Situated legal systems and their operational semantics

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Abstract This work adopts H. Kelsen's concept of *legal system*, proposes a formal definition for such notion, and introduces an *operational semantical framework* for legal systems that are (structurally and operationally) situated in *agent societies*. Agent societies are defined. Relevant formal properties of *situated legal systems* (action-based dynamics; orthogonality between the operational semantics and the processes of legal reasoning and decision-making; validity of norms; and completeness) are discussed; the way they are exposed in the operational semantical framework is explained, and their truth formally proved. Also, for the sake of a better understanding of the legal-theoretic assumptions of the paper, recurring issues regarding Kelsen's theory of law (namely, his "positivism", the attribution of a plain deductive nature to legal reasoning and decision-making, and the notions of *basic norm*, *authorization*, and *discretion*) are briefly reviewed. They are put in confrontation with the points of view of R. Dworkin, H. Hart, and J. Raz, and an attempt is made to clarify them from the perspective of the provided formalization. A brief case study in *agent-based modeling and simulation* of *public policy processes* is presented, as an illustration of the way of using *situated legal systems*, and the proposed *operational semantical framework*, in a practical application.

Keywords Situated legal systems · Kelsen's theory of legal systems · Agent societies · Operational semantics of situated legal systems · Formal analysis of the Kelsen-Dworkin-Hart-Raz debate

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1 Introduction

1.1 Aim and methodological choices

This work concerns the possibility of using *operational semantical frameworks* for modeling the *structure* and *dynamics* of legal systems.

   Operational semantics is a traditional approach to the formal modeling of various types of information-processing systems (Wegner 1968; Plotkin 1981), and in this work we adopt the general principles of the *transition systems* approach to operational semantics (Plotkin 1981), to model the structure and dynamics of *situated legal systems*.

   We take H. Kelsen's *Pure Theory of Law* (Kelsen 2009), and its notion of *legal systems*, as the theoretical basis of the work.[1] This allows us to concentrate on the *structural* and *operational* aspects of legal systems, while leaving aside "political" or "sociological" issues concerning the *contents* of the legal systems (e,g., the issue of the imports of *morality* or *natural rights*). Also, in a crucial way, we leave aside issues related to the *legal reasoning* and *legal decision-making* processes, which usually tend to lead the analysis away from the structural and operational aspects of legal systems, toward the *logical aspects* related to their contents.

   To support such *separation of concerns*, and to justify the approach taken in the work, we argue in the paper that the formal modeling of the latter issues, which are tightly related to the *scope of discretion* of legal organs,[2] should proceed within semantical frameworks that are *functionally orthogonal* to the structural and operational one introduced here (*orthogonality* which, we claim, is precisely the idea underlying Kelsen's conception of a *pure* theory of law Kelsen 2009).

   Regarding motivations, the following two issues motivated the present work:

1. The research area of *Multiagent Systems* has advanced constantly, since the 1980s, to clarify the possible ways to use the notion of *organization* in the specification, design and operation of agent systems, but as we argued elsewhere (Costa 2014a, b, c), it seems that that work has restricted itself just to the *intra-organizational level* of concerns, having left almost untouched the *inter-organizational level*. This seems to have prevented a deeper exploration of the notion of agent *society*, in a more encompassing, sociological sense, as envisaged in the present work. We think that one of the reasons for the *lack of attention* to such inter-organizational (and other higher-level) issues may be the lack of a suitable formal notion of *legal system* for agent societies. Kelsen's notion of *legal system* seems to fulfill such purpose, when construed as a *situated legal system*, as proposed in the present paper.
2. However, Kelsen's theory of *legal system* has constantly been contested by rival theories, most notably those by R. Dworkin and H. Hart, but also the one by J. Raz. We also intend the paper, then, to show how the formalization introduced here may help both to clearly present the main concepts of Kelsen's theory and to explain

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[1] In Costa (2014d), we introduced a first version of the formalization of Kelsen's concept of *legal system*, which we use, slightly revised, in the present paper.

[2] See Davis (1976), or Sect. 10, for the concept of *discretion*.

away some of the main issues raised in those contestations. For such purpose, the paper goes into some close analysis of certain central aspects of those rival theories.

1.2 Structure of the paper

The paper is structured as follows. Section 2 summarizes the concept of *agent society* that we use to situate legal systems.

Section 3 summarizes Kelsen's informal concept of *legal system*.

Section 4 introduces informally the concept of *situated legal system*.

Section 5 presents our formalization of Kelsen's concept of *legal system*.

Section 6 formally introduces the general features of the *operational semantical framework* that we later adopt to characterize the operational aspects of situated legal systems.

On the basis of the formalization of Kelsen's concept of legal system (presented in Sect. 5) and of the general features of the operational semantical framework (presented in Sect. 6), Sect. 7 presents our formal concept of *situated legal system*.

Section 8 introduces, then, the *operational semantical framework* for situated legal systems.

Section 9 shows how the operational semantical framework introduced in Sect. 8 exposes (and supports the proof of) some of the main *formal properties* that situated legal systems inherit from Kelsen's concept of legal system, namely: the *action-based* nature of the dynamics of legal systems; the *orthogonality* between structural and operational features, and the processes of legal reasoning and decision-making; the *validity* of norms; and the *completeness* of legal systems.

Section 10 briefly discusses certain misunderstandings that usually arise, concerning Kelsen's notion of legal system, often motivated by confrontations with rival conceptions of legal systems, such as those by R. Dworkin and H. Hart (themselves self-declared to be "opposed" to each other).

Section 11 discusses those two rival conceptions in more detail, and also J. Raz's meta-theoretical conception of what a *theory of law* should be.

Section 12 considers very briefly three complementary topics, relevant for the subject of the *situated legal systems*, namely, the issue of the formalization of *history-savvy* situated legal systems, the research on *normativity* in the Muiltiagent Systems area, adn the possibility of formalizing Kelsen's concept of legal system in terms of the concepts developed in the *theories of processes*.

Section 13 illustrates the use of the proposed *operational semantical framework* in the context of agent-based social simulation studies.

Section 14 brings the Conclusion, and some prospects for the proposed notion of *situated legal systems* and for the proposed approach to their *operational semantics*.

1.3 A *Caveat*

This paper takes Kelsen's concept of legal system "as is" [meaning, essentially as first presented in Kelsen (2009)], and attempts to give it a formalization that, to the best of our ability, is faithful to its original, informal presentation.

Thus, it is not the purpose of the present paper neither to "defend" Kelsen's theory of law and the concept of legal system that it embeds, from the list of criticisms that have been raised against it, nor to "improve" it according to any more recently proposed conceptions (formal or informal).

That is, if any of those criticisms really apply to any of Kelsen's theory, clearly it sould continue to apply to that theory when it is presented formally, for formalization is no salvation.

What we claim in this paper, and hope to have shown, even if preliminarily, in a sound way, is precisely that, in spite of whatever weaknesses it may have, Kelsen's theory is strong enough to allow for a *formal operational theory* of legal systems, based on situating them in agent societies.

Thus, when "hunting faults" in the present paper, care should be taken to distinguish between faults in Kelsen's theory (which are, of course, of Kelsen's own responsibility, not ours) and faults in the proposed formalization (which are, of course, of our own exclusive responsibility, not Kelsen's).

2 Agent societies

2.1 Tfhe concept of agent society

In the *Multiagent Systems* research area, the term *organization* seems to have been introduced by taking as a conceptual basis the seminal work by Fox (1979), who proposed that complex software systems be thought of in "organizational terms", that is, in terms of the concepts of *Organization Theory*.

However, comparative theoretical social studies, like that by Burrell and Morgan (1979), have shown that there is a strong correlation between the conceptual paradigms that have traditionally been used in *Sociology* (i.e., the study of modern human societies) and in *Organization Theory* (i.e., the study of corporate organizations, typical of modern human societies).

Thus, it is no surprise that the study of the organization of multiagent systems has been loosely and indistinguishably phrased both in terms of *agent organizations* and in terms of *agent societies*, as any quick browse of the literature of the area can show.

In the former case, one can say that a *multiagent system* is construed as a *corporation* of agents, aiming to achieve a definite set of corporate goals, often stated from the point of view of the users of such agent corporation.

In the latter case, on the other hand, one can say that a *multiagent system* is construed as a *society* of agents, aiming to empower both the *individual agents* and the *agent corporations* that inhabit it, toward the achievement of their individual goals.

In our work (Costa and Dimuro 2009; Costa et al. 1994; Demazeau and Costa 1996) , we have engaged on the latter trend, and have tried to consistently use the term *agent society* with the following definite meaning (which, in connection to the architectural style presented in the next subsection, tries to capture the sense of the term "society" as it is commonly used in sociology Turner 2010):

An **agent society** is a multiagent agent system that is:

- **open**: the agents can enter and leave the system freely;
- **organized**: the working of the society is performed by articulated (possibly, hierarchically structured) *organizational units* (social groups, corporations, etc.), so that each agent may (potentially) distinguish between the *individual processes* and the *social processes* occurring in the society (i.e., between the processes performed by *single agents* and processes performed by *sets of agents*, e.g., organizational units);
- **persistent**: the *organizational forms* of the organizational units that perform social processes, and the *organizational form* of the society as a whole, are able to *persist* in time, independently of which agents enter or leave the society (possibly, subject to a requirement of minimal population size);
- **situated**: the agents operate in a definite (real or simulated) environment, whose objects may be involved in the individual and social processes of the society.

2.2 An architectural style for agent societies

Figure 1 illustrates an *architectural style* for agent societies Costa (2014c),[3] appropriate for *formally situating* legal systems, as explained below.

There are four main *architectural components* in this style of architecture for agent societies: the *populational structure*, the *organizational structure*, the *material environment*, and the *ideological structure*.

The main features of these components are:

1. The *populational structure*, denoted by *Pop*, encompasses the *individual agents* that inhabit the society, together with their *behaviors*, *interactions*, and *personal cognitive structures*.
2. The *organizational structure*, denoted by *Org*, includes three *organizational levels*:

   (a) the *micro-organizational level*, denoted by Org_ω, which encompasses the *social roles* that the individual agents may enact, when operating in the organizational units of the meso-organizational level;

   (b) the *meso-organizational level*, denoted by Org_μ, which encompasses the *organizational units* that structure social roles in functional ways, some of which constitute the social groups, corporations, etc., of the agent society;

   (c) the *macro-organizational level*, denoted by Org_Ω, which encompasses *social systems* that structure organizational units in *functional* ways.

   So that $Org = (Org_\omega, Org_\mu, Org_\Omega)$.

3. A set of *implementation relations* between the components of the different levels, denoted by *Imp*, which determine:

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[3] Compare with the *XYZ* model in e.g., (Costa and Dimuro 2009), from which it developed.
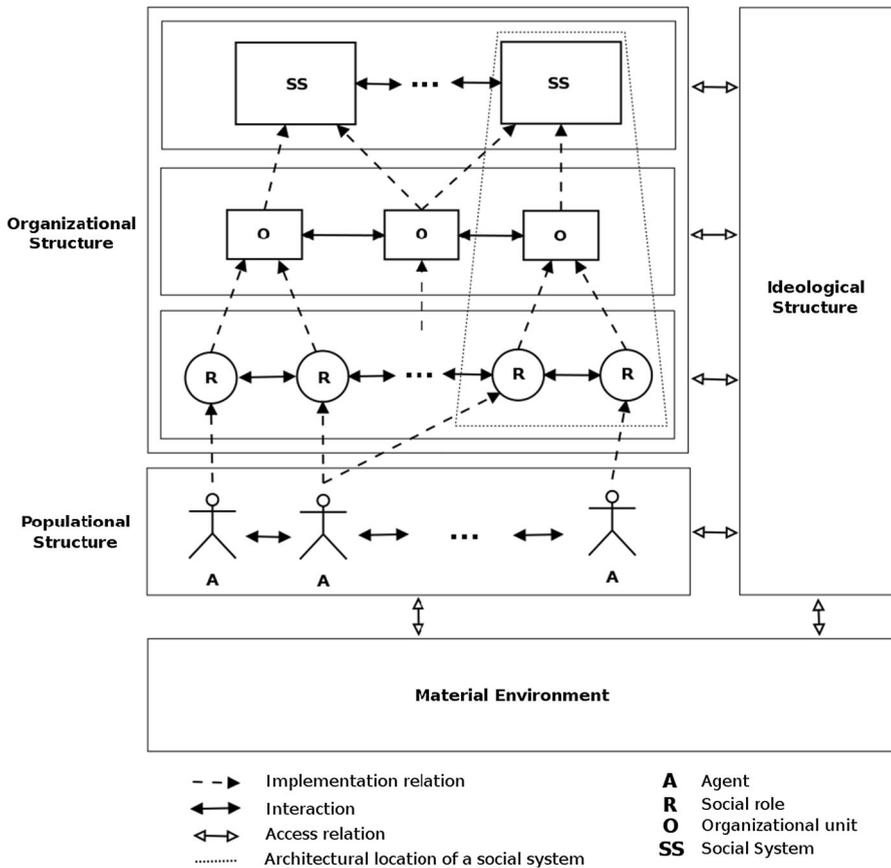
Fig. 1 An architectural style for agent societies

    (a)   how components (and their behaviors and interactions) at each organizational level, participate in the implementation of the components (and their behaviors and interactions), at the next higher organizational level;

    (b)   how individual agents (and their behaviors and interactions) participate in the implementation of the social roles (and their behaviors and interactions), at the micro-organizational level.

