ON CHARGED ANALOGUES OF BUCHDAHL'S TYPE FLUID SPHERES

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Abstract. In this article the charged analogues of recently derived Buchdahl's type fluid spheres have been obtained by considering a particular form of electric field intensity. In this process, Einstein–Maxwell field equations yield eight different classes of solutions, joining smoothly with the exterior Reissner–Nordstrom metric at the pressure free intersurface. Out of the eight solutions only seven could be utilized to represent superdense star models with ultrahigh surface density of the order 2×10^{14} gm cm^{-3}. The maximum masses of the star models were found to be $8.223931 M_{\Theta}$ and $8.460857 M_{\Theta}$ subject to strong and weak energy conditions, respectively, which are much higher than the maximum masses $3.82 M_{\Theta}$ and $4.57 M_{\Theta}$ allowed in the neutral cases. The velocity of sound seen to be less than that of light throughout the star models.

Keywords: Einstein–Maxwell equations, relativistic charged stars

1. Introduction

Since the inception of Reissner–Nordstrom metric, research workers have been busy in deriving interior regular charged perfect fluid solutions. A good account of the same can be had from the work of B.V. Ivanov (Ivanov, 2002a, b). The relevance of the study of charged fluid distributions is connected with the following interesting facts such as:

(i) Charge dust (CD) (pressure free distribution) may be realized in the slight ionization of neutral hydrogen.
(ii) CD may possess arbitrary mass and radius, can attain very large redshifts, their exteriors can be made arbitrarily near to the exterior of an extreme charged black hole.
(iii) A classical model of an electron is likely to be represented by CD if many of its characteristics remain finite and non-trivial while the junction radius shrinks to zero.
(iv) Besides many other speciality, the charge in the fluid distribution helps in countering the gravitational collapse by means of the coulombian repulsion together with the pressure gradient.

Many workers have studied the number of charged fluids in different contexts in the past and recently too (Treves and Turolla, 1999; Bonnor, 1960, 1980, 1998;

Bonner and Wickramasuria, 1975, Felice et al., 1995, 1999; Ray et al., 2003, 2004; Anninos and Rothman, 2001). Some others (Patel et al., 1997; Tikekar and Singh, 1998; Patel and Pandya, 1986) have considered the charged analogues of Vaidya-Tikekar types of fluid spheres, then utilized to depict the uncharged superdense star models with ultrahigh surface density (Vaidya and Tikekar, 1982). In the recent past (Sharma et al., 2001) have presented very general class of charged analogues of Vaidya-Tikekar type fluid spheres, which contains all such solutions obtained by others as special cases. In fact Vaidya–Tikekar fluid spheres are described by space-time with the hypersurfaces $t = \text{const}$ as 3-spheroids. The said space–time can easily be seen as special case of what had been chosen by Buchdahl, which includes the hypersurfaces as 3-hyperboloids too (Buchdahl, 1959). All the Buchdahl's type fluid spheres have already been obtained and analysed by Gupta and Jasim (Gupta and Jasim, 2000, 2003). In the present article the authors have obtained most general class of charged Buchdahl's type fluid spheres joining smoothly with the Reissner-Nordstrom Metric at the pressure free boundary and utilized them to represent superdense star with surface density of the order 2×10^{14} gm cm^{-3}. The later are analysed numerically subject to the energy conditions throughout the star. Out of the eight solutions so obtained only seven could satisfy the required conditions. Consequently the maximum masses of the star models were found to be $8.223931 M_\Theta$ and $8.460857 M_\Theta$ subject to strong and weak energy conditions respectively. It is worth pointing out here that the maximum masses obtained by Gupta and Jasim (2000, 2003) were $3.82 M_\Theta$ and $4.57 M_\Theta$ respectively in the neutral cases. All the solutions obtained by others in the prevailing circumstances and conditions, can be seen as special cases of the present work.

2. Field Equations

Let us take the following spherically symmetric metric to describe the space–time of a charged fluid sphere

$$ds^2 = -e^\lambda dr^2 - r^2(d\theta^2 + \sin^2\theta \, d\phi^2) + e^\upsilon dt^2 \tag{1}$$

The functions $\lambda(r)$ and $\upsilon(r)$ are to satisfy the Einstein–Maxwell equations

$$R^i_j - \frac{1}{2} R \delta^i_j = -\kappa \left[(c^2\rho + p) v^i v_j - p\delta^i_j + \frac{1}{4\pi}\left(-F^{im}F_{jm} + \frac{1}{4}\delta^i_j F_{mn}F^{mn} \right) \right], \tag{2}$$

where $\kappa = \frac{8\pi G}{c^4}$, ρ, p and v^i denote matter density, fluid pressure and the unit time-like flow vector of the fluid, respectively and F_{ik} being the skew symmetric electromagnetic field tensor satisfying the Maxwell equations

$$F_{ik;j} + F_{kj;i} + F_{ji;k} = 0, \tag{3}$$

$$\frac{\partial}{\partial x^k}(\sqrt{-g}F^{ik}) = -4\pi\sqrt{-g}\,j^i, \tag{4}$$

where $j^i = \sigma v^i$ represents the four-current vector of charged fluid while the charged density is denoted by σ.

