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ORIGINAL RESEARCH

Two factor-based models of precedential constraint: a comparison and proposal

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Abstract
The article considers two different interpretations of the reason model of precedent pioneered by John Horty. On a plausible interpretation of the reason model, past cases provide reasons to prioritize reasons favouring the same outcome as a past case over reasons favouring the opposing outcome. Here I consider the merits of this approach to the role of precedent in legal reasoning in comparison with a closely related view favoured by some legal theorists, according to which past cases provide reasons for undercutting (or 'excluding') reasons favouring the opposing outcome. After embedding both accounts within a general default logic, I note some important differences between the two approaches that emerge as a result of plausible distinctions between rebutting and undercutting defeat in formal models of legal reasoning. These differences stem from the 'preference independence' of undercutting defeat . Undercutting reasons succeed in defeating opposing reasons irrespective of their relative strength. As a result, the two accounts differ in their account of the way in which precedents constrain judicial reasoning. I conclude by suggesting that the two approaches can be integrated within a single model, in which the distinction between undercutting and rebutting defeat is used to account for the distinction between strict and persuasive forms of precedential constraint.

Keywords Legal reasoning · Non-monotonic logic · Default logic · Precedent

1 Introduction

A distinguished tradition in the study of non-monotonic reasoning has focused on the development of models that are explicitly based on the legal domain. This tradition can be traced back to the work of Ronald Louie and his coauthors (1993), and is further represented in the work of Thomas Gordon (1993), Jaap Hage (1997), and Henry Prakken and Giovanni Sartor (1996; 1998). In many respects, the significance

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of legal argument to formal accounts of non-monotonic reasoning is unsurprising. After all, through the work of Hart (1949), legal philosophy is often credited with adding defeasibility to the vernacular of logicians and argumentation theorists. Law provides theorists with a well-structured domain that instantiates many patterns of practical reasoning that should be captured by any desirable framework for the representation of non-monotonic reasoning. For instance, legal reasoning often explicitly invokes consideration of the relative priority of norms in order to resolve normative conflicts. It is no surprise, then, that some of the earliest formal systems for representation of reasoning about priorities were explicitly modelled on the legal domain (Gordon 1993; Prakken and Sartor 1996). But the influence of these accounts of legal reasoning on more general accounts of non-monotonic reasoning is curious in at least one respect: unlike other forms of practical reasoning, legal reasoning is highly institutionalized. Forms of argument that are acceptable in the legal domain, such as those that involve appeals to authority, precedent or custom, are often unacceptable in non-legal settings.

The reasoning patterns considered in this paper concern the interaction between institutionalized legal reasoning and what might be called 'natural' or 'common sense' practical reasoning–ordinary practical reasoning that is unmediated by authoritative interventions. In particular, I consider the role of precedent, derived from past cases, in constraining a court or tribunal's decision-making based on their own view of the merits of a case. The paper draws on the factor-based, 'reason model' of precedential constraint pioneered by John Horty (2011; 2012), and discussed further in the context of AI and Law by Adam Rigoni (2015), Gabriel Broughton (2019), and Henry Prakken (2021). According to the reason model, a decision-maker (usually a court) is bound to make a finding for one of the parties to the dispute in cases where the opposite decision would introduce inconsistency into a priority order over reasons, interpreted as sets of factors, that is derived from an existing case base.

On a plausible interpretation of the reason model, past cases provide reasons for a court to prioritize reasons that favoured the the outcome of a past case over reasons that supported the opposing outcome (Prakken and Sartor 1998; Horty 2016). In more abstract terms, precedents support the *rebutting defeat* of reasons supporting the opposing outcome that were present in the past case. Here I consider the merits of this approach to the role of precedent in legal reasoning, particularly in comparison with an alternative view favoured by some legal theorists, according to which past cases provide reasons for undercutting (or 'excluding') reasons for the opposing outcome (Lamond 2005; Mullins 2020). Initially, I present the two approaches as rival accounts of precedential constraint. I note some important differences between the two accounts that emerge as a result of the distinction between rebutting and undercutting defeat. (John Pollock (1970) is generally credited with introducing the distinction between rebutting defeat and undercutting defeat into argumentation theory.) These differences stem from the fact that, undercutting defeat, unlike rebutting defeat, is 'preference independent' (Modgil and Prakken 2013). Undercutting reasons succeed in defeating opposing reasons irrespective of their relative strength. As a result, the two accounts differ in their treatment of the interaction between precedent and the priorities generated by a court's values or preferences. I conclude by

suggesting that the two approaches can be integrated within a single model, in which the distinction between undercutting and rebutting defeat is used to account for the distinction between strict and persuasive forms of precedential constraint.

The paper begins with a brief overview of the reason model of precedent, accompanied by an illustration of the model involving an interpretation of the 'wild animals' cases, which have been widely discussed in AI and Law . I note that the reason model is also consistent with an interpretation that views past cases as providing reasons to exclude or undercut reasons for the opposing outcome. The paper then embeds both models of precedent within a more structured account of legal reasoning, which takes Horty's (2012) default theory as its underlying formalism. I discuss some significant points of distinction between the two interpretations that arise in this setting with the introduction of priorities that are derived from the values or preferences of the decision-maker. I do not attempt to adjudicate between the two accounts. Instead, the final section of the paper is constructive. I suggest that the two accounts need not be interpreted as rival interpretations of the reason model of precedent. They are complementary. Where undercutting reasons offer a suitable account of what is sometimes called 'strict' or 'authoritative' precedential constraint, prioritizing reasons are needed to offer a suitable account of the use of precedent that, while not strictly binding, is regarded as having some degree of persuasive weight or strength.

2 The reason model

Horty's reason model of precedent is well known in the AI and Law community. I will set out the formal details of the account only briefly. Further details and discussion are contained in Horty and Bench-Capon (2012). I will ignore the problems presented by expanding the model to accommodate dimensions, rather than factors, in precedent cases (cf. Rigoni 2018; Horty 2019, 2021; Prakken 2021).

The reason model adopts the familiar factor-based approach to representing case-based reasoning. The set of all possible relevant factors F divides into two subsets of plaintiff and defendant favouring factors: F^{π} and F^{δ}. We stipulate that $F = F^{\pi} \cup F^{\delta}$. All factors favour the plaintiff or the defendant. A fact scenario X is a subset of F that represents factors applicable in a given case. Let $X^{\pi} = F^{\pi} \cap X$ and $X^{\delta} = F^{\delta} \cap X$. The variable s will be used to range over the two possible outcomes π and δ, with $\bar{s}$ representing its negation. If $s = \pi$ then $\bar{s} = \delta$, and *vice versa*. Reasons are sets of factors. We say that a fact scenario X *satisfies* a reason R just in case $R \subseteq X$.

Cases are then represented in terms of structures that involve a set of factors X that represents the relevant fact scenario in which the case was decided, a privileged reason R that represents the court's stated reason for the outcome of the case–its *ratio decidendi*, or ratio–and an outcome s. We stipulate that court's reason must be a subset of the set of factors supporting the outcome s in the relevant fact scenario.

Definition 1 (Cases and Casebases) A case is a triple $\langle X, R, s\rangle$ where X is a set of factors representing the facts of the case, s is the outcome, and R is a reason supporting the outcome of the case such that $R \subseteq X^s$. A casebase Γ is any set of cases.

The function *Ratio* will be used to identify the set of reasons that represents the ratio of the case. Where $c = \langle X, R, s\rangle$, $Ratio(c) = R$. The function $Con(c)$ identifies the set of all factors that support the opposing outcome to that reached in a given case. Where $c = \langle X, R, s\rangle$, $Con(c) = X^{\bar{s}}$. The basic idea motivating the reason model of precedent is that a past decision reflects a court's decision to prioritize the reason stated in the ratio of the decision over any reason for the opposing outcome that was satisfied in the same case. Moreover, any reason that contains the same set of factors as the ratio of the case will stronger than or equal in strength to the reason representing the ratio, and will therefore also be stronger than any reason for the opposing outcome that was satisfied in the past case. This idea is reflected in the definition below, which is then lifted to define a priority order derived from a casebase.

Definition 2 (Derived Priority Order for a Case, Casebase) Where $c = \langle X, R, s\rangle$ is a case, a priority order is derived from c by taking $R <_c R'$ just in case $Ratio(c) \subseteq R'$ and $R \subseteq Con(c)$. Where Γ is a casebase, a priority order is derived from Γ by taking $R <_\Gamma R'$ just in case $R <_c R'$ for some $c \in \Gamma$.

Note that there are no consistency constraints on what comprises a casebase, and thus a casebase can contain cases with conflicting derived priority orders, so that both $R <_\Gamma R'$ and $R' <_\Gamma R$. Consistency across a case-base is maintained by precedential constraint, understood as the requirement that a decision in a new case be consistent with the priority-information contained in the background casebase. The reason based account of precedential constraint is captured by the following two definitions.

Definition 3 (Inconsistency) A casebase Γ is inconsistent just in case there are two reasons R and R' such that $R <_\Gamma R'$ and $R' <_\Gamma R$. A casebase is consistent just in case it is not inconsistent.

Definition 4 (Precedential Constraint) Let X be a new fact scenario and Γ a consistent casebase, a decision in X based on reason R justifying outcome s is permissible just in case the casebase $\Gamma \cup \{\langle X, R, s\rangle\}$ is consistent.

The factor-based account will be illustrated in a running example involving the so-called 'wild animals' cases first introduced to AI and Law by Donald Berman and Carole Hafner (1993). The illustrations in the following discussion will involve some degree of stipulation and idealisation. In particular, I will assume that the three cases were considered by a single court, bound by its own past decisions, and not (as they were in fact) issued by a series of different courts belonging to different jurisdictions. But I hope the example is less artificial than relying on imagined cases. The illustration here involves the original three cases discussed by Berman and Hafner:

Table 1 Factors associated with the wild animals cases

Factor notation	Factor
f_1^π	The plaintiff was hunting on their own land
f_2^π	The plaintiff was engaged in activity to earn their living
f_3^π	The plaintiff was engaged in a socially valuable activity
f_1^δ	Animal not in possession of the plaintiff
f_2^δ	The defendant was engaged in activity to earn their living
f_3^δ	The defendant was in competition with the plaintiff

Table 2 Wild Animals Casebase

Case name	Case	Summary of ratio
Keeble v Hickeringill	$c_1 = \langle \{f_1^\pi, f_2^\pi, f_1^\delta\}, \{f_1^\pi\}, \pi \rangle$	Plaintiff was hunting on his own land
Pierson v Post	$c_2 = \langle \{f_3^\pi, f_1^\delta\}, \{f_1^\delta\}, \delta \rangle$	Plaintiff never had possession
Young v Hitchens	$c_3 = \langle \{f_2^\pi, f_1^\delta, f_2^\delta, f_3^\delta\}, \{f_1^\delta\}, \delta \rangle$	Plaintiff never had possession

Pierson v Post, *Young v Hitchens*, and *Keeble v Hickeringill*. Summaries of these cases and my interpretation of their rationales are offered below.

Modifying Berman and Hafner's initial account (which used four factors), Bench-Capon (2012) suggests that the reasoning in these cases involves five factors, three of which are defendant favouring and the remaining two of which are plaintiff favouring. These five factors are detailed in Table 1. I have added a sixth factor, representing the fact that the plaintiff was engaged in activity that is nonetheless socially useful. As Bench-Capon notes, this factor appears to have been invoked by the dissenting judge in *Pierson v Post*. The six factors are utilized in a casebase containing the three cases outlined above, which is represented in Table 2.

The decision in *Keeble v Hickeringill* (1707) concerned the defendant Hickeringill's action in firing guns, out of malice, in order to scare ducks away from the plaintiff Keeble's land.[1] The case involved three plaintiff-favouring factors: that the plaintiff was hunting on his own land, that the plaintiff was acting in pursuit of his own livelihood, and that the plaintiff was pursuing a socially useful activity. The case involved the single defendant favouring factor that the plaintiff did not have possession of the ducks. I follow Bench-Capon (2012) in interpreting the case as one in which the majority made a finding for the plaintiff on the basis that he owned the land on which he was hunting. The case is represented as c_1 in Table 2.