4.   The *material environment*, denoted by *Env*, which encompasses the set of *material objects* (and their causal interconnections) available for use in the *material actions* of the agents of the society.[4]

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[4] See Lehmann et al. (2004) for an analysis of the import of the *material environment* of agent societies for their legal systems, a subject we don't treat explicitly here.

5. The *ideological structure*, denoted by *Ideo*, which encompasses *beliefs*, *values*, *norms*, and *customs* that are collectively established, and that the individual agents and organizational units may adopt (either *individually* or *collectively*), when inhabiting the society.

In the present work, we take this architectural style for agent societies, but in its *time-variant* version (Costa and Dimuro 2009). Thus, an *agent society* is taken here to be a time-indexed structure

$$AgSoc = (Pop^t, Org^t, Imp^t, Env^t, Ideo^t)_{t \in T}$$

such that the contents of the components of the agent society vary with time (e.g., Pop^t is the *populational structure* at time $t \in T$, where T is the *time-structure*, see Sect. 5).

2.3 Core operational structure of an agent society

In Fig. 1, one may note what we call the *core operational structure* of an agent society, namely, the *set of processes* (behaviors and interactions) occurring both *within* each organizational level (micro, meso, macro), and *between* these different organizational levels.

Such processes constitute the *core operational structure* of an agent society, in the sense that they are the processes that *organizationally support* the accesses of the individual agents and organizational units to the *public ideological structure*, and to the *material environment*.

2.4 Connection with situated legal systems

Notice that, regarding the structure of the *situated legal systems* introduced in Sect. 7, the set of *legal norms* (i.e., the legal order $LOrd^t$) belongs to the *ideological structure* of the agent society where the legal system is situated.

The set of *social roles* and *organizational units* involved in the operation of the legal system, the set of so-called *legal organs* ($LOrg^t$), as well as the *legal system* itself, taken as a *functional system* of the agent society, belong to the *core operational structure* of that society.

The *record of legal facts* ($LFact^t$), as an auxiliary component that is internal to the legal system, and that (in principle) should be functionally accessible only to the *legal organs*, may also be taken to belong (in the form of, e.g., an *organizational artifact* Hübner et al. 2010) to the *core operational structure* of the agent society.

3 A summary of Kelsen's concept of legal system

We take as the main reference for the concept of a *legal system* situated in an agent society the general concept of legal system proposed by Kelsen (2009) (see also Costa 2014d), who states that a *legal system* is a structure that, at each instant of time, encompasses:

– a set of *valid legal norms* (called a *legal order*), i.e., a set of statements *validly* determining *obligations*, *prohibitions*, *permissions*, and *authorizations* of conducts for its *legal subjects*[5];
– a set of *legal subjects*, i.e., a set of agents that are subject to the legal norms of the legal system;
– a set of possible *legal acts*, i.e., a set of acts legally performed, and through which the set of valid legal norms may evolve in time (through *norm creations* and *derogations*), and the *sanctions* enforcing the valid legal norms may be performed;
– a set of *legal organs*, i.e., the subset of legal subjects that are legally authorized to perform legal acts.

The *basic form of a legal norm*, which Kelsen calls an *imputation* (Kelsen 2009), is one where the norm indicates both:

– a *material sphere of validity*, i.e., a *conduct* (or, respectively, an *omission of a conduct*) that ought *not be performed* by the legal subjects of the legal system;
– a *sanction*, which ought to be *executed* by a legal organ, in case a legal subject *does perform* the conduct (or, respectively, *does perform* the omission of the conduct).

The understanding of the notion of *imputation* is crucial for the proper understanding of Kelsen's conception of law.[6] What he aims with this notion is the determination of a distinction between two ways of presenting legal norms[7]:

– the *usual* way of presenting legal norms, namely, through texts written in standard natural language;
– the *formal* way of presenting legal norms, namely, through legal norms construed as *imputations*.

One can try to understand such distinction in analogy with the distinction introduced by N. Chomsky in the realm of linguistics (Chomsky 2002), between the *surface structure* of a linguistic expression (the one that people ordinarily employ in their conversations) and the *deep structure* of the linguistic expression (the one that the linguist makes use of to analyse that linguistic expression).[8]

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[5] Kelsen (2009) uses the term *legal norm* both in this wider sense (including the idea of *authorizations*) and in a narrower sense, distinguishing the former senses (*obligation*, *prohibition*, and *permission*) from that of *authorization*. In the formalization introduced in the next sections, we use the term *legal norm* in its narrower sense, captured in the formal concept of *imputation*. That is, the formal concept of *imputation* does not capture the idea of *authorization*, which receives a separate formal treatment.

[6] Section 5 presents our formalization of Kelsen's concept of legal system, detailing the main aspects of the notion of *imputation* and other related notions.

[7] Kelsen's expressions for these two types of presentations of legal norms are, respectively, "figure of speech" and "unambiguous description" (Kelsen 2009). Raz (1970), on the other hand, frames this distinction in terms of what he calls the *problem of the individuation* of legal norms (see also Sect. 12).

[8] Or else, one can understand it in the terms of a similar distinction made in the realm of *programming languages*, between the *concrete syntax* and the *abstract syntax* of a programming language expression (Fernández 2014).

Kelsen's notion of *imputation* is the operational means he has found to allow for the presentation of the *deep structure* of legal norms.

As a simple example: any two laws presented in the *concrete* form

L_1: It is forbidden to perform a conduct of type C_1.

L_2: Any conduct not complying with law L_1 is liable to be sanctioned by a conduct of type C_2.

may be expressed, in an *abstract* form, by a single imputation that, in the formal notation introduced in Sect. 5, is given by[9]: $C_1 \Downarrow C_2$.

4 The informal concept of situated legal system

In this section, we deal with the concept of *sjituated legal system* in a preliminary, informal way, reserving the development of the formal presentation for the next sections.

We call *situated legal system* any legal system *operationally coupled* to the *core operational structure* of an agent society, and through the latter to the individual and social processes performed by the individual agents and organizational units that inhabit that society.

The *essential requirement* of the operational coupling of a *legal system* to the *core operational structure* of a target agent society is that there are enough *individual agents* and *organizational units* in the agent society capable of performing the behaviors and interactions required by the *legal organs* of the legal system, so that those individual agents and organizational units can *implement* the legal functions performed by those legal organs.

In particular, such *functional implementation*[10] should allow for the performance of the *legal monitoring processes* (through which the *legal organs* of the legal system monitor the conducts of the *legal subjects*) and of the *legal enforcement processes* (through which those legal organs *act on* the legal subjects and their conducts).

Figure 2 illustrates the main components and interactions of a *situated legal system*.

More specifically, the figure shows:

– the legal system *LS*;
– the agent society *AgSoc* and its *social agents* (individuals and organizational units), which constitute the *set of legal subjects* of *LS*;
– the *social interactions* between the social agents;
– the legal order *LOrd* and set of legal organs *LOrg*;

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[9] Compare the simple form of Kelsen's *abstract syntax* of legal norms with the complex form resulting from Bentham's (1970a) logical analysis of the *concrete syntax* of legal norms, as formally presented by Raz (1970). See also Alchourrón and Bulygin (1971), where Alchourrón and Bulygin also aim to formalize the complex logical structure of the *concrete* language of law.

[10] See (Costa and Dimuro 2009) for the detailed formal operational requirements for the *proper realization* of a functional implementation.

Fig. 2 Basic structure of a
situated legal system
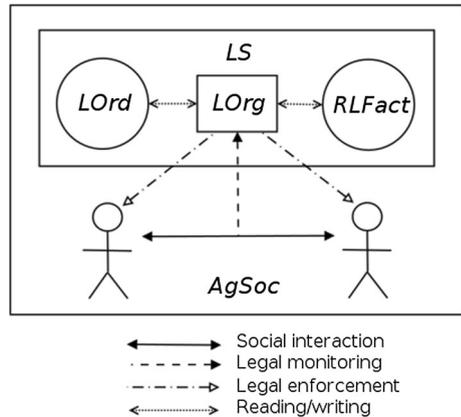
– the record of legal facts *RLFact*[11];
– the *reading* and *writing* processes that the legal organs may perform to manage
  the set of recorded *legal facts*;
– the *legal monitoring processes* and the *legal enforcement processes*.

Other important, but secondary, components of a situated legal system are not
shown in Fig. 2:

– a way for the *legal subjects* to have reading access to the legal order *LOrd* and to
  the record of legal facts *RLFact*;
– given the fact that legal organs may have to abide by legal norms concerned
  with their official conducts, a way to allow some legal organs to monitor the
  conducts of other legal organs, to check such abiding.

Such secondary components are not shown in the figure because they are relevant
only for the reasoning and decision-making processes of the (social or legal)
individual agents or social groups, or for complementary features of the operation of
situated legal systems, not for their basic mode operation, which is the one we are
concerned here.

The record of legal facts *RLFact* is essential for the *transition systems* approach
adopted in the present work, because it is in the form of *legal facts* that the legal
organs keep a record of the evolution of the operational state of the situated legal
system, and of its legal processes.[12]

In the next section, we present our formalization of Kelsen's concept of *legal
system* (Kelsen 2009), on which we base our definition of the *structural* aspects of *situated
legal systems*. The *operational* aspects of *situated legal systems* are defined in Sect. 8.

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[11] We use the expression *legal fact* to denote either a record of the performance of a *legal* or *social act*, or
a record of a *factual ascertainment* that a legal organ may make, when trying a concrete legal case, but
with the formal requirement that such records be performed only when *legally authorized*, see Sect. 8.

[12] See Sect. 12 for a brief comment on alternative possibilities of formalization of situated legal systems,
based on *process-theoretic* approaches.

5 A formal presentation of Kelsen's concept of legal system

The formalization of Kelsen's concept of *legal system*, presented below,[13] allows us to construe *legal systems* as *action systems* (with *legal acts* and *social acts* taken as *actions*), and *core operational structures* of agent societies as *operational semantical domains* for legal systems (see Sect. 8).

For simplicity, we take into account, in the present section, only the *personal* and *material* spheres of validity of *legal norms*. The *temporal* and *spatial* spheres are taken into account only in the illustrative case presented in Sect. 13.

The formal definition of *legal systems* proceeds through the following steps:

– *basic concepts*: the universes, variables and operations used in the definition;
– *basic constructs*: imputations and authorizations;
– *legal norms*
– *legal orders, legal organs, legal acts, and legal facts*
– *concept of legal system*
– *temporal evolution of legal systems*

5.1 Basic concepts

1. **Universes of basic elements:**
    – Agents (both individuals and organizational units): **Ag**.
    – Conducts: **Cd**.
    – Legal acts: **LAct**.
    – Legal norms: **LNrm**.
    – Legal orders: **LOrd**.
    – Systems of legal organs: **LOrg**.
    – Legal systems: **LSys**.
    – Legal facts: **LFact**.
    – Records of legal facts: **RLFact**.
    – Authorizations: **Auth**.
    – Basic imputations: **BasImpt**.
    – Addressed imputations: **AdImpt**.
    – Individualized imputations: **IndImpt**.
    – General imputations: **Impt** = **BasImpt** ∪ **AdImpt** ∪ **IndImpt**.
    – States of agent societies: **StAgSoc**.
    – Time: $T = \{0, 1, 2, \ldots\}$ (linearly ordered).

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[13] The presentation is based on the formalization of Kelsen's notion of legal system that we have introduced in Costa (2014d). We revise such formalization here, specially regarding *authorizations* (see Sect.10) and the concept of *temporal evolution* of situated legal systems.

2. **Universes of types**[14]:

    – Agent types: $\wp(\mathbf{Ag})$.
    – Conduct types: $\wp(\mathbf{Cd})$.

3. **Variables running on universes and types:**

    – Agents: $ag \in \mathbf{Ag}$.
    – Agent types: $Ag \in \wp(\mathbf{Ag})$.
    – Legal acts: $lact \in \mathbf{LAct}$.
    – Conducts: $c \in \mathbf{Cd}$.
    – Conduct types: $Ct \in \wp(\mathbf{Cd})$.
    – Legal orders: $LOrd \in \mathbf{LOrd}$.
    – Systems of legal organs: $LOrg \in \mathbf{LOrg}$.
    – Legal systems: $LS \in \mathbf{LSys}$.
    – Legal facts: $LFact \in \mathbf{LFact}$.
    – Sets of legal facts: $LFact \in \wp(\mathbf{LFact})$
    – Records of legal facts: $RLFact \in \mathbf{RLFact}$.
    – States of agent societies: $StAgSoc \in \mathbf{StAgSoc}$.
    – Time instants: $t \in T$.

4. **Operations on universes:**

    – Omission of conduct:
    An operation $^-: \mathbf{Cd} \to \mathbf{Cd}$ such that conduct $\bar{c}$ is the *omission* of the conduct $c \in \mathbf{Cd}$.[15] In particular, it is assumed that $\bar{\bar{c}} = c$. Also, we state that $\overline{Ct} = \mathbf{Cd} - Ct$ is the type of omitted conducts of type Ct.

5.2 Basic constructs

In Kelsen's theory of norms (Kelsen 2009), *legal norms* are formally construed as *imputations*.[16] *Authorizations*, on the other hand, are construed as *legal acts* (implicitly or explicitly) performed through the issuing of *legal norms*.[17]

5.2.1 Authorizations

An (explicit) *authorization* is a construct of the form $(Ag, Ct) \in \wp(\mathbf{Ag}) \times \wp(\mathbf{Cd})$ which states that agents of type Ag are authorized to perform conducts of type Ct.

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[14] $\wp(X)$ is the powerset of the set X.

[15] Following the notation in Raz (1970).