The field Eq. (2) with respect to the metric Eq.(1) reduce to (Dionysiou, 1982)

$$-\frac{\upsilon'}{r}e^{-\lambda} + \frac{(1-e^{-\lambda})}{r^2} = -\kappa p + \frac{q^2}{r^4} \tag{5}$$

$$-\left[\frac{\upsilon''}{2} - \frac{\lambda'\upsilon'}{4} + \frac{\upsilon'^2}{4} + \frac{\upsilon' - \lambda'}{2r}\right]e^{-\lambda} = -\kappa p - \frac{q^2}{r^4} \tag{6}$$

$$\frac{\lambda'}{r}e^{-\lambda} + \frac{(1-e^{-\lambda})}{r^2} = \kappa c^2 \rho + \frac{q^2}{r^4} \tag{7}$$

where

$$q(r) = 4\pi\int_0^r \sigma r^2 e^{\lambda/2} dr = r^2\sqrt{-F_{14}F^{14}} = r^2 F^{41} e^{(\lambda+\upsilon)/2} \tag{8}$$

represents the total charge contained with in the sphere of radius r. The Eq. (4) reduces to

$$\frac{\partial}{\partial r}\left(e^{(\lambda+\nu)/2} r^2 F^{41}\right) = -4\pi e^{(\lambda+\nu)/2} r^2 j^4 \tag{9}$$

Beyond the pressure free interface $r = a$ the charged fluid sphere is expected to join with the Reissner–Nordstrom metric:

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) + \left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right) dt^2 \tag{10}$$

where m is the gravitational mass of the distribution such that

$$m = \mu(a) + \varepsilon(a)$$

while

$$\mu(a) = \frac{\kappa}{2}\int_0^a \rho r^2 dr, \varepsilon(a) = \frac{\kappa}{2}\int_0^a r\sigma q e^{\lambda/2} dr, \quad e = q(a) \tag{11}$$

$\varepsilon(a)$ is the mass equivalence of the electromagnetic energy of distribution while $\mu(a)$ is the mass and e is the total charge inside the sphere (Florides, 1983).

In the present article we propose a charged fluid distributions by considering the electric field intensity

$$\frac{q^2}{r^4} = \frac{C^2 r^2 \beta^2}{2(Cr^2+1)^2} \tag{12}$$

and the metric potential

$$e^{\lambda} = \frac{K(1+Cr^2)}{K+Cr^2}, \quad C > 0 \tag{13}$$

where C, K, β being constants.

The later was proposed by Buchdahl to ensure the density gradient negative for the uncharged case. However the expression considered for electric intensity has already been considered by the other workers (Patel and Koppar, 1987; Sharma et al., 2001; Patel et al., 1997) and is such that the Einstein-Maxwell field equation reduces to hypergeometric equation after some appropriate substitutions. Also the electric intensity so assumed has negative gradient with zero value at the center.

The consistency of the field Eqs. (5)–(7) using Eqs. (12) and (13) yield the hypergeometric equation

$$(1-X^2)\frac{d^2y}{dX^2} + X\frac{dy}{dX} + (1-K+K\beta^2)y = 0, \tag{14}$$

where

$$X = \sqrt{\frac{K}{K-1}}\sqrt{1+\frac{Cr^2}{K}}, \quad K < 0 \text{ or } K > 1$$

and

$$e^{\nu} = y^2$$

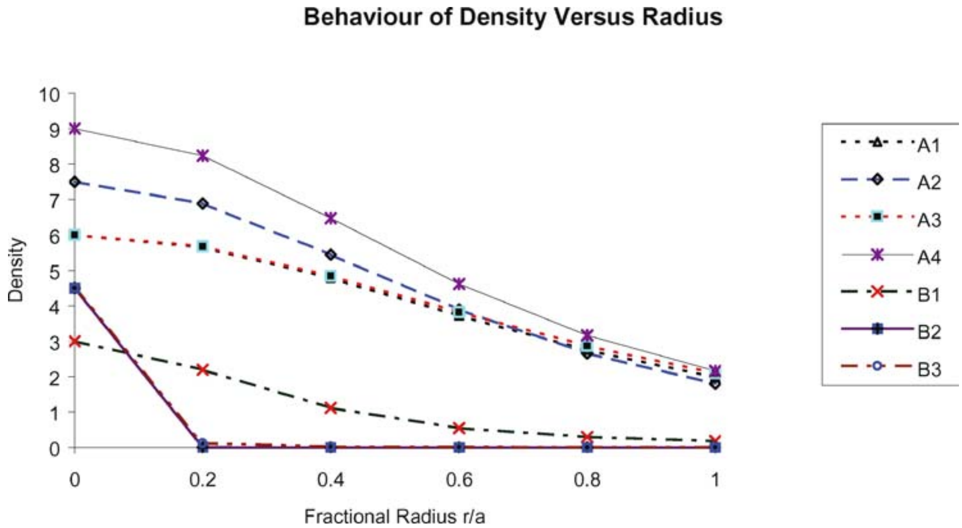
The range $0 < K < 1$ has been left as it corresponds to negative density at the centre.

The same set up with $K < 0$ and $C = -K/R^2$ has been used by Vaidya and Tikekar (1982) to describe the uncharged ($\beta = 0$) isentropic superdense star models. Also they had shown that the hypersurfaces $t =$ const are spheroids in this case. It is trivially easy to prove that the hypersurfaces $t =$ const turns out to hyperboloids if one considers $K > 1$ and $C = K/R^2$ (Figure 1).

