Astrophys Space Sci (2006) 306:171–174
DOI 10.1007/s10509-006-9210-0

ORIGINAL ARTICLE

A Cosmological Model with Negative Constant Deceleration Parameter in a Scalar-Tensor Theory

D. R. K. Reddy · M. V. Subba Rao · G. Koteswara Rao

Received: 8 February 2006 / Accepted: 3 July 2006

Abstract With the help of a special law of variation for Hubble's parameter presented by Bermann [Nuovo Cimento B (1983), 74, 182], a cosmological model with negative constant deceleration parameter is obtained in the framework of Saez-Ballester [Phys. Lett (1985), Al 13, 467] scalar – tensor theory of gravitation. Some physical and kinematical properties of the model are, also, discussed.

Keywords Scalar-tensor theory · Constant deceleration parameter · Cosmological model

1. Introduction

In recent years there has been a considerable interest in scalar – tensor theories of gravitation proposed by Brans and Dicke (1961), Nordtvedt (1970), Lyra (1951), Sen and Dunn (l971) and Saez and Ballester (1985). Brans-Dicke theory includes a long range scalar field interacting equally with all forms of matter (with the exception of electromagnetism) while in Saez-Ballester scalar – tensor theory the metric is coupled with a dimensionless scalar field in a simple manner. For the benefit of the readers a brief review of the Saez-Ballester scalar-tensor theory is given in the Appendix.

The field equations given by Saez and Ballester (1985) for the combined scalar and tensor fields are

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\,\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\,\phi^{,k}\right) = -T_{ij} \tag{1}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n\phi^{;i}_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0 \tag{2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}\ R$ is the Einstein tensor, ω and n are constants, T_{ij} is stress tensor of the matter and comma and semicolon denote partial and covariant differentiation respectively.
Also

$$T^{ij}_{;j} = 0 \tag{3}$$

is a consequence of the field Equations (1) and (2).

The study of cosmological models in the frame work of scalar - tensor theories has been the active area of research for the last few decades. Singh and Rai (1983) gives a detailed discussion of Brans-Dicke cosmological models while Rahaman and Bera (2001, 2002), Rahaman and Chakraborty (2002), Soleng (1987), Reddy and Venkateswarlu (1987), Rahaman et al. (2002) and Rahaman et al. (2003) are some of the authors who have investigated several aspects of cosmology with in the frame work of Lyra (1951) geometry. Singh and Agrawal (1991), Shri Ram and Singh (1995), Shri Ram and Tiwari (1998) and Reddy and Rao (2001) have studied Saez-Ballester scalar – tensor theory with special reference to cosmological models.

Very recently, Rahaman et al. (2004) have obtained some cosmological models with negative constant deceleration parameter with in the frame work of Lyra geometry. In this paper, we present an axially symmetric Bianchi-I cosmological model with negative constant deceleration parameter in Saez-Ballester scalar tensor theory using the special law of variation of Hubble's parameter proposed by Bermann (1983). This study is important because of the fact

D. R. K. Reddy (✉)
Department of Mathematics, Maharaja Vijayaram Gajapathi Raj College of Engineering, Chintalavalasa, Vizianagaram
e-mail: reddy_einstein@yahoo.com

M. V. S. Rao · G. K. Rao
Department of Mathematics, Nagarjuna University, Nagarjuna Nagar, Guntur

that the recent observations show that our universe is accelerating.

2. Field equations and the model

We consider an axially symmetric Binachi-I metric given by

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2 \pm dz^2) \tag{4}$$

where A and B are functions of time t only. The axially symmetry assumed implies that the scalar field ϕ, is also function of t only.

The energy – momentum tensor T_{ij} for perfect fluid distribution is given by

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij} \tag{5}$$

together with

$$g_{ij}\, u^i u_j = 1 \tag{6}$$

where u_i is the four-velocity vector of the fluid and p and ρ are the proper pressure and energy density respectively.

By adoption of commoving coordinates the field equations (1)–(3), with the help of Equations (5) and (6) for the metric (4), can be written as

$$\frac{2A_4B_4}{AB} + \frac{B_4^2}{B^2} = \rho - \frac{\omega}{2}\phi^n\phi_4^2 \tag{7}$$

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} = -p + \frac{\omega}{2}\phi^n\phi_4^2 \tag{8}$$

$$\frac{B_{44}}{B} + \frac{A_4B_4}{AB} + \frac{A_{44}}{A} = -p + \frac{\omega}{2}\phi^n\phi_4^2 \tag{9}$$

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{2B_4}{B}\right) + \frac{n}{2}\frac{\phi_4^2}{\phi} = 0 \tag{10}$$

$$\rho_4 + (\rho + p)\left(\frac{A_4}{A} + \frac{2B_4}{B}\right) = 0 \tag{11}$$

where the suffix 4 following an unknown function denotes ordinary differentiation with respect to time t.