[16] Kelsen (1991, 2009) was one of the first to make clear the distinction between *norms* (expressions of norms themselves) and *norm propositions* (statements about the existence and properties of norms), a conception largely amplified by C. Alchourrón in the 1960's, and later, through the distinction between the *logic of norms* and the *logic of normative propositions* (Alchourrón 1969; Alchourrón and Bulygin 1971).

[17] For more on the relation between *legal norms* and *authorizations*, and specially about the difference between *explicit* and *implicit* authorizations, see the discussion in Sect. 10.

The universe of all authorizations is denoted by **Auth**. We denote $(Ag, Ct) \in$ **Auth** by $Auth(Ag, Ct)$.

Remark the difference of treatment between the *operational* and the *legal* conditions for the *efficacy* of authorizations. The construct above is the *operationally unconditional* form of the *authorization* construct, which captures acts of authorization that are *operationally efficacious* in an unconditional way. *Operationally conditional* authorizations may be defined by extending the construct thus: $Auth(Ag, Ct, Cond)$, where $[Cond$ indicates the *operational condition* under which the act of *authorizing Ag* to perform conducts of type Ct can be operationally efficacious. The *legal* conditions of the efficacy of *authorizations*, on the other hand, are determined by the (conditional or unconditional) character of the *legal norms* from which such authorizations derive.

In the *operational semantical framework* introduced below (Sect. 8), the performance of an act of *authorization* may have to be recorded as a *legal fact*, in the record of *legal facts* RLFact that is part of the *configuration* of the legal system, as it is modeled by the framework. We use, then, the same construct, $Auth(Ag, Ct)$, to denote such *recorded legal fact*.

5.2.2 Imputations

Imputations may be *basic*, *addressed*, or *individualized*.

– **BasImpt** is the set of basic imputations.
   A *basic imputation* is a pair of types of conducts $impt \in \wp(\mathbf{Cd}) \times \wp(\mathbf{Cd})$, so that if $impt = (Ct_1, Ct_2)$ one says that *impt* establishes the type of conducts Ct_1 as the type of its *condition*, and the type of conducts Ct_2 as the type of its *consequence*.
   Whenever $impt = (Ct_1, Ct_2)$ we denote *impt* by $Ct_1 \Downarrow Ct_2$.[18]
   The legal reading of $Ct_1 \Downarrow Ct_2$ is: The performance of a conduct of type Ct_1 by a legal subject has, as a consequence, that a conduct of type Ct_2 *ought* to be performed, by a legal organ, as a sanction for that performance.[19]
– **AdImpt** is the set of addressed imputations.
   An *addressed imputation* is a tuple $adimpt \in \wp(\mathbf{Cd}) \times \wp(\mathbf{Cd}) \times \wp(\mathbf{Ag}) \times \wp(\mathbf{Ag})$, so that if $adimpt = (Ct_1, Ct_2, Ag_1, Ag_2)$ we say that the basic imputation $Ct_1 \Downarrow Ct_2$ is addressed to agents of type Ag_1 and Ag_2.
   That means: if $adimpt = (Ct_1, Ct_2, Ag_1, Ag_2)$ then a conduct of type Ct_1 is a condition for the addressed imputation *adimpt* only if it is performed by an agent of type Ag_1, and a conduct Ct_2 is a consequence of the addressed imputation *adimpt* only if it is performed by an agent of type Ag_2.

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[18] Note that $\Downarrow$ aims to capture Kelsen's idea that *imputations* are not logical *implications*, since they do not connect *propositions*, but two *types of conducts*, thus being unsuitable to an assignment of *truth-values*: imputations can not be *true* or *false*, only *valid* or *non valid*, in a given legal system (see Kelsen 1991, and Sect. 9).

[19] That Ag_2 should be a type of *legal organ* is a requirement imposed by the fact that legal systems restrict the performance of legal sanctions, in a monopolistic way, to their *legal organs* (i.e., to the *state* Kelsen 2009).

The legal reading is: If a legal subject of type Ag_1 performs a conduct of type Ct_1, then a legal organ of type Ag_2 *ought* to perform a conduct of type Ct_2, as a sanction for that performance.

If $adimpt = (Ct_1, Ct_2, Ag_1, Ag_2)$ we denote *adimpt* by $(Ag_1, Ct_1) \Downarrow (Ag_2, Ct_2)$.

– **IndImpt** is the set of individualized addressed imputations.

To *individualize* an addressed imputation is to instantiate at least one of its *addressees* to an individual agent (or, individual organizational unit) of the corresponding type.

An imputation, thus, may be individualized either in its condition, or in its consequence, or in both its condition and its consequence. An imputation individualized in both its addressees is called a *completely individualized imputation*.

For example, the addressed imputation $(Ag_1, Ct_1) \Downarrow (Ag_2, Ct_2)$ may be individualized either as $(ag_1, Ct_1) \Downarrow (Ag_2, Ct_2)$, or as $(Ag_1, Ct_1) \Downarrow (ag_2, Ct_2)$, or else as $(ag_1, Ct_1) \Downarrow (ag_2, Ct_2)$, for $ag_1 \in Ag_1$ and $ag_2 \in Ag_1$.

5.2.3 Addressing and individualization as operations

The following operations may be defined, for convenience:

The operation of *addressing a basic imputation* is the operation addr : **Impt** $\times$ $\wp(\mathbf{Ag}) \times \wp(\mathbf{Ag}) \to \mathbf{AdImpt}$, given by $\mathrm{addr}(Ct_1 \Downarrow Ct_2, Ag_1, Ag_2) = (Ag_1, Ct_1) \Downarrow (Ag_2, Ct_2)$.

The operation of *complete individualization* of an addressed imputation is the operation indiv : **AdImpt** $\times \mathbf{Ag} \times \mathbf{Ag} \to \mathbf{IndImpt}$, given by $\mathrm{indiv}((Ag_1, Ct_1) \Downarrow (Ag_2, Ct_2), ag_1, ag_2) = (ag_1, Ct_1) \Downarrow (ag_2, Ct_2)$, for $ag_1 \in Ag_1$ and $ag_2 \in Ag_2$.

Analogous operations can be defined for the *partial individualization* of addressed imputations.

An operation of *individualization of authorizations* may also be defined.

5.3 Legal norms

We make use of the *basic constructs* as means for the definition of legal norms. In particular, *legal norms* are based on *addressed imputations* (Kelsen 2009).

5.3.1 General form of legal norms

In the process of construing combinations of *basic constructs* as legal norms, the notion of *imputation* goes through a slight specialization, essential to Kelsen's concept of *legal norm* (Kelsen 2009): for an *addressed imputation* of the form $(Ag_1, Ct_1) \Downarrow (Ag_2, Ct_2)$ to be taken as a *legal norm*, its consequence should be construed as an *authorization* for an agent of type Ag_2 (to be individualized at the moment of the application of the legal norm) to perform a conduct of type Ct_2.

We thus write the *general form of legal norms* as imputations of the form

$$(Ag_1, Ct_1) \Downarrow Auth(Ag_2, Ct_2)$$

for some authorization $Auth(Ag_2, Ct_2) \in$ **Auth**.

The legal reading of such a formula is: A performance of a conduct of type Ct_1 by a legal subject of type Ag_1 has as a consequence that a legal organ of type Ag_2 *ought* to be *authorized* to perform a conduct of type Ct_2, as a sanction for that performance.

This general form of legal norms indicates a very important and typical characteristic of Kelsen's view of law: a *legal norm* should be seen not as the statement of a prohibition or an obligation, directly addressed to the legal subjects whose behaviors it intends to control, but as an *authorization* addressed to a legal organ, namely, an *authorization* to apply, as a sanction, a conduct of the type specified as the consequence of the imputation that formally represents the legal norm.

That is, from Kelsen's point of view, a legal system is a *social control system* that operates by *controling directly*, through the *authorizations* stipulated in the legal norms, the *system of legal organs* responsible for its operation, to the effect that it controls *only indirectly*, through the *possibility of sanctions* by those legal organs, the system of social agents legally subject to it.[20]

5.3.2 Main forms of legal norms

There are three main specific forms that legal norms may assume in legal systems: *prohibition*, *imposition*, and *strong permission*. A fourth specific form, *weak permission*, is defined by the *non-existence* of any legal norm of any kind involving the concerned conduct. *Obligations* are defined on the basis of *impositions* (Kelsen 2009).

Only prohibitions and impositions, however, can be defined independently of a definite notion of *validity* of legal norms. Strong and weak permissions can only be defined in the context of such a definite notion. So, this sub-section limits itself to *prohibitions* and *impositions*. Strong an weak permissions are defined only after a notion of *validity of legal norms* is presented, in the next sub-section.

– A *prohibition* is a legal norm of the form $(Ag_1, Ct_1) \Downarrow Auth(Ag_2, Ct_2)$ in which the consequence should be understood as an *authorization* for a legal organ of type Ag_2 (to be properly individualized at the moment of the application of the legal norm) to perform a conduct of type Ct_2, as a sanction for the performance of a conduct of type Ct_1 (taken then as a *prohibited conduct*) by a legal subject of type Ag_1.

   Clearly, *prohibitions* are the essential meaning of *imputations*.

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[20] Which represents a significant departure from J. Bentham and J. L. Austin's view of legal systems as systems of norms directly commanding their subjects (Bentham 1970b; Austin 1995), a point rightly indicated by Hart (2012).

– The *imposition* of a conduct is the *prohibition of the omission* of the conduct: an *imposition* is a legal norm of the form $(Ag_1, \overline{Ct_1}) \Downarrow Auth(Ag_2, Ct_2)$, meaning that a legal organ of type Ag_2 (to be properly individualized at the moment of the application of the norm) ought to be authorized to perform a conduct of type Ct_2 as a sanction for the performance of a conduct of type $\overline{Ct_1}$ (that is, for an *omission* of the performance of a conduct of type Ct_1) by a legal subject of type Ag_1.

In general, if conducts of type Ct are imposed on legal subjects of type Ag, one says that the performance of conducts of type Ct is a *legal obligation* of the legal subjects of type Ag.

We illustrate the use of the formal notions of *prohitions* and *impositions* by formalizing the distinction between *fines* and *taxes*.

A *fine* is a sanction, in the form of the charge of an amount of money, applied to a legal subject for his/her performance of a prohibited type of conduct. Thus, a legal norm determining a fine of value v for the performance of a conduct of type C can be formalized, in a schematic way, as the prohibition: $(Ag_1, C) \Downarrow Auth(Ag_2, fine(v))$.

On the other hand, a *tax* is an amount of money, charged from a legal subject, for some good he has (or, service he performs or receives), but with such possession or performance not taken to be legally prohibited, so that the charging of the tax should not be taken as a sanction.

As the charging of a tax usually happens to be efficacious, however, only if a sanction is determined for the avoidance of the payment of the tax, we need to express a legal norm determining the payment of a tax of value v, regarding a good or service g, subject to a sanction S in case of evation, as: $(Ag_1, \overline{tax(v, g)}) \Downarrow Auth(Ag_2, S)$.

The *formal difference* between the two imputations

$$(Ag_1, C) \Downarrow Auth(Ag_2, fine(v)) \quad \text{and} \quad (Ag_1, \overline{tax(v, g)}) \Downarrow Auth(Ag_1, S)$$

illustrates not only the *legal difference* between the *sanctioning through a fine* and the *charging of a tax*,[21] but also the general idea of how the formal notion of *imputation* may be used to clarify concepts involving legal norms.

5.3.3 Conditional legal norms

The above forms of legal norms are *unconditional* forms. Often, legal norms are defined to be applicable only under certain definite conditions (either regarding the underlying *agent society* or regarding the *legal system* itself).

We adopt the following schematic form, to denote *conditional legal norms*:

$$\frac{Cond_1 \quad \ldots \quad Cond_n}{(Ag_1, Ct_1) \Downarrow Auth(Ag_2, Ct_2)}$$

where $Cond_1, \ldots, Cond_n$ are the *conditions* to which the legal norm is subject.

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[21] A difference that, according to Hart (1962–1963), can not be clarified within the Kelsenian conceptual framework.

5.4 Legal orders, legal organs, legal acts, and the record of legal facts

5.4.1 Legal orders

A *legal order* is a time-indexed family of sets of legal norms, that is, $LOrd : T \rightarrow \wp(\mathbf{LNrm})$. For any legal order $LOrd \in \mathbf{LOrd}$, $LOrd^t \in \wp(\mathbf{LNrm})$ denotes the set of legal norms of the legal order $LOrd$, at the time t.

The set of legal norms $LOrd^t$ that belong to a legal order $LOrd$ at a given time t is the set of legal norms that are said to be *valid*, at that time.

5.4.2 Spheres of validity of legal norms

In this section, we have omitted the specification of the spheres of validity *space* and *time*, in the forms of legal norms that we have defined above.

They can be be seamlessly introduced in those forms, however, by introducing temporal and spatial parameters in the formulation of norms. For instance, we may define the *time t* and *space s* of validity of a prohibition by

$$(Ag_1, Ct_1) \Downarrow^{t,s} Auth(Ag_2, Ct_2)$$

5.4.3 Legal organs

Legal organs are agents that are authorized by valid legal acts of *authorization* to perform certain *legal acts*.

A *system of legal organs* is a time-indexed family of sets of legal organs $LOrg : T \rightarrow \wp(\mathbf{Ag})$. For any system of legal organs $LOrg \in \mathbf{LOrg}$, $LOrg^t$ denotes the set of legal organs of $LOrg$ at the time t.

The definition of the set of legal organs of a given legal system, at any time, depends on what *legal authorizations* are valid in that legal system at that time. Thus, it happens that $ag \in LOrg^t$ if and only if there is, at time t, a valid individualized authorization $Auth(ag, Ct)$, for some type of conduct Ct.