The expression for energy density and pressure can be had from (5), (7), (12) and (13) as

$$\kappa c^2 \rho = \frac{4C(K-1)(3+Cr^2) - K\beta^2 C^2 r^2}{4K(1+Cr^2)^2} \tag{15}$$

$$\kappa p = -\frac{2y'}{ry}\frac{(K+Cr^2)}{K(1+Cr^2)} - \frac{C(K-1)}{K(1+Cr^2)} + \frac{C^2 r^2 \beta^2}{2(Cr^2+1)^2} \tag{16}$$



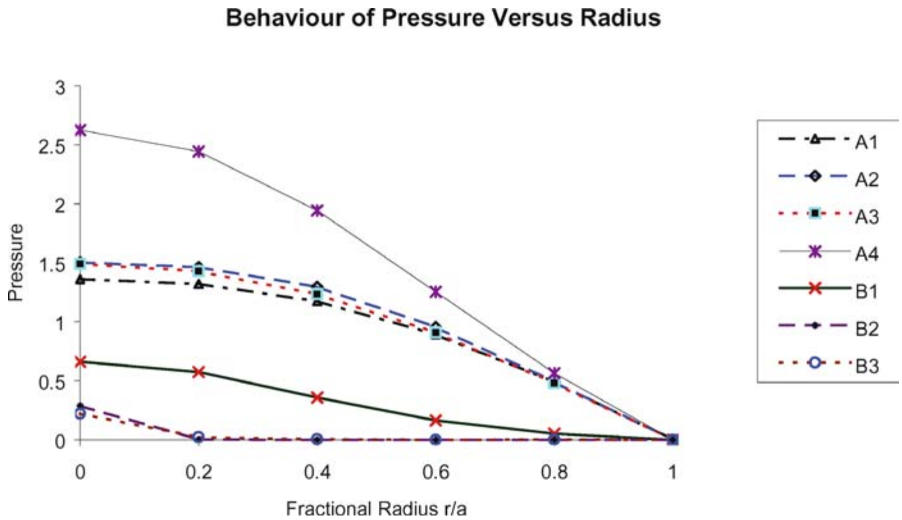


Figure 1. Plots for density, pressure, electric intensity and velocity of sound subject to the SEC for maximum masses (bold data in the Table I) corresponding to the Cases: A1, A2, A3, A4, B1, B2 and B3, respectively. (In the above plot, density for the Case B1 is multiplied by 10^{-3}).

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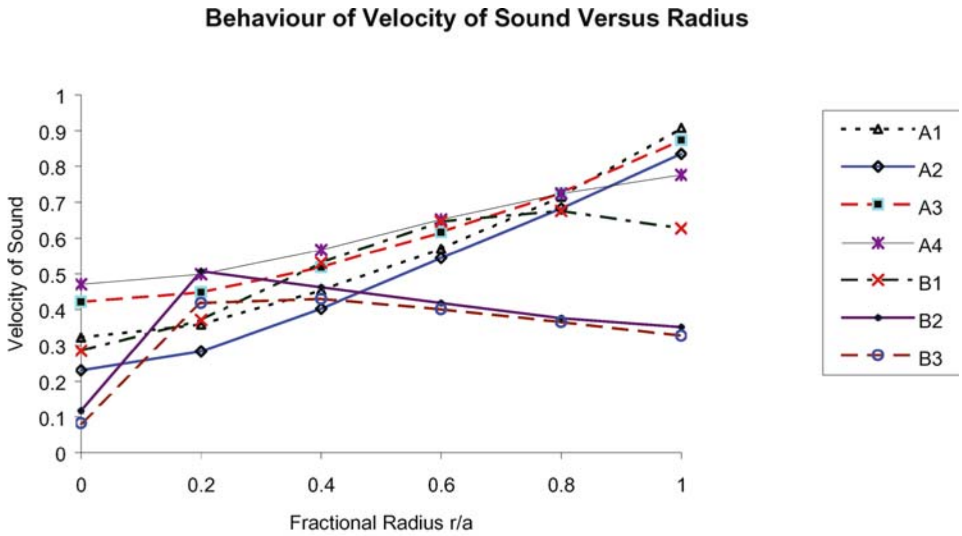

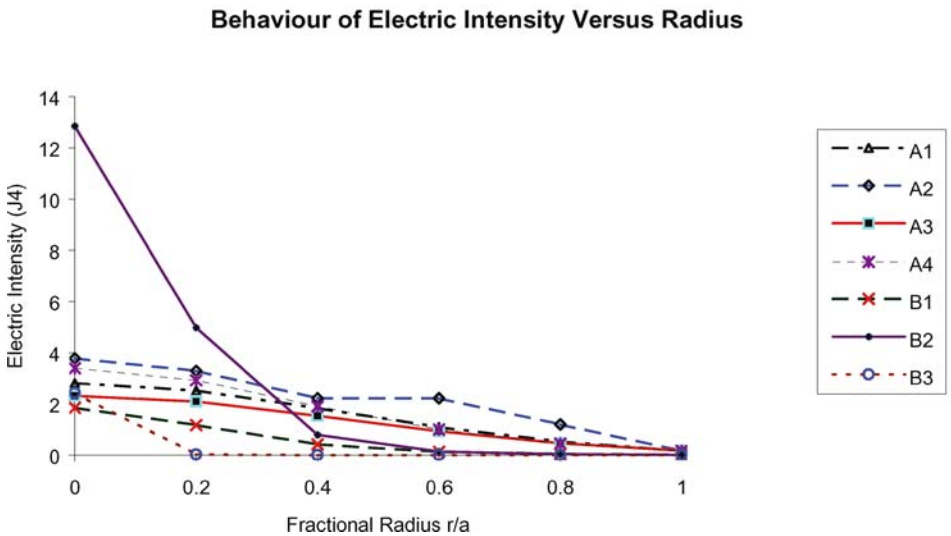

Figure 1. (*Continued*).

