Astrophys Space Sci (2007) 307:365–367
DOI 10.1007/s10509-006-9282-x

ORIGINAL ARTICLE

A Cosmological Model with a Negative Constant Deceleration Parameter in Scale-Covariant Theory of Gravitation

D. R. K. Reddy · R. L. Naidu · K. S. Adhav

Received: 14 May 2006 / Accepted: 27 November 2006 / Published online: 26 January 2007

Abstract An axially symmetric Bianchi type-I space-time is considered in the presence of perfect fluid source in the scale-covariant theory of gravitation formulated by Canuto et al. [1977a, Phys. Rev. Lett. **39**, 429]. With the help of special law of variation for Hubble's parameter proposed by Bermann [1983, Nuovo Cimento **74B**, 182] a cosmological model with a negative constant declaration parameter is obtained in this theory. Some physical properties of the model are also discussed.

Keywords Deceleration parameter · Cosmological model · Scale-covariant theory

1 Introduction

Alternative theories of gravity have been extensively studied in connection with their cosmological applications. Noteworthy among them are scalar-tensor theories of gravitation formulated by Brans and Dicke (1961), Nordvedt (1970), Sen (1957), Sen and Dunn (1971) and Saez and Ballester (1985). In Brans-Dicke theory there exists a variable gravitational parameter G. Canuto et al. (1977a) formulated a scale-covariant theory of gravitation which is a viable alternative to general relativity (Wesson, 1980; Will, 1984). In the scale-covariant theory Einstein's field equations are valid in gravitational units where as physical quantities are measured in atomic units. The metric tensors in the two systems of units are related by a conformal transformation

$$\bar{g}_{ij} = \phi^2(x^k) g_{ij} \tag{1}$$

where in latin indicies take values 1,2,3,4, bars denote gravitational units and unbar denotes atomic quantities. The gauge function ø, $(0 < ø < \infty)$ in its most general formulation is function of all space-time coordinates. Thus using the conformal transformation of the type given by Equation (1), Canuto et al. (1977a) transformed the usual Einstein equations into

$$R_{ij} - \frac{1}{2} R g_{ij} + f_{ij}(\phi) = -8\pi G(\phi) T_{ij} + \Lambda(\phi) g_{ij} \tag{2}$$

where

$$\phi^2 f_{ij} = 2\phi\phi_{i;j} - 4\phi_i\phi_j - g_{ij}\left(\phi\phi^k_{;k} - \phi^k\phi_k\right) \tag{3}$$

Here R_{ij} is the Rici tensor, R the Ricci scalar, Λ the cosmological 'constant' G the gravitational 'constant' and T_{ij}, the energy momentum tensor. *A* semicolon denotes covariant derivative and $ø_i$ denotes ordinary derivative with respect to x^i. A particular feature of this theory is that no independent equation for ø exists. The possibilities that have been considered for gauge function ø are (Canuto et al., 1977).

$$\phi(t) = \left(\frac{t_o}{t}\right)^{\varepsilon}, \quad \varepsilon = \pm 1, \pm 1/2 \tag{4}$$

where t_o is a constant. The form

$$\phi \sim t^{1/2} \tag{5}$$

D. R. K. Reddy (✉)
Department of Mathematics, Maharaj Vijayaram Gajapathi Raj Engineering College, Chintalavalasa, Vizianagaram, India
e-mail: reddy_einstein@yahoo.com

R. L. Naidu
Department of Mathematics, GMR Institute of Technology, Rajam, India

K. S. Adhav
Department of Mathematics, Amravathi University, Amravathi, India

is the one most favoured to fit observations (Canuto and Goldman, 1983a,b).

In recent years there has been a lot of interest in the study of alternative theories of gravitation with the perfect fluid matter distribution as source. Reddy and Rao (2006) and Reddy et al. (2006a) have investigated axially symmetric perfect fluid cosmological models in Brans-Dicke theory while Reddy et al. (2006b) have discussed the same in Saez-Ballester scalar-tensor theory. In particular Rahaman et al. (2005) have obtained Bianchi type-I and Kantowski-Sachs cosmological models in Lyra (1951) manifold. Very recently, Reddy et al. (2006c,d) presented models of the universe with a negative constant deceleration parameter in Saez-Ballester and Brans-Dicke Scalar-tensor theories.