In *Pierson v Post* (1804), Post (assigned the role of plaintiff for our purposes) was hunting a fox with his hounds. [2]The defendant Pierson intercepted and killed the fox. The case involves only two factors. The sole reason favouring the defendant was

[1] Keeble v Hickeringill (1707) 103 ER 1127.

[2] Pierson v Post 3 Cai. R. 175 (N.Y. Sup. Ct. 1804).

that the plaintiff, Post, did not have possession of the fox. Arguably, the case also involved a plaintiff favouring reason, which was that fox-hunting was a socially useful activity. The clear ratio of the case is that the defendant succeeded because Post never had possession of the animal. The case is represented as c_2 in Table 2.

In the later case of *Young v Hitchens* (1844), Young (the plaintiff) was a commercial fisherman fishing for pilchards with a net in open water.[3] When the net was nearly closed, Hitchens rowed up and disturbed the fish so that they escaped and captured them himself. The case is represented with a single plaintiff favouring factor: the plaintiff was engaged in pursuit of their living. All three defendant favouring factors were present, but the court found for the defendant solely on the basis of the reason that the plaintiff never had possession of the fish. The case is represented as c_3 in Table 2.

These three cases are contained in the casebase $\Gamma_1 = \{c_1, c_2, c_3\}$. Note that the priority order derived from Γ_1 includes $\{f_1^\delta\} <_{\Gamma_1} \{f_1^\pi\}$ (from c_1), and that $\{f_2^\pi\} <_{\Gamma_1} \{f_1^\delta\}$ (from c_3). It is easy to verify that the casebase is consistent and that therefore all the decisions were permissible on the reason model of precedential constraint. On the interpretation pursued here, *Young v Hitchens* is a strengthening of the ratio offered in *Pierson v Post*. Although there were a variety of distinguishing factors that the court could have relied upon in order to avoid the precedent, the court reapplied the ratio of *Pierson v Post*.

3 The Protected Reason Model

In its canonical form, the reason model offers a relatively austere account of precedential constraint. Precedent cases are used to derive a priority order. Judges in subsequent cases are then constrained by the need to ensure consistency with this priority order. As Horty (2016) argues, a great strength of the reason model is that it offers an account of precedent as a form of 'constrained natural reasoning'. In cases in which courts are constrained, they must follow the priorities derived from the precedents in question. In cases where courts are unconstrained, they have the discretion to decide according to their own assessment of underlying priorities. My present aim is to explore the implications of the reason model when it is embedded in a more general framework of practical reasoning–one that allows us to offer a more structured account of the interaction between precedent and the unconstrained, discretionary reasoning of the decision-maker. On the interpretation of the reason model that I wish to explore, past cases provide a defeasible 'higher-order' reason for favouring the reasons cited as the ratio of the case over any reason for the opposing outcome that was satisfied in the same case (Horty 2016). On this interpretation, the constraint associated with precedent is

[3] Young v Hitchens (1844) 6 QB 606.

defeasible. The reason for following the priorities associated with the precedent may be defeated if the court takes itself to have sufficient reason for following its own priorities.[4]

Consider a dispute between plaintiff and defendant in the context of a new wild animal case that takes place against the background of the casebase Γ_1, involving the fact scenario $X_1 = \{f_2^{\pi}, f_1^{\delta}\}$. The sole applicable reason to find for the defendant, represented by the set $\{f_1^{\delta}\}$, is that the plaintiff never had possession or control, and the sole applicable reason to find for the plaintiff, represented by the set $\{f_2^{\pi}\}$, is that the defendant interfered with the plaintiff's pursuit of their living. On the canonical interpretation of the reason model, the precedent in *Young v Hitchens* provides a reason for prioritizing the reason to find for the defendant, $\{f_1^{\delta}\}$, over the reason to find for the plaintiff, $\{f_2^{\pi}\}$. In other-words, the precedent in *Young v Hitchens* does not give the court a reason to find of the plaintiff or defendant; it gives the court a reason for favouring one defendant-favouring reason over another plaintiff-favouring reason.

Something like this account of precedential reasoning is captured in existing formalizations of case-based reasoning. Henry Prakken and Giovanni Sartor (1998) showed that *a fortiori* reasoning from past cases can be modelled along these lines within an argument based system that accommodates arguments about the priorities assigned to defaults. In more recent work Horty (2016) advances a similar interpretation of the reason model within a default logic with variable priorities, while also allowing for a further distinction between precedential reasoning and *a fortiori* reasoning.

My purpose here is to compare this approach with an alternative interpretation of the reason model–one that interprets precedential constraint, not in terms of reasons for prioritizing some sets of factors over others, but as reasons for concluding that in those cases in which binding precedent applies certain reasons are excluded from providing a sound basis for decision. Consider, by way of comparison, a different line of reasoning that a court could follow in the scenario X_1, involving the same plaintiff and defendant favouring reasons, which again invokes the decision in *Young v Hitchens*. The court could reason that while it is true that the plaintiff was pursuing their livelihood, the decision in *Young v Hitchens* gives them good reason not to regard the fact that the plaintiff was pursuing their livelihood as providing a reason for finding for the defendant in the present case. Here the court's invocation of precedent does not invoke the relative weight or priority of the reasons for deciding for or against the plaintiff. Instead, the court reasons that due to the effect of the precedent in *Young v Hitchens*, the fact that the plaintiff was pursuing their livelihood cannot provide a proper basis for finding for the plaintiff. In other words, the precedent in *Young v Hitchens* undercuts the only applicable reason for finding for the plaintiff. Something like this

[4] It is worth noting that there are other plausible interpretations of the norm that requires judges to preserve consistency in the priority order. The requirement for courts to maintain consistency with the priority order derived from the casebase could also be interpreted as strict or exceptionless. In his forthcoming book on common law reasoning, Horty explores the idea that precedent can be represented as a 'hard constraint' along these lines, by embedding the priorities derived from a casebase in the hard information of a default theory. While this proposal is intriguing, I will leave full consideration of its merits for another occasion.

account of precedential constraint is popular among legal theorists. Drawing on Joseph Raz's (1975) influential work, precedential authority can be interpreted in terms of 'protected reasons'–reasons for an outcome that are coupled with reasons for 'excluding' or undercutting reasons for the opposing outcome. (On the relation between exclusionary reasons and undercutting defeat, see Horty (2012).) Support for this interpretation of precedential constraint can be found, not only in Raz's work, but in Grant Lamond's (2005) work on common law reasoning, which itself partly inspired Horty's development of the reason model.

Rather than interpreting cases as providing reasons for prioritizing reasons over others, the 'protected reason' model interprets past cases as providing information about what reasons should be excluded from providing a valid basis for a decision in a new fact scenario. Both accounts interpret past cases in terms of a relation between subsets of reasons. On the canonical interpretation of the reason model, any reason that satisfies the ratio of a past case is *stronger* than any reason for the opposing outcome that was satisfied in that past case. On the protected reason model, the ratio of a past case provides a reason for excluding any reason for the opposing outcome that was satisfied in the past case. This idea is captured by the following definition.

Definition 5 (Exclusionary Reasons Derived from a Case) Where $c = \langle X, R, s \rangle$, the reason identified by $Ratio(c)$ is a reason for excluding any reason for the opposing outcome that was satisfied in c. More formally, $Ratio(c)$ is a reason for excluding reason R' just in case $R' \subseteq Con(c)$.

Precedential constraint is then captured by the idea that a court cannot rely on a reason R^s as the basis for an outcome in any case where there is an applicable reason $R^{\bar{s}}$ supporting the opposing outcome that was recognised as a reason for excluding R^s in a previous case. This idea motivates the following revised definitions of consistency and precedential constraint.

Definition 6 (Excluded Reasons Consistency) A case base Γ is excluded reason consistent just in case there are no two cases in Γ, c and c', such that (i) $Ratio(c)$ is a reason for excluding $Ratio(c')$ and (ii) $Ratio(c) \subseteq Con(c')$.

Definition 7 (Precedential Constraint, Protected Reasons Interpretation) Let X be a new fact scenario and Γ an excluded reason consistent casebase, a decision in X based on *reason* R^s justifying outcome s is permissible just in case the casebase $\Gamma \cup \{\langle X, R, s \rangle\}$ is excluded reasons consistent.

Interestingly, the protected reason model is formally compatible with the reason model. As demonstrated in Mullins (2020), the two models of precedential constraint are equivalent. A decision will be permissible on the protected reason model just in case it is permissible on the version of the reason model that invokes priorities. I will spare the reader the proof of this observation, which follows directly from the models' use of set-inclusion to derive either a priority order or set of excluded reasons for a case.

In order to distinguish it from the protected reason model, I will now refer to the more canonical version of the reason model as the *prioritized* reason model. While the two models of precedent are formally equivalent, I think their equivalence masks some important differences in the way in which the two models represent precedential reasoning as a constraint on courts' discretionary reasoning. These differences emerge more clearly if they are embedding in a framework for representing practical reasoning that accommodates both prioritizing and exclusionary reasons. In the remainder of this paper, I embed both models of protected reasoning within a framework of default logic, in order to facilitate a more detailed comparison of the two models.

Unfortunately, since any framework suitable for comparing these two approaches has to accommodate variable priorities and undercutting defeat, the details of the theory are somewhat complicated. I will set them out in the next section of this article. Readers who are uninterested in the specific details of the framework, and who are happy to proceed with an intuitive characterisation of the interaction between defaults, may wish to ignore these details and proceed to the section following.

4 Default logic with variable priorities and exclusion

In order to compare the two models of precedential constraint, I will rely on Horty's (2012) default logic with variable priorities and 'exclusion' (undercutting). I have chosen Horty's framework because it accommodates both variable priorities and undercutting defeat, and because, unlike other frameworks, such as Prakken and Sartor's (1996) argument-based approach, it allows us to characterize extensions directly, in terms of sets of defaults, or 'proper scenarios', rather than requiring the construction of arguments prior to evaluation. In the relatively simple cases I will be considering, which do not require the chaining-together of default rules, I have found it more straightforward to use Horty's framework than an argument-based approach. Just about any framework for representing reasoning that accommodates both undercutting and variable priorities would be suitable for my purposes. There are interesting points of distinction between Horty's default theory and Prakken and Sartor's argumentation framework, for instance, but these distinctions are not relevant to the relatively straightforward reasoning contexts we will be considering.

Where X and Y are sentences in an underlying language, a set of normal defaults of the form $d : X \rightarrow Y$ capture the idea that X is a defeasible reason for Y. As in earlier systems for default reasoning with variable or 'dynamic' priorities proposed by Prakken and Sartor (1996) and Gerhard Brewka (1996), we extend the underlying language of the default logic by stipulating that each default d_i has a unique name n_i, and that the the language contains a predicate $\prec$ expressing the priority of one named default over another. Read $n \prec n'$ as indicating that the default named by n' has higher priority than the default named by n.

Following Horty, we also stipulate that the language contains the predicate *Out*, which, as we will see, allows the framework to accommodate a kind of undercutting

defeat. Defaults of the form $d : X \rightarrow Out(n_i)$ capture the idea that the proposition X is a reason for excluding or undercutting the default d_i.

Horty's approach to defining extensions for a variable priority default theory depends upon the characterisation of proper scenarios for a *fixed priority* default theory, so we begin by defining a fixed priority default theory with exclusion.

Definition 8 (Fixed Priority Default Theory With Exclusion) A fixed priority default theory is a structure $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ where $\mathcal{W}$ is a set of sentences in a background language, $\mathcal{D}$ is a set of normal default rules, $<$ is a strict partial order over $\mathcal{D}$, where there is a unique name n_i in $\mathcal{W}$ for each default $d_i \in \mathcal{D}$, and where the background language of the default theory contains the predicate *Out*.