Bermann (1983) presented a law of variation of Hubble's parameter that yields constant deceleration parameter models of the universe. With the help of this special law of variation for Hubble's parameter, we obtain an exact solution of the field equations, which represents a cosmological model with negative constant deceleration parameter in this theory.

We consider the constant deceleration parameter defined by

$$q = -R\, R_{44}/R_4^2 \tag{12}$$

where $R = (AB^2)^{1/3}$ is the over all scale factor. Here the constant is taken as negative (i.e.; it is an accelerating model of the universe).

The solution of Equation (12) is

$$R = (\mathrm{at} + b)^{1/1+q} \tag{13}$$

where a and b are constants of integration. This equation implies that the condition of expansion is $1 + q > 0$.

From Equations (8) and (9), we get

$$\frac{B_{44}}{B} + \frac{A_{44}}{A} + \left(\frac{B_4}{B}\right)^2 - \left(\frac{A_4B_4}{AB}\right) = 0 \tag{14}$$

which yields, on integration,

$$B^2A_4 - ABB_4 = h \tag{15}$$

where h is an integration constant,

From Equation (13), we get

$$\frac{A_4}{A} + \frac{2B_4}{B} = 3\frac{R_4}{R} = [3a/(1+q)](at+b)^{-1} \tag{16}$$

From Equation (15) we get

$$\left(\frac{A_4}{A}\right) - \left(\frac{B_4}{B}\right) = h/R^3 = h(at+b)^{-3/1+q} \tag{17}$$

Solving Eqs. (16) and (17), we get

$$A = A_o[at+b]^\alpha \exp[2\delta(at+b)^\gamma] \tag{18}$$

$$B = B_o[at+b]^\alpha \exp[-\delta(at+b)^\gamma] \tag{19}$$

where

$$\begin{aligned}\alpha &= 1/(q+1)\\ \delta &= [h(1+q)/3a(q-2)]\\ \gamma &= (q-2)/(q+1)\end{aligned} \tag{20}$$

and A_o, B_o are constants of integration.

In view of Equations (18) and (19) the metric (4), through a proper choice of constants and coordinates, takes the form (i.e. taking $A_o = B_o = 1$, $at + b = T$)

$$\begin{aligned}ds^2 = dT^2 - T^{2\alpha}&[\exp(4\delta T^\gamma)]dX^2\\ &+ \exp(-2\delta T^\gamma)(dY^2 - dZ^2)\end{aligned} \tag{21}$$

3. Some physical properties

The model (21) represents an exact perfect fluid cosmological model, with negative constant deceleration parameter, in the scalar tensor theory of gravitation formulated by Saez and Ballester (1985).

For the model (21) the physical and kinematical variables which are important in cosmology are

$$\rho = \frac{3\alpha^2}{T^2} - 3\gamma^2\delta^2 T^{2\gamma-2} + \frac{\omega}{2}T^{-6\alpha} \tag{22}$$

$$p = \frac{\omega}{2}T^{-6\alpha} - \frac{\alpha^2}{T^2} - 3\gamma^2\delta^2 T^{2\gamma-2} \tag{23}$$

$$-T^{\gamma-2}[\gamma\delta(\gamma-1) + 3\alpha\gamma\delta]$$

$$\phi^{(n/2)+1} = \left(\frac{n+2}{a}\right)T^{\gamma} \tag{24}$$

$$\text{Spatial volume } V^3 = T^{3\alpha} \tag{25}$$

$$\text{Expansion scalar } \theta = 3a\alpha/T \tag{26}$$

$$\text{Shear Scalar } \sigma^2 = \frac{h^2}{6}T^{-6\alpha} \tag{27}$$

$$\text{Hubble's parameter } H = a\alpha/T \tag{28}$$

Where α, γ, δ are given by Equation (20)

The model (21) has a singularity at $T = 0$. At this instant all the physical quantities diverge. Thus the universe starts with an infinite rate of expansion and measure of anisotropy. So this is consistent with the big-bang model. Also, as $T \to \infty$, the proper volume becomes infinitely large and the other physical quantities such as density, pressure, shear etc become insignificant. It can, also, be observed that shear tends to zero faster than the expansion.

In this model, the scalar field has no initial singularity and becomes infinitely large for large values of T (i.e. $T \to \infty$).

