3.3) become zero and the spatial volume V increases as the time increases.

4 Conclusions

Here we have presented a spatially homogeneous Bianchi type-V cosmological model in the scalar-tensor theory proposed by Saez and Ballester (1986) in the presence of perfect fluid. The volume increases as the time increases i.e., the model is expanding. The model is anisotropic and the scalar field, energy density and the pressure of the model have singularity. The shear dies out as t increases.

It is very interesting to observe that this is the only possible exact solution for this system.

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