Two functions *Premise* and *Conclusion* identify the premises and conclusions of default rules, respectively. I will overload notation by allowing these functions to apply to sets of defaults, as well as individual default rules (i.e. $Conclusion(\mathcal{S}) = \{Conclusion(d) : d \in \mathcal{S}\}$).

Unlike other approaches to undercutting defeat, which allow for arguments to be constructed prior to evaluation of their status. Horty's approach defines a set of undercut or 'excluded' defaults, which are then prevented from being applicable if the set other defaults supporting their exclusion is already accepted.

Definition 9 (Excluded defaults) Where $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ is a fixed priority default theory with exclusion, a set of excluded defaults $Excluded_{\mathcal{S}}$ is defined, relative to a subset of defaults $\mathcal{S} \subseteq \mathcal{D}$, as the set of all d_i such that $\mathcal{W} \cup Conclusion(\mathcal{S}) \vdash Out(n_i)$, where n_i is the unique name for d_i in $\mathcal{W}$. More formally,

$$Excluded_{\mathcal{S}} = \{d_i \in \mathcal{D} : \mathcal{W} \cup Conclusion(\mathcal{S}) \vdash Out(n_i)\}$$

A default is thus applicable relative to a subset of defaults $\mathcal{S} \subseteq \mathcal{D}$ just in case two conditions obtain. First, its premise must be entailed by a combination of the hard information $\mathcal{W}$ of the default theory and the conclusions of the defaults in $\mathcal{S}$. Second, the default must not be excluded relative to the same set of defaults $\mathcal{S}$.

Definition 10 (Applicable defaults) Where $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ is a fixed priority default theory, a default is applicable, relative to a subset of defaults $\mathcal{S} \subseteq \mathcal{D}$ just in case it belongs to the set

$$Applicable_{\mathcal{W},\mathcal{D}}(\mathcal{S}) = \{d \in \mathcal{D} : \ Conclusion(\mathcal{S}) \cup \mathcal{W} \vdash Premise(d) \text{ and } d \notin Excluded_{\mathcal{S}}\}$$

Conflicted defaults are then defined, relative to a subset of defaults $\mathcal{S} \subseteq \mathcal{D}$, as those defaults with conclusions that are inconsistent with the hard information of the default theory taken together with the conclusions of the defaults in $\mathcal{S}$.

Definition 11 (Conflicting Defaults) Where $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ is a fixed priority default theory, a default is conflicted, relative to a subset of defaults $\mathcal{S} \subseteq \mathcal{D}$ just in case it belongs to the set

$$Conflicted_{\mathcal{W},\mathcal{D}}(\mathcal{S}) = \{d \in \mathcal{D} : \mathcal{W} \cup Conclusion(\mathcal{S}) \vdash \neg Conclusion(d)\}$$

Finally, defaults are defeated, relative to a subset $\mathcal{S} \subseteq \mathcal{D}$, if they belong to a set of defaults that could be replaced in $\mathcal{S}$ by a stronger set of defaults that are applicable in $\mathcal{S}$. Using the notation $\mathcal{S}^{\mathcal{D}'/\mathcal{S}'}$ to represent $(\mathcal{S} - \mathcal{S}_{\simeq}) \cup \mathcal{D}'$ (i.e the replacement of $\mathcal{S}'$ in $\mathcal{S}$ by $\mathcal{D}'$), we offer the following definition of defeated defaults.

Definition 12 (Defeated defaults) Where $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ is a fixed priority default theory, a default is defeated, relative to a subset of defaults $\mathcal{S} \subseteq \mathcal{D}$ just in case it belongs to the set

$$\begin{aligned} Defeated_{\mathcal{W},\mathcal{D},<}(\mathcal{S}) = \{d \in \mathcal{D} : & \text{ there is a set: } \mathcal{D}' \subseteq Applicable(\mathcal{S}) \text{ such that} \\ & (a)\ \{d\} < \mathcal{D}', \text{ and} \\ & (b) \text{ there is a set } \mathcal{S}' \subseteq \mathcal{S} \text{ such that } \mathcal{S}' < \mathcal{D}', \text{ and} \\ & \mathcal{W} \cup Conclusion(\mathcal{S}^{\mathcal{D}'/\mathcal{S}'}) \nvdash \bot, \text{ and} \\ & \mathcal{W} \cup Conclusion(\mathcal{S}^{\mathcal{D}'/\mathcal{S}'}) \vdash \neg Conclusion(d)\} \end{aligned}$$

I will set to one side the question of lifting a priority order from individual defaults to sets of defaults. According to Horty's definition, $\mathcal{D} < \mathcal{D}'$ just in case for all defaults $d \in \mathcal{D}$ and $d' \in \mathcal{D}'$, $d < d'$. Although there are plausible alternatives, here I will limit myself to cases that involve comparison over singleton sets of defaults. Since any plausible ordering over sets of defaults lifted from an ordering over individual defaults will satisfy the constraint that $\{d\} < \{d'\}$ just in case $d < d'$, an alternative ordering could just as easily be adopted.

Extensions are based on 'proper scenarios', which are privileged sets of non-conflicted and non-defeated defaults. These are identified by adapting Reiter's (1980) 'quasi-inductive' construction to the current framework. We start with the following definition of a proper scenario.

Definition 13 (Proper Scenarios) $\mathcal{S}$ is a proper scenario based on $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ just in case $\mathcal{S} = \bigcup_{0 \leq i} \mathcal{S}_i$, where:

$$\mathcal{S}_0 = \emptyset$$

and where:

$$\begin{aligned} \mathcal{S}_i = \{d \in \mathcal{D} : & \ d \in Applicable_{\mathcal{W},\mathcal{D},<}(\mathcal{S}_{i-1}), \text{ and} \\ & d \notin Conflicted_{\mathcal{W},\mathcal{D},<}(\mathcal{S}_{i-1}), \text{ and} \\ & d \notin Defeated_{\mathcal{W},\mathcal{D},<}(\mathcal{S}_{i-1})\}. \end{aligned}$$

We then define an extension as the set of propositions generated by a proper scenario, noting that a default theory may have multiple proper scenarios and thus multiple extensions.

Definition 14 (Extension) Where $\Delta = \langle \mathcal{W}, \mathcal{D}, < \rangle$ is a fixed priority default theory and $\mathcal{S}$ is any proper scenario based on Δ, then $\mathcal{E}$ is an extension of Δ just in case $\mathcal{E} = \{X : \mathcal{W} \cup Conclusion(\mathcal{S}) \vdash X\}$

Variable priority default theories with exclusion are default theories in which information about the relative priority of defaults can be expressed in the underlying language of the default theory, in addition to information about the exclusion of defaults. We impose the requirement that the hard information of the default theory contains each relevant instance of the of the antisymmetry schema: $(n \prec n') \supset \neg(n' \prec n)$.

Definition 15 (Variable Priority Default Theory With Exclusion, or VPE Default Theory) A variable priority default theory with exclusion, or a VPE default theory, is a structure $\Delta = \langle \mathcal{W}, \mathcal{D} \rangle$ where $\mathcal{W}$ is a set of sentences in a background language and $\mathcal{D}$ is a set of normal default rules, which is subject to the requirements that (1) there is a unique name n_i in $\mathcal{W}$ for each default $d_i \in \mathcal{D}$; (2) $\mathcal{W}$ contains each instance of the anti-symmetry schema for named defaults in $\mathcal{D}$; and (3) $\mathcal{W}$ contains the predicates *Out* and $\prec$.

While variable priority default theories usually include each ground instance of the transitivity and anti-reflexivity schema for each named default in $\mathcal{D}$, Horty (2011, 2016) notes that this may be too strong in the context of common law reasoning. I will return to discussion of this point, but for the time being we impose the weaker requirement.

In order to define the proper scenarios, and thus the extensions, of a VPE default theory, we first derive a priority order from the conclusions of a set of defaults.

Definition 16 (Derived Priority Order) Let $\Delta = \langle \mathcal{W}, \mathcal{D} \rangle$ be a VPE default theory and let $\mathcal{S} \subseteq \mathcal{D}$, then the ordering $<_{\mathcal{S}}$ derived from $\mathcal{S}$ is defined as $<_{\mathcal{S}} = \{d < d' : \mathcal{W} \cup Conclusion(\mathcal{S}) \vdash n \prec n'\}$

Proper scenarios for VPE default theory are then defined as those sets of defaults that are proper scenarios for the corresponding fixed priority default theory, where the priority order is derived from the same set of defaults. Intuitively, a proper scenario is a set of defaults that can be used to provide priority information that justifies its own acceptance.

Definition 17 (Proper Scenario, VPE Default Theory) Let $\Delta = \langle \mathcal{W}, \mathcal{D} \rangle$ be a VPE default theory and let $\mathcal{S} \subseteq \mathcal{D}$, then $\mathcal{S}$ is a proper scenario based on Δ just in case $\mathcal{S}$ is a proper scenario based on the fixed priority default theory with exclusion $\langle \mathcal{W}, \mathcal{D}, <_{\mathcal{S}} \rangle$.

With this approach in hand, we can proceed to embed the two models of precedential constraint in our default theories. Before proceeding it is important to

reiterate the point, noted above, that VPE default theories can support multiple proper extensions. For the purpose of modeling precedential reasoning, I will suppose that an outcome s is permissible just in case it is supported by some extension of a default theory. For example, if a theory Δ supports two extensions, $\mathcal{E}_1$ and $\mathcal{E}_2$, with $\pi \in \mathcal{E}_1$ and $\delta \in \mathcal{E}_2$, then this will be taken to support a conclusion that it is permissible to decide for either plaintiff or defendant in the case represented by a default theory.[5]

5 Comparing the two models

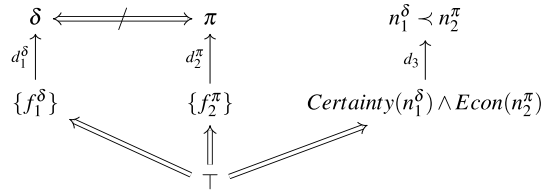
5.1 Factor-based reasoning as natural reasoning

Before we examine the two models of precedential constraint, it will be useful to illustrate how default theories can be used to represent the ordinary discretionary reasoning of a court, in cases in which it is unconstrained by precedent. The same factor-based reasoning of the sort that underlies the reason model of precedent can be used to represent the position of a court or decision-maker when they are unconstrained by precedent. The sets of factors satisfied in a given fact scenario continue to be associated with reasons supporting one of two possible outcomes in the case. In cases in which they are unconstrained, a courts assign priorities over reasons according to their assessment of the merits.

In order to accommodate this form of unconstrained factor-based reasoning within our underling default theory, we will suppose that the background information $\mathcal{W}$ of the default theory contains information about the applicable factors in a reasoning context (represented by the set of factors X), and past cases (represented by the casebase Γ). We will also suppose that the hard information contains the proposition $\neg(\delta \wedge \pi)$, telling us that a finding for the plaintiff is inconsistent with a finding for the defendant. I will use the notation $\mathcal{W}_{X,\Gamma}$ to indicate that the hard information in $\mathcal{W}$ conforms to these constraints. We will then suppose that each factor-based reason is represented by a default rule which has, as its premise, the set of factors that represents the reason in question, and, as its conclusion, a finding for the plaintiff or defendant. (For simplicity, I will ignore the empty set of factors as a reason for finding for the plaintiff or defendant.) I will follow the notational convention of labeling factor-based defaults so that both their label and their corresponding name in the underlying language default theory reflect the factors contained in the premise of the default. For instance, the reason $\{f_1^\delta\}$, to find for the defendant on the basis that the animal was not in the possession of the plaintiff, and the reason $\{f_2^\pi\}$, to find for the plaintiff on the basis that they were acting in pursuit of their livelihood, will be represented by the following defaults:

[5] More precisely: we adopt the so-called 'skeptical' approach to entailment of a required outcome, so that an outcome s is required just in case it is contained in all extensions. Since requirement and permission are duals, an outcome s will therefore be permissible if it is contained in some extension.

