

Synchrotron emission in the fast cooling regime: which spectra can be explained?

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Abstract We consider the synchrotron emission from relativistic shocks assuming that the radiating electrons cool rapidly (either through synchrotron or any other radiation mechanism). It is shown that the theory of synchrotron emission in the fast cooling regime can account for a wide range of spectral shapes. In particular, the magnetic field, which decays behind the shock front, brings enough flexibility to the theory to explain the majority of gamma-ray burst spectra even in the parameter-free fast cooling regime. Also, we discuss whether location of the peak in observed spectral energy distributions of gamma-ray bursts and active galactic nuclei can be made consistent with predictions of diffusive shock acceleration theory, and find that the answer is negative. This result is a strong indication that a particle injection mechanism, other than the standard shock acceleration, works in relativistic shocks.

Keywords Radiation mechanisms: non-thermal · Gamma rays: bursts · Galaxies: active

1 Introduction

The synchrotron radiation is a very common emission mechanism among various astrophysical sources: supernova remnants, active galactic nuclei (AGNs), gamma-ray bursts (GRBs), etc. It allows to explain a wide range of different broad-band spectra by adjusting distribution functions of radiating particles. The models employing synchrotron

radiation are more constrained in the so-called fast cooling regime, where the bulk of particles radiate away their energy before escaping the emitting region or losing energy through adiabatic cooling. The fast cooling regime is desirable for very luminous objects like AGNs and GRBs since the energy limitations of their central engines imply rather high radiative efficiency. Also, this regime is what one expects in these objects theoretically, based on the standard assumption that the magnetic field strength is close to the equipartition value, which is enough to ensure fast cooling even through synchrotron radiation alone.

In this paper we put emphasis on AGNs and GRBs, investigating general characteristics of their spectral energy distributions (SEDs), such as location of the peak and the spectral indices below and above the peak. We analyze whether the observed spectra can be made consistent with the standard particle acceleration models or in principle explained by the synchrotron radiation (including into consideration models with non-uniform magnetic field distribution).

2 Relation of particle injection to the observed spectra

In the generally accepted model, the observed emission of AGNs and GRBs comes from a succession of multiple mildly relativistic shocks, which form within continuous outflow of magnetized plasma with the bulk Lorentz factor $\Gamma_b \gg 1$. These internal shocks accelerate electrons, which are eventually advected downstream of the shock front, where they produce synchrotron radiation.

The schematic distribution of electrons injected at the shock front, $f(\gamma)$ (where γ is the Lorentz factor of an electron), is presented in Fig. 1. Most of the electrons belong to “thermal” population with average energy of the order of the average energy of shocked protons $\Gamma m_p c^2$ (Γ is

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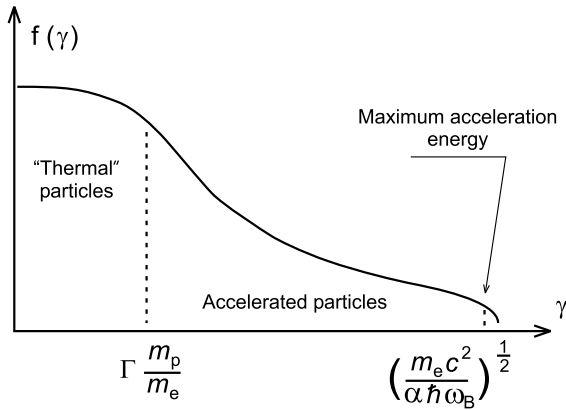


Fig. 1 Distribution of particles injected at the shock front as a result of diffusive shock acceleration. The cut-off energy is defined assuming that the synchrotron radiation is the only energy loss mechanism. Here Γ is the shock Lorentz factor, $\omega_B = eB/m_e c$ —electron’s gyrofrequency, and α —the fine structure constant

the shock Lorentz factor). At a certain (low) level there is a smooth transition from the “thermal” distribution to a power-law non-thermal one formed by shock-accelerated electrons. The power-law cuts off at an energy where the radiative losses start to prevail over the acceleration energy gain.