The expression for the pressure gradient and density gradient can be written as:

$$\kappa c^2 \frac{d\rho}{dr} = \frac{C^2 r[2(1-K)(5+Cr^2) - K\beta^2(1-Cr^2)]}{K(1+Cr^2)^3} \tag{17}$$

$$\kappa \frac{dp}{dr} = \frac{C^2\beta^2 r(Cr^2+3)}{(Cr^2+1)^3} - \frac{1}{2}e^{\lambda}(c^2\rho + p)\left[\frac{m(r)}{r^2} - \frac{q^2}{r^3} + \frac{\kappa p r}{2}\right] \tag{18}$$

where

$$m(r) = \frac{Cr^3[2(K-1)(1+Cr^2)+CK\beta^2r^2]}{4K(1+Cr^2)^2}$$

such that

$$e^{-\lambda} = 1 - \frac{2m(r)}{r} + \frac{q^2}{r^2}$$

The speed of sound

The sphere being charged, it is not possible on the present naive phenomenological level to say what the speed of propagation of a sound wave of arbitrary frequency will be. However, if the frequency is sufficiently large the adiabatic speed of propagation by the fluid is presumably by (Buchdahl, 1979)

$$\begin{aligned} v^2 &= \frac{dp}{d\rho} \\ &= \frac{c^2\left\{KC^2\beta^2(Cr^2+3) - Ke^{\lambda}\kappa(c^2\rho+p)\left(\frac{M(r)}{r^3} - \frac{q^2}{r^4} + \frac{\kappa p}{2}\right)(1+Cr^2)^3\right\}}{C^2[2(1-K)(5+Cr^2) - K\beta^2(1-Cr^2)]} \end{aligned} \tag{19}$$

Also if the ratio of the surface density ρ_a to central density ρ_0 is $\lambda = \frac{\rho_a}{\rho_o}$ then using (15) we get

$$1 + Ca^2 = \frac{S + \sqrt{\{S^2 + 24\lambda(6-S)\}}}{12\lambda}, \quad (1 + Ca^2 > 0)$$

while surface density ρ_a is taken out to be 2×10^{14} gm cm^{-3}. Where $S = 2 - \frac{K\beta^2}{(K-1)}$ and $C = \frac{8\pi GK\rho_0}{3c^2(K-1)} = \frac{1.24479\times10^{-13}K}{(K-1)\lambda}$, however the expression for the pressure can be derived as follows:

Case A: For $K < 0$, $X = \sin\theta$, is an appropriate choice (spheroidal case).
Differentiating (14) with respect to X and then letting $dy/dX = G$, we get

$$\frac{d^2G}{d\theta^2} + (2 - K + K\beta^2)G = 0 \tag{20}$$

which yields solutions for various G depending upon the nature of the parameter K and β and the corresponding solution of the Eq. (20) can be furnished as:

$$G = A[\cosh(m\theta) + B\ \sinh(m\theta)], \quad 2 - K + K\beta^2 = -m^2, \tag{21}$$

$$G = A[\theta + B], \quad 2 - K + K\beta^2 = o \tag{22}$$

$$G = A[\sin(m\theta + B)], \quad 2 - K + K\beta^2 = m^2(\neq 1), \tag{23}$$

$$G = A[\cos(\theta) + H\sin(\theta)], \quad 2 - K + K\beta^2 = 1 \tag{24}$$

inserting Eqs. (21)–(24) into Eqs. (14) and (16) we get the following expressions for y and pressure corresponding to various cases:

A(1): For $2 - K + K\beta^2 = -m^2$,

$$\begin{aligned} y &= \frac{A}{(m^2+1)}[m\cos\theta\,\sinh(m\theta) + \sin\theta\,\cosh(m\theta) \\ &\quad + B(m\cos\theta\,\cosh(m\theta) + \sin\theta\,\sinh(m\theta))] \\ \kappa p &= \frac{2C(m^2+1)}{(1-K)K\cos^2\theta} \\ &\quad \times \left[\frac{\cosh(m\theta) + B\sinh(m\theta)}{m\cot\theta\,\sinh(m\theta) + \cosh(m\theta) + B(\cot\theta\,\cosh(m\theta) + \sinh(m\theta))}\right] \\ &\quad + \frac{C}{K\cos^2\theta} + \frac{C^2r^2\beta^2}{2(Cr^2+1)^2} \end{aligned} \tag{21a}$$