Fig. 1 Factor-Based Reasoning as Value-Based Reasoning



$$d_1^\delta : \{f_1^\delta\} \rightarrow \delta,$$
$$d_2^\pi : \{f_2^\pi\} \rightarrow \pi.$$

We will postulate a set of factor defaults $\mathcal{D}_F$ that contains all possible instances of these factor-based defaults.

In the fact scenario $X_1 = \{f_2^\pi, f_1^\delta\}$, represented in the hard information of a default theory $\mathcal{W}_{X_1,\Gamma}$, both defaults d_1^δ and d_2^π will be applicable, and provide conflicting reasons for defendant and plaintiff favoring outcomes. In the absence of any binding precedent, we will imagine that the court is invited to decide based on their own assignment of priorities over these factors. I will suppose that in their ordinary, discretionary, reasoning, courts appeal to underlying 'values' as a basis for prioritizing one reason over another. These need not be values in the strict sense that would satisfy an axiologist, but they are abstract considerations which the court can appeal to in justifying their decision to prioritize one reason over another–the promotion of freedom of contract, for instance, or the desire to resolve a dispute in a manner that promotes legal certainty. Courts do not always make explicit appeals to values in their discretionary judgements, but it is often possible interpret their judgements as relying on an implicit desire to promote underlying values. It is common for scholars in AI and Law to incorporate values of this sort in their model in order to explain courts' decisions to prioritize certain factors over others (cf. Bench-Capon 2002; Bench-Capon and Sartor 2003).

Suppose, for instance, that the court wished to prioritize the reason to find for the plaintiff represented by d_2^π over the reason represented by d_1^δ on the basis that d_2^π is associated with the promotion of greater economic value, and that the promotion of this values takes priority over the promotion legal certainty associated with d_1^δ. Provided we ignore the process (surely quite complex, see Prakken (2002)), by which a decision-maker reasons from abstract values through to priorities over reasons, we can represent this reasoning in a fragmentary form with a general default like the following, which applies to each relevant factor-based default in $\mathcal{D}_F$:

$$Certainty(n) \wedge Econ(n') \rightarrow n \prec n'.$$

We will postulate a set of value defaults $\mathcal{D}_V$ that apply in a given reasoning context, noting that in different reasoning contexts different values will be associated with different priority orderings over factor-based defaults, reflecting the different priorities assigned to those abstract values by individual courts. (A different court may well prioritize certainty over economic value.) I will use the notation $\mathcal{D}_{F,V}$ to indicate the union $\mathcal{D}_F \cup \mathcal{D}_V$, and will generalize this notation to apply to other unions of sets of defaults below.

The reasoning scenario presented to our unconstrained decision-maker can then be represented as a very simple default theory $\Delta_1 = \langle \mathcal{W}_{X_1,\Gamma}, \mathcal{D}_{F,V_1} \rangle$. The background information $\mathcal{W}$ contains the fact scenario $X_1 = \{f_2^\pi, f_1^\delta\}$, an empty case-base $\Gamma = \emptyset$, and the information that d_1^δ promotes certainty while d_2^π promotes the economic value of the plaintiff pursuing their own living, represented by the sentence $Certainty(n_1^\delta) \wedge Econ(n_2^\pi)$. The set of value defaults and factor defaults $\mathcal{D}_{V_1,F} = \{d_1^\delta, d_2^\pi, d_3\}$ where $d_3 : Certainty(n_1^\delta) \wedge Econ(n_2^\pi) \rightarrow n_1^\delta \prec n_2^\pi$ is an instance of the general default above concluding that d_1^δ has lesser priority than d_2^π. This theory is represented as an inference graph in Fig. 1, with double arrows used to signify deductive inference and strike-through arrows indicating negation. It is easy to see that the default theory Δ_1 supports the sole proper scenario $\mathcal{S} = \{d_2^\pi, d_3\}$, which supports a a unique extension containing conclusion π, so that the case should be found for the plaintiff. The default d_1^δ is defeated by d_2^π based on the priority information derived from d_3, preventing the construction of a proper scenario supporting a finding for the defendant.

With this account of unconstrained factor-based reasoning as a form of value-based reasoning in hand, we can now consider the different role played by precedent in constraining this form of value-based reasoning on the two different accounts. Both models interpret precedential cases as defeasible constraints on the discretionary reasoning of the court, but they do so in different ways.

Before we proceed to consider the two models, it is worth noting that it is also possible to represent this sort of discretionary reasoning by invoking exclusionary reasons rather than prioritizing reasons. In the default theory Δ_1, for instance, the default d_3, which represents the court's reason to prioritize d_2^π over d_1^δ because the promotion of economic value is more important than promoting certainty, could be replaced with the default:

$$d_3' : (Certainty(n_1^\delta) \wedge Econ(n_2^\pi)) \wedge \{f_2^\pi\} \rightarrow Out(n_1^\delta),$$

which here represents the court's reason to exclude d_1^δ in circumstances in which the premise of d_2^π is satisfied, given the relevant values promoted by the two defaults. From an abstract perspective, prioritizing reasons and exclusionary reasons are formally interchangeable. The choice between the two forms of reasons is one of design. My own view is that the sort of unconstrained reasoning discussed here is best represented in terms of a court's decision to prioritize one factor-based reason over others. As I will discuss further below, the distinction between prioritizing reasons and exclusionary reasons is motivated by a desire for descriptive adequacy. The recognition of the possibility of undercutting (or exclusionary) reasons is motivated by the recognition of a class of reasons, which unlike ordinary rebutting reasons, appear to attack other reasons without supporting an opposing conclusion (cf. Pollock 1995, p. 41). It appears to me to be descriptively inaccurate to represent the discretionary reasoning of courts in terms of a decision to exclude reasons for the opposing outcome. When courts are unconstrained, they take themselves to be free to decide on the merits of the case. They weigh or evaluate the relative merits of

the reasons in favour of or against an outcome. In the absence of authoritative constraints, a court's decision to rule in favour of side s on the basis of a reason R^s does not reflect their judgement that opposing factor-based reasons cannot support the opposing outcome.

5.2 The prioritized reason model

According to the interpretation of the reason model of precedent that I wish to consider, the ratio of a past case provides reasons for prioritizing any reason that is at least as strong as the ratio of the case over reasons for the opposing outcome that were applicable in the same past case. In order to represent this form of defeasible precedential constraint, we thus postulate a set $\mathcal{D}_\Gamma$ of prioritizing case defaults, which represent the priority information derived from past cases. We stipulate that for each case $c \in \Gamma$ there is a set of defaults in $\mathcal{D}_\Gamma$ of the form:

$$c \rightarrow n^{\bar{s}} \prec n^s$$

Where c is the case from which the priority is derived, where n^s names the factor-default representing the ratio of c, and where $n^{\bar{s}}$ names a factor default supporting the opposing outcome that was applicable in c. Informally, the default represents the idea that the decision in c is a reason for prioritizing the factor-default representing the ratio of the case over another factor-default supporting the opposing outcome that was satisfied in c. We suppose that there is an instance of this default prioritizing the factor representing the ratio of the case for every factor default $d^{\bar{s}}$ supporting the opposing outcome in c. So, for example, a hypothetical case $c_4 = \langle \{f_2^\pi, f_3^\pi, f_1^\delta\}, \{f_1^\delta\}, \delta \rangle$, involving a finding for the plaintiff on the basis of the reason $\{f_1^\delta\}$, will be associated with three case-based defaults in $\mathcal{D}_\Gamma$, each of which prioritizes a the reason for finding for the plaintiff cited in the ratio of the case over one of the factor-defaults that were applicable in c_4:

$$\begin{aligned} d^1_{c_4} &: c_4 \rightarrow n^\pi_2 \prec n^\delta_1, \\ d^2_{c_4} &: c_4 \rightarrow n^\pi_3 \prec n^\delta_1, \\ d^3_{c_4} &: c_4 \rightarrow n^\pi_{2,3} \prec n^\delta_1. \end{aligned}$$

Taken together, these case-based defaults indicate that the defaults based on the sets $\{f_2^\pi\}$, $\{f_3^\pi\}$ and $\{f_2^\pi, f_3^\pi\}$ all have lower priority than the default based on the set $\{f_1^\delta\}$, which represents the ratio of the case c_4

Since priorities derived from past cases can conflict with the priorities that courts would assign if they were free to decide on the merits, we capture the sense in which a decision-maker is *bound* to follow a precedential case by postulating that $\mathcal{D}_\Gamma$ contains each relevant ground instance of the general precedential default which prioritizes the prioritizing reasons in $\mathcal{D}_\Gamma$ over prioritizing reasons based on the decision-maker's own value-based priorities in $\mathcal{D}_V$:

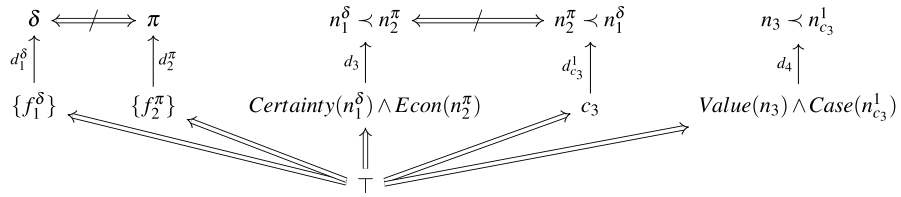


Fig. 2 The Prioritized Reason Model

$$Value(n) \wedge Case(n') \rightarrow n \prec n'$$

This general precedential default is needed in order to capture the distinction between binding and persuasive precedent. Without it, the court's reason to follow the priorities derived from a previous case could be defeated by the court's assessment of the underlying priorities. As I note below, omission of these precedential defaults is one way in which we can formalise cases in which the court treats past precedents as merely persuasive rather than binding or authoritative.

Letting $\mathcal{D}_{F,V_1,\Gamma_1} = \mathcal{D}_F \cup \mathcal{D}_{V_1} \cup \mathcal{D}_{\Gamma_1}$, the prioritized reason model of precedential constraint can now be illustrated with a relatively straightforward default theory $\Delta_2 = \langle \mathcal{W}_{X_1,\Gamma_1}, \mathcal{D}_{F,V_1,\Gamma_1} \rangle$, against the background of the same factual scenario considered above, X_1, in which the same two opposing reasons are satisfied, and the casebase Γ_1. Suppose that, again, $\mathcal{D}_{F,V_1,\Gamma_1}$ contains the factor reasons d_1^δ, and d_2^π, and the default d_3, expressing the court's value-based reason for prioritizing d_2^π over d_1^δ. In this reasoning, context, however, we will suppose that the court is constrained by the decision in c_3 (*Young v Hitchens*) from following their own value-based priorities. We now also have the set of case defaults $\mathcal{D}_{\Gamma_1}$ derived from the casebase Γ_1, which includes the default $d_{c_3}^1 : c_3 \rightarrow n_2^\pi \prec n_1^\delta$, derived from the decision in *Young v Hitchens*, as well as the following ground instance of the general precedent default, $d_4 : Value(n_3) \wedge Case(n_{c_3}^1) \rightarrow n_3 \prec n_{c_3}^1$, so that $\mathcal{D}_{F,V_1,\Gamma_1}$ includes the defaults $d_1^\delta, d_2^\pi, d_3, d_{c_3}^1$, and d_4. We also stipulate that, in addition to relevant information about the casebase Γ_1 and fact scenario X_1, the hard information in $\mathcal{W}_{X_1,\Gamma_1}$ includes the information that default d_1^δ promotes certainty and d_2^π promotes economic value (expressed by the sentence $Certainty(n_1^\delta) \wedge Econ(n_2^\pi)$) and that d_3 reflects the decision-maker's value-based priorities, while $d_{c_3}^1$ derives from precedent (expressed by the sentence $Value(n_3) \wedge Case(n_{c_3}^1)$). The theory Δ_2 is represented as an inference graph in Fig. 2.