Formally: $ag \in LOrg^t$ if and only if $Auth(ag, Ct) \in RLFact^t$, that is, if and only if $Auth(ag, Ct)$ is a *legal fact* at time t (see item (d), below).

5.4.4 Legal acts and the record of legal facts

A *legal act* is a conduct for which there is a *valid authorization* authorizing an agent (then said to be a *legal organ*) to perform it.

Thus, the performance of a conduct of type Ct at the time t, by an agent ag is the performance of a *legal act* if and only an individualized authorization $Auth(ag, Ct)$ is a legal fact at time t, that is, $Auth(ag, Ct) \in RLFact^t$.

There are two *general types of conducts* that may be authorized as *legal acts* (see Sect. 7):

– *External conducts*:

  – That is, conducts impacting the agent society where the legal system is situated, including the performance of *sanctions*.

– *Internal conducts*:

  – That is, conducts impacting the internal structure and operation of the legal system itself, including the *creation* and *derogation* of legal norms,[22] *creation* and *cancellation* of legal authorizations, and the *recording* and *deletion* of legal facts.

As will be seen below, the *creation* and *cancellation* of *authorizations*, including the creation and cancellation of *authorizations to perform authorizations*, are treated as special cases of *creation* and *deletion* of legal facts.

We focus, first, on the two main types of *internal conducts*, namely, creation and derogation of *legal norms*:

– *Creation of a legal norm* in a legal order *LOrd* is the operation that *includes* a legal norm in the given legal order, at a given time. *Creation of a legal norm* at a time t is, thus, the operation *createlnrm* : **LNrm** × **LOrd** → **LOrd** defined by $createlnrm(nrm, LOrd^t) = LOrd^t \cup \{nrm\}$, for any legal norm $nrm \in$ **LNrm**, such that $nrm \notin LOrd^t$.
  An important condition for the *validity* of the legal norms of a given legal system, at a given time t, is that legal norms be created (i.e., added to the *legal order*) only in a legal way, that is, through *legal acts*. Thus, if it happens that a legal norm nrm and a legal order *LOrd* are such that $nrm \notin LOrd^{t''}$ but $nrm \in LOrd^t$, for some $t'' < t$, then it must have happened a legal act of creation $createlnrm(nrm, LOrd^{t'})$, at some time t', with $t'' \leq t' < t$, and it must have occured no derogation of nrm between t' and t.

– *Derogation of a legal norm* is the operation that *removes* a legal norm from a given legal order, at a given time. *Derogation of a legal norm* from a legal order *LOrd*, at a time t, is the operation *deroglnrm* : **LNrm** × **LOrd** → **LOrd** defined by $deroglnrm(nrm, LOrd^t) = LOrd^t - \{nrm\}$, for any legal norm $nrm \in LNrm$, such that $nrm \in LOrd^t$.
  An important condition for the *valid derogation* of *valid legal norms* is that valid legal norms be derogated from legal orders in a legal way, that is, through legal acts. Thus, if it happens that a legal norm nrm and a legal order *LOrd* are such that $nrm \in LOrd^{t''}$, but $nrm \notin LOrd^t$, for some $t'' < t$, then it must have happened a legal act of derogation $deroglnrm(nrm, LOrd^{t'})$ at some time t', with $t'' \leq t' < t$, and it must have occurred no creation of nrm between t' and t.

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[22] Notice how the operational semantics allows to account for the *derogation* of legal norms in terms of the performance of *legal acts*, not in terms of the validity of *derogation norms*. From the operational point of view, Kelsen's account of *norm derogation* in terms of the validity of *derogation norms* (Kelsen 2009), is—still—given in terms of the *surface* structure of law (see Sect. 3).

– As a consequence, a legal norm $N \in \mathbf{LNrm}$ is *valid* in a legal order *LOrd*, at a given time t, if and only $N \in LOrd^t$. Thus, given a legal order *LOrd*, we say that $LOrd^t$ is the *set of valid legal norms* of *LOrd*, at time t.

We focus, next, on the auxiliary *internal conducts* of creation and deletion of *legal facts*:

– A *record of legal facts* is a time-indexed family of sets $RLFact : T \rightarrow \wp(\mathbf{LFact})$, so that for any $t \in T$, $RLFact^t$ is the set of legal facts that are recorded in *RLFact*, at time t.
– We denote each operation of *recording* and *deletion* of legal facts in (from) *RLFact* by $recordlfct : LFact \times \wp(\mathbf{LFact}) \rightarrow \wp(\mathbf{LFact})$ and $deletelfct : LFact \times \wp(\mathbf{LFact}) \rightarrow \wp(\mathbf{LFact})$, so that $recordlfct(lfact, RLFact^t) = RLFact^t \cup \{lfact\}$, whenever $lfact \notin RLFact^t$, and $deletelfct(lfact, RLFact^t) = RLFact^t - \{lfact\}$, whenever $lfact \in RLFact^t$, for any time t.
– Often, the *recording* and *deletion* of legal facts are implict consequences of a legal act, so that the operations *recordlfct* and *deletelfct* need not always be explicitly indicated.
– The record of legal facts *RLFact* is an *auxiliary device* introduced in the formalization of Kelsen's concept of *legal system* to allow the presentation of the dynamics of such systems in terms of *transition systems*.[23] The justification of our using of *transition systems* for the formalization of Kelsen's concept of *legal system*, and the possibility of formalizing it in an alternative way, in terms of *process-based models*, where *events* are taken as the basic notions, is briefly discussed in Sect. 12.
– We determine no specific syntactical requirements for the denotation of *legal facts*, except for the requirement that such denotation should accomodate constructs equivalent to those used in the present paper, for the types of legal facts that it mentions.

Finally, we focus on the *internal conducts* of creation and cancellation of *authorizations*:

– *Creation of an authorization* is the operation that *includes* an authorization into a given record of legal facts. *Creation of an authorization* is thus the operation $createlauth : \mathbf{Auth} \times \wp(\mathbf{LFact}) \rightarrow \wp(\mathbf{LFact})$ defined, for the time t and record of legal facts *RLFact*, by $createlauth(auth, RLFact^t) = record(auth, RLFact^t)$.
– *Cancellation of authorization* is the operation that *removes* an authorization from a given record of legal facts. *Cancellation of an authorization* is thus the operation $cancellauth : \mathbf{Auth} \times \wp(\mathbf{LFact}) \rightarrow \wp(\mathbf{LFact})$ defined, for the time t and record of legal facts *RLFact*, by $cancellauth(auth, RLFact^t) = delete(auth, RLFact^t)$.

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[23] Kelsen himself makes little or no theoretical use at all of the notion of *legal fact* in the way we are using it here (Kelsen 2009). His account of the operation of legal systems is given mainly in terms of the *conducts* of legal organs, that is, in terms of the *performance of legal actions* (or, *legal events*), thus in purely *process-theoretic* terms, as suggested in Sect. 12.

- An authorization created in a legal way is a *legal authorization*. Similarly with the case of the creation and derogation of *valid legal norms*, then, an important condition for the creation and cancellation of *legal authorizations* is that they occur in a legal way, that is, under the condition of being *legally authorized*.
- If necessary, one may define the *set of legal authorizations* of a given legal system, at a time t, by $LAuth^t = \{Auth(ag, Ct) \mid Auth(ag, Ct) \in RLFact^t\}$.

5.5 Permissions and subjective rights

In this section, we present the difference between *strong* and *weak* permissions, which was delayed to this point due to its dependence on the definition of the *validity* of legal norms. In addition, we present the notion of *subjective right*, which also depends on the notion of *validity* of legal norms.

5.5.1 Strong permissions and weak permissions

Given the definition of the set of *valid legal norms* of a given legal order at a given time, we can now define the two special types of legal norms, whose definitions we have postponed: *strong permissions* and *weak permissions* (Kelsen 2009).

- The *strong permission* of a conduct is the partial derogation of an existent *prohibition* of the conduct, so that the conduct becomes *permitted* for a particular subtype of agents after that partial derogation occurs.
  Formally: a conduct of type $Ct_1 \in \wp(\mathbf{Cd})$ is *strongly permitted* in a legal order *LOrd* at the time t for the type of agents Ag'_1 if and only if it happens that:

  - at time $t'' < t$, Ct_1 is prohibited for a class of agents Ag_1 that is a super-class of Ag'_1, i.e.: $Ag'_1 \subseteq Ag_1$ and $(Ag_1, Ct_1) \Downarrow Auth(Ag_2, Ct_2) \in LOrd^{t''}$, for some type of agents Ag_2 and conduct Ct_2;
  - at time t', with $t'' < t' < t$, there is a derogation $deroglnrm((Ag_1, Ct_1) \Downarrow Auth(Ag_2, Ct_2), LOrd^{t'})$, and at the same time t' a creation operation $createlnrm((Ag_1 - Ag'_1, Ct_1) \Downarrow Auth(Ag_2, Ct_2), LOrd^{t'})$, which together reduce the scope of the prohibition of Ct_1 to the agents of type $Ag_1 - Ag'_1$, i.e.: so that $(Ag_1 - Ag'_1, Ct_1) \Downarrow Auth(Ag_2, Ct_2) \in LOrd^{t'+1}$;
  - no other creation or derogation operation concerning Ct_1 and Ag'_1 (or its subclasses) occurs between t' and t.

- The *weak permission* of a conduct is the lack of any authorization, prohibition, imposition, or strong permission, involving that conduct.

5.5.2 Subjective rights

Whenever an imposition $(Ag_1, \overline{Ct_1}) \Downarrow Auth(Ag_2, Ct_2)$ is valid in a legal order, at a time t, so that the performance of conducts of type Ct_1 is an *obligation* of the agents

of type Ag_1, at time t, one says that the performance of conducts of type Ct_2 is a *subjective right* of the agents of type Ag_2, at time t.

A typical example would be: the right to sue a merchant for not delivering a good that has been paid.

5.6 Concept of legal system

Formally, we state that a *legal system* over the universes of legal norms **LNrm**, agents **Ag**, and conducts **Cd**, is a *time-indexed structure*[24] $LS = (\{LOrd^t\}_{t \in T},$ $\{LOrg^t\}_{t \in T}, \{RLFact^t\}_{t \in T}, createlnrm, deroglnrm, createlauth, cancellauth,$ $recordlfct, deletelfct)$ where:

- $LOrd^t$ is the *legal order* at time t;
- $LOrg^t$ is the *system of legal organs* at time t;
- $RLFact^t$ is the *record of legal facts* at time t;
- $LOrd^0 \neq \emptyset \neq LOrg^0$ [a requirement emphasized by Raz (1970)];
- *createlnrm, deroglnrm, createlauth, cancellauth, recordlfct* and *deletelfct* are as defined above.

5.7 Temporal evolution of legal systems

A legal system *functions*, at each time t, through the performance of *legal acts* by the legal organs that exist in the system at that time t.

The *temporal evolution* of a legal system is given by the temporal evolution of its *legal state* (i.e., the structure composed of its *legal order*, *system of legal organs*, and *record of legal facts*), according to the legal acts validly performed, at each time t, by the legal organs that exist in the system at that time t.

We write $LS^t = (LOrd^t, LOrg^t, RLFact^t)$ for the *legal state* of a legal system LS at time t, so that the *temporal evolution* of the legal system LS is given by the sequence of legal states

$$LS = LS^0, LS^1, \ldots$$

that is,

$$LS = (LOrd^0, LOrg^0, RLFact^0), (LOrd^1, LOrg^1, RLFact^1), \ldots$$

where legal orders evolve according to the legal performance of legal acts, under the constraint that:

$$\begin{aligned} LOrd^{t+1} &= createlnrm(LOrd^{t+1} - LOrd^t, deroglnrm(LOrd^t - LOrd^{t+1}, LOrd^t)) \\ &= deroglnrm(LOrd^t - LOrd^{t+1}, createlnrm(LOrd^{t+1} - LOrd^t, LOrd^t)) \end{aligned}$$

and similarly for $LOrg^{t+1}$ and $RLFact^{t+1}$.

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[24] The use of a *time-indexed structure* allows us to capture, in one single formal construct, the *synchronic* and the *diachronic* aspects of the *historical evolution* of legal systems, an idea that (e.g., Raz 1970) considered in terms of two distinct informal concepts: *legal systems* and *momentary legal systems*.

The temporal evolution of the *system of legal organs* is determined by the temporal evolution of the *set of legal authorizations* in the legal system, in the sense that:

$$LOrg^t = \{ag \in \mathbf{Ag} \mid \exists Auth(ag, Ct) \in LFact^t\}$$

so that an agent is a *legal organ*, at a time t if and only if it is *legally authorized* to perform some specific conduct (thus, not by any statement that explicitly declares it to be a legal organ, but does not authorize it to perform any specific conduct).

5.8 An aside: why legal norms are not recorded as legal facts?

A quick comparison of the definitions of the operations of *creation* and *derogation* of legal norms with the operations of *recording* and *deletion* of legal facts (including the operations of *creation* and *cancellation* of legal authorizations) will show an isomorphism between the operatory structures they determine on the sets they respectively operate (legal norms, legal facts, and legal authorizations).

The question then arises: Why not consider that legal systems are endowed with one single *record of legal facts*, where valid legal norms, legal authorizations, and all other sorts of legal facts are seamlessly recorded, so that we could formally say that "the validity of a legal norms is determined by the issuing of the norm being recorded as a legal fact"?

Although such formal possibility exists, we preferred to keep separate the *legal order* (i.e., the *set of valid legal norms*) and the *record of legal facts* (legal authorizations, included), to keep visible the distinction between them.