A(2): For $2 - K + K\beta^2 = 0$,

$$\begin{aligned} y &= A(\cos\theta + \theta\sin\theta + B\sin\theta), \\ \kappa p &= \frac{2C}{(1-K)K\cos^2\theta}\left[\frac{\sin\theta(\theta + B)}{(\theta\sin\theta + \cos\theta + B\sin\theta)}\right] \\ &\quad + \frac{C}{K\cos^2\theta} + \frac{C^2r^2\beta^2}{2(Cr^2+1)^2} \end{aligned} \tag{22a}$$

A(3): For $2 - K + K\beta^2 = m^2(\neq 1)$,

$$\begin{aligned} y &= \frac{A}{(1-m^2)}[m\cos\theta\,\cos(m\theta + B) + \sin\theta\,\sin(m\theta + B)] \\ \kappa p &= \frac{2C(m^2-1)}{(K-1)K\cos^2\theta}\left[\frac{\sin\theta\,\sin(m\theta + B)}{m\cos\theta\,\cos(m\theta + B) + \sin\theta\,\sin(m\theta + B)}\right] \\ &\quad + \frac{C}{K\cos^2\theta} + \frac{C^2r^2\beta^2}{2(Cr^2+1)^2} \end{aligned} \tag{23a}$$

A(4): For $2 - K + K\beta^2 = 1$,

$$\begin{aligned} y &= \frac{A}{2}(\theta + \sin\theta\,\cos\theta + B) \\ \kappa p &= \frac{4C}{(1-K)K}\left[\frac{\tan\theta}{\theta + \sin\theta\,\cos\theta + B}\right] + \frac{C}{K\cos^2\theta} + \frac{C^2r^2\beta^2}{2(Cr^2+1)^2} \end{aligned} \tag{24a}$$

Case B: For $K > 1$, $X = \cosh\theta$, is an appropriate choice (hyperboloidal case).

Differentiating (14) with respect to X and then letting $dy/dX = G$, we get

$$\frac{d^2G}{d\theta^2} - (2 - K + K\beta^2)G = 0, \tag{25}$$

which yields solutions for various G depending upon the nature of the parameter K and β and the corresponding solution of the Eq. (25) can be furnished as:

$$G = A[\cos(m\theta + B)], \qquad 2 - K + K\beta^2 = -m^2 \tag{26}$$

$$G = A[\theta + B], \qquad 2 - K + K\beta^2 = o \tag{27}$$

$$G = A[\cosh(m\theta) + B\ \sinh(m\theta))], \qquad 2 - K + K\beta^2 = m^2(\neq 1) \tag{28}$$

$$G = A\left[\sinh(\theta) + H\ \cosh(\theta)\right], \qquad 2 - K + K\beta^2 = 1 \tag{29}$$

inserting the Eqs. (26)–(29) into Eqs. (14) and (16) we get the following expressions of y and the corresponding expressions for pressure for the respective cases furnished as:

B(1): For $2 - K + K\beta^2 = -m^2$,

$$\begin{aligned} y &= \frac{A}{(m^2+1)}[m\sinh\theta\sin(m\theta + B) + \cosh\theta\cos(m\theta + B)] \\ \kappa p &= \frac{2C(m^2+1)}{(K-1)K\sinh^2\theta}\left[\frac{1}{1 + m\tanh\theta\tan(m\theta + B)}\right] \\ &\quad - \frac{C}{K\sinh^2\theta} + \frac{C^2r^2\beta^2}{2(Cr^2+1)^2} \end{aligned} \tag{26a}$$

B(2): For $2 - K + K\beta^2 = 0$,

$$\begin{aligned} y &= A(\theta\cosh\theta - \sinh\theta + B\cosh\theta), \\ \kappa p &= \frac{2C}{(K-1)K\sinh^2\theta}\left[\frac{(\theta + B)}{(\theta + B)\cosh\theta - \sinh\theta}\right] \\ &\quad - \frac{C}{K\sinh^2\theta} + \frac{C^2r^2\beta^2}{2(Cr^2+1)^2} \end{aligned} \tag{27a}$$

B(3): For $2 - K + K\beta^2 = m^2(\neq 1)$,

$$\begin{aligned} y &= \frac{A}{(m^2-1)}[m\sinh\theta\{\sinh(m\theta) + B\cosh(m\theta)\} \\ &\quad - \cosh\theta\{\cosh(m\theta) + B\sinh(m\theta)\}] \end{aligned}$$

$$\kappa p = \frac{2C(m^2-1)}{(K-1)K\sinh^2\theta} \times \left[\frac{\cosh(m\theta)+B\sinh(m\theta)}{m\tanh\theta\{\sinh(m\theta)+B\cosh(m\theta)\}-\{\cosh(m\theta)+B\sinh(m\theta)\}}\right] - \frac{C}{K\sinh^2\theta} + \frac{C^2r^2\beta^2}{2(Cr^2+1)^2} \tag{28a}$$

B(4): For $2-K+K\beta^2=1$,

$$y = \frac{A}{2}(\sinh\theta\cosh\theta - \theta + B)$$

$$\kappa p = \frac{4C}{(K-1)K}\left[\frac{\coth\theta}{\sinh\theta\cosh\theta-\theta+B}\right] - \frac{C}{K\ \sinh^2\theta} + \frac{C^2r^2\beta^2}{2(Cr^2+1)^2} \tag{29a}$$

3. Physical Analysis of the Solutions

The physical validity of the charged fluid sphere (CFS) depends upon the following conditions (called reality conditions or energy conditions) inside and on the the sphere $r = a$ such that

(i) $\rho > o, 0 \leq r \leq a$,
(ii) $p > o, r < a$,
(iii) $p = o, r = a$
(iv) $dp/dr < 0, d\rho/dr < 0, 0 < r < a$
(v) $c^2\rho \geq p$ weak energy condition (WEC) or $c^2\rho \geq 3p$ strong energy condition (SEC)
(vi) The velocity of sound $(dp/d\rho)^{1/2}$ should be less than that of light throughout the CFS $(0 \leq r \leq a)$.