In this paper, we study an axially symmetric Bianchi type-I cosmological model with a negative constant deceleration parameter with the help of Hubble's special law of variation proposed by Bermann (1983). This study is relevant in view of the recent scenario of the accelerating of universe.

2 Metric and field equations

We consider the axially symmetric and spatially homogenous Bianchi type-I metric in the form

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)(dy^2 + dz^2) \tag{6}$$

The energy momentum tensor for a perfect fluid distribution is

$$T^i_j = (\rho + p)u_i u_j - p\, g_{ij} \tag{7}$$

together with

$$u^i u_i = 1, \quad u^i u_j = 0 \tag{8}$$

where ρ is the energy density, p is the isotropic pressure and u^i is the four-velocity of the fluid. Here ρ, p and ø are homogeneous functions of cosmic time t.

Using commoving coordinates, the field Equations (1) and (2) for the metric (6) can be written as

$$\frac{2B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 + \frac{\phi_{44}}{\phi} - \frac{A_4\phi_4}{A\phi} + \frac{2B_4\phi_4}{B\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = -8\pi Gp \tag{9}$$

$$\frac{B_{44}}{B} + \frac{A_{44}}{A} + \frac{A_4B_4}{AB} + \frac{\phi_{44}}{\phi} + \frac{A_4\phi_4}{A\phi} - \left(\frac{\phi_4}{\phi}\right)^2 = -8\pi Gp \tag{10}$$

$$\frac{2A_4B_4}{AB} + \left(\frac{B_4}{B}\right)^2 - \frac{\phi_{44}}{\phi} + \frac{A_4\phi_4}{A\phi} + \frac{2B_4\phi_4}{B\phi} + 3\left(\frac{\phi_4}{\phi}\right)^2 = 8\pi G\rho \tag{11}$$

where a subscript 4 after an unknown function denotes differentiation with respect to t.

Also, the energy conservation equation which is a consequence of the field Equations (1) and (2), in this theory, is (Canuto et al., 1977a)

$$\rho_4 + (\rho + p)u^k_{;k} = -\rho\frac{(G\phi)_4}{G\phi} - 3p\frac{\phi_4}{\phi} \tag{12}$$

For the metric (6), this takes the form

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{2B_4}{B}\right) + \rho\left(\frac{G_4}{G} + \frac{\phi_4}{\phi}\right) + 3p\frac{\phi_4}{\phi} = 0 \tag{13}$$

3 Cosmological model

In this section, we obtain cosmological model with a negative constant deceleration parameter.

Bermann (1983) proposed a special law of variation for Hubble's parameter which yield constant deceleration parameter models of the universe. Here we consider constant deceleration parameter defined by

$$q = -\left(R_{44}R/R_4^2\right) = \text{constant} \tag{14}$$

where $R = (\mathrm{AB}^2)^{1/3}$ is the overall scale factor. Here the constant is taken as negative (i.e. it is an accelerating model of the universe)

Equation (14) gives the solution

$$R = (\mathrm{AB}^2)^{1/3} = (at + b)^{1/1+q} \tag{15}$$

where a and b are integration constants. This equation implies that the condition of expansion is $1 + q > 0$.

Equations (9) and (10) yield

$$\frac{B_{44}}{B} - \frac{A_{44}}{A} + \left(\frac{B_4}{B}\right)^2 - \frac{A_4B_4}{AB} - \frac{2A_4\phi_4}{A\phi} + \frac{2B_4\phi_4}{B\phi} = 0 \tag{16}$$

which admits an exact solution

$$A = \alpha B \tag{17}$$

Where α 0 is a constant

From Equations (15) and (17) we have

$$A = (at + b)^{1/1+q}$$
$$B = (1/\alpha)(at + b)^{1/1+q} \tag{18}$$

Hence the cosmological model in scale covariant theory can be written as (through a proper choice of integration constants, i.e., $a = 1$, $b = 0$)

$$ds^2 = dt^2 - t^{2/1+q} dx^2 - (1/\alpha)^2 t^{2/1+q}(dy^2 + dz^2) \tag{19}$$

4 Some physical properties of the model

The cosmological model given by Equation (19) represents a homogeneous universe in scale-covariant theory of gravitation. The model has no singularities at $t = 0$.