The default theory Δ_2 supports the unique proper scenario $\mathcal{S} = \{d_1^\delta, d_{c_3}^1, d_4\}$, generating a unique extension that contains the propositions δ, $n_2^\pi \prec n_1^\delta$ and $n_3 \prec n_{c_3}^1$,

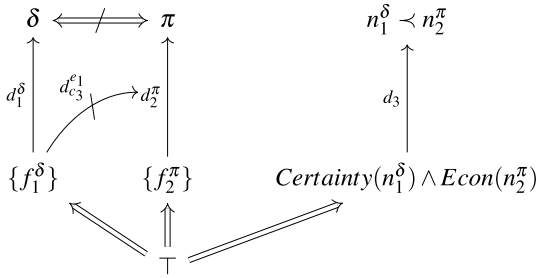


Fig. 3 The Protected Reason Model

where the priority information derived from d_4 enables the default $d_{c_3}^1$ to defeat d_3, and where the priority information derived from $d_{c_3}^1$ then enables the default d_1^δ to defeat d_2^π, so that the theory supports the conclusion δ, that a finding should be made for the defendant.

5.3 The protected reason model

It is also relatively straightforward to adapt this framework to accommodate the idea that, rather than reasons for prioritizing certain factors over others, *rationales* provide reasons for excluding factors from providing the basis for an outcome in the present case. Rather than a set of prioritizing case defaults, we instead postulate a set of exclusionary case defaults $\mathcal{D}_{\Gamma^E}$ that contains all relevant *exclusionary* case defaults for each case c, of the form

$$R^s \rightarrow Out(n),$$

where R^s is the set of factors comprising the ratio of c and where n is any of the factor defaults supporting the opposing outcome that were applicable in c. Thus the hypothetical case $c_4 = \langle \{f_2^\pi, f_3^\pi, f_1^\delta\}, \{f_1^\delta\}, \delta \rangle$ associated with a finding for the plaintiff on the basis of the factor $\{f_1^\delta\}$ will now be associated with the three case-based defaults:

$$\begin{aligned} d_{c_4}^{e_1} &: \{f_1^\delta\} \rightarrow Out(n_2^\pi), \\ d_{c_4}^{e_2} &: \{f_1^\delta\} \rightarrow Out(n_3^\pi), \\ d_{c_4}^{e_3} &: \{f_1^\delta\} \rightarrow Out(n_{2,3}^\pi). \end{aligned}$$

Taken together, these case-based defaults indicate that the set of factors $\{f_1^\delta\}$, representing the ratio of c_4, provides a reason for excluding defaults based on the sets $\{f_2^\pi\}$, $\{f_3^\pi\}$ and $\{f_2^\pi, f_3^\pi\}$. In other words, the account provides that $\{f_1^\delta\}$ is a protected reason–a reason supporting a finding for the defendant accompanied by a set of reasons for excluding reasons that support a finding for the plaintiff.

The protected reason model and prioritized reason model produce the same result, at least with respect to their support of an outcome for plaintiff or defendant.

Consider, for example, the default theory $\Delta_3 = \langle \mathcal{W}_{X,\Gamma_1}, \mathcal{D}_{F,V_1,\Gamma_1^E} \rangle$, which is the equivalent of the theory Δ_2, introduced above to illustrate the prioritized reason model, but where the previous set of prioritizing case defaults $\mathcal{D}_{\Gamma_1}$ is replaced with the the set of exclusionary case defaults $\mathcal{D}_{\Gamma_1^E}$, so that we have $\mathcal{D}_{F,V_1,\Gamma_1^E} = \{d_1^\delta, d_2^\pi, d_3, d_{c_3}^{e_1}\}$, where $d_{c_3}^{e_1} : \{f_1^\delta\} \rightarrow Out(n_2^\pi)$. This theory is represented in Fig. 3, where a negated single arrow between a node and a default name is used to represent an exclusionary reason (as in Horty (2012)).

The unique proper scenario for Δ_3 is $\mathcal{S} = \{d_1^\delta, d_3, d_{c_3}^{e_1}\}$, generating a unique extension that contains the propositions δ and $Out(n_2^\pi)$. Like Δ_2, the theory Δ_3 supports a unique extension that contains the proposition that the court should find for the defendant.

6 Preference independence and transitivity

The two models of precedential constraint will produce equivalent results in these simple cases. The choice between the two models seems largely to be motivated by concern for descriptive adequacy. In previous work (Mullins 2020), I have argued that the protected reason model better conforms with the phenomenology of precedential constraint, since it reflects the sense in which the judge's obligation to follow precedent is independent of her assessment of the underlying priorities. I will not recapitulate this argument here. Instead I will focus on some technical points of distinction that seem to arise between the two models if we treat them as offering differing descriptive accounts of of precedential constraint. These points of distinction relate, in one way or another, to what Modgil and Prakken (2013) call the 'preference independent' character of undercutting defeat.

The preference independence of exclusionary reasons can be illustrated by a comparison between the default theories used to illustrate the two models above, Δ_2 and Δ_3. The two theories both supported an extension containing the proposition that the case should be resolved for the defendant. In addition, however, Δ_3 supported an extension that contained the proposition $n_1^\delta \prec n_2^\pi$, that the defendant favouring reason represented by d_1^δ was of lower priority than the defendant favouring reason contained in d_2^π. We can interpret this result as indicating a finding that, though the priority of reasons supports a finding for the plaintiff, the justified arguments support a finding for the defendant due to the effect of precedential constraint. This is, I think, a desirable feature of the protected reason model. English judges have been known to lament the fact that they are bound by precedent to make a ruling that they regard as 'fallacious',[6] or 'bordering on the absurd',[7] indicating the irrelevance of the weight or priority that the court would afford to the reasons at hand. The protected

[6] Radcliffe v Ribble Motor Services Ltd (1939), at 228 (per Lord Atkin).

[7] R. v Pigg (1982)74 Cr. App. Rep. 352. .

reason model's invocation of exclusionary reasons is motivated by the insight that courts bound to follow authoritative precedent regard the relative priority of reasons as irrelevant to the outcome. As we saw, this aspect of precedential constraint is captured on the prioritized reason model in a different way, by stipulating that the decision-maker has reason to prioritizing the priorities derived from past cases over those based in their own values. The two approaches thus differ, somewhat subtly, in the way in which they they handle the conflict between precedent and the decision-maker's value-based priorities. These subtle differences can be further exposed if we alter our underlying default theory to preserve the transitivity of priorities. On the prioritized reason model, past cases are used to derive information about the relative priority of factor based reasons. The principle of transitivity can thus be used to derive further information about the the relative priority of different factor-based reasons, beyond those derived directly from a case in the casebase. The protected reason model cannot plausibly accommodate this sort of extension without abandoning the phenomenology of precedential constraint on which it is based.

6.1 Transitivity on the prioritized reason model

As Horty (2011) notes, transitivity can be used to strengthen the account of precedent offered on the prioritized reason model. Suppose that we have a casebase $\Gamma_2 = \Gamma_1 \cup \{c_5\}$, where $c_5 = \langle \{f_1^\pi, f_3^\delta\}, \{f_3^\delta\}, \delta \rangle$ is a (fictional) case in which the court found for the defendant on the basis that the defendant was in direct competition with the plaintiff, prioritizing this reason over the reason to find for the plaintiff because they were hunting over their own land. From the decision in *Keeble v Hickeringill* we have that $\{f_1^\delta\} <_{\Gamma_2} \{f_1^\pi\}$, from the decision in *Pierson v Post* we have that $\{f_3^\pi\} <_{\Gamma_2} \{f_1^\delta\}$, by transitivity we therefore have that $\{f_3^\pi\} <_{\Gamma_2} \{f_1^\pi\}$. Now suppose that the decision-maker is faced with a new fact scenario $X_2 = \{f_3^\pi, f_3^\delta\}$, in which the sole reason to find for the plaintiff, represented by the set $\{f_3^\pi\}$, is that they were engaged in a socially valuable activity, and the sole reason for the defendant, $\{f_3^\delta\}$, is that they were in competition with the plaintiff. Though there is no individual case in Γ_2 telling us that $\{f_3^\pi\}$ has lower priority than $\{f_3^\delta\}$, we are able to reach this conclusion by applying the relevant instances of the transitivity schema to the priority information contained in the casebase. From the ratio in c_5 we have that $\{f_1^\pi\} <_{\Gamma_2} \{f_3^\delta\}$. Since we have already concluded that $\{f_3^\pi\} <_{\Gamma_2} \{f_1^\pi\}$, we can further conclude by transitivity that $\{f_3^\pi\} <_{\Gamma_2} \{f_3^\delta\}$. The decision-maker must therefore find for the defendant, since a finding for the plaintiff on the basis of $\{f_3^\pi\}$ would introduce inconsistency into the casebase.

In order to accommodate the effects of transitivity in the context of a default theory, we must first suppose that the background information $\mathcal{W}$ of our default theories contains each ground instance of the transitivity schema, in addition to the antisymmetry schema:

$$((n \prec n') \wedge (n' \prec n'')) \supset (n \prec n'').$$

We can now illustrate the effect of extending the language of our default theory with transitivity on the prioritized reason model by considering a default theory $\Delta_4 = \langle \mathcal{W}_{X_2,\Gamma_2}, \mathcal{D}_{F,V,\Gamma_2} \rangle$ which is based on the set of factors F, the casebase Γ_2 and the new fact scenario $X_2 = \{f_3^\pi, f_3^\delta\}$. We suppose, for the purpose of illustration, that the set of value based defaults $\mathcal{D}_V$ is empty. Noting that only d_3^δ and d_3^π are applicable in this scenario, the relevant factor defaults are:

$$\begin{aligned} d_1^\delta &: \{f_1^\delta\} \rightarrow \delta \\ d_1^\pi &: \{f_1^\pi\} \rightarrow \pi \\ d_3^\delta &: \{f_3^\delta\} \rightarrow \delta \\ d_3^\pi &: \{f_3^\pi\} \rightarrow \pi \end{aligned}$$

The defaults d_1^δ and d_1^π, while not applicable, are used to derive priority information. The relevant case defaults are thus:

$$\begin{aligned} d_{c_1}^1 &: c_1 \rightarrow n_1^\delta \prec n_1^\pi, \\ d_{c_2}^1 &: c_2 \rightarrow n_3^\pi \prec n_1^\delta, \\ d_{c_5}^1 &: c_5 \rightarrow n_1^\pi \prec n_3^\delta. \end{aligned}$$

The theory supports the unique proper scenario $\mathcal{S} = \{d_3^\delta, d_{c_1}^1, d_{c_2}^1, d_{c_5}^1\}$, which generates a unique extension containing the propositions δ and $n_3^\pi \prec n_3^\delta$, with the latter proposition derived from applying relevant instances of the transitivity schema to the conclusions of the case defaults $d_{c_1}^1$, $d_{c_2}^1$, and $d_{c_3}^1$. This priority information supports the defeat of d_3^π by d_3^δ and ensures that the theory supports a finding for the defendant.

In addition to strengthening the account of precedent offered by the prioritized reason model, the account also allows decision-makers to derive new priority information by applying the transitivity schema to a combination of the conclusions of case defaults and conclusions of value-based defaults. This feature of the account is particularly attractive if we assume that an explicit goal of precedential reasoning is to promote the coherence between priorities over factors derived from a body of case law and the priorities over abstract values that these factors are taken to represent. Bench-Capon and Sartor's (2003) 'theory construction' approach to representing case-based reasoning offers a particularly compelling illustration of an approach along these lines. On the theory construction approach, the priority order over factors derived from a case-based is taken to reveal an underlying set of value priorities, which can then be used to derive priority information over sets of factors in new cases even where those factors have not been considered together by a previous court.