Beside the above the CFS is expected to join smoothly with the Nordstrom metric, which requires the continuity of e^λ, e^ν and q across the pressure free interface $r = a$.

$$\frac{(K+Ca^2)}{K(1+Ca^2)} = 1 - \frac{2m(a)}{a} + \frac{e^2}{a^2} \tag{30}$$

$$y^2 = \left(1 - \frac{2m(a)}{a} + \frac{e^2}{a^2}\right) \tag{31}$$

$$q(a) = e \tag{32}$$

$$p_{(r=a)} = 0 \tag{33}$$

The conditions (30) and (32) are automatically satisfied subject to the preposition (11) however (31) and (33) can provide the unique values of arbitrary constants A and B in each of the cases A(1) to A(4) and B(1) to B(4).

4. Numerical Results for Various Cases

From the expressions of $m(r)$ and C, it can easily be derived that $\frac{\partial m}{\partial \lambda} < 0$ and $\frac{\partial m}{\partial \beta^2} > 0$.

However the physical constraints in terms of the reality conditions have severely restricted the magnitude of mass. An attempt has been made to get maximum mass (effectively M/M_Θ) in each case. M and M_Θ denote mass of the model and solar mass respectively.

Various physical quantities such as ρ, p, dp/dr, $d\rho/dr$, $c^2\rho - p$ (WEC), $c^2\rho - 3p$ (SEC), $(dp/d\rho)^{1/2}$ (velocity of sound), j^4 (the only surviving component of the 4-current vector) and M/M_Θ have been calculated numerically for $o \leq x \leq 1$, $(x = r/a)$ and the data so obtained is analysed subject to the reality conditions.

The data for the various cases reveals the following informations:

Case A(1): $(\beta^2 > 1 - \frac{2}{K})$,

It is observed that the mass increases with the increase of charge for $o < \lambda \leq 0.5$, however it decreases with the increase of charge when $0.5 < \lambda < 1$, (let us call this behavior as S). Physically valid solutions are possible only for $-3 \leq K < 0$. This case provides the maximum mass e.g. $8.4609 M_\Theta$ (WEC) for $\lambda = .32$, $\beta = 1.83$, $K = -1$ and $8.2239\ M_\Theta$ (SEC) for $\lambda = .33$, $\beta = 1.82$, $K = -1$.

Case A(2): $(\beta^2 = 1 - \frac{2}{K})$,

Physical solutions are valid only in the $-15 \leq K < 0$. Behavior S is still true. The maximum mass for (SEC) $7.8221 M_\Theta$ for $\lambda = .25$, $\beta = 1.5275$, $K = -1.5$ and $8.0297 M_\Theta$ (WEC) for $\lambda = .25$, $\beta = 1.5275$, $K = -1.5$.

Case A(3): $(\beta^2 < 1 - \frac{2}{K})$,

This is the case when the charged model is reducible its neutral counterpart in the absence of charge. Physically valid solutions are available for all K however the mass continues to be same for $|K| \geq 500$. Behaviour S holds good. The maximum mass turns out to be $8.0115 M_\Theta$ (WEC) for $K = -1$, $\lambda = .34$, $\beta = 1.73$ and $7.7808 M_\Theta$ (SEC) for $K = -1$, $\lambda = .35$, $\beta = 1.73$.

Case A(4): $(\beta^2 = 1 - \frac{1}{K})$,

Physically valid cases are possible for all K however the mass continues to be same for $|K| \geq 500$. Case is similar to A(3). The maximum mass is found to be $6.8211 M_\Theta$ (SEC) for $K = -2$, $\beta = 1.2247$, $\lambda = .24$ and $7.1753 M_\Theta$ (WEC) for $K = -2$, $\beta = 1.2247$, $\lambda = .22$.

Case B(1): $(\beta^2 < 1 - \frac{2}{K})$, Physically valid sets of results are possible for all $K > 1$ however the mass does not change after $K = 1000$, $\beta \leq .98$ otherwise pressure turnout to be negative. The maximum mass is $5.9451 M_{\Theta}$ (WEC) for $\lambda = .024$, $\beta = .99$, $K = 1000$ and $5.5410 M_{\Theta}$ (SEC) for $\lambda = .06$, $\beta = .85$, $K = 1000$.

Case B(2): ($\beta^2 = 1 - \frac{2}{K}$),

Physical situation allows $2 \leq K \leq 2.5$. Naturally for $K = 2$ uncharged Case is recovered. Same maximum mass $2.7454 M_{\Theta}$ is obtained for (WEC) as well as (SEC) for $K = 2.5$, $\beta = .4473$ and $\lambda = 10^{-4}$. For each K, β lies in the interval (0,1) for the physically valid case.