6 Main features of the adopted approach to operational semantics

We take *operational semantics* as a reference for our *semantics of situated legal systems*.

We format our *operational semantical framework* following the general principles of the *transition systems*-based approach to operational semantics introduced in Plotkin (1981).

In this section, we summarize the main ideas of the transition systems-based approach to operational semantics, and we apply them to situated legal systems in the next sections.

6.1 Operational interpretation of action systems

A *transition system* is a structure $S = (\Gamma, \longrightarrow, D_S)$, where[25]:

- Γ is the set of all *configurations* that the system may possibly assume, at each instant of time;

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[25] Clearly, D_S can be deduced from $(\Gamma, \longrightarrow)$. It is made explicit here for convenience.

- the set Γ is structured by the *transition relation* $\longrightarrow \subseteq \Gamma \times \Gamma$, determining how each configuration $\gamma \in \Gamma$ may evolve toward other(s) configuration(s) γ', with $(\gamma, \gamma') \in \longrightarrow$ denoted by $\gamma \longrightarrow \gamma'$;
- D_S is the set of all *possible sequences of configurations*, where γ_0, $\gamma_1, \ldots, \gamma_i, \gamma_{i+1}, \ldots \in D_S$ if and only if $\gamma_i \longrightarrow \gamma_{i+1}$, for all $i = 0, 1, 2, \ldots$.
  Also, we construe the idea of an *action system* as a structure $A = (Act, D_A)$ where:
- *Act* is a set of actions;
- $D_A \subseteq T \rightarrow \wp(Act)$ is the set of possible *processes* of A, where each process is a function $d : T \rightarrow \wp(Act)$.
- each process $d \in D_A$, with $d(0) = A_0$, $d(1) = A_1, \ldots$, for $A_i \in \wp(A)$, is denoted by $d = A_0, A_1, \ldots$.

We say that an *operational interpretation* of an action system $A = (Act, D_A)$ over a transition system $S = (\Gamma, \longrightarrow, D_S)$ is a function $\rho : \wp(Act) \rightarrow \wp(\longrightarrow)$, where we denote each possible *configuration transition* $(\gamma, \gamma') \in \rho(A)$ by $\gamma \xrightarrow{A} \gamma'$, for all sets of actions $A \in \wp(Act)$.

Note that for any set of actions $A \in \wp(Act)$, the operational interpretation $\rho(A)$ is a set of *possible* transitions, meaning that A may be put to operate in different systems configurations, including the possibility of *non-deterministic* configuration transitions.

We say that the transition system S is an *operational semantical domain* for the action system A whenever it is possible to define an operational interpretation ρ for A over S.

The *operational semantics* of A over S is, thus, characterized by the pair $(\rho, \longrightarrow)$, given by *configuration transitions* of the form $\gamma \xrightarrow{A} \gamma'$.

A process d is said to be *sequential* if $d = \{\alpha_0\}, \{\alpha_1\}, \ldots$, for $\alpha_i \in Act$. We may also denote it by $d = \alpha_0, \alpha_1, \ldots$, so that we may take it be a function $d : T \rightarrow Act$.

Clearly, action systems that have only *sequential* processes, may have their operational intepretations over transition systems characterized by configuration transitions of the form $\gamma \xrightarrow{\alpha} \gamma'$.

For simplicity, we restrict ourselves to action systems with only *sequential* processes, in the operational semantics of *legal systems* introduced below.

6.2 Validity of configuration transitions in operational semantics

A *configuration transition* $\gamma \xrightarrow{A} \gamma'$ is said to be a *valid configuration transition* in an operational interpretation $\rho : D_A \rightarrow \wp(D_S)$ of an action system $A = (Act, D_A)$
over a transition system $S = (\Gamma, \longrightarrow, D_S)$ if and only if $(\gamma, \gamma) \in \rho(A)$.

One may want to have, for any given interpretation of an action system over a given transition system, a means to determine, through a systematic process, the validity of any given configuration transition. One may use *derivation systems* for such purpose (Plotkin 1981).

A *derivation system* for the *validation of configuration transitions* is a set of two types of *derivation rules*:

1. *Unconditional rules*:

    $$\frac{}{\gamma \xrightarrow{\alpha} \gamma'} \, R$$

    where R is the rule name, and the action α transforms the pre-configuration γ into the post-configuration γ'.
2. *Conditional rules*:

    $$\frac{Cond_1 \quad \ldots \quad Cond_n}{\gamma \xrightarrow{\alpha} \gamma'} \, R$$

    where the conditions $Cond_i$, in the *premiss* part of the rule, are said to be the *enabling conditions* of the action α, and should be true of the pre-configuration γ, if the configuration transition determined by the action α is to occur.

We say that a *configuration transition* $\gamma_t \xrightarrow{\alpha_t} \gamma_{t+1}$, occurring at time $t \in T$ in the dynamics of a given *action system*, operationally interpreted over a given *transition system*, is a *valid configuration transition* if and only if the *temporal evolution* of the action system over the transition system (i.e., the combined sequences of actions and configurations) is a *validating history* for that configuration transition.

That is, if and only if the *temporal evolution* can be given as a *sequence* of the form

$$\gamma_0 \xrightarrow{a_0} \gamma_1 \xrightarrow{a_1} \gamma_2 \xrightarrow{a_2} \ldots \gamma_t \xrightarrow{a_t} \gamma_{t+1}$$

where, at each time $i \in T$, the transition $\gamma_i \xrightarrow{a_i} \gamma_{i+1}$ transforms the pre-configuration γ_i into the post-configuration γ_{i+1}, according to a *derivation rule* of the derivation system.

The *operational semantical framework* introduced in Sect. 8 allows for the definition of *derivation systems* that can support the formal determination of the validity of the legal acts performed along the temporal evolution of *situated legal systems*.[26]

6.3 Operational couplings between action systems and transition systems

Whenever there is an operational interpretation for an action system over a transition system, we say that there is an *operational coupling* between the action system and the transition system, and that the action system is *operationally coupled* to the transition system *through* that operational interpretation.

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[26] *General methods* for defining the operational semantics of action systems over transition systems were introduced (Plotkin 1981), but their exposition is out of the scope of the present paper.

7 Formal concept of situated legal system

In this section, *situated legal systems* are defined on the bases of *operational couplings* between *legal systems* (taken as *action systems*, with *legal actions* as the system's actions), and *core operational structures* of agent societies (taken as *transitions systems*, with combinations of *states of agent societies* and *states of legal systems* as configurations).

Thus, we say that a legal system *LS* is *situated* in an agent society *AgSoc* whenever an *operational interpretation* $(\rho, \longrightarrow)$ has been defined for *LS* over the core operational structure of *AgSoc*.

That is, a *situated legal system* is a structure

$$LS_{AgSoc} = (LS, AgSoc, ag_0, \rho, \longrightarrow)$$

where:

– $LS = (\{LOrd^t\}_{t \in T}, \{LOrg^t\}_{t \in T}, \{LFact^t\}_{t \in T}, createlnrm,$
  $deroglnrm, createlauth, cancellauth, recordlfct, deletelfct, ag_0)$ is a *legal system*;
– $AgSoc = (Pop^t, Org^t, Imp^t, Env^t, Ideo^t)$ is an *agent society*;
– $ag_0 \in \mathbf{Ag}$ is the said to be the *founder agent* of the legal system;
– $(\rho, \longrightarrow)$ is an *operational interpretation* of *LS* over *AgSoc*, whose formal definition is given in terms of the operational semantical framework introduced in Sect. 8.

We note that, from the point of view of the architectural style for agent societies presented in Sect. 2, situated legal systems should be construed as *social systems*, located within the macro-organizational level of agent societies, for they are *systems of legal organs*, each legal organ architecturally being either a *social role* (at the micro-organizational level) or an *organizational unit* (at the meso-organizational level).[27]

The location of a situated legal system within an agent society is exemplified in the schema of Fig. 1 by the dotted trapezoid, which delimits the organizational units and social roles that constitute the legal organs of the hypothetical legal system.

On the other hand, note that an *operational semantics* for a situated legal system allows for the placement of only the *legal organs* of the legal system within the core operational structure of the underlying agent society.

The other main structural components of the situated legal system, namely, the *legal order* and the *record of legal facts*, are to be located in other parts of the structure of the society:

– the *legal order*, in the *ideological structure* (as the *legal order* is, in general, presumed to be for public access, both for the *legal organs* and the *social agents*, either individual agents or organizational units);

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[27] See Kelsen (2009) for the clarification of the relation between *legal organs* and the individuals and organizational units (Kelsen says *corporations*) that implement them in a society.

– the *record of legal facts*, in the *private stores* of the legal organs (as the *record of legal facts* is, in general, presumed to be of restricted access, only for *legal organs*).

8 An operational semantical framework for situated legal systems

We introduce here a framework for giving the transition systems-based semantics of situated legal systems, that is, of legal systems operationally coupled to agent societies.

The framework is given in terms of (example patterns for) *derivation rules*, each derivation rule determining the (internal or external) *effect* of the (legal or social) act that it envolves and, at the same type, in case the act is a the legal act, determining the conditions for its *validity*.

The definition of the framework proceeds according to the following steps:

– *general features* of the semantical framework;
– *derivation rules* (rule parameters may require adaptation, in concrete applications):

– a derivation rule for the *basic norm*;
– a set of derivation rules for the performance of *internal legal acts*: creation of authorization, derogation of norm, cancellation of authorization, imputation of a sanction, registration of legal fact by legal organ, deletion of legal fact by legal organ;
– a derivation rule for the performance of *external legal acts* (in particular, performance of *sanctions*);
– a derivation rule for *social acts*.

8.1 General features

8.1.1 Configurations of situated legal systems

For any situated legal system $LS_{AgSoc} = (LS, AgSoc, ag_0\rho, \longrightarrow)$, the configurations operated by the derivation rules are called *legal configurations*, and are taken to have the form $\langle \text{LOrd}, \text{LOrg}, \text{RLFact}, \text{StAgSoc} \rangle$ where:

– $(\text{LOrd}, \text{LOrg}, \text{RLFact})$ is the *state* of the legal system LS at that configuration, with:

– LOrd, the set of *valid legal norms* of LS;
– LOrg, the set of *legal organs* of LS;
– RLFact, the set of *legal facts* of LS;

– StAgSoc is the *state* of the agent society $AgSoc$ at that configuration.

8.1.2 Functional types of conducts

We distinguish the following functional types of conducts (*functional acts*):[28] legal (internal and external), and social.

The set of *legal acts* is a set **LAct** such that **ILAct** $\subseteq$ **LAct**, where **ILAct** is the set of *internal legal acts* (legal acts whose effects are *internal* to the legal system), given by **ILAct** $= \{fndAct, createlnrm, createlauth, recordlfctderoglnrm, cancellauth, deletelfct\}$.

The set **EAct** $=$ **LAct** $-$ **ILAct** is the set of *external legal acts*, which refer to legal acts that have effects that are external to the legal system, i.e., acts performed by the legal organs (usually in the form of sanctions) that affect the society where the legal system is situated.

Any act in **Act** $-$ **LAct** is said to be a *social act*, that is, an act performed by a *social agent*, that is, an agent (individual agent or organizational unit) that is not functioning as a legal organ, when performing that act.

8.1.3 Notation for the performance of a conduct by an agent

We use expressions of the form $ag \in Ag \rhd c \in Ct$ to denote that the agent ag, of type Ag, performs the conduct c, of type Ct. We often omit the type information, writing simply $ag \rhd c$.

8.2 Derivation rules

8.2.1 The basic norm

The following is the derivation rule that gives an operational account of the *legal validity of the founding act*, which is performed by the *founder agent* of the legal system:

– Rule **Basic Norm**:

$$\langle \emptyset, \emptyset, \emptyset, \mathrm{StAgSoc}_0 \rangle \xrightarrow{\mathrm{ag}_0 \rhd fndAct} \langle \mathrm{LOrd}_0, \mathrm{LOrg}_0, \mathrm{RLFact}_0, \mathrm{StAgSoc}_0 \rangle$$

where:

– $\mathrm{ag}_0 \in \mathbf{Ag}$ is the *founder agent* of the legal system;
– $fndAct \in \mathbf{LAct}$ is the *founding act*, determining the *initial legal order* LOrd_0, the *initial set of legal organs* LOrg_0, and the *initial record of legal facts* RLFact_0, with $fndAct \in \mathrm{RLFact}_0$.

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[28] Distinguished by the *legal function* they perform in the system, not by their *form* or *content*. See the example distinction between *tax* and *fine*, in Sect. 5, where both acts have the same form and content (charge of an amount of money), but different *legal functions*.

Notice how the *basic norm* of a legal system states that a founder agent ag_0 is authorized in an (for the semantical framework) *unconditional* way to create the legal system.

The founder agent ag_0 freely creates, at time $t = 0$, a non-empty set of the *legal norms*, a non-empty set of *legal organs*, and a non-empty sect of *legal facts* (including *legal authorizations*), which then respectively become, at time t, the *initial set of valid legal norms* $LOrd^1 = \text{LOrd}_0$, the *initial set of legal organs* $LOrg^1 = \text{LOrg}_0$, and the *initial set of legal facts* $RLfact^1 = \text{RLFact}_0$ (including the initial set of *legal authorizations*, in particular *fndAct*, see Sect. 10).

This is so, because Kelsen's idea about the *founding act* is that the effective support for the realization of the founding act is a set of *subjective* facts (political, moral, ideological, etc.; see Sects. 10 and 11), not *positive* legal facts (i.e., facts legally recorded in RLFact), since the legal system is yet to be created, when the founding act is performed.