The role of transitivity in facilitating consistency between abstract priorities of this sort can be illustrated by considering the values associated with the wild animals cases. It seems plausible, for instance, that the decision in *Young v Hitchens* revealed the priority of factors that promote certainty over factors that promote economic value, since the court prioritized the reason to find for the defendant because he was not in possession of the fish over reasons to find for the plaintiff because they were pursuing their living. Allowing, again, for oversimplification, we can suppose for the purpose of illustration that the case c_3 is now associated with a general case-value default of the form

$$d_{v_{c_3}} : Econ(n) \wedge Certainty(n') \rightarrow n \prec n'$$

which will apply to each relevant factor default in $\mathcal{D}_F$.

We now suppose a default theory $\Delta_5 = \langle \mathcal{W}_{X_3,\Gamma_1}, \mathcal{D}_{F,V_2,\Gamma_1} \rangle$, which is based on the set of factors F, casebase Γ_1 and the new fact scenario $X_3 = \{f_1^\pi, f_2^\delta\}$, which satisfies the sole plaintiff-favouring reason, $\{f_1^\pi\}$, that the plaintiff was hunting on his own land, and the sole defendant-favouring reason $\{f_2^\delta\}$, that the defendant was engaged in an activity to earn their living. The relevant factor-defaults in $\mathcal{D}_F$ are therefore:

$$d_1^\delta : \{f_1^\delta\} \rightarrow \delta,$$
$$d_1^\pi : \{f_1^\pi\} \rightarrow \pi,$$
$$d_2^\delta : \{f_2^\delta\} \rightarrow \delta.$$

The only relevant case-based default in $\mathcal{D}_{\Gamma_1}$ is based on the decision in *Keeble v Hickeringill*, represented here in terms of the default

$$d_{c_1}^1 : c_1 \rightarrow n_1^\delta \prec n_1^\pi,$$

where n_1^δ names the default d_1^δ, which is not satisfied in the current factual scenario. At first glance, then, the default in $d_{c_1}^1$ appears irrelevant. Suppose, however, that the decision-maker is persuaded by the suggestion that the best interpretation of the body of case law represented by Γ_1 is that the defendant favoring factor-default d_2^δ is weaker than the defendant favouring d_1^δ, because d_1^δ promotes certainty while d_2^δ only promotes economic value. For illustrative purposes, I will represent this line of reasoning by supposing that the set of defaults $\mathcal{D}_{F,V_2,\Gamma_1}$ also contains a specific instance of the general default considered above:

$$d_{v_{c_3}}^1 : Econ(n_2^\delta) \wedge Certainty(n_1^\delta) \rightarrow n_2^\delta \prec n_1^\delta.$$

Supposing that $\mathcal{W}_{X_3,\Gamma_1}$ contains each relevant ground instance of the transitivity schema, as well as the statement $Econ(n_2^\delta) \wedge Certainty(n_1^\delta)$, the theory now supports

the unique proper scenario $\mathcal{S} = \{d_1^{\pi}, d_{c_1}^1, d_{v_{c_3}}^1\}$ supporting the conclusion π, that the court find for the plaintiff, where the additional information that $n_2^{\delta} \prec n_1^{\pi}$, obtained by applying an instance of the transitivity schema to the conclusions of $d_{c_1}^1$ and $d_{v_{c_3}}^1$, supports d_1^{π} in defeating d_2^{δ}.

Even on the prioritized reason model, there are good reasons for eschewing an approach that strengthens precedent with an assumption of transitivity. As Horty (2011) notes, the assumption of transitivity over the priority order derived from a casebase is problematic when viewed from a social choice perspective. The priority order derived from the casebase is derived from a body of collective decisions. An assumption of transitivity allows us to strengthen the information derived from this collected body of decisions so that it applies to sets of factors that have never been considered together by a single court. Moreover, while the sort of reasoning represented above, which associates cases, not just with priorities over factor-defaults but with general priorities derived from values, strikes me as a very plausible form of legal reasoning, it does not seem to form part of what we should properly regard as a doctrine of precedent. For this reason, Horty and Bench-Capon (2012) suggest that theory-construction approaches centred on promoting the coherence of a body of case-law with a set of underlying values are better suited to modelling periods of 'fluctuation and reinterpretation' in the development of case-law. In cases where they are unconstrained, courts may well have *persuasive* reasons to align their decision with the value priorities revealed by a body of case law, but they are not constrained to do so. I will return to this point in Sect. 7.

In spite of these reservations, I feel the issues surrounding transitivity in the reason model of precedent deserve further consideration, and it is certainly a technical advantage of the prioritized reason model that it can accommodate the patterns of reasoning that I have considered here.[8]

6.2 Transitivity on the protected reason model: downward closure of exclusion

On the protected reason model, case-based defaults do not encode information about the priorities of factor-based reasons. The model therefore does not allow a court to strengthen a body of precedent through an application of transitivity, as occurred in the theory Δ_4, above. It would be possible to capture a similar pattern of reasoning without appealing to any priority ordering over factor defaults. To do so, we would need to adopt a general schema to the effect that where any default d concludes that another default d' is excluded, then any default d'' that is excluded by d' will also be excluded. But there is no good reason for thinking that exclusionary reasons conform to this schema. The protected reason model captures the binding effect of a precedent by stipulating a relationship between the ratio R^s of a past case and a set

[8] As a reviewer notes, transitivity seems to be a necessary assumption if we wish to capture the effect of precedent on dimensional reasoning. The need to accommodate dimensional reasoning complicates the protected reason model, and seems to make a unified treatment of precedent based on priorities more plausible. I leave discussion of this problem for future work.

of factor defaults that were applicable for outcome $\bar{s}$ in the past case. The conclusions of these exclusionary reasons do not express any general relationship between factor-defaults that could be extended by the assumption of some sort of closure property analogous to the closure of priorities under transitivity.

As I noted above, the assumption of transitivity also allowed for an account of precedential reasoning that combined priorities over factor-defaults with value priorities derived from a body of case law. This allowed us to represent the process by which a court could reason that, since one reason R was lower in priority than another R' according a body of case law, and R' was lower in priority than R'' according to the set of value priorities derived from the same body of case law, R would also be lower in priority than R''. If we continue to represent values as providing reasons for prioritizing one set of factor-defaults over another, an analogous pattern of reasoning could be captured by stipulating that there is a general schema

$$(Out(n) \wedge (n' \prec n)) \supset Out(n')$$

in the hard information $\mathcal{W}$ of the default theory, which applies to each factor-based default. In other words, the theory could be extended by an assumption of what Horty (2011) calls 'downward closure of exclusion'. Alternatively, we could assume something like 'defeasible downward closure of exclusion', by stipulating that there is a general default with the schema as its conclusion:

$$\top \rightarrow (Out(n) \wedge (n' \prec n)) \supset Out(n').$$

Either approach would allow a court to reason that because a certain factor default is excluded, and because that default is higher in priority (according to the value priorities derived from the body of case law) than another applicable factor default supporting the same outcome, the weaker default should also be excluded.

I feel, however, that adopting downward closure of exclusion, in either its strict or defeasible form, would be a mistake in the context of representing the protected reason model of precedent. To begin with, the motivation underlying the protected reason model is the idea that the priority of the reasons excluded by a certain past outcome is irrelevant to the outcome of the present case. Consideration of the reason's strength is improper. Indeed, in practical philosophy, Raz's (1975) idea of an exclusionary reason is motivated by the desire to capture the sense in which certain reasons are defeated by considerations other than those of weight or priority. Raz explicitly contemplates the possibility that we might have good reason to exclude one reason in favour of following other, less weighty reasons. On this interpretation of undercutting defeat, it does not follow, in any logical sense, from the fact that one reason is excluded that all reasons weaker than it will be excluded. Working within a framework of default logic inspired by Raz's account, Horty (2012) offers a plausible argument that downwards closure of exclusion results in unwanted exclusion of reasons in certain contexts. Horty offers the example of an order by a Colonel to a soldier to disregard the order of a Major, because the Major is intoxicated, who then receives an opposing order from their Captain, who is lower in rank than the Major. Intuitively, without formalizing the example, it seems clear that the soldier would be mistaken if they took

themselves to have a reason to disregard the orders of their Captain. The intuition behind this example can be generalized: the existence of a good reason to exclude another reason tells that if certain circumstances obtain, they support a conclusion about the unreliability or inappropriateness of reasoning with a particular reason (or particular set of reasons). It is not, as a general rule, safe to reason from the conclusion that one reason is excluded to the conclusion that any other reasons that bare some relevant relation to the excluded reasons (that they are of lower priority, for example) are also excluded.

In spite of its similarities with the prioritized reason model, the protected reason model is motivated by a different understanding of the phenomenology of precedential constraint. On the protected reason model, the consistency, or coherence, of a body of case law with any set of underlying priorities is a by-product of precedential constraint, and not its primary motivation (see discussion in Mullins (2020)). Although the protected and prioritized reason models of precedent produce equivalent results in simple cases, there are important points of distinction between the two models that reflect differences in their underlying accounts of precedential constraint. On the prioritized reason model, courts are constrained by reasons to follow the priority order derived from past cases rather than assigning their own priorities. On the protected reason model, courts are constrained by reasons to disregard certain reasons for the opposing outcome. The behaviour of the two models on the assumption of a transitive priority order exposes these points of distinction.

The failure of downward closure of exclusion formally distinguishes the protected reason model from the prioritized reason model. Otherwise, it would be possible to view exclusionary reasons as the limit case of a reason for assigning a lower priority to other reasons. Although he later abandoned it, Horty (2007) initially developed a formal framework along these lines, echoing insights developed informally in moral philosophy by Jonathan Dancy (2004) and Mark Schroeder (2011), and in legal philosophy by Stephen Perry (1987). Perry's discussion of this idea is particularly useful in the present context. Working on the assumption that reasons can be assigned a weight or degree of strength, Perry treats prioritizing reasons as 'reweighting' reasons: reasons for altering the degree of strength assigned to a reason. On Perry's model, exclusionary reasons are just the limit case of a reweighting reason: a reason for assigning zero weight to another reason (compare Schroeder (2011)). Interestingly, Perry motivates this account of practical reasoning by considering the differing levels of strength that courts assign to different forms of precedent. Adopting what he describes as a 'strong Burkean' view of precedent, Perry suggests that precedent is associated with different levels of strength or weight. Stricter precedential authority is associated with reweighting reasons that come close to being exclusionary–they are reasons for assigning close to zero weight to other reasons, where precedential authority that is only weakly persuasive is associated with reasons for assigning only slightly lower weight to other reasons. Since it appears to validate downwards closure of exclusion, Perry's account is not compatible with my own. As I note below, I am also disinclined to follow him in thinking of reasons as having numerical weights or 'degrees' of strength. Nonetheless, I think the idea that the difference between exclusionary reasons and prioritizing reasons might explain the distinction between strict and persuasive precedent is a promising one. I will explore it in the next section.

7 Modelling the distinction between strict and persuasive precedent

Having introduced these formal accounts of precedential reasoning as alternatives, I would like to suggest an account that allows for them to be reconciled with one another. My suggestion, in short, is that the prioritized reason model and the protected reason model offer complementary accounts of different aspects of precedential reasoning. Where prioritized reasons can be used to account for precedent that is treated as having a certain level of persuasive strength, protected reasons can be used to account for strict, or 'authoritative' precedent.

The distinction between strict and persuasive precedent that I have in mind is not the distinction between accounts of precedential reasoning in terms of strict or exceptionless legal rules and accounts of precedential reasoning that allow for courts to rely on distinguishing factors.[9] As we have seen, both the prioritized reasons account and the protected reasons account allow for courts to rely on distinguishing factors.