In the fast cooling regime, the electrons’ distribution function changes as the electrons are advected away from the shock front and cool: the cut-off shifts towards progressively smaller energies. However, the observed luminosity is the integral of emissivity along the line of sight, so that in the one-zone model with constant magnetic-field strength downstream one only needs to know the integral of the distribution function along the shock normal, $N(\gamma)$, to calculate the observed spectrum (due to the effect of relativistic beaming, only those portions of the shock whose normal is closely aligned with the line of sight contribute to the observed emission). In effect, the problem is reduced to the case of uniform injection.

The electron distribution function can be found from the continuity equation in the energy-momentum space

$$\frac{\partial N}{\partial t} + \text{div}(\dot{\gamma} N) = f(\gamma), \tag{1}$$

which gives stationary solution

$$N(\gamma) = -\frac{1}{\dot{\gamma}} \int_{\gamma}^{\infty} f(\gamma') d\gamma'. \tag{2}$$

The corresponding SED (assuming the radiation is monochromatic with frequency $\nu \propto \gamma^2$) is:

$$\nu F_{\nu} \propto \frac{dF}{d \ln \gamma} \propto \eta \gamma \int_{\gamma}^{\infty} f(\gamma') d\gamma', \tag{3}$$

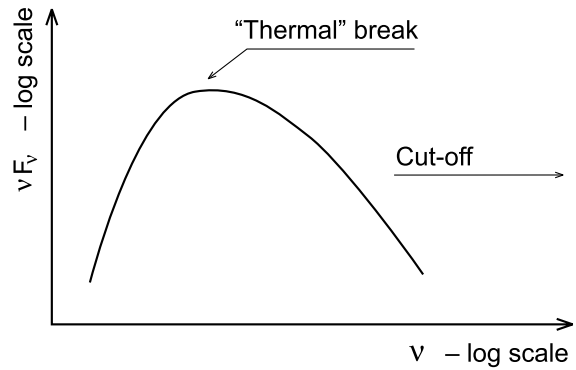


Fig. 2 Standard assignment of spectral features

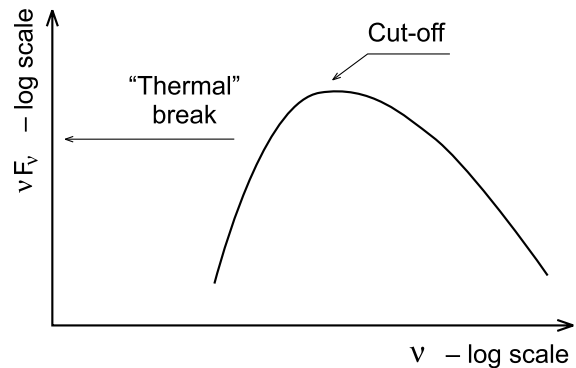


Fig. 3 Alternative assignment of spectral features

where $\eta(\gamma)$ is the fraction of electron’s energy transferred to the synchrotron radiation.

Apparently, there are two prominent features in the distribution function given by (2), and the peak in an observed SED can be related to either of them. One, more or less standard assignment (see Fig. 2), links the peak to the break at the transitional region between “thermal” and non-thermal electrons. Alternatively, one may link the peak to the cut-off region of the electron distribution (see Fig. 3).

The alternative assignment of the SED peak has more explanatory power as far as spectral index above the peak is considered. The predicted cut-off shape for a particle distribution originating from acceleration at a relativistic shock is model-dependent and very different from a simple exponential cut-off (Zirakashvili and Aharonian 2006), in accordance with the observed spectral indices, which are indeed different for different sources. On the contrary, the standard assignment of the SED peak has difficulty in explaining this difference since diffusive shock acceleration gives a universal power-law $f(\gamma) \propto \gamma^{-2.2}$ (e.g., Keshet and Waxman 2005),¹ resulting in very hard ($\nu F_{\nu} \propto \nu^{-0.1}$) and universal spectra above the peak.