Case B(3): ($\beta^2 > 1 - \frac{2}{K}$),

The case is valid for $1 < K < 2.5$. Behaviour S is valid. Beyond $K > 2.5$ the pressure is negative. The maximum mass is given by $2.7437 M_{\Theta}$ for (SEC) and (WEC) given $K = 2.5$, $\lambda = .001$ and $\beta = .45$.

Case B(4): ($\beta^2 = 1 - \frac{1}{K}$),

No physically reasonable set of data is available for any K. The pressure continues to be negative for all the combination of K, β and λ.

Analytical base of the result can be understood by the following analysis:

Expression for the pressure reads as

$$\kappa p = \frac{4C}{(K-1)K}\left[\frac{\coth\theta}{\sinh\theta\,\cosh\theta - \theta + B}\right] - \frac{C}{K\,\sinh^2\theta} + \frac{C^2 r^2 \beta^2}{2(Cr^2+1)^2}$$

while pressure at the center (p_0) is given by

$$\kappa p_0 = \frac{4C}{(K-1)K}\left[\frac{\coth\theta_0}{\sinh\theta_0\cosh\theta_0 - \theta_0 + B}\right] - \frac{C}{K\,\sinh^2\theta_0} \tag{34}$$

Let us say $p_0 > 0$ i.e.

$$\frac{4C}{(K-1)K}\left[\frac{\coth\theta_0}{\sinh\theta_0\cosh\theta_0 - \theta_0 + B}\right] - \frac{C}{K\,\sinh^2\theta_0} > 0 \tag{35}$$

owing the data,

$$\sinh\theta_a = \sqrt{\frac{1+c_1}{K-1}}, \quad B = \theta_a + D\sqrt{\frac{K}{K-1}} \quad \text{and } c_1 = Ca^2.$$

and

$$D = \frac{c_1(9 - K) + 10 - 2K}{(K - 1)(c_1 + 2)} \tag{36}$$

we get

$$\frac{4}{\{K + \sqrt{K}(K - 1)(B - \theta_0)\}} > \frac{(K - 1)}{K} \tag{37}$$

It is obvious that right hand side quantity of the above expression is positive (for $K > 1$), therefore the left hand side of the above expression must also be positive. Now if the quantity

$$K + \sqrt{K}(K - 1)(B - \theta_0) < 0 \tag{38}$$

or

$$\coth\left[\frac{8(c_1 + 1)\sqrt{K}}{(K - 1)^2(c_1 + 2)}\right] > \left[\frac{\sqrt{(c_1 + K)}\sqrt{K} - \sqrt{(c_1 + 1)}}{\sqrt{(c_1 + K)} - \sqrt{K(c_1 + 1)}}\right] \tag{39}$$

We see that the numerator of RHS is positive while denominator is negative and hence the RHS is negative. Also $\coth\theta > 0$ for $\theta > 0$, so the above inequality (39) and hence (38) is also true. Which implies that (37) is not true and therefore the preposition made in (35) (i.e. $p_0 > 0$) is wrong and so $p_o < 0$ (pressure is negative at the center).

5. Conclusions

All the exact solutions of charged Fluid spheres described by spherically symmetric space–time proposed by Buchdahl, have been obtained by considering the particular form of electric intensity. The class of solutions represented by the Case A(3) have already been derived and discussed by Sharma et al. (2001). While all the other workers mentioned in the introduction have derived the special cases of A(3). The Cases B(1), B(2) and B(3) contain the solutions which are reducible to the uncharged fluid spheres in the absence of charge (i.e. $\beta = 0$). It is worth pointing out here that the cases e.g. A(1), A(2), A(4), B(1), B(2), B(3) and B(4) are appearing at the very first time as for as authors are aware. All the eight solutions derived above are analysed numerically after joining them smoothly with the Reissner–Nordstrom Metric at the pressure free boundary. And hence discussed the limitations on the guiding parameters K, β and λ subject to the prescribed energy conditions. The sample values of mass M/M_{Θ}, and radii are depicted in the Table I

TABLE I

The star of maximum mass with its radius corresponding to various values of the guiding parameters K, β and λ