Lawyers and theorists use the term 'persuasive authority' to refer to a variety of disparate forms of reasoning,[10] For present purposes it will be useful to offer a more precise account of what I have in mind when I refer to the persuasive use of precedent. It is, of course, completely compatible with the reason model of precedent that past cases can be used persuasively in future cases. There are several forms of persuasive case-based reasoning that are already accommodated by the reason model. In a factual scenario X in which a precedent case c applies to but does not bind the court or decision-maker, because there are distinguishing factors available, the court may nonetheless decide to follow the ratio of the past case, thereby strengthening the precedent. More fundamentally, the reason model allows that courts reason by analogy with past cases even where the rationales of past cases do not directly apply to the present factual scenario. Along these lines, Prakken and Sartor (1998) present a framework that allows for parties to argue that the rationales of past case should be broadened in order to apply to a factual scenario to which they do not directly apply. Finally, the set of factors F can itself be viewed as information, extracted from a body of authoritative case law, concerning legally relevant bases that a court can rely upon in forming their judgement. Even when they do not determine the outcome, precedents provide important forms of epistemic guidance to courts. Past cases can assist courts in identifying the particular facts of a case with certain reasons for judgement. Even in a case in which *Pierson v Post* is not binding authority, a court may still cite the dissenting opinion as authority for the proposition that the eradication of vermin is 'socially valuable' and thus identify a reason for finding

[9] This distinction is used, quite plausibly, by Gabriel Broughton (2019) in his modification of Horty's model of precedent to account for the distinction between vertical and horizontal precedent in U.S. courts. I do not intend to present my account as a rival to Broughton's. Broughton may be right that lower courts in the US are required to treat higher court's precedent as creating strict legal rules. If this is the case, he has identified a valuable point of distinction between two kinds of strict precedential authority–those which allow for consideration of distinguishing factors and those that do not. This distinction does not challenge the further distinction between strict and persuasive precedent considered here.

[10] For illustrative discussions of persuasive authority in legal theory see Bronough (1987) Lamond (2010), and Schauer (2008).

for the defendant. Existing factor-based approaches, such as CATO (Aleven 1997), incorporate this form of epistemic guidance by recognizing the existence of hierarchies of factors, in order to represent the process by which a court reasons from the simple facts of a case (such as the fact that the defendant was eradicating vermin) through to the recognition of higher-level 'abstract' reasons supporting an outcome (that the defendant was engaged in a socially valuable activity, for example). Rigoni (2015) offers some useful reflection on the relation between this form of reasoning and precedential reasoning.

The kind of persuasive use of *precedent* that I have in mind differs from these other forms of persuasive use of cases, since it arises in cases where courts are permitted to depart from the applicable ratio of a past case even in the absence of a relevant distinguishing factor. The relevant difference between persuasive precedent and strict precedent is evident in cases in which no distinguishing factors are present, but where the court has good reasons for departing from the ratio of a past case. As Cross and Harris suggest, 'a precedent may be persuasive authority even though it does not persuade' (Cross and Harris 1991, p. 9). Consider, for example, the factual scenario $X_1 = \{f_2^{\pi}, f_1^{\delta}\}$. According to the precedent in *Young v Hitchens*, the court in X_1 is forced to make a finding for the defendant. If the decision in *Young v Hitchens* is treated as having persuasive, rather than strict precedential authority, then the court is permitted to find for the defendant in cases where they take themselves to have a sufficiently strong reason for departing from the precedent.

If a precedent has strict authority, then it binds the court irrespective of the court's assessment of the merits of the case. In the absence of distinguishing factors, courts can only decline to follow strict precedents on exceedingly narrow grounds: in English law, for instance, the Court of Appeal can only depart from a past precedent of its own in circumstances where there are conflicting authorities, where the past decision conflicts with a higher authority or statute, or where the decision was given *per incurium*, in ignorance of some applicable law. (As I note in further detail below, these are cases where, properly construed, the precedent in question has no authority at all.)

Persuasive precedents appear to be associated with differing levels of strength. In some jurisdictions, persuasive precedents merely provide a reason that the court must take into account in determining the outcome of the case. In other cases, the court must cite strong reasons for departing from persuasive precedents–it must treat them as possessing a higher level of strength or weight, even where it is not bound to follow them. This is arguably the case in the Supreme Court of the United Kingdom. Though empowered by its 1966 Practice Statement to depart from precedent in situations in which it is 'right to do so', the Supreme Court has frequently held that the mere conclusion that a previous judgement was 'wrong' on its merits is not sufficient reason to depart from a precedent.[11] Departure from precedent must be a clear improvement, and requires 'much more than doubts' about the correctness of a past decision.[12]

[11] See the summary in Cross and Harris (1991, pp. 135–143).

[12] Fitzleet Estates Ltd v Cherry (1977), at 999 (per Lord Wilberforce).

There are several ways in which we might model the distinction between strict and persuasive precedent, some of which would not require reliance on both prioritizing and exclusionary reasons. In the version of the prioritized reason model I considered in Sect. 5, the constraining effect of precedent was captured by postulating that there was a general precedential default which prioritizes the prioritizing reasons in $\mathcal{D}_\Gamma$ over prioritizing reasons based on the decision-maker's own value-based priorities in $\mathcal{D}_V$:

$$Value(n) \wedge Case(n') \rightarrow n \prec n'.$$

For those who are not convinced of the merits of the protected reason model, the distinction between strict and persuasive precedent could be captured by including or omitting this constraining default. (Another possibility would be identify strict precedent with cases in which courts have a protected reason to follow the priorities derived from a past case.) The method of representing the distinction between strict and persuasive precedent that I will present involves a straightforward combination of the two models. The insight motivating my approach is that both strict and persuasive precedents constrain subsequent courts, although they do so in different ways. Strict precedents constrain courts by excluding certain reasons from providing a basis for judgement. Persuasive precedents provide reasons (of varying strength) for resolving a dispute in a manner that is consistent with the priorities they recommend.

In order to accommodate both strict and persuasive precedential constraint within a single model, we now postulate that decisions take place against the background of two distinct casebases, a casebase Γ, which contains all strict precedential authorities, and a casebase γ, which contains persuasive precedential authorities. We assume that there is a corresponding set of prioritizing case defaults $\mathcal{D}_\gamma$ for the cases in γ and a corresponding set of exclusionary case defaults $\mathcal{D}_\Gamma$ for the cases in Γ. However, we will no longer assume that the set of prioritizing case defaults contains a general default of the type considered above, which provides a reason for prioritizing precedential defaults over defaults based on the reasoner's own values. Instead, the strength of a court's reasons for accepting prioritizing defaults that are in the casebase will vary depending on the reasoning context, in order to reflect the different levels of strength or priority carried by persuasive precedent. Priorities derived from strongly persuasive precedents will tend to prevail over priorities derived from other reasons, but they will not necessarily do so, and they can be overcome by a sufficiently strong reason. Our case-based default theories will now take the form $\Delta = \langle \mathcal{W}_{X,\Gamma,\gamma}, \mathcal{D}_{F,V,\Gamma,\gamma} \rangle$, where the hard information in the default theory, $\mathcal{W}_{X,\Gamma,\gamma}$, contains relevant information about the factual scenario X and the two casebases Γ and γ, and where $\mathcal{D}_{F,V,\Gamma,\gamma}$ is the union of factor defaults, values-based defaults, and strict and persuasive case defaults representing a particular reasoning context.

7.1 The effect of persuasive precedent

The effect of persuasive precedent is best illustrated by a case in which the precedent provides the court or tribunal with a strong reason for prioritizing one factor

over another that the court decides not to follow. Suppose that a court is faced with the fact scenario X_1, where Γ is empty and where $\gamma_1 = \{c_3\}$, so that there is a single case that is treated as having persuasive value. (Recall that the case c_3 represents *Young v Hitchens*). We will also suppose that the court takes itself to have good reason to depart from the precedent in c_3. Let us represent this reasoning context with the theory $\Delta_6 = \langle \mathcal{W}_{X_1,\Gamma,\gamma_1}, \mathcal{D}_{F,V_3,\Gamma,\gamma_1} \rangle$, so that there are two applicable factor defaults:

$$d_1^\delta : \{f_1^\delta\} \rightarrow \delta,$$
$$d_2^\pi : \{f_2^\pi\} \rightarrow \pi.$$

There is a single case default in $\mathcal{D}_{\gamma 1}$:

$$d_{c_3}^1 : c_3 \rightarrow n_2^\pi \prec n_1^\delta.$$

We suppose, also, that there are two value defaults in $\mathcal{D}_{V_3}$:

$$d_3 : Econ(n_2^\pi) \wedge Certainty(n_1^\delta) \rightarrow n_1^\delta \prec n_2^\pi,$$
$$d_4 : \top \rightarrow n_{c_3}^1 \prec n_3.$$

As in previous examples, the first value default, d_3 represents the court's reason for prioritizing the plaintiff favouring factor-default over the defendant favouring factor-default. The second value-based default, d_4 represents the court's reason to follow their own value-based priorities over the precedential default. (For the sake of simplicity, I will assume that the default takes $\top$ as its premise and ignore the process of reasoning by which the court reaches this conclusion.)

In this case, the sole proper scenario for the theory Δ_6 is $\mathcal{S} = \{d_2^\pi, d_3, d_4\}$, supporting an extension that contains the propositions $\pi, n_1^\delta \prec n_2^\pi$, and $n_{c_3}^1 \prec n_3$, and therefore supporting the conclusion that the court is required to find for the plaintiff. The court's value-based defaults give them sufficient reason to depart from the persuasive precedent in *Young v Hitchens*.

As I noted above, persuasive precedents appear to be associated with different levels of strength depending on the type of precedent and the jurisdiction in which it is considered. In the previous example, involving the theory Δ_6, the court simply took their own value defaults to be sufficient reason for departing from the persuasive precedent. But there are many cases where courts will be reluctant to depart from a precedent–even a persuasive precedent–simply because they have a reason to do so. A commitment to the ideal that like cases should be decided alike, for example, may compel subsequent courts to apply a precedent in the same way as a previous court even where the precedent does not have strict authority. These cases can be modelled in terms of a further form of value default, which reflects deference to a persuasive authority in terms of a reason to defer to a previous court's judgement even where it goes against the current court's assessment of the relative priorities.

Suppose, for example that a court is again faced with the fact scenario X_1against the background of an empty strict casebase Γ, where once more $\gamma_1 = \{c_3\}$, and where the court takes itself to have reason to defer to the precedent in c_3 in spite of

their values-based reasons to depart from it. This context can be represented with the default theory $\Delta_7 = \langle \mathcal{W}_{X_1,\Gamma,\gamma_1}, \mathcal{D}_{F,V_4,\Gamma,\gamma_1} \rangle$, where there are again two applicable factor defaults

$$d_1^\delta : \{f_1^\delta\} \rightarrow \delta,$$
$$d_2^\pi : \{f_2^\pi\} \rightarrow \pi,$$

and a single relevant case default

$$d_{c_3}^1 : c_3 \rightarrow n_2^\pi \prec n_1^\delta.$$

But where the set $\mathcal{D}_{V_4}$ now contains three value defaults:

$$d_3 : Econ(n_2^\pi) \wedge Certainty(n_1^\delta) \rightarrow n_1^\delta \prec n_2^\pi,$$

$$d_4 : \top \rightarrow n_{c_3}^1 \prec n_3$$
$$d_5 : Persuasive(n_{c_3}^1) \wedge Value(n_3) \rightarrow n_3 \prec n_{c_3}^1$$

The additional default d_5 represents the court's reasons to defer to persuasive precedent in spite of their own value judgement. The default theory Δ_7 now supports two stable scenarios, $S_1 = \{d_2^\pi, d_3, d_4\}$ and $S_2 = \{d_1^\delta, d_{c_3}^1, d_5\}$, which support either an extension $\mathcal{E}_1$ that contains the propositions $\pi, n_1^\delta \prec n_2^\pi$, and $n_{c_3}^1 \prec n_3$ or the extension $\mathcal{E}_2$, which contains the propositions $\delta, n_2^\delta \prec n_1^\pi$, and $n_3 \prec n_{c_3}^1$. The court can therefore permissibly find for either the defendant or the plaintiff.