Thus, such subjective conditions, not being *legal* ones, should not be taken into account in the *legal* characterization of the founding act (and, so, neither in the conditions of the *derivation rule*, which aims to capture the idea of the basic norm).

The net result is, from the point of view of the operational semantics, that the *legal effects* of the founding act have to be incorporated into the situated legal system in an *unconditional* way, as exposed by the rule.

Notice, finally, that no requirement of *adequacy*[29] is imposed neither on the contents of the various initial sets defined by the founding agent, nor on the relations implied by such contents.

In other words, the *initial adequacy* of the legal system is taken to be the sole responsibility of the founder agent ag_0, while the subsequent adequacy of the legal system is, of course, the responsibility of the *legal organs* that the legal system gets along its history.

8.2.2 Internal legal acts

The following are the derivation rules that give the operational semantics of the legal acts that affect the *legal order*, the *system of legal organs*, and the *record of legal authorizations*:

(a)   Rule **Creation of Norm**:

$$\frac{nrm \notin \text{LOrd} \qquad Auth(Ag, createlnorm(nrm)) \in \text{RLFact}}{\langle \text{LOrd, LOrg, RLFact, StAgSoc} \rangle \xrightarrow{ag \in Ag \triangleright createlnorm(nrm)} \langle \text{LOrd}', \text{LOrg, RLFact, StAgSoc} \rangle}$$

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[29] *Adequacy* regarding the *function* that the legal system is to perform in the agent society where it is situated.

where:

– LOrd$'$ = LOrd $\cup$ {*nrm*};

(b) Rule **Creation of Authorization**:

$$\frac{Auth(Ag, createlauth(Auth(Ag', Ct))) \in \text{RLFact}}{\langle\text{LOrd}, \text{LOrg}, \text{RLFact}, \text{StAgSoc}\rangle \xrightarrow{ag \in Ag \rhd createlauth(Auth(ag', Ct))} \langle\text{LOrd}, \text{LOrg}', \text{RLFact}', \text{StAgSoc}\rangle}$$

where:

– $ag' \in Ag'$;
– RLFact$'$ = RLFact $\cup$ {*Auth(ag', Ct)*};
– LOrg$'$ = LOrg $\cup$ {*ag'*}, if *Ct* is a *legal act*;
– = LOrg, otherwise.

(c) Rule **Derogation of Norm**:

$$\frac{nrm \in \text{LOrd} \qquad Auth(Ag, deroglnorm(nrm)) \in \text{RLFact}}{\langle\text{LOrd}, \text{LOrg}, \text{RLFact}, \text{StAgSoc}\rangle \xrightarrow{ag \in Ag \rhd deroglnorm(nrm)} \langle\text{LOrd}', \text{LOrg}, \text{RLFact}, \text{StAgSoc}\rangle}$$

where:

– LOrd$'$ = LOrd $-$ {*nrm*}.

(d) Rule **Cancellation of Authorization**:

$$\frac{Auth(ag', Ct) \in \text{LOrd} \qquad Auth(Ag, cancellauth(Auth(Ag', Ct)) \in \text{RLFact}}{\langle\text{LOrd}, \text{LOrg}, \text{RLFact}, \text{StAgSoc}\rangle \xrightarrow{ag \in Ag \rhd cancellauth(Auth(ag', Ct))} \langle\text{LOrd}, \text{LOrg}', \text{RLFact}', \text{StAgSoc}\rangle}$$

where:

– $ag' \in Ag'$;
– RLFact$'$ = RLFact $-$ {*Auth(ag', Ct)*};
– LOrg$'$ = LOrg $-$ {*ag'*} if *onlylauth(ag', Auth(ag', Ct))*;
– = LOrg, otherwise.

where *onlylauth(ag', Auth(ag', Ct))* is true only if the legal organ *ag'* has the cancelled authorization as its only legal authorization (so that it should be excluded from the set of legal organs, when *Auth(Ag', Ct)* is cancelled).

(e) Rule **Imputation of Sanction**:

$$\frac{did(ag,c) \in \text{RLFact} \quad (Ag, Ct) \Downarrow Auth(Ag'', Ct'') \in LOrd \quad Auth(Ag', Aut(Ag'', Ct'')) \in \text{RLFact}}{\langle \text{LOrd, LOrg, RLFact, StAgSoc} \rangle \xrightarrow{ag' \in Ag' \triangleright createlauth(Auth(ag'', Ct''))} \langle \text{LOrd, LOrg, RLFact}', \text{StAgSoc} \rangle}$$

where:

– $ag \in Ag$
– $c \in Ct$
– $ag'' \in Ag''$
– $Auth(ag'', Ct'') \notin \text{RLFact}$
– $\text{RLFact}' = \text{RLFact} \cup \{Auth(ag'', Ct'')\}$

which is the rule by which, given the legal fact that an agent ag of type Ag has performed a conduct c of type Ct, an agent of type Ag'' may be authorized by an agent ag' of type Ag', to perform a conduct of type Ct'', as a sanction for the performance of c.

Notice the essential role played by the legal organ of type Ag', which has the authorization to authorize the legal organ of type Ag'' to perform the sanction, if the performance of the conduct of type Ct is determined as a legal fact. The mere validity of the legal norm $(Ag, Ct) \Downarrow Auth(Ag'', Ct'')$ is not enough, by itself, to authorize the legal organ of type Ag'' to perform the sanction. It must be explicitly authorized to do that, by a legal organ legally authorized to authorize it.

In a typical situation, the legal organ of type Ag' would be a *judge*, the legal organ of type Ag'' would be a *sanction executor*.

The essential kelsenian idea, here, is that legal norms may "validate" acts (i.e., *authorize* them, determining them as *legal*), but may not directly "cause" legal acts: legal acts can be "legally caused" only by other legal acts, through the creation of *legal authorizations*, not by the mere validity of legal norms.[30]

8.2.3 Legal facts

The following are rules giving the operational semantics of legal acts that *explictly* affect the *record of legal facts*[31]:

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[30] That is why the main result in the conclusion of the derivation rule for the *basic norm* is the *legal authorization* for the founder agent to create the *initial configuration* of the legal system (determining such creation as a *legal act*), which *initial configuration* should include *legal authorizations* for the *initial legal organs* to perform their *inital legal acts* (otherwise the thus created legal system will not be able to start to operate legally).

[31] There are legal acts *implicit* in other derivation rules, like the rule for the *imputation of sanctions*, that also affect the *record of legal facts*.

(a)   Rule **Registration of Legal Fact by Legal Organ**:

$$\frac{lfact \notin \mathrm{RLFact} \qquad Auth(Ag, recordlfct(lfact)) \in \mathrm{RLFact}}{\langle \mathrm{LOrd}, \mathrm{LOrg}, \mathrm{RLFact}, \mathrm{StAgSoc}\rangle \;\; \overset{ag \in Ag \rhd recordlfct(lfact)}{\longrightarrow} \;\; \langle \mathrm{LOrd}, \mathrm{LOrg}, \mathrm{RLFact}', \mathrm{StAgSoc}\rangle}$$

where:

– RLFact$' =$ RLFact $\cup \{lfact\}$

(b)   Rule **Deletion of Legal Fact by Legal Organ**:

$$\frac{lfact \in \mathrm{RLFact} \qquad Auth(Ag, deletelfct(lfact)) \in \mathrm{RLFact}}{\langle \mathrm{LOrd}, \mathrm{LOrg}, \mathrm{RLFact}, \mathrm{StAgSoc}\rangle \;\; \overset{ag \in Ag \rhd deletelfct(lfact)}{\longrightarrow} \;\; \langle \mathrm{LOrd}, \mathrm{LOrg}, \mathrm{RLFact}', \mathrm{StAgSoc}\rangle}$$

where:

– RLFact$' =$ RLFact $- \{lfact\}$

8.2.4 External legal acts

The following is the rule that gives the operational semantics of *external legal acts*, that is, legal acts that (as, e.g., the performance of a *sanction*) affect the state of the society:

(a)   Rule **Execution of External Legal Act by Legal Organ**:

$$\frac{c \in \mathbf{ELAct} \qquad Auth(Ag, c) \in \mathrm{RLFact}}{\langle \mathrm{LOrd}, \mathrm{LOrg}, \mathrm{RLFact}, \mathrm{StAgSoc}\rangle \;\; \overset{ag \in Ag \rhd c}{\longrightarrow} \;\; \langle \mathrm{LOrd}, \mathrm{LOrg}, \mathrm{RLFact}', \mathrm{StAgSoc}'\rangle}$$

where:

– StAgSoc$' = apply(c, \mathrm{StAgSoc})$
– RLFact$' =$ RLFact $\cup \{did(ag, c)\}$

8.2.5 Social acts

The following is the rule giving the operational semantics of a *social act*, that is, an act that is not a *legal act* (so that no agent is required to have a *legal authorization* to perform it):

(a)   Rule **Execution of Social Act**:

$$\frac{c \notin \mathbf{LAct}}{\langle \text{LOrd}, \text{LOrg}, \text{RLFact}, \text{StAgSoc} \rangle \overset{ag \rhd c}{\longrightarrow} \langle \text{LOrd}, \text{LOrg}, \text{RLFact}, \text{StAgSoc}' \rangle}$$

where:

– StAgSoc$'$ = $apply(c, \text{StAgSoc})$

9 Some properties of situated legal systems, exposed by the operational semantics

In this section, we look at the main formal properties that *situated legal systems* inherit from Kelsen's concept of *legal system*.

9.1 Action-based dynamics

The operational semantics introduced above builds on the idea that legal systems operate under an *action-based dynamics*, that is, that the configuration transitions are determined by the performance of actions by the *legal organs* and *legal subjects* of the legal system.

That is the idea expressed by the above rules, which define the *pre-conditions* for the realization of *configuration transitions* $\gamma \overset{ag \rhd \alpha}{\longrightarrow} \gamma'$.

Notice that the operational rules only indicate conditions for the *legal enabling* of the execution of configuration transitions, not their *inevitability*.

That is, the derivation rules leave to the *discretion* of the agents responsible for the execution of the actions, the *deliberation* about their execution, or not, in each circumstance.[32]

9.2 Separation of concerns between the operational rules of the legal systems and the reasoning rules of their legal organs

The fact that *legal norms* (and the *derivation rules* that capture them operationally) determine just the *legal enabling* of conducts, not their *inevitability*, establishes the *relative independence* between the rules that drive the *dynamics* of the legal system, and the rules (if any) that drive the *legal reasonings* and *deliberations* of the legal organs.

This is reflected in an important characteristic of Kelsen's *Pure Theory of Law* (Kelsen 2009), namely, that it focuses on the overall *structure* and *operation* of legal systems, taken as a whole, not on the *contents* of their norms, nor on the rules

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[32] But the *obligation* of their execution can be determined by sanctioning their omission.

for the individual *reasoning* and *decision-making* of the system's *legal organs* (which support their deliberations concerning the way the system should operate, given their own *interests* and the *contents* of the legal norms).

This *separation of concerns* is one of the central ideas in Kelsen's theory of law, and is possible only because of the *relative independence* between those two types of aspects of the legal systems.

We say that the *relative independence* of the contents of those two types of rules (*dynamical* and *deliberative* rules) generate two *orthogonal* types of processes in the legal system: *macro-level processes*, encompassing the overall structure and operation of the legal system, and *meso* and *micro-level* processes, in and between the legal organs, concerning their interactions, reasonings and deliberations.

We call *point of discretion*, any point in the dynamics of the legal system where a legal organ can exercise its *discretion*, regarding a legal act that it may perform at that point. *Points of discretion* are associated, in the derivation rules, with the places where the "▷" symbol appears, in expressions of the form $ag \triangleright \alpha$.

Such symbol gives no indication, however, either about the *motivation* that may lead *ag* to decide to perform (or, not to perform) the action α, or about the *legal argument* that *ag* may use to justify that decision, so that such *deliberation* and *argument* rest, in the dynamics of the system, as a processes that are *internal* to the legal organ.

We picture in Fig. 3, in a general way,[33] how the orthogonality between the two types of processes articulates the *sequence of configuration transitions*, at the macro-level of a situated legal system, with the corresponding *sequence of deliberation processes*, at the meso and micro-levels of the system.

Clearly, then, it is only natural that the formal expression of the *deliberation processes* of the legal organs requires a style of formal system (say, some for of *logical* or *decision-theoretic* formalism), different from the dynamical *system-theoretic* formalism that we are studying in the present paper.

In addition, one may see that the *separation of concerns* (between the *dynamics* of the system and the *internal operation* of the legal organs), allowed by the *orthogonality* of their operational interrelation, should be closely related to the highly debated issue of the possible connections between *law* and *morality* (Kelsen 1991), or *law* and *politics* Kelsen (2009) (see also Hart 1958; Dworkin 1977), as discussed in Sect. 11.

$$\frac{\vdots D_0}{\gamma_0 \xrightarrow{ag_0 \triangleright \alpha_0} \gamma_1} \quad , \quad \frac{\vdots D_1}{\gamma_1 \xrightarrow{ag_1 \triangleright \alpha_1} \gamma_2} \quad , \quad \frac{\vdots D_2}{\gamma_2 \xrightarrow{ag_2 \triangleright \alpha_2} \gamma_3} \quad , \quad \cdots$$

Fig. 3 The articulation of the *sequence of configuration transitions* $\gamma_0 \to \gamma_1, \gamma_1 \to \gamma_2, \gamma_2 \to \gamma_3, \ldots$ with the *sequence of deliberation processes* $D_0, D_1, D_2, \ldots$, of the legal organs $ag_0, ag_1, ag_2 \ldots$

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[33] Figure 3 builds on the general idea of the *small-steps semantics* of action systems (Plotkin 1981).

