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ORIGINAL ARTICLE

Five dimensional string cosmological models in a scalar-tensor theory of gravitation

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Abstract Five dimensional Kaluza-Klein space-time is considered in the presence of cosmic string source in the frame work of scalar-tensor theory of gravitation proposed by Saez and Ballester (*Phys. Lett. A* **113**, 467 (1985)). Exact cosmological models, which represent Nambu, Takabayasi and Reddy strings are presented. Some physical and kinematical properties of the models are also discussed.

Keywords Five dimensional · String models · Scalar-tensor theory

1 Introduction

In recent years there has been a multitude of efforts to construct alternative theories of gravitation. The most popular among them are scalar-tensor theories of gravitation proposed by Brans and Dicke (1961), Nordtvedt (1970), Ross (1972) and Dunn (1974). Saez and Ballester (1985) have formulated a scalar-tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. Inspite of the dimensionless character of the scalar field an antigravity regime appears. This theory, also, suggests a possible way to solve missing matter problem in non-flat FRW cosmologies. The scalar-tensor theories of gravitation are important in unified theories of gravitation and to remove the possible "graceful exit" problem (Pimentel 1997) in inflationary era.

Considerable interest has been focussed on the scalar-tensor theory developed by Saez and Ballester. Saez (1985), Singh and Agrawal (1991), Shri Ram and Tiwari (1998) and Reddy and Venkateswara Rao (2001) are some of the authors who have investigated several aspects of the theory with perfect fluid as a source.

It is still a challenging problem to know the exact physical situation at very early stages of the formation of our universe. At the very early stages of evolution of the universe, it is generally assumed that during the phase transition (as the universe passes through its critical temperature) the symmetry of the universe is broken spontaneously. It can give rise to topologically stable defects such as strings, domain walls and monopoles (Kibble 1976). Of all these cosmological structures, cosmic strings have excited the most interest. The present day configurations of the universe are not contradicted by the large scale network of strings in the early universe. Moreover, they may act as gravitational lenses (Vilenkin 1981) strings possess stress energy and are coupled to the gravitational field. Letelier (1979) and Stachel (1980) have given detailed general relativistic treatment of strings. Letelier (1983), Krori et al. (1990), Banerjee et al. (1990), Myung et al. (1986), Hogan and Rees (1984) and Yavuz and Yilmaz (1996) are some of the workers who have obtained various string cosmological models in general relativity.

Very recently, the study of string cosmological models in alternative theories of gravitation is gaining momentum. Sen (2000), Barros et al. (2001), Sen et al. (1997), Gundalach and Ortiz (1990), Barros and Romero (1995), Bhattacharjee and Baruah (2001), Rahaman et al. (2003) and Reddy

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(2005) have presented string cosmological models in alternative theories of gravitation. In particular, Reddy (2003a, 2006) and Reddy et al. (2006) have discussed some string cosmological models in Saez-Ballester scalar-tensor theory of gravitation in four dimensions.

The study of higher dimensional cosmological models is motivated mainly by the possibility of geometrically unifying the fundamental interactions of the universe. In the context of the Kaluza-Klein and super string theories higher dimensions have, recently, acquired much significance. Also, the higher dimensional theory is important at the early stages of the evolution of the universe (Applequist et al. 1987). Rahaman et al. (2002), Chatterjee (1993) and Khadekar et al. (2005) have investigated higher dimensional string cosmological models in general relativity.

In this paper, we discuss string cosmological models in Kaluza-Klein five dimensional space-time in the framework of Saez and Ballester (1985) scalar-tensor theory of gravitation. We have discussed Nambu, Takabayasi and Reddy strings in this theory. As far as our information goes, there has not been much work in literature on five dimensional cosmic string models in scalar-tensor theories of gravitation.

2 Metric and field equations

We consider five dimensional Kaluza-Klein metric in the form

$$ds^2 = dt^2 - R^2(dx^2 + dy^2 + dz^2) - A^2 dm^2. \tag{1}$$

Unlike Wesson (1983), the fifth coordinate is taken to be space-like and the metric coefficients are assumed to be functions of time only, i.e. A and R are functions of time only. Here the spatial curvature has been taken as zero (Grøn 1988).

The field equations given by Saez and Ballester (1985) for the combined scalar and tensor fields are

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) = T_{ij} \tag{2}$$

and the scalar field satisfies the equation

$$2\phi^n\phi^{i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{3}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, ω and n are constants, T_{ij} is the energy tensor of the matter and comma and semicolon denotes partial and covariant differentiation respectively.

Also

$$T^{ij}_{;j} = 0 \tag{4}$$

is a consequence of the field equations (2) and (3).