4. Conclusions

Cosmological models with in the frame work of scalar-tensor theories play a vital role for a better understanding of the early stages of evolution of the universe. Here we have discussed a cosmological model with a negative constant deceleration parameter using Bermann's (1983) Hubble's law of variation. The model, obtained, has initial singularity and expands indefinitely with acceleration while all the physical parameters diverge at the initial epoch. Our model is quite similar to the model obtained, recently, by Rahaman et al. (2004) within the framework of Lyra geometry.

Appendix

A brief review of Saez-Ballester theory

Several physically acceptable scalar-tensor theories of gravitation have been proposed and widely studied so far by many workers. There are two categories of gravitational theories involving a classical scalar field ϕ . In the first category the scalar field ϕ, has the dimension of inverse of the gravitational constant G among which the Brans-Dricke (1961) theory is of considerable importance and the role of the scalar field is confined to its effects on gravitational field equations. In the second category the theories involve a dimensionless scalar field.

Saez and Ballester (1985) developed a theory in which the metric is coupled with a dimensionless scalar field. This coupling gives satisfactory descriptions of weak fields. Inspite of the dimensionless character of the scalar field an antigravity regime appears.

Saez and Ballester (1985) assumed the Lagrangian.

$$L = R - \omega\phi^n(\phi_{,\alpha}\,\phi^{,\alpha}) \tag{A1}$$

where R is the scalar curvature, ϕ, is the dimensionless scalar field, ω and n are arbitrary dimensionless constants and $\phi^{,\alpha} = \phi_{,\alpha}g^{\gamma\alpha}$. For scalar field having the dimension ϕ G^{-1}, the Lagrangian given by Equation (A1) has different dimensions. However it is a suitable Lagrangian in the case of a dimensionless scalar field.

From the above Lagrangian one can build the action,

$$I = \int_{\Sigma}(L + GL_m)(-g)^{1/2}dx\,dy\,dz\,dt \tag{A2}$$

where L_m is the matter Lagrangian, $g = |g_{ij}|$, Σ is an arbitrary region of integration and $G = -8\bar{\wedge}$. By considering arbitrary independent variations of the metric and the scalar field vanishing at the boundary of Σ, the variational Principle

$$\delta I = 0 \tag{A3}$$

leads to the Saez-Ballester field Equations (1), (2) and (3).

Saez (1985) discussed the initial singularity and inflationary universe in this theory. He has shown that there is an antigravity regime which could act either at the beginning of the inflationary epoch or before. He has also obtained non-singular FRW model in the case $k = 0$. Also this theory suggests a possible way to solve the missing matter problem in non-flat FRW cosmologies.

Acknowledgements The authors are grateful to the referee for his constructive comments.

References

Bermann, M.S.: Nuovo Comento **74B**, 182 (1983)

Brans, C., Dicke, R.H.: Phys. Rev. **124**, 925 (1961)

Halford, W.D.: Aust. J. Phys. **23**, 863 (1970)

Lyra, G.: Math. Z. **54**, 52 (1951)

Nordtvedt, K., Jr.: Astrophys, J. **161**, 1069 (1970)

Rahaman, F., Bera, J.: Int. J. Mod. Phys. **D10**, 729 (2001)

Rahaman, F., Bera, J.: Astrophys. Space Sci. **281**, 595 (2002)

Rahaman, F., Chakraborty, S., Bera, J.: Int. J. Mod. Phys. **D11**, 1501 (2002)

Rahaman, F. Begum, N., Bag, G., Bhui, B.C.: Astrophys. Space Sci. **299**, 211 (2005)

Rahaman, F., Das, S., Begum, N., Hossain, M.: Pramana-J. Phys. **61**, 153 (2003)

Rahaman, F., Chakraborty, S., Begum, N., Hossain, M., Kalam, M.: Fizika **B11**, 57 (2002)

Reddy, D.R.K., Venkateswarlu, R.: Astrophys. Space Sci. **136**, 191 (1987)

Saez, D., Ballester, V.J.: Phys. Lett. **A113**, 467 (1985)

Saez, D.: A Simple coupling with cosmological implications, (a preprint) (1985)

Sen, D.K., Dunn, K.A.: J. Math. Phys. **12**, 578 (1971)

Shri Ram, Singh, J.K.: Astrophys. Space Sci. **234**, 325 (1995)

Shri Ram, Tiwari, S.K.: Astrophys. Space Sci. **277**, 461 (1998)

Singh, T., Agrawal, K.: Astrophys. Space Sci. **182**, 289 (1991)

Soleng, H.H.: Gen. Relativ. Gravitation **19**, 1213 (1987)

Songh, T., Rai, L.N.: Gen. Relativ. Gravitation **15**, 875 (1983)