BIANCHI TYPE-V BULK VISCOUS FLUID STRING DUST COSMOLOGICAL MODEL IN GENERAL RELATIVITY

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(Received 15 June 2004; accepted 4 April 2005)

Abstract. Bianchi Type-V bulk viscous fluid string dust cosmological model in General Relativity is investigated. It has been shown that if coefficient of bulk viscosity (ζ) is inversely proportional to the expansion (θ) in the model then string cosmological model for Bianchi Type-V space-time is possible. In absence of bulk viscosity (ζ), i.e. when $\zeta \to 0$, then there is no string cosmological model for Bianchi Type-V space-time. The physical and geometrical aspects of the model are also discussed.

Keywords: Bianchi Type V, bulk viscous string dust, cosmological

1. Introduction

Bianchi Type-V space-times are interesting to the study because of their richer structure both physically and geometrically than standard Friedmann–Robertson–Walker (FRW) models. These models represent the open FRW cosmological model with $k = -1$ where k is the curvature of three-dimensional space at any time t. Nayak and Sahoo (1989) have investigated Bianchi Type-V models for a matter distribution admitting anisotropic pressure and heat flow. Ram (1990) has obtained Bianchi Type-V cosmological model for perfect fluid distribution. He has given a new method to generate exact solution of Einstein field equation in Bianchi Type-V space-times. Viscosity is important for number of reasons. Heller and Klimek (1975) have investigated viscous fluid cosmological model without initial singularity. They have shown that the introduction of bulk viscosity effectively removes the initial singularity. Roy and Singh (1983) have investigated LRS Bianchi Type-V cosmological model with viscosity. Santos et al. (1985) investigated isotropic homogeneous cosmological model with bulk viscosity assuming viscous coefficient as power function of mass density. Banerjee and Sanyal (1988) have investigated Bianchi Type-V cosmological models with viscosity and heat flow. Coley (1990) investigated Bianchi Type-V imperfect fluid cosmological models for equation of state $p = (\gamma - 1)\rho$ where ρ is the energy density, p the pressure and $0 \leq \gamma \leq 2$. Bali and Yadav (2002) have investigated an LRS Bianchi Type-V viscous fluid

Astrophysics and Space Science **300:** 387–394, 2005.
DOI: 10.1007/s10509-005-0152-8

cosmological model assuming the condition $\sigma \ \alpha \ \theta$, where σ is the shear and θ the expansion in the model.

Linde (1979) has conjectured that after the big-bang, the universe might have experienced a number of phase transitions. The phase transitions produce vacuum domain structure such as domain walls, strings and monopoles (Kibble, 1980; Zel'dovich, 1980). Cosmic strings have excited considerable interest as these act as gravitational lenses and give rise to density perturbations leading to the formation of galaxies (Vilenkin, 1981a,b; Kibble and Turok, 1982). Letelier (1983) has done a pioneering work in formulation of energy-momentum tensor for classical massive strings. Stachel (1980) has considered a massless (geometric string) models. Banerjee et al. (1990) have obtained an axially symmetric Bianchi Type-I string dust cosmological model in presence and absence of magnetic field using a supplementary condition $\alpha = a\beta$ between metric potentials where a is a constant. Roy and Banerji (1995) have investigated some LRS Bianchi Type-II string cosmological models for cloud of geometrical and massive strings using the condition $\varepsilon = \lambda$ for geometric string and $\sigma/\theta =$ constant for massive string. Patel et al. (1996) investigated the integrability of cosmic strings in the context of Bianchi Type-II, VIII and IX space-times. Recently Bali and Dave (2001) have investigated some special string solutions for Bianchi Type-IX space-time using the condition $\rho = \lambda$ and $a = e^{\alpha t}$ where a is metric potential. Bali et al. (2003) have also obtained LRS Bianchi Type-II cosmological model filled with string dust as the source for gravitational field and assuming the condition $\varepsilon = \lambda$ and where ε is the rest energy density and λ the string tensor density. Singh and Agrawal (1993), Saha (2004) have studied Bianchi models including Bianchi Type-V models in details.

In this paper, we have investigated Bianchi Type-V bulk viscous fluid string dust cosmological model in General Relativity. To get a determinate model, we have assumed that the coefficient of bulk viscosity (ζ) is inversely proportional to the expansion (θ) in the model. It has been shown that string dust cosmological model for Bianchi Type-V space-time in absence of bulk viscosity is not possible. The physical and geometrical aspects of the model in presence and absence of bulk viscosity are also discussed.

We consider the Bianchi Type-V metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2x} dy^2 + C^2 e^{2x} dz^2 \quad (1.1)$$

where A, B and C are functions of t-alone.