The energy density, pressure, the gauge function ø and the gravitational 'constant' G are obtained as

$$8\pi G\rho = \left[\frac{2q^2 + 7q + 11}{2(1+q)^2}\right]\frac{1}{t^2} \tag{20}$$

$$8\pi G p = \left[\frac{q^2 + 3q - 2}{2(1+q)^2}\right]\frac{1}{t^2} \tag{21}$$

$$\phi = \phi_0/t^{1/2}, \quad \phi_0 = \text{constant} \tag{22}$$

$$8\pi G = \left[\frac{q^3 - 12q^2 - 27q - 28}{2(1+q)(2q^2 + 7q + 11)}\right] t \tag{23}$$

For the model (19), the expressions for the spatial volume V, scalar expansion θ, shear scalar σ and the Hubble's parameter H are

$$V^3 = (-g)^{1/2} = (1/\alpha^2) t^{3/1+q} \tag{24}$$

$$\theta = \frac{1}{3} u^i_{;i} = \left(\frac{1}{1+q}\right) t^{2-q/1+q} \tag{25}$$

$$\sigma^2 = \sigma^{ij}\sigma_{ij} = \frac{1}{6}\theta^2 = \frac{1}{6(1+q)^2} t^{4-q/1+q} \tag{26}$$

$$H = R_4/R = \left(\frac{1}{1+q}\right)\frac{1}{t} \tag{27}$$

At the initial moment $t = 0$, the spatial volume is zero and the energy density, pressure and the scalar fielddiverge while θ, σ^2 and G become zero and the Hubble's parameter tends to ∞. As $t \to \infty$, V, θ, σ^2 tend to infinity while H, ρ and p become zero. Also $Lt_{t\to\infty}(\frac{\sigma^2}{\theta^2}) = \frac{1}{6} \neq 0$ and hence the model does not approach isotropy for large values of t. However, for $\alpha = 1$, the model becomes spatially isotropic.

In this model, particle horizon exists because

$$\int_{t_0}^{t} dt/V(t) = \left(\frac{q+1}{q}\right)[t^{q/1+q}]_{t_0}^{t}$$

is a convergent integral.

5 Conclusions

In this paper, we have considered the field equations of scale covariant theory of gravitation proposed by Canuto et al. (1977a) for an axially symmetric Bianchi-I space-time. The field equations being highly non-linear, we have obtained a cosmological model using special law of variation of Hubble's parameter proposed by Bermann (1983). The model obtained represents an expanding, accelerating and spatially homogenous anisotropic universe in the theory.

References

Bermann, M.S.: Nuovo Cimento **74B**, 182 (1983)

Brans, C.H., Dicke, R.H.: Phys. Rev. **24**, 925 (1961)

Canuto, V.M., Goldman, I.: Nature **304**, 311 (1983a)

Canuto, V.M., Goldman, I.: in G.O. A bell and Chincarini (eds.), D. Reidel Pub. Co. Dordrecht, Holland, p. 485 (1983b)

Canuto, V.M., Hsieh, S.H., Adams, P.J.: Phys. Rev. Lett. **39**, 429 (1977a)

Canuto, V.M., Hsieh, S.H., Tsiang, E.: Phys. Rev. **D16**, 1643 (1977b)

Lyra, G.: Math. Z. **54**, 52 (1951)

Nordtvedt, K. Jr.: Astrophys. J. **161**, 1069 (1970)

Reddy, D.R.K., Rao, M.V.S.: Astrophys. Space Sci. **305**, 183 (2006)

Reddy, D.R.K., Rao, M.V.S., Kelkar, I.: Astrophys. Space Sci. **306**, 1 (2006a)

Reddy, D.R.K., Naidu, R.L., Rao, V.U.M.: Astrophys. Space Sci. **306**, 185 (2006b)

Reddy, D.R.K., Rao, M.V.S., Rao, G.K.: Astrophys. Space Sci. **306**, 171 (2006c)

Reddy, D.R.K., Naidu, R.L., Rao, V.U.M.: Int. J. Theor. Phys. DOI 10.1007/s10773-006-9283-0 (2006d)

Saez, D., Ballester, V.J.: Phys. Lett. **A113**, 467 (1985)

Sen, D.K.: Z. Phys. **149**, 311 (1957)

Sen, D.K., Dunn, K.A.: J. Math. Phys. **12**, 578 (1971)

Wesson, P.S.: Gravity, Particles and Astrophysics, D. Reidel Publ. Co. Dordrecht, Holland (1980)

Will, C.M.: Phys. Rep. **113**, 345 (1984)