9.3 Discretion in the imputation and execution of sanctions

To make more concrete the above discussion about *points of discretion* and *deliberation processes*, we analyse here their presence inside processes of *imputation and execution of sanctions*, as allowed by the derivation rules.

The delibaration processes of legal organs, concerning the imputation and execution of sanctions, *do not need* to have the form of *logical deductions*, because deliberations about the imputation of sanctions, and about the execution of sanctions, are *discretionary*, as is any other legal reasoning and legal decision-making process (Kelsen 2009).

Clearly, if one takes the operational rule for the *imputation of sanctions*, given above:

$$\frac{did(ag,c) \in \text{RLFact} \quad (Ag,Ct) \Downarrow Aut(Ag'',Ct'') \in LOrd \quad Aut(Ag', Aut(Ag'',Ct'')) \in \text{RLFact}}{\langle \text{LOrd, LOrg, RLFact, StAgSoc}\rangle \xrightarrow{ag' \in Ag' \triangleright createlauth(Auth(ag'',Ct''))} \langle \text{LOrd, LOrg, RLFact}', \text{StAgSoc}\rangle}$$

One can see that the *main condition* for a legal organ of type Ag' to perform the imputation of a sanction of type Ct'' for the performance of a conduct of type Ct' by the agent *ag*, is an *authorization* for the agent of type Ag' to authorize an agent of type Ag'' to perform the sanction of type Ct'', and that this *authorization* is precisely that: an *authorization*, not an *obligation*, to authorize the performance of the sanction.

In addition, one also sees that, when the imputation of the sanction is performed, the agent of type Ag'' is also just *legally authorized* to perform the sanction Ct'', not *obliged* to do it.

That is, both the agents involved in the application of the sanction (namely, the agent of type Ag' that authorizes the performance of the sanction, and the agent of type Ag'' that performs the sanction) are just *authorized*, not *obliged*, to do what the rules specify (if no sanctions are applicable, of course, but even then the agents may deliberate not to comply with the obligations).

Thus, to deliberate on the realization of their respective actions (authorization of the sanction and execution of the sanction), a reasoning has to be performed by each of those legal organs, a reasoning that, as we have already mentioned, is of a "practical", discretionary character [Kelsen's term is "political character" (Kelsen 2009)], not of a purely "deductive" character.

The following scheme shows the articulation between the *deliberation processes* of the involved legal organs (denoted by D_1 and D_2), and the *configuration transitions* that the deliberated actions may give rise to (denoted by $C_0 \rightarrow C_1$ and $C_2 \rightarrow C_3$):

$$\frac{\begin{array}{c} did(ag,c) \in \text{RLFact} \\ (Ag,Ct) \Downarrow Auth(Ag'',Ct'') \in LOrd \\ Auth(Ag', Auth(Ag'',Ct'')) \in \text{RLFact} \end{array}}{C_0 \xrightarrow{ag' \in Ag' \triangleright createlauth(Auth(ag'',c''))} C_1} \vdots D_1 \quad , \quad \ldots \quad , \quad \frac{Auth(ag'',c') \in \text{RLFact}}{C_2 \xrightarrow{ag'' \triangleright c''} C_3} \vdots D_2$$

- The first *deliberation process*, D_1, concerns the action *createlauth(Auth(ag'', Ct''))*, which causes the transition from the configuration C_0 (that allows both the

determination of the ocasion for the sanction of type Ct'' and the availability of the *authorization* for the agent of type Ag' to authorize the sanction) to the configuration C_1 (where it is recorded as a legal fact that the agent ag'', of type Ag'', has been authorized to execute the sanction).

– The second *point of discretion*, D_2, concerns the action $ag'' \triangleright c''$, which causes the transition from the configuration C_2 (a configuration that may be the configuration C_1 itself, or some later configuration, but that is such that the authorization for ag'' has not been cancelled) to the configuration C_3 (where it is recorded as a legal fact that the sanction c'', of type Ct'', has been executed).

One sees that the *points of discretion* embedded in the process of imputation and execution of sanctions dwell on at least the following four different deliberations that the agents ag' and ag'' have to take, when performing the reasonings that "fill the dots" in the figure:

1. The first, within D_1, concerns the deliberation about the ascertainment (or denial) that $ag \in Ag$ and that $c \in Ct$, thus the possibility of justifying (or not) the deliberation about the issuing (or not) of the authorization for ag'' to perform the sanction c''.[34]
2. The second, still within D_1, concerns how long ag' wil take to issue (if it happens that ag' has really deliberated to issue) the authorization for ag'' to perform the sanction c''.
3. The third, within D_2, concerns how long ag'' will take to decide to execute the sanction c'', and to effectively execute it.
4. The fourth, still within D_2, concerns the specific way in which ag'' will execute c'' (crucial if the specification of the type Ct'' is a loose one).

In addition, previously to the configuration C_0 (thus, previously to the application of the rule of *imputation of sanction*, but relating to the pre-conditions for its application), at least two additional *points of discretion* exist, for the legal organ that is responsible for the determination of the truth of those pre-conditions:

– The first concerns the acceptance of $did(ag, c)$ as a legal fact, acceptance that is usually subject to a debatable proof, thus usually subject to the discretionary acceptation by the legal organ responsible for its registration as a legal fact.
– The second concerns the choice of the norm to be taken into account in the process (shown as $Ag : Ct \Downarrow Ag'' : Ct''$, in the above scheme), which is usually subject to the discretionary choice of the legal organ responsible for determining both if the conduct c was a delict and, in the positive case, which should be the corresponding sanction. Such choice is specially discretionary if there are different norms that, even with the same material sphere of validity, specify different additional pre-conditions (e.g., concerning details of the performance

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[34] Of course, if the norm also makes explicit its *temporal* and *spatial* spheres of validity, additional facts concerning such spheres of validity have to be determined and taken into account in the legal reasoning, thus giving rise to additional *points of discretion*, within this *deliberation process*.

of the conduct c) and, on that account, different types of sanctions for the performance of that same conduct.

9.4 Validity of legal norms

As explained in Sect. 5, *validity* is a property that legal norms inherit through the *valid* execution of *norm creation* operations.

That is, *validity* is an *inductively inherited* property:

Validity Property: For any time t, the legal order $LOrd^t$ is a set of valid legal norms.

Proof

– **Basis:** $LOrd^0$ is a set of valid legal norms.
  Proof: $LOrd^0$ is created by the *founder legal organ* ag_0. By the *basic norm*

$$\frac{}{\langle \emptyset, \emptyset, \emptyset, \text{StAgSoc}_0 \rangle \overset{ag_0 \triangleright fndAct}{\longrightarrow} \langle \text{LOrd}, \text{LOrg}, \emptyset, \text{StAgSoc}_0 \rangle}$$

  ag_0 is *assumed* to be legally authorized, in an unconditional way, to create $LOrd^0 = \text{LOrd}$. So, $LOrd^0$ is a set of valid legal norms.

– **Induction Step:** Assume that $LOrd^t = \text{LOrd}$ is a set of valid legal norms. One notes that the only way to have $LOrd^{t+1} = LOrd^t \cup \{nrm\}$ is through the *Norm Creation Rule*:

$$\frac{N \notin \text{LOrd} \qquad Auth(Ag, createlnrm(nrm)) \in \text{LOrd}}{\langle \text{LOrd}, \text{LOrg}, \text{RLFact}, \text{StAgSoc} \rangle \overset{ag \in Ag \triangleright createlnrm(nrm)}{\longrightarrow} \langle \text{LOrd}', \text{LOrg}, \text{RLFact}, \text{StAgSoc} \rangle}$$

where $LOrd^{t+1} = \text{LOrd}' = \text{LOrd} \cup \{nrm\} = LOrd^t \cup \{nrm\}$, where nrm is created by a legal organ legally authorized to created it, being, thus, a valid legal norm. So, $LOrd^{t+1}$ is a set of valid legal norms.

The only other operation that may affect $LOrd^t$, namely, *deroglnrm*, can only *remove* legal norms from $LOrd^t$, so that it also can only make of $LOrd^{t+1}$ a set of valid legal norms.

– **Conclusion:**
  For any time t, the set $LOrd^t$ is a set of valid legal norms.

9.5 Completeness of legal orders

9.5.1 Completeness in Kelsen's concept of legal systems

Completeness of a legal order (the property that legal orders have no "gaps" Kelsen 2009) is the property according to which for each conduct (social or legal) brought

to trial by a legal organ, there remains no doubt as to whether a legal act should be perfomed, or not, as a sanction for the occurrence of the conduct being tried.

Kelsen's position about the property of *completeness* is clear: *legal systems are complete*, for *completeness* means that:

- either there is a legal norm such that the conditions of the case satisfy the condition of the legal norm and, then, the sanction determined by the legal norm ough to be applied;
- or, there is *no* legal norm that applies to the case in a straightforward way, but the legal organ responsible for the trial *has a legal authorization* to create a *new* legal norm, to deal with the case: the case may, then, be tried on the basis of this new legal norm, created specifically to deal with it;
- or else, both there is *no* legal norm that applies to the case in a straightforward way and the legal organ responsible for the trial has *no* legal authorization to create a new legal norm to deal with such case: the case should, then, be dismissed, due to the lack of a legal norm straightforwardly applicable to it and to the legal impossibility of creating a specific legal norm.

Whatever the case, one sees that the *completeness* property guarantees that the case being tried has a *definite* legal solution.

The issue of the *completeness* of legal systems, and the problem of the so-called *hard cases*,[35] which usually rise such issue, is a highly debatable one, as will be discussed in Sect. 11.

9.5.2 Completeness of legal orders in situated legal systems

Completeness property For any time t, any agent ag, and any conduct c, an occurrence of which was registered as a legal fact (i.e., $did(ag, c) \in \text{RLFact}^t$), either the occurrence of the conduct c ought to be sanctioned or the occurrence of the action c ought not to be sanctioned.

Proof

- In the operational semantics, *completeness* can be verified by analyzing the rule for the *imputation of a sanction*:

$$\frac{did(ag, c) \in \text{RLFact} \quad Ag : Ct \Downarrow Ag'' : Ct'' \in LOrd \quad Aut(ag', aut(Ag'', Ct'')) \in LOrd}{\langle LOrd, LOrg, RLFact, StAgSoc \rangle \xrightarrow{ag' \triangleright aut(Ag'', Ct'')} \langle LOrd', LOrg, RLFact, StAgSoc \rangle}$$

in which one notice that, for each imputation $(Ag : Ct \Downarrow Ag'' : Ct'')$ that one can take into consideration:

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[35] Dworkin (1977) made extensive use, in his argumentations, of the expression *hard case*. The meaning he assigns to that expression is identical to that which (Hart 2012) assigns, namely, a case to which no legislation applies in a straightforward way, either because of the plain *absence* of a pertinent legislation, or because of a *lack of clarity* in the extant legislation, regarding the concrete circumstance involved in the case.

- either the conduct c that is being judged is of the type Ct of conducts sanctioned by the norm;
- or the conduct c being confronted is not of the type Ct of conducts sanctioned by the norm.

Thus, we see that whenever an occurrence of conduct c is registered as a legal fact, at any time t, if there is at least one imputation $(Ag : Ct \Downarrow Ag'' : Ct'')$ for which c is of the type Ct, then there is a derivation rule that can be applied to the case, and the occurrence of the conduct c ought to be sanctioned. Otherwise, the occurrence of the conduct c ought not to be sanctioned.

Notice, however, that it is often subject to debate if the conduct c is (or, is not) of the type of conduct Ct indicated as a condition of a given imputation. Thus, the mere recording of the legal fact $did(ag, c) \in$ RLFact is no guarantee that a single conclusion can be achieved, as if by deduction, about the applicability or not of some sanction to the performance of that conduct.

That is, the issue of the *mechanical undecidability* of the *legal typification of conducts* preserves a space for the *discretion* of the legal organs, in that respect. But that *mechanical undecidability* does not obviate the property of completeness, for whatever the solution the debate may find for the issue of the applicability of the derivation rule, either the derivation rule will be determined as applicable, and the sanction authorized, or the derivation rule will be determined as non applicable, and the sanction not authorized, so that the case, in either alternative, will have a definite solution.

On the other hand, in a situation where there is no imputation $Ag : Ct \Downarrow Ag'' : Ct''$ straightforwardly applicable to the conduct c, and no debate can determine its applicability (and, thus, no derivation rule is determined as applicable), but a legal organ is legally authorized to create a new legal norm, specifically applicable to the case (and, thus, a new derivation rule), the situation turns out to become like the first one, for then the creation of the new imputation (and derivation rule) fulfills what is required by the completeness property, namely, that the case has a definite solution. In a similar way, the requirement for the completeness property is fulfilled if the no imputation (and no derivation rule) is straightforwardly applicable to c, and no debate can determine its applicability, and no legal organ is legally authorized to create a new imputation (new rule) specifically applicable to c, for then the rule can not be applied, and no sanction can be authorized, so that again the case has a definite legal solution.

10 Recurring misunderstandings

In this section, we briefly treat some of the misunderstandings about Kelsen's theory of law, which have recurrently appeared since its formulation in Kelsen (2009). The reason for dealing with such issues in the present paper (given that, in strict sense,

they are unrelated to our work) is prophylactic: to try to avoid that they plague the notion of *situated legal systems* that we are introducing here.

This will help us, also, to put into proper perspective, in the next section, some of the attacks that Kelsen's concept of legal system has suffered, from rival theories, and which may also end up by turning upon *situated legal systems*.