¹It has been claimed in a number of recent papers (Niemiec and Ostrowski 2006; Lemoine et al. 2006), that more realistic models of par-

3 Location of the peak

In the internal shock model, the comoving-frame Lorentz factor of thermal electrons is $\gamma \sim m_p/m_e$, so that the standard assignment of SED peak implies that it is observed at the energy

$$\varepsilon_{\text{peak}} \sim \Gamma_b \left(\frac{m_p}{m_e} \right)^2 \frac{\hbar e B}{m_e c}, \tag{4}$$

where B is the magnetic field strength in the comoving frame.

The internal shocks form at a distance $\sim \Gamma_b^2 t_v c$ from the central engine, where t_v is the source’s variability timescale, which is related to the size of central engine. Assuming equipartition between the magnetic-field and the radiation-field energy densities, we find that the magnetic field strength is

$$B \sim \frac{L^{1/2}}{\Gamma_b^3 t_v c^{3/2}}, \tag{5}$$

where L is the apparent luminosity of a source. Substituting (5) into (4), we get the location of SED peak:

$$\varepsilon_{\text{peak}} \sim \left(\frac{m_p}{m_e} \right)^2 \frac{\hbar e L^{1/2}}{\Gamma_b^2 t_v m_e c^{5/2}} \sim \frac{L_{51}^{1/2}}{\Gamma_3^2 t_{-3}} \text{ MeV}. \tag{6}$$

Here L_{51} is the luminosity in units 10^{51} erg/s, t_{-3} the variability timescale in units 10^{-3} s, and Γ_3 the bulk Lorentz factor in units 10^3 . These units are chosen because they are standard parameters of a typical gamma-ray burst. Equation (6) predicts that GRBs have their spectral energy distributions peaked at roughly 1 MeV, in accordance with observations. However, a typical AGN with luminosity $L \sim 10^{45}$ erg/s, variability timescale $t_v \sim 10^4$ s, and the bulk Lorentz factor $\Gamma_b \sim 10$ should have an SED peaked at around 1 eV—hardly enough to explain IR-peaked AGNs, and far too low for MeV-peaked blazars.

Assigning the SED peak to the cut-off region of the electron injection function poses difficulties as well. Indeed, in the case of Bohm diffusion the acceleration rate at a relativistic shock is $m_e c^2 \dot{\gamma} \simeq e B c$ and hence the maximum acceleration energy for electrons, determined from the balance between energy gain and radiative losses,

$$m_e c^2 \dot{\gamma} = - \frac{1}{\eta(\gamma)} \frac{4}{9} \left(\frac{e^2}{m_e c^2} \right)^2 \gamma^2 B^2 c, \tag{7}$$

particle scattering lead to softer and model-dependent spectra of accelerated particles. On the other hand, the acceleration is inefficient (or even absent) in these models. Thus, wherever relativistic shocks efficiently accelerate electrons, the injected particle distribution must be close to $f(\gamma) \propto \gamma^{-2.2}$.

equals to

$$\gamma_{\text{max}} \simeq \frac{3}{2} \frac{[\eta(\gamma_{\text{max}})]^{1/2} m_e c^2}{\sqrt{e^3 B}}, \tag{8}$$

and the associated SED peak of their synchrotron emission is at

$$\varepsilon_{\text{peak}} \simeq \Gamma_b \gamma_{\text{max}}^2 \frac{\hbar e B}{m_e c} \simeq \frac{9}{4} \Gamma_b \eta(\gamma_{\text{max}}) \frac{m_e c^2}{\alpha_f}, \tag{9}$$

where α_f is the fine structure constant. The location of SED peak given by (9) does not explicitly depend on the magnetic field strength (although depends on it implicitly through $\gamma_{\text{max}}(B)$ and then $\eta(\gamma_{\text{max}})$) and even for moderate values of the bulk Lorentz factor appears to be in the GeV range, unless the synchrotron efficiency η is unreasonably low.