The energy–momentum tensor for bulk viscous string dust is given by Letelier (1983), Landau and Lifshitz (1963) as

$$T_i^j = \varepsilon v_i v^j - \lambda x_i x^j - \zeta v^{\ell}_{;\ell}\left(g_i^j + v_i v^j\right) \quad (1.2)$$

together with

$$v_i v^i = -x_i x^i = -1 \tag{1.3}$$
$$v^i x_i = 0, \quad x_1 \neq 0, \quad x_2 = 0 = x_3 = x_4 \tag{1.4}$$
$$v^1 = 0 = v^2 = v^3, \quad v^4 = 1 \tag{1.5}$$

where $\varepsilon = \varepsilon_p + \lambda$, is the rest energy density for cloud of strings with particles attached to them, λ the string tension density, v^i the flow vector, x^i the direction of string and ζ the coefficient of bulk viscosity.

The Einstein field equation

$$R_i^j - \frac{1}{2} R\, g_i^j = -8\pi\, T_i^j \quad \text{(using the units in which } c = G = 1) \tag{1.6}$$

for the metric (1.1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{1}{A^2} = 8\pi \left(\lambda + \zeta v^\ell_{;\ell}\right) \tag{1.7}$$
$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = 8\pi \zeta v^\ell_{;\ell} \tag{1.8}$$
$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = 8\pi \zeta v^\ell_{;\ell} \tag{1.9}$$
$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} - \frac{3}{A^2} = 8\pi \varepsilon \tag{1.10}$$
$$\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} = 0 \tag{1.11}$$

where

$$\theta = v^\ell_{;\ell} = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \tag{1.12}$$

2. Solutions of Field Equations

From Eq. (1.11), we have

$$A^2 = \ell BC \tag{2.1}$$

where ℓ is constant of integration.

Equations (1.8) and (1.9) lead to

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = 0 \tag{2.2}$$

which after using (1.11), leads to

$$\frac{(CB_4 - BC_4)_4}{CB_4 - BC_4} = -\frac{1}{2}\frac{(BC)_4}{BC} \tag{2.3}$$

This after integration leads to

$$C^2\left(\frac{B}{C}\right)_4 = \frac{L}{\sqrt{BC}} \tag{2.4}$$

where L is constant of integration.

Using Eq. (1.11) in Eq. (1.8), we have

$$\frac{2B_{44}}{B} + \frac{6C_{44}}{C} - \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} + \frac{2B_4C_4}{BC} - \frac{4}{\ell BC} = 32\pi\zeta\theta \tag{2.5}$$

Let

$$BC = \mu, \frac{B}{C} = \nu \tag{2.6}$$

Using these in Eqs. (2.4) and (2.5), we get

$$\frac{\nu_4}{\nu} = L\mu^{-3/2} \tag{2.7}$$

and

$$\frac{4\mu_{44}}{\mu} - \frac{2\nu_{44}}{\nu} - \frac{\mu_4^2}{\mu^2} + \frac{3\nu_4^2}{\nu^2} - \frac{3\mu_4\nu_4}{\mu\nu} - \frac{4}{\ell\mu} = 32\pi\zeta\theta \tag{2.8}$$

Using (2.7) in (2.8), we get

$$4\mu\ \mu_{44} + \frac{L^2}{\mu} - \mu_4^2 - \frac{4\mu}{\ell} = 32\pi\zeta\theta$$

which implies that

$$\mu_{44} - \frac{1}{4\mu}\mu_4^2 = \frac{1}{\ell} - \frac{L^2}{4\mu^2} + \frac{8\pi\zeta\theta}{\mu} \tag{2.9}$$

To get a determinate model, we assume that coefficient of bulk viscosity (ζ) is inversely proportional to expansion (θ). Thus we have

$$8\pi\zeta\theta = \gamma \quad \text{(constant)} \tag{2.10}$$

Now Eq. (2.9) leads to

$$\mu_{44} - \frac{\mu_4^2}{4\mu} = \frac{1}{\ell} - \frac{L^2}{4\mu^2} + \frac{\gamma}{\mu} \tag{2.11}$$

Let $\mu_4 = f(\mu)$, then Eq. (2.11) leads to

$$2f\ f' - \frac{1}{2\mu} f^2 = \frac{2}{\ell} - \frac{L^2}{2\mu^2} + \frac{2\gamma}{\mu} \tag{2.12}$$

which again leads to

$$\frac{\mathrm{d}}{\mathrm{d}\mu}(f^2) - \frac{1}{2\mu}(f^2) = \frac{2}{\ell} - \frac{L^2}{2\mu^2} + \frac{2\gamma}{\mu} \tag{2.13}$$

From Eq. (2.13), we have

$$f^2 = \frac{4\mu}{\ell} + \frac{L^2}{3\mu} - 4\gamma + K\sqrt{\mu} \tag{2.14}$$

where K is constant of integration.

Equations (2.7) and (2.14) lead to

$$\log\ \nu = \int \frac{L\mu^{-3/2}\mathrm{d}\mu}{\sqrt{\frac{4\mu}{\ell} + \frac{L^2}{3\mu} - 4\gamma + K\sqrt{\mu}}} \tag{2.15}$$

Hence the metric (1.1) reduces to the form

$$\mathrm{d}s^2 = -\frac{\mathrm{d}T^2}{\left[\frac{4T}{\ell} + \frac{L^2}{3T} + 4\sqrt{T} - 4\gamma\right]} + \ell T \mathrm{d}X^2 + T\nu\ \mathrm{e}^{2X}\ \mathrm{d}Y^2 + \frac{T}{\nu}\mathrm{e}^{2X}\ \mathrm{d}Z^2 \tag{2.16}$$

where $\mu = T$ and ν can be determined by (2.15).