The energy-momentum tensor for cosmic strings is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j. \tag{5}$$

Here ρ is the rest energy density of the system of strings with massive particles attached to the strings and λ the tension density of the system of strings. As pointed out by Letelier (1983), λ may be positive or negative, u^i describes the system four velocities and x^i represents direction of anisotropy, i.e. The direction of strings which is taken to be along fifth dimension.

We have

$$u^i u_i = -x^i x_i = +1 \quad \text{and} \quad u^i x_i = 0. \tag{6}$$

We consider

$$\rho = \rho_p + \lambda \tag{7}$$

where ρ_p is the rest energy density of particles attached to the strings. Here, we consider ϕ, ρ and λ are functions of t only.

The field equations (2), (3) and (4) for the metric (1) with the help of (5) and (6) can, explicitly, be written as

$$\frac{3R_4^2}{R^2} + \frac{3R_4A_4}{RA} - \frac{\omega}{2}\phi^n\phi_4^2 = \rho, \tag{8}$$

$$\frac{2R_{44}}{R} + \frac{R_4^2}{R^2} + \frac{2R_4A_4}{RA} + \frac{A_{44}}{A} + \frac{\omega}{2}\phi^n\phi_4^2 = 0, \tag{9}$$

$$\frac{R_{44}}{R} + \frac{R_4^2}{R^2} + \frac{\omega}{2}\phi^n\phi_4^2 = \lambda, \tag{10}$$

$$\phi_{44} + \frac{\phi_4A_4}{\phi} + \frac{3\phi_4R_4}{R} + \frac{n}{2}\frac{\phi_4^2}{\phi} = 0, \tag{11}$$

$$\rho_4 + 3\rho\frac{R_4}{R} + (\rho - \lambda)\frac{A_4}{A} = 0 \tag{12}$$

where a suffix 4 after an unknown function denotes differentiation with respect to t.

3 Solutions and the models

The field equations (8–12) are four independent equations in five unknowns R, A, ϕ, ρ and λ. Hence to get a determinate solution one has to assume a physical or mathematical condition. In the literature, we have equations of state for string model (Letelier 1983),

$\rho = \lambda$ (geometric string or Nambu string),

$\rho = (1 + w)\lambda$ (P-string or Takabayasi string).

In addition to the above, recently, Reddy (2003a, 2003b) and Reddy and Rao (2006) have obtained inflationary string cosmological models in Brans and Dicke (1961), Saez and Ballester (1985) and Lyra (1951) scalar-tensor theories of gravitation assuming a relation

$$\rho + \lambda = 0 \quad \text{(Reddy string)} \tag{13}$$

i.e. the sum of rest energy density and tension density for a cloud of strings vanishes. The relation (13) is analogous to $\rho + p = 0$ in general relativity with perfect fluid as source which represents false vacuum case.

Here we find string cosmological models corresponding to (i) $\rho = \lambda$ (ii) $\rho = (1+w)\lambda$ and (iii) $\rho + \lambda = 0$ in five dimensions in Saez-Ballester scalar-tensor theory.

Case (I): geometric string ($\rho = \lambda$)

Here we also assume the relation between metric coefficients, i.e. $A = R^n$ because of the fact that the field equations are highly non-linear. Using this relation, the field equations (8–12) admit the exact solution

$$R = [N(k_1 t + k_2)]^{\frac{1}{N}},$$

$$A = [N(k_1 t + k_2)]^{\frac{n}{N}},$$

$$\phi = \left[\left(\frac{k_3}{2k_1}\right)\frac{N(n+2)}{(N-n-3)}(k_1 t + k_2)^{-(N-n-3)} + k_4\right]^{\frac{2}{n+2}},$$

$$\rho = \lambda = \frac{k_1^2(5 - N + 3n)}{2N^2(k_1 t + k_2)^2} \quad \text{where } N = \frac{2n^2 + 7n + 7}{2n+3} \tag{14}$$

and k_1, k_2, k_3 are constants of integration. After a suitable choice of coordinates and constants of integration, the model corresponding to the solution (14) can be written as

$$ds^2 = dt^2 - (NT)^{\frac{2}{N}}(dx^2 + dy^2 + dz^2) - (NT)^{\frac{2n}{N}}\, dm^2 \tag{15}$$

where $N = \frac{2n^2+7n+7}{2n+3}$.

Some physical features of the model

The model (15) represents five dimensional geometric or Nambu string in Saez-Ballester scalar-tensor theory of gravitation.