However, a diffusion faster than the Bohm one results in a smaller peak energy, in better agreement with observations. For example, if the plasma in the bulk outflow can sustain magnetic-field inhomogeneities with sizes $\geq \ell_c$, then the electron scattering length can be made no larger than

$$\ell_s = \ell_c \left(\frac{r_g}{\ell_c} \right)^2 = \frac{(\gamma m_e c^2)^2}{\ell_c e^2 B^2}, \tag{10}$$

where $r_g = \gamma m_e c^2 / e B$ is the electron’s gyroradius. The value of the scattering length given by the above equation minimizes acceleration rate, which becomes equal to

$$\dot{\gamma} \simeq \frac{1}{\gamma} \frac{\ell_c e^2 B^2 c}{(m_e c^2)^2}. \tag{11}$$

Radiative losses terminate acceleration at

$$\gamma_{\text{max}} \simeq \left(\eta(\gamma_{\text{max}}) \frac{\ell_c}{r_e} \right)^{1/3}, \tag{12}$$

where r_e is the classical electron radius. Consequently, the peak of synchrotron SED is at

$$\varepsilon_{\text{peak}} \simeq \Gamma_b \left(\eta(\gamma_{\text{max}}) \frac{\ell_c}{r_{g0}} \right)^{2/3} \left(\frac{\alpha_f B}{B_{\text{cr}}} \right)^{1/3} \frac{m_e c^2}{\alpha_f}, \tag{13}$$

where $B_{\text{cr}} \simeq 4.4 \times 10^{13}$ G is the Schwinger magnetic field and $r_{g0} = m_e c^2 / e B$ the “cold” gyroradius.

Pushing theory to the limits given by (10) and (11) requires that electron’s motion is random small-angle scattering, that implies the magnetic field is effectively uncorrelated on scales larger than ℓ_c or—more precisely—that the power spectrum of the magnetic field B_k^2 peaks at $k = 1/\ell_c$ and decreases towards smaller wavenumbers faster than $B_k^2 \propto k$. Under such circumstances, the electrons emit mostly due to their interaction with small-scale magnetic field inhomogeneities. If $\ell_c < r_{g0}$, then electrons radiate in the undulator regime (Toptygin and Fleishman 1987;

Medvedev 2000) and the typical frequency of their emission increases with decreasing magnetic-field scale.

Thus, the factor ℓ_c/r_{g0} in (13) can be made as small as unity, and prevalence of inverse-Compton radiative losses ($\eta \ll 1$) further decreases $\varepsilon_{\text{peak}}$. In the case of GRBs, where $B \sim 10^5\text{--}10^6$ G, we find from (13) that the peak of synchrotron SED can be located at just few MeV, roughly in agreement with observations. For AGNs, whose typical value of the magnetic field strength is ~ 0.1 G, it is not possible to push the location of SED peak significantly below 1 keV while keeping the synchrotron efficiency at an acceptable level. This agrees with observations for many blazars, but cannot explain IR-peaked AGNs.

4 Low-frequency spectral index

One of the problems in the interpretation of GRB emission as the synchrotron radiation is that the low-frequency spectral index in the fast-cooling regime is too soft. The hardest possible injection $f(\gamma) = \delta(\gamma - \gamma_0)$ gives (see (3)) $\nu F_\nu \propto \gamma \eta$ below the SED peak (whose position in this case corresponds to γ_0), that is $\nu F_\nu \propto \nu^{1/2}$ if the synchrotron efficiency η is constant.