10.1 Kelsen's "positivism"

The most common misunderstanding about the "positivist" stance adopted by Kelsen is to interpret his "positivism" strictly in the sense that the term usually has in the philosophy of science, that is, as the point of view that admits the possibility of a purely "descriptive" scientific attitude toward an object of study, which a scientist could take in neat separation from any voluntary "evaluative" attitude toward that object (Ladyman 2001).

Clearly, Kelsen is a "positivist", in this sense (as is also Hart 2012, see the confrontation in Sect. 11).

A strong version of this positivistic stance, however, would also imply a refuse of the possibility of a *scientic* treatment of the (then called) "subjective", "metaphysical", "speculative" or "qualitative" aspects of the object being studied, accepting as the only aspects amenable to science those that, in some sense, can be "measured" or "formalized".

To such strict point of view, however, Kelsen should not be associated: it should be clear from all exempt reading of Kelsen (2007, 2009), that he does not refuse the possibility of the scientific treatment of the *non-positive* aspects of law (political, ideological, moral, etc.), but just that he considers them to be *non pertinent* to a *pure* theory of law.

But, then, Kelsen should be considered a "positivist" also in this other, more specialized and crucial sense: that he considers that the *pure* the theory of law should concentrate just on the *positive* aspects of law, and leave the *non-positive* aspects to their corresponding specialized sciences (political science, sociology, etc.).

This second sense of Kelsen's "positivism" is the one that should be put in frontal opposition to, e.g., Dworkin's point of view (Dworkin 1977, 1986), that the *core* of the theory of law should be *interpretive* of those non-positive aspects, and thus also *evaluative* and *justificatory* of them (see Sect. 11).

From the misunderstanding about this second, more specific sense of Kelsen's "positivism", seems to follow all the other misunderstandings about Kelsen's theory of law, some of which we examine presently.

10.2 The "non-sense" of the basic norm

In Sect. 8, we stated the derivation rule for *basic norm* as follows:

$$\langle \emptyset, \emptyset, \emptyset, \mathrm{StAgSoc}_0 \rangle \xrightarrow{\mathrm{ag}_0 \triangleright \mathit{fndAct}} \langle \mathrm{LOrd}_0, \mathrm{LOrg}_0, \mathrm{RLFact}_0, \mathrm{StAgSoc}_0 \rangle$$

where:

- $\mathrm{ag}_0 \in \mathbf{Ag}$ is the *founder agent* of the legal system;
- *fndAct* $\in \mathbf{LAct}$ is the *founding act*.

Two common misunderstandings about the idea of the *basic norm* are the following.

First, that the *basic norm* is the *constitution*, not a separate, specific norm: in fact, the *constitution* should be considered to be just the *initial* part of the legislation, the part directly issued by the founder agent, but it is not the *basic norm*, which is a norm *assumed* to be legally valid in order to allow for the *legal validity* of that initial part of the legislation (the constitution).

Second, that the basic norm is no more than a *fiction* of Kelsen's mind, since in no legal order one can find a *positive* formulation of the basic norm: in fact, the basic norm *can not be* a positive part of the legal order, for it is precisely the norm that, assumed to be valid *previously* to the creation of the legal order, allows for its establishment as a set of *legally valid* norms.

Thus, it should be no surprise that, in no legal order $LS = (\{LOrd^t\}_{t \in T}, \{LOrg^t\}_{t \in T}, \{LFact^t\}_{t \in T}, createlnrm, deroglnrm, createlauth,$ $cancellauth, recordlfct, deletelfct, ag_0)$, one can find a time $t \in T$ for which there is a norm $nrm \in LOrd^t$ that is the *basic norm* of *LS*.

In case one insists in having, in the notation we are using, a formal expression for the *basic norm* of a legal system, even if acknowledging that it is not a norm pertaining to the legal order of the legal system, one could try this:

$$circ_0 \Downarrow Auth(\mathrm{ag}_0, \mathit{fndAct})$$

where $circ_0$ indicates the *initial circumstance*, for which no particular conduct is specified as a condition for the imputation, but which represents the circumstance where the decision to create the legal system is taken.

This formulation makes explicit the peculiar situation in which the founder agent ag_0 finds itself: in need of receiving a *legal authorization* for the performance of the *founding act*, but an authorization that can not be given through a *legal act* in the legal system (precisely because the legal system has not been founded yet).

Which means that the "triggering" act, providing the initial authorization, required by the founder agent, has to be of a different nature. Clearly, it can only be a *political authorization*.

To articulate an appropriate formal solution to this peculiar situation (the need of giving *legal validity*, in the legal system being created, to a *political act* performed before the creation of the system) is that the *basic norm* is *assumed* to be legally valid.

More specifically: to allow that the *political authorization* for the founder agent to found the legal system, be construed also as a *legal authorization* for the founder agent to determine the *initial legal configuration* of the legal system that is being

founded, so that such initial configuration can occur, from the start, as *legally valid* in the legal system being created.[36]

As a consequence, in any *situated legal system* one should find that the *initial legal configuration*, resulting from the (in itself, just political) *founding act*, can only be a configuration

$$\langle \text{LOrd}_0, \text{LOrg}_0, \text{RLFact}_0, \text{StAgSoc}_0 \rangle$$

where the *initial record of legal facts* is such that $Auth(\text{ag}_0, fndAct) \in \text{RLFact}_0$.

One can clearly see this effect in a classical constitutional example: in the *legal system* of the United States,

the *initial legal fact*, legally authorizing the *founder agent* to found the legal system, which we formally denote, in our *abstract syntax*, by

$$Auth(\text{ag}_0, fndAct) \in \text{RLFact}_0$$

is *concretely* recorded, in the beginning of the *Preamble* to the Constitution, as:

> We the People of the United States, in Order to form [...], do ordain and establish this Constitution for the United States of America.

clearly indicating the (representatives of) the *People of the United States* as the founder agent ag_0, determining that the *ordainment* of the Constitution is, at the same time, the (then, legally authorized) establishment (*founding act*) of the legal system, as well as nominating *United States of America* the corresponding country and its *state*.

10.3 The "philosophical poverty" of the positivist theory of law

Another misunderstanding about Kelsen's "positivism" concerns not the coherence of his conception, but a fear that a certain "philosophical impoverishment" will be imposed on the *theory of law* by that conception. A fear that may be expressed by questions such as:

– Isn't it conceptually too restrictive to say that "political" issues as central as, e.g., the political and moral origin of the right of the founder agent to found the legal system, should not be treated by the theory of law?
– Isn't the theory of law giving up to other fields of knowledge a subject as essential to law as this?
– Isn't the theory of law impoverishing itself too much by saying "this is not a subject that should concern us"?

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[36] Notice the double point of view required by this account of the founding act of a legal system: a combination of a socially *causal* element (the *political authorization* for the founding act), and a *normative* element (the *assumed validity* of the basic norm). From this perspective, Kelsen's solution to the problem of the founding of legal systems is analogous to that by Hart, who similarly states that the *ultimate rule of recognition*, which determines the identity of the legal system, has to be seen from this double, political and normative, point of view (Hart 2012).

Kelsen's point of view seems to be: Yes, it is conceptually impoverishing to keep such issues outside the scope of the theory of law, but this is the price to be payed to have a minimally "positive" theory of law, that is, one that is minimally resistant to become overtaken by subjective (and, thus, ideologically debatable) principles.

10.4 The "reducibility" of authorizations to coercive norms

The misunderstanding about the issue of *authorizations* concerns the way that they are related to *imputations*, which are a basic form of *coercive norms*: Is it possible to reduce *authorizations* to *coercive norms*? Are they to be treated as two *distinct types* of legal norms? Are authorizations *legal norms*?

As is well known, it is a point strongly made by Hart (2012), that *authorizations* and *coercive norms* are *two different types* of norms.

Unfortunately, while arguing in favor of such difference, Hart states that Kelsen's view is that *authorizations* are reducible to *coercive norms*, and this "reading" of Kelsen seems to have propagated there on.

Hart's reading of Kelsen, though, seems not to be a correct reading. In fact, what Kelsen says (Kelsen 2009) is the following.

An expression like "authorized conduct" should be taken in the *technical* sense of "conduct allowed by a legal norm to be considered as a condition or a consequence of an imputation".

In other words, the expression "authorization" should be understood in one of two senses: either in the sense of an *explicit* authorization (i.e., the determination of the *legality of the result* of the conduct, be such result a new legal norm, or a loss or injury caused by a sanction), or in an *implicit* sense (i.e., the determination that certain conduct is "authorized" to be sanctioned).[37]

Thus, if a legal norm *prohibits* that certain individuals perform a certain conduct, that conduct should be considered as "authorized" by that legal norm, in the specific technical sense of being a conduct "authorized" to be sanctioned.

One sees, then, that the reading of Kelsen that attributes to him the idea of reducing *authorizations* to "coercive norms" (i.e., *imputations*) is not a correct one.

In fact, for Kelsen, *authorizations* are not *imputations*, and to him there is no sense in trying to reduce them to *imputations*. From his point of view, the correct statement is that both *conditional* and *unconditional authorizations* are established, not directly in the form of norms, but indirectly, *through* imputations, either *explictly* (as consequences of imputations) or *implictly* (as conditions of imputations).

Thus, Kelsen's point of view about expressions like *authorizing norm*, *competence norm*, or *power confering norm* (that Hart takes as a central aspect of his own theory Hart 2012), is that they should be considered as part of the *surface structure* of the legal discourse, not part of its *deep structure*.

In the formalization that we are giving in the present paper, we introduce formal denotations just for *authorizations* taken in the *explicit* sense, through expressions of

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[37] In either case, the conduct must be admitted by the legal system in its *material sphere*.

the form $Auth(Ag, Ct)$. *Authorizations*, taken in the *implict* sense are not explicitly represented, in our formalization.

10.5 The "logic" of legal reasoning

Another misunderstanding is that "positivism" implies the idea that *legal reasoning* is a plain *deductive* process.

This idea is clearly refused by Kelsen (2009), the most emphatic refusal concerning the idea that the *validity* of legal norms can be determined by purely logical means.[38]

As discussed above, the idea that the *validity* of norms can be determined by purely deductive means should be refused for the simple reason that any operation of *creation* of legal norms is based on a *decision* by a legal organ, and any decision involves the *discretion* of the legal organ, which can not be treated in any plain logical way, so that *no pure deduction* (which disconsiders the *discretion* of the legal organs responsible for the creation of legal norms) can determine the set of logical norms that are *valid* in a legal order, at any given time.

By the same token, the idea of the logical deductibility of the validity of an *application* of a legal norm should be refused, for such application also involves the *discretion* of the legal organ that applies the law.

In other words, *legal reasoning* [as strongly emphasized, e.g., by Dworkin (1986), see Sect.11] is not a plain *deductive process*, and Kelsen readily agrees with that (Kelsen 2009; Kelsen and Klug 1981).

Clearly, however, this rejection of the possibility of a plain logical treatment of legal reasoning does not imply that legal reasoning can not be formally presented by means of *non-standard* logical formalisms (like non-monotonic, defeasible, etc).

The fact that legal systems have a *dynamic* structure, determined by the presence of *norm-changing norms* that give to legal reasoning a *non-standard* logical form (to include the effects of norm derogation and norm conflicts), implies that, as shown in Governatori and Rotolo (2010), the *temporal structure* necessary for modeling the dynamics of legal reasoning in an adequate non-standard logical way may happen to be different from the usual, and more convenient, linear time structure (which is enough for modeling the dynamics of the situated legal system as an organized whole, as shown in the present paper).

This makes clear, thus, another advantage of the *separation of concerns* supported by Kelsen's theory of law: the dynamics of a situated legal system (its temporal evolution, together with the temporal evolution of the agent society in which it is situated), may be formalized in a way (e.g., with an *operational semantical model*, based on a *single, linear time structure*, as in the present paper) that is independent of the ways in which are formalized the legal reasoning processes performed by the legal organs that are acting within that situated legal system.

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[38] See Kelsen and Klug (1981), specially the chapter with Kelsen's manuscript about *Law and Logic*.

Such possibility allows, for instance, that different logical models of legal reasoning be used, by different legal organs, in their legal working, while the semantical model of the situated legal system, which may be common to all the agents of the society, is kept one and the same for all legal organs, independently of the logic model they use for their individual legal reasonings.

That should be contrasted with alternatives like, e.g., Governatori and Rotolo (2010), where a *double time structure* was required to support, integrated in one single model (based on *Defeasible Logic*), both the formalization of the dynamics of the legal system and one particular legal reasoning model, which is, thus, required to be adopted by all legal organs operating in the society.

10.6 Discretion and the institutional appropriation of social facts as legal facts, through *counts-as* rules

Finally, an interesting issue to be clarified concerns the distinction between approaches, like Kelsen's, that take *discretion* as the basis for the operation of *legal* (or, more generally, *institutional*) *appropriation* of social facts, and their recording as *legal* (and *institutional*) *facts*, and approaches, like that in Governatori and Rotolo (2008), that follow J. Searle's proposal (Searle 1995), of basing such kind of institutional appropriation on *institutional rules* of the form $X\,counts-as\,Y\,in\,C$ (that is, in Searle's terms, "brute fact X counts as institutional fact Y in the context C").

Regarding *legal systems*, the discretion-based approach, adopted by Kelsen, presents the advantage, over the "counts-as"-based approach, of allowing for the *individualization* of the legal decision-making about the appropriation of *social facts* as *legal facts*, giving space for the *legal organs* responsible for such type of decision to take into account values, maxims, and inclinations that are not explicitly acknowledged in the legal order, when deliberating if a "brute fact" should, or should not, be