The physical and kinematical parameters for the model (15) are

$$\rho = \lambda = \left(\frac{5 - N + 3n}{2N^2}\right)\left(\frac{1}{T^2}\right), \tag{16}$$

$$\phi = \left[\frac{N(n+2)}{2(N-n-3)}\frac{1}{T^{(N-n-3)}}\right]^{\frac{2}{n+2}}. \tag{17}$$

Spatial volume

$$V^3 = (NT)^{\frac{n+3}{N}}. \tag{18}$$

Scalar expansion

$$\theta = \frac{1}{3T}. \tag{19}$$

Shear scalar

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{54T^2}.$$

Deceleration parameter (Feinstein et al. 1995)

$$q = -\frac{3}{\theta^2}\left[\theta_\alpha u^\alpha + \frac{1}{3\theta^2}\right] = 8. \tag{20}$$

The model (15) has no initial singularity, while the energy density, the tension density of the string given by (16) and the scalar field ϕ given by (17) possess initial singularities. However as T increases these singularities vanish. The spatial volume of the model given by (17) shows the anisotropic expansion of the universe (15) with time. For the model (15) the expansion scalar θ and shear scalar σ tend to zero as $T \to \infty$. The positive value of the deceleration parameter indicates that the model decelerates in the standard way.

Also, since

$$\operatorname*{Lim}_{T\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0. \tag{21}$$

The model does not approach isotropy for large values of T.

Case (II): Takabayasi string [$\rho = (1+w)\lambda$, $w \geq 0$]

In this case, again, assuming the relation between metric coefficients i.e. $A = R^n$. We obtain the five dimensional Takabayasi string (p-string) model in Saez-Ballester scalar-tensor theory as

$$ds^2 = dt^2 - \left[\left(\frac{N_1 + N_2}{N_1}\right)T\right]^{\frac{2N_1}{N_1+N_2}}(dx^2 + dy^2 + dz^2) - \left[\left(\frac{N_1 + N_2}{N_1}\right)T\right]^{\frac{2nN_1}{N_1+N_2}} dm^2, \tag{22}$$

where $N_1 = w + 3 + n(w+2)$, $N_2 = 4 + n(2w+7) + n(n-1)(w+2)$.

It may be observed that the model (22) is similar to the model (15) but for the constants.

The string density, tension density and the scalar field in the universe (22) are

$$\rho = (1+w)\lambda = \frac{N_1(1+w)[(3n+4)N_1 - N_2]}{(2+w)(N_1+N_2)^2 T^2}, \tag{23}$$

$$\phi = \left[\frac{(N_1+N_2)(n+2)}{N_2 - (n+2)N_1}T^{-\frac{(n+2)N_1+N_2}{N_1+N_2}}\right]^{\frac{2}{n+2}}. \tag{24}$$

It can be easily seen that, as in case (I), the physical and kinematical parameters have an identical behaviour in the universe given by (22).

Case (III): Reddy string ($\rho + \lambda = 0$)

In this case, again, assuming $A = R^n$, the five dimensional Reddy string model can be written as

$$ds^2 = dt^2 - [(3n+5)T]^{\frac{2}{3n+5}}(dx^2 + dy^2 + dz^2) - [(3n+5)T]^{\frac{2n}{3n+5}}\,dm^2. \quad (25)$$

Which is, again, similar to the model (15) with similar behaviour of the physical and kinematical parameters and in this model

$$\rho = -\lambda = \frac{3n+3}{(3n+5)^2T^2} - \frac{w}{2}[(3n+5)T]^{-\frac{2n+6}{3n+5}} = \frac{3n+3}{(3n+5)^2T^2} - \frac{w}{2}[(3n+5)T]^{-\frac{2(n+3)}{3n+5}}. \quad (26)$$

Comparing the cosmic string models (15), (22) and (25) with the string cosmological models obtained in four dimensions by Reddy (2003a) in Brans-Dicke theory and Reddy and Rao (2006) in Lyra manifold, we observe that the model in Brans-Dicke theory inflates while the models in this theory decelerate in the standard way while the models obtained here have similar behaviour as the models obtained in Lyra manifold. Also, the physical quantities like energy density, tension density and the scalar field diverge in this theory while they do not in Brans-Dicke theory.

4 Conclusions

We have studied five dimensional cosmological models generated by a cloud of strings with particles attached to them in Kaluza-Klein space-time in the frame work of Saez and Ballester (1985) scalar-tensor theory of gravitation. The models obtained represent Nambu string, p-string and Reddy string in this theory in five dimensions. The models are free from initial singularities and they are expanding, anisotropic, shearing, non-rotating and decelerate in the standard way. Also, we find that all the physical quantities like energy density, tension density of the string and the scalar field diverge at the initial moment of creation and we do not have any knowledge of the cosmic strings at that instant. However, the four dimensional string models in general relativity obtained by Bhattacharya and Karade (1993) and models presented by Reddy and Rao (2006) in Lyra manifold are quite similar to the five dimensional string models discussed here.

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