However, in the synchrotron-self-Compton (SSC) model the synchrotron efficiency is, generally speaking, a rising function of the electron Lorentz factor. Due to the Klein–Nishina effect, only photons whose frequency is less than $m_e c^2 / h\gamma$ can significantly contribute to the effective energy density of seed radiation:

$$w_{\text{ph}}(\gamma) \simeq \int_0^{\frac{m_e c^2}{h\gamma}} w_\nu d\nu. \quad (14)$$

Therefore, relative weight of synchrotron energy losses, $\dot{\gamma} \propto -\gamma^2 B^2$, compared to the inverse-Compton energy losses, $\dot{\gamma} \propto -\gamma^2 w_{\text{ph}}$, increases for electrons with larger Lorentz factors. In a consistent SSC model with prevalence of inverse Compton radiative losses in the Klein–Nishina regime, the synchrotron efficiency can rise as fast as $\eta(\gamma) \propto \gamma$, leading to a rather hard low-frequency spectrum of synchrotron radiation $\nu F_\nu \propto \nu$ (Derishev et al. 2001). This can be a remedy for the synchrotron model of GRB emission for the majority of bursts,² although at a price of decreased synchrotron efficiency.

In the internal shock model of GRBs applicability of this recipe is limited by the fact, that the comptonization proceeds not very deep in the Klein–Nishina regime. Moreover, in some bursts the bulk of radiating electrons comptonize

²A small number of bursts has still harder low-frequency spectral index (Preece et al. 1998), and may require employment of undulator emission in addition to the synchrotron emission (Medvedev 2000).

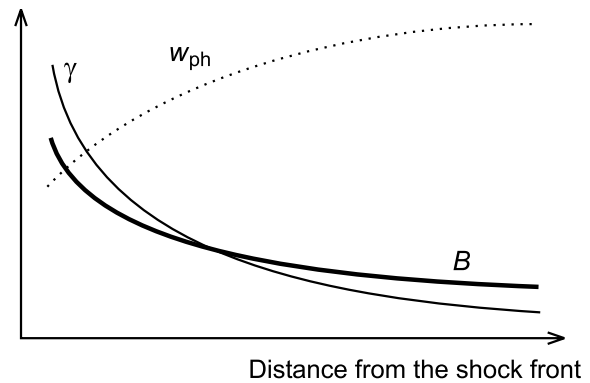


Fig. 4 The Lorentz factor of cooling electrons γ (thin line), the magnetic field strength B (thick line), and the effective energy density of seed photons w_{ph} (dotted line), as functions of distance from the shock front

their own synchrotron radiation in the Thomson regime. In this paper we propose another way to obtain hard low-frequency spectra, which is also suitable for SSC models in the Thomson regime. To reach this goal we note, that the magnetic field behind the front of a relativistic shock is produced by various plasma instabilities rather than by simple MHD compression. This field is not frozen-in and must decay as the shocked plasma moves away from the shock front. It means that the different parts of the electron distribution function, which was treated as a single integrated distribution earlier in this paper, in fact “feel” different magnetic field strength (see Fig. 4), whereas the spectrum and energy density of seed photons do not significantly change inside a thin slab occupied by radiating electrons.

To investigate the main features of the proposed model, we start with delta-functional injection $f(\gamma) = \delta(\gamma - \gamma_0)$ at the shock front. While cooling, the electrons are advected downstream with constant velocity $v = c/3$, so that the spatial derivative of their Lorentz factor is related to the time derivative:

$$\frac{d\gamma}{dr} = \frac{3}{c} \frac{\partial \gamma}{\partial t} = -4\gamma^2 \sigma_{\text{T}} \left(w_{\text{ph}} + \frac{B^2}{8\pi} \right), \quad (15)$$

where r is the distance from the shock front and σ_{T} the Thomson cross-section.