It is easy to imagine even higher levels of complexity in a court's treatment of persuasive precedent on this account. The value default reflecting a court's deference to another court's judgement could be reinforced by a further default reflecting a preference for following this default over the court's own judgement. This default could be rebutted by a good reason for declining to follow this prioritizing reason, and so on. I will not illustrate these levels of complexity here, but it is interesting to consider their implications for our understanding of the representation of persuasive precedent. The general intuition underlying this interpretation is that persuasive precedent provides reasons for prioritizing factor defaults over others, so that the differing strengths of persuasive precedent is reflected in terms of the differing strengths of the reasons needed to justify departure from them.

I have not formalized the idea that a reason must reach some sufficient level of strength before it can justify departure. In the default theory Δ_7, for instance, there is no reason for the default d_4 to posses some requisite level of strength before it supports departure from the persuasive precedent in c_3. The need for a reason to meet a certain threshold of strength before it can justify departure from precedent could be captured relatively straightforwardly by associating each precedent with a cardinal weight, so that conflicting reasons must match or exceed this weight in order to justify departure (Perry 1987). A cardinal representation of the strength of precedents strikes me as highly implausible. Nonetheless, the idea that precedents are

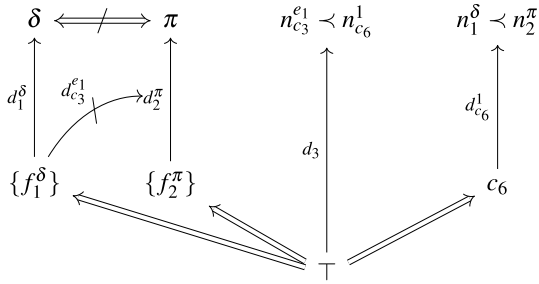


Fig. 4 A Conflict Between Strict and Persuasive Precedent

associated with varying strengths is more difficult to capture on an ordinal approach. One possible response would be to associate case defaults with different threshold values in the variable priority order, encoding the default assumption that certain sets of reasons for departure are below these thresholds (cf. Horty (2007)). Another possibility would be to enrich the structure of the underlying set of value-based defaults to encode the information that only defaults that are justified by some limited set of values can justify departure from precedent. A related possibility, which I hope to explore in future work, is that the priority courts assign to a persuasive precedent can be derived from a further body of case law, in a manner analogous to the procedure we have used to derive of a priority order over sets of factors.

7.2 Conflicting strict and persuasive authorities

Note that on this model, strict precedential authorities will always prevail over inconsistent persuasive precedent, even if there are good arguments for preferring the persuasive precedent over the strict precedent. Suppose for example, that our reasoner is again faced with the fact scenario X_1, but this time where the decision in *Young v Hitchens*, is contained in the strict casebase $\Gamma_3 = \{c_3\}$, and a persuasive casebase $\gamma_2 = \{c_6\}$,which contains the (fictional) persuasive precedent $c_6 = \langle\{f^\pi_2, f^\delta_1\}, \{f^\pi_2\}, \pi\rangle$ directly conflicts with c_3. A conflict between strict and persuasive precedent can then be represented by the default theory $\Delta_8 = \langle \mathcal{W}_{X_1,\Gamma_3,\gamma_2}, \mathcal{D}_{F,V,\Gamma_3,\gamma_2}\rangle$. The relevant factor defaults are again

$$d^\delta_1 : \{f^\delta_1\} \rightarrow \delta,$$
$$d^\pi_2 : \{f^\pi_2\} \rightarrow \pi.$$

The relevant case defaults are:

$$d^1_{c_6} : c_6 \rightarrow n^\delta_1 \prec n^\pi_2,$$
$$d^{e_1}_{c_3} : \{f^\delta_1\} \rightarrow Out(n^\pi_2).$$

And where the further default

$$d_3 : \top \rightarrow n^{e_1}_{c_3} \prec n^1_{c_6}$$

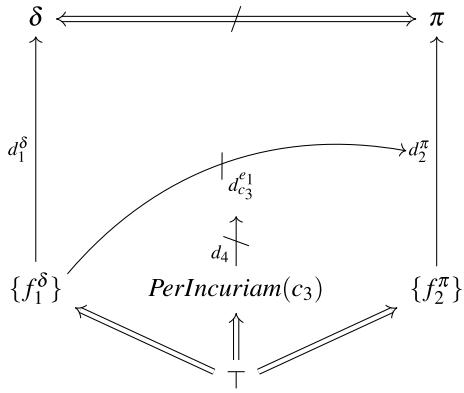


Fig. 5 Overruling Strict Precedent

represents the decision-maker's reason to prioritize the persuasive precedent over the strict precedent. (For simplicity, I ignore the process of reasoning by which the decision maker reaches this conclusion, and stipulate that the default has ⊤ as its premise.)

Figure 4 represents the theory Δ_8 as an inference graph. The theory generates the unique proper scenario $\mathcal{S} = \{d_1^\delta, d_{c_3}^{e_1}, d_{c_6}^1, d_3\}$ and the corresponding extension $\mathcal{E}$ containing the propositions $\delta, Out(n_2^\pi), n_1^\delta \prec n_2^\pi$, and $n_{c_3}^{e_1} \prec n_{c_6}$. $\mathcal{E}$ includes the priority information derived from the conclusions of the defaults d_3 and $d_{c_6}^1$, but since the default $d_{c_3}^{e_1}$ excludes the default d_1^δ, there is be no applicable default supporting an outcome for the defendant. The unique extension supports a finding for the plaintiff, based on the binding authority in c_3 (*Young v Hitchens*). It is not so much that the case c_6 fails to persuade, but that its persuasive strength is irrelevant to the outcome of the case.

7.3 Overruling strict precedent

Even strict systems of precedent allow for courts to depart from precedent in certain very restricted scenarios. Again, the position of the Court of Appeal of England and Wales is instructive. The Court of Appeal is permitted to depart from its own precedents only in certain restricted cases, namely: where there are two conflicting precedents, each of which strictly applies, where there is a conflicting higher authority, and where the decision was made *per incurium*, with lack of regard for an important legal fact that would have changes the outcome. Because undercutting defeat is preference independent, the protected reason model makes the priorities of the underlying factors, and even the priorities assigned to those prioritized defaults, irrelevant to the outcome. It would be possible to represent reasons to overrule strict precedent in terms of rebutting defaults to the effect the reasons excluded by precedent are not excluded. That is, if the exclusionary defaults representing the precedent take the form

$$R^S \rightarrow Out(n^{\bar{s}}),$$

we could represent reasons to overrule as reasons supporting the conclusion that each named factor-default $n^{\bar{s}}$ is not excluded. The reason to overrule a case because it was decided *per incurium*, for example, could be represented in terms of a set of defaults like

$$PerIncurium(c) \rightarrow \neg Out(n^{\bar{s}}).$$

When supported by relevant reason for prioritizing the reason to overrule, reasons of this sort would generate the correct result in the restricted scenarios in which a court is permitted to overrule. I feel, however, that this would be the wrong design choice with respect to representing the narrow cases in which courts are permitted to depart from strict precedent. These cases are, I think, better interpreted as cases in which the precedents are not reasons for exclusion at all. In other words, the reasons for exclusion are themselves excluded. They are cases where putative precedents fail to possess any authority over the present court. On this interpretation, we should interpret reasons for overruling strict precedent in terms of defaults of the form

$$PerIncurium(c) \rightarrow Out(n^e_c).$$

where n^e_c names an exclusionary case-default associated with the case to be overruled. Consider, for example, the default theory $\Delta_9 = \langle \mathcal{W}_{X_1,\Gamma_3,\gamma}, \mathcal{D}_{F,V,\Gamma_3,\gamma} \rangle$, which represents a fictional context, based on the factual scenario X_1, an empty persuasive casebase γ, and the strict persuasive casebase $\Gamma_3 = \{c_3\}$ in which it is decided that the decision in *Young v Hitchens* (c_3) is no longer a binding precedent because it was decided *per incurium*. In addition to the usual factor and case information, then, $\mathcal{W}_{X,\Gamma_3,\gamma}$ also contains the proposition $PerIncuriam(c_3)$, telling us that the case c_3 was decided *per incurium*. The set of defaults $\mathcal{D}_{F,V,\Gamma_3,\gamma}$ includes the two factor defaults, d^δ_1 and d^π_2, as well as the following two defaults:

$$d^{e_1}_{c_3} : \{f^\delta_1\} \rightarrow Out(n^\pi_2),$$
$$d_4 : PerIncuriam(c_3) \rightarrow Out(n^{e_1}_{c_3}).$$

$d^{e_1}_{c_3}$ is an exclusionary case default. The default d_4 excludes $d^1_{c_3}$ because c_3 was decided *per incurium*.

It is easy to see that the theory Δ_9 (represented in Fig. 5) will support two proper scenarios $\mathcal{S}_1 = \{d^\delta_1, d_4\}$ and $\mathcal{S}_2 = \{d^\pi_2, d_4\}$, both of which support an extension containing the proposition $Out(n^{e_1}_{c_3})$, that the strict precedent is excluded, but which support the conflicting conclusions π and δ. The court is no longer bound by the strict precedent, and is permitted to rule for either plaintiff or defendant.

7.4 The role of transitivity in strict and persuasive precedent

The inclusion of each instance of the transitivity schema for named defaults in $\mathcal{D}$ reveals important points of formal distinction between the prioritized and protected

reason models. In Sect. 6, I argued that the assumption of transitivity was problematic in the context of the protected reason model, which represents strict precedential constraint in terms of the court's reasons to exclude certain reasons for the opposing outcome, and to disregard priorities entirely.

Nonetheless, the assumption of transitivity seems particularly natural in the context of teleological case-based reasoning, where the priority assigned to a set of factors corresponds to an underlying comparison of the values that it promotes (Berman and Hafner 1993; Prakken 2002). Though there are some theorists who are willing to argue against the transitivity of comparative value (most prominently Larry Temkin (2011)), I assume that this is still a minority position. Given the role that value-based reasoning seems to play in legal discourse, it would be surprising if appeals to transitivity were entirely foreign to legal argument. I think, in fact, that a role for transitivity in factor-based reasoning can be preserved once the distinction between strict and persuasive precedent has been incorporated in the model. Priorities derived through application of the transitivity schema can still have persuasive value in the context of case-based reasoning.

By using the prioritized reason model to represent persuasive–rather than strict–precedent, it is possible to capture the sense in which the transitivity schema can be applied to priorities derived from past cases (or the underlying values reflected in past cases) in legal reasoning. The theories Δ_4 and Δ_5, discussed in Sect.6.1, can be reinterpreted as involving persuasive, rather than strict, precedential authority. If priorities extracted from past cases are regarded as having persuasive value, then inclusion of the transitivity schema seems to me to be less objectionable. Outside of periods in which legal doctrine is relatively settled, courts may take themselves to have reasons to decide in a way that promotes the coherence among the values and priorities revealed by a body of case-law.

8 Conclusion

Viewed from an abstract perspective, the protected reasons and prioritized reason models of precedent are equivalent. Both embody precedential reasoning in terms of an argumentative structure that allows a set of factors that satisfied the ratio of a past case to defeat any set of factors supporting the opposing outcome in the same case. But once we adopt a more structured approach to legal reasoning, some important points of difference between the two accounts are revealed. The choice between the two models of precedential constraint therefore raises important questions about the nature of precedential constraint, its relationship with value-based reasoning, and the extent to which it promotes the coherence of judicial decisions with the values and priorities of an underlying body of case-law. I have suggested that the two models can be incorporated in an account that offers a unified treatment of strict and persuasive precedent. I believe that this proposal is attractive, and merits further consideration. Even if my proposal is unsuccessful, however, I hope that I have at least helped to draw attention to the some important questions that will surely arise for anyone seeking to embed the factor-based models of precedent within a formal account of legal reasoning.

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