3. Some Physical and Geometrical Features

The energy density (ε) and string tension (λ) for the model (2.16) are given by

$$\varepsilon = \frac{3}{8\pi T^2}[k\sqrt{T} - \gamma] \tag{3.1}$$

$$\lambda = \frac{\gamma}{8\pi}\frac{(1 - T^2)}{T^2} \tag{3.2}$$

The scalar of expansion (θ), shear (σ) and the spatial volume R^3 for the model (2.16) are given by

$$\theta = \frac{3}{2T^{3/2}}\sqrt{\frac{4T^2}{\ell} + KT^{3/2} - 4\gamma T + \frac{L^2}{3}} \tag{3.3}$$

$$\sigma = \frac{L}{2T^{3/2}} \tag{3.4}$$

$$R^3 = \ell T^2 e^{2X} \tag{3.5}$$

The reality condition $\varepsilon > 0$ given by Ellis (1971), leads to $\sqrt{T} > \gamma/K$.

The non-vanishing components of conformal curvature tensor (C_{hijk}) are given by

$$C_{2323} = -\frac{L^2}{6T^3} \tag{3.6}$$

$$C_{1212} = \frac{L\sqrt{\frac{4T}{\ell} + \frac{L^2}{3T} + k\sqrt{T} - 4\gamma}}{4T^{5/2}} + \frac{L^2}{12T^3} \tag{3.7}$$

$$C_{1313} = -\frac{L\sqrt{\frac{4T}{\ell} + \frac{L^2}{3\mathrm{T}} + k\sqrt{T} - 4\gamma}}{4T^{5/2}} + \frac{L^2}{12\mathrm{T}^3} \tag{3.8}$$

$$C_{3134} = \frac{L}{2\sqrt{\ell}T^2} \tag{3.9}$$

The spatial volume $R^3 \to \infty$ as $T \to \infty$. The model (2.16) starts with a big-bang at $T = 0$ and the expansion in the model decreases as time increases.

The energy density $\varepsilon \to \infty$ as $T \to 0$ and $\varepsilon \to 0$ when $T \to \infty$. The string tension density $\lambda \to \infty$ when $T \to 0$ and $\lambda \to 0$ when $T \to \infty$. Since $\lim_{T\to\infty} \sigma/\theta = 0$. Hence the model isotropizes for large values of T. The model (2.16) has real physical singularity at $T = 0$. The space-time (2.16) is conformally flat for large values of T.

In absence of bulk viscosity, i.e. when $\gamma = 0$ then energy density (ε), the string tension density (λ), the expansion (θ) and shear (σ) are given by

$$\varepsilon = \frac{3k}{8\pi T^{3/2}} \tag{3.10}$$

$$\lambda = 0 \tag{3.11}$$

$$\theta = \frac{3}{2T^{3/2}}\sqrt{\frac{4T^2}{\ell} + kT^{3/2} + \frac{L^2}{3}} \tag{3.12}$$

and

$$\sigma = \frac{L}{2T^{3/2}} \tag{3.13}$$

Thus

$$\theta = \frac{3\sigma}{L}\sqrt{\frac{4T^2}{\ell} + kT^{3/2} + \frac{L^2}{3}} \tag{3.14}$$

In absence of bulk viscosity, i.e. when $\gamma = 0$ then the non-vanishing components of conformal curvature tensor (C_{hijk}) are given by

$$C_{1212} = \frac{L}{4T^{5/2}}\sqrt{\frac{4T}{\ell} + \frac{L^2}{3T} + k\sqrt{T}} + \frac{L^2}{12T^3} \tag{3.15}$$

$$C_{1313} = -\frac{L}{4T^{5/2}}\sqrt{\frac{4\mathrm{T}}{\ell} + \frac{L^2}{3\mathrm{T}} + k\sqrt{T}} + \frac{L^2}{12\mathrm{T}^3} \tag{3.16}$$

$$C_{2323} = -\frac{L^2}{6\mathrm{T}^3} \tag{3.17}$$

$$C_{3134} = \frac{L}{2\sqrt{\ell}T^2} \tag{3.18}$$

In absence of bulk viscosity, i.e. when $\gamma \to 0$ then the reality condition $\varepsilon > 0$, is satisfied, when $T \to 0$ then $\varepsilon \to \infty$ and when $T \to \infty$ then $\varepsilon \to 0$. The model in absence of bulk viscosity, starts with a big-bang at $T = 0$ and the expansion in the model decreases as time increases. The space-time is conformally flat for large values of T. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} = 0$. Hence the model isotropizes for large values of T. The spatial volume $R^3 \to \infty$ as $T \to \infty$. Since in absence of bulk viscosity, i.e. when $\gamma \to 0$ then the string tension density $\lambda = 0$. Thus the string cosmological model in absence of bulk viscosity for Bianchi Type-V metric is not possible.

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