Let us assume that the spectrum of seed radiation is a power-law $w_\nu \propto \nu^q$, where $-1 < q < 0$, so that $w_{\text{ph}}(\gamma) \propto \gamma^{-1-q}$. Then, the solution to the continuity equation (2) gives

$$N(\gamma) \propto \gamma^{q-1} \quad \text{for } \gamma < \gamma_0. \quad (16)$$

In the case of low synchrotron efficiency $\eta \ll 1$ we obtain

$$\frac{d\gamma}{dr} \propto -\gamma^{1-q} \Rightarrow \gamma = \left(\frac{r}{r_0} \right)^{1/q} \quad \text{for } \gamma \ll \gamma_0. \quad (17)$$

Here r_0 is the radiation length of an electron with the Lorentz factor γ_0 . Assuming that the magnetic field strength is a power-law function of the distance from the shock front, $B \propto r^{-y}$, we find the synchrotron efficiency

$$\eta(\gamma) = \frac{B^2}{8\pi w_{\text{ph}}} \propto r^{-2y} \gamma^{1+q} \propto \gamma^{1+q-2qy} \quad (18)$$

and the dependence of typical synchrotron frequency on the Lorentz factor of radiating electrons

$$\nu(\gamma) \propto \gamma^2 B \propto \gamma^{2-2qy}. \quad (19)$$

Substituting (18) and (19) into the right-hand-side of (3), we find the emerging spectrum:

$$\nu F_\nu \propto \gamma \eta \propto \nu^{\frac{2+q-2qy}{2-2qy}}. \quad (20)$$

There are two cases, which deserve particular attention. First of all, choosing $q = -1$ in (20) models comptonization in the Thomson regime: the effective energy density of seed photons very weakly (logarithmically) depends on the electron Lorentz factor and this dependence can be ignored when calculating power-law indices. In this case we get

$$\nu F_\nu \propto \nu^{\frac{1+2y}{2+y}}. \quad (21)$$

In principle, this model allows for spectra as hard as $\nu F_\nu \propto \nu^2$, which is even harder than the low-frequency asymptotic in the synchrotron spectrum of an individual electron. However, the low-frequency spectrum remains softer than $\nu F_\nu \propto \nu$, unless $y \geq 1$.

The self-consistent SSC model with comptonization in the Klein–Nishina regime and $q = 0$ eventually brings no difference from its one-zone counterpart with constant magnetic field strength:

$$\nu F_\nu \propto \nu. \quad (22)$$

The above model is easy to generalize for the case of purely synchrotron radiation ($\eta = 1$) in non-uniform magnetic field. The resulting spectrum is

$$\nu F_\nu \propto \nu^{\frac{1-2y}{2-3y}}, \quad (23)$$

which gives the familiar $\nu F_\nu \propto \nu^{1/2}$ outcome for the constant magnetic field strength, and even softer spectra for any decaying magnetic field. However, this case may be interesting if for some reason the magnetic field strength increases behind the shock front, so that y is negative; such a situation results in relatively hard low-frequency spectra.

5 Conclusion

We show that the low-energy (below the SED peak) part of AGN and GRB spectra can be adequately described by synchrotron emission models, which allow for a broad range of spectral indices. The low-energy synchrotron spectra can be as hard as $\nu F_\nu \propto \nu$ for the synchrotron-self-Compton model with comptonization in the Klein–Nishina regime. Even harder spectra are possible in the case, where the magnetic field decays behind the shock front, and this result holds true even if comptonization proceeds in the Thomson regime. However, this flexibility of theory always comes at a price of low synchrotron efficiency.

Comparison of observed high-energy spectra above the SED peak to the predictions of diffusive shock acceleration theory favors assignment of the peak to the cut-off in the injected electron distribution, whereas the standard assignment of the SED peak to the thermal break produces too hard and invariable high-energy spectral index.

Both assignments are consistent with the SED peak location observed in GRBs, although the agreement is only marginal and may be merely a chance coincidence. Furthermore, none of them can explain the position of SED peaks in all of the observed AGNs or even in the majority of them. We consider this result as a strong indication that a particle injection mechanism, other than the standard diffusive shock acceleration, is at work in relativistic shocks.

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