K	λ	β	Radius (Km)	Max. M/M_Θ	λ	β	Radius (Km)	Max. M/M_Θ
Case (A1)								
−3	.125	1.291	17.134	7.601	.13	1.295	17.162	7.516
−2.5	.15	1.35	17.646	7.725	.155	1.342	17.707	7.633
−2	.175	1.45	18.201	8.077	.18	1.45	18.220	7.978
−1.5	.235	1.53	19.271	8.135	.25	1.53	19.250	7.822
−1.4	.25	1.56	19.497	8.160	.26	1.56	19.474	7.946
−1	**.32**	**1.83**	**20.357**	**8.461**	**.33**	**1.82**	**20.326**	**8.224**
−.8	.4	1.88	21.097	7.966	.4	2.00	21.803	7.919
−.45	.6	2.34	21.610	6.528	.6	2.34	21.610	6.528
−.20	.71	5.10	22.756	7.815	.71	5.10	22.756	7.815
Case (A2)								
−15	.04	1.065	14.273	6.427	–	–	–	–
−10	.05	1.095	14.721	6.625	–	–	–	–
−5	.08	1.183	15.861	7.142	–	–	–	–
−3	.13	1.291	17.179	7.515	–	–	–	–
−2	.18	1.414	18.349	7.975	.19	1.414	18.377	7.781
−1.5	**.24**	**1.528**	**19.274**	**8.030**	**.25**	**1.528**	**19.257**	**7.822**
−1	.34	1.732	20.497	8.011	.35	1.732	20.424	7.780
−.8	.4	1.871	21.119	7.969	.41	1.871	21.013	7.722
−.5	.55	2.236	21.902	7.229	.56	2.236	21.709	6.947
−.25	.77	3.0	20.979	4.926	.77	3.000	20.979	4.926
Case (A3)								
−500	.032	.95	14.056	5.906	.05	.9	14.817	5.706
−100	.034	.95	14.201	5.958	.055	.9	14.997	5.718
−50	.033	1	13.965	6.066	.058	.9	15.149	5.767
−25	.04	1	14.393	6.178	.045	1	14.538	6.127
−15	.05	1	14.922	6.304	.058	1	15.105	6.211
−7	.07	1.1	15.560	6.757	.1	1	16.424	6.297
−5	.1	1.1	16.521	6.817	.11	1.1	16.625	6.669
−3	.14	1.2	17.606	7.315	.163	1.2	17.715	6.930
−2	.199	1.35	18.610	7.599	.204	1.35	18.615	7.505
−1	**.34**	**1.73**	**20.503**	**8.012**	**.35**	**1.73**	**20.429**	**7.781**
Case (A4)								
−250	.026	1.002	13.516	5.977	–	–	–	–
−100	.026	1.005	13.556	6.030	–	–	–	–

(*Continued on next page*)

TABLE I
(*Continued*)

K	λ	β	Radius (Km)	Max. M/M_Θ	λ	β	Radius (Km)	Max. M/M_Θ
−50	.03	1.010	13.790	6.092	–	–	–	–
−25	.036	1.020	14.147	6.220	–	–	–	–
−15	.045	1.033	14.612	6.370	–	–	–	–
−10	.06	1.049	15.215	6.494	–	–	–	–
−5	.1	1.095	16.540	6.815	.11	1.095	16.644	6.667
−3	.15	1.155	17.823	7.126	.17	1.155	17.891	6.797
−2	**.22**	**1.225**	**18.999**	**7.175**	**.24**	**1.225**	**18.958**	**6.821**
−1	.39	1.414	20.807	6.942	.45	1.414	20.096	5.758
Case (B1)								
3	.03	.1	13.754	2.885	.06	.1	13.986	2.732
8	.04	.6	14.738	4.576	.06	.6	15.013	4.401
15	.06	.65	15.398	4.874	.08	.65	15.601	4.679
25	.07	.65	15.761	4.975	.10	.65	15.989	4.673
50	.02.	.97	13.212	5.799	.05	.85	14.849	5.495
100	027	.96	13.690	5.844	.06	.85	15.127	5.469
150	.024	.98	13.467	5.895	.06	.85	15.159	5.496
500	.024	.99	13.447	5.936	.06	.85	15.203	5.533
1000	**.024**	**.99**	**13.457**	**5.945**	**.06**	**.85**	**15.213**	**5.541**
Case (B2)								
2.1	.001	.218	11.573	2.148	.003	.218	11.631	2.138
2.2	.001	.302	11.574	2.317	.002	.302	11.576	2.317
2.3	.001	.361	11.574	2.472	.001	.361	11.574	2.472
2.4	.001	.408	11.574	2.615	.001	.408	11.574	2.615
2.5	**.001**	**.447**	**11.574**	**2.745**	**.001**	**.447**	**11.574**	**2.745**
Case (B3)								
1.1	.001	.05	4.908	.153	.001	.05	4.908	.153
1.5	.001	.2	9.177	1.096	.001	.2	9.177	1.096
1.7	.001	.28	10.010	1.525	.001	.28	10.010	1.525
1.9	.001	.325	10.640	1.892	.001	.325	10.640	1.892
2.0	.001	.36	10.822	2.064	.001	.36	10.822	2.064
2.1	.001	.367	11.113	2.209	.001	.367	11.113	2.209
2.3	.001	.42	11.333	2.500	.001	.42	11.333	2.500
2.5	**.001**	**.45**	**11.590**	**2.744**	**.001**	**.45**	**11.590**	**2.744**

For $0 \leq p \leq c^2\rho$ & $dp/d\rho \leq c^2$ (WEC).
For $0 \leq 3p \leq c^2\rho$ & $dp/d\rho \leq c^2$ (SEC).
$M_\Theta = 1.475$ Km, $G = 6.673 \times 10^{-8}$ cm^3/gm sec^2, $c = 2.997 \times 10^{10}$ cm/sec.

under the heading strong and weak energy conditions for the various values of K, β and λ. The graphs for pressure, density, J^4 and velocity of sound are being traced for each case. Overall the charged Fluid spheres of maximum mass M belongs to the spheroidal case A(1) are found to be $8.223931 M_\Theta$ and $8.460857 M_\Theta$ for strong and weak energy conditions, respectively. The corresponding radii are given as 20.3261 Km. and 20.3569 Km, respectively. It is already known that in the absence of charge, the maximum mass contained by the neutral star models were found to be $3.82 M_\Theta$ and $4.57 M_\Theta$ for strong and weak energy conditions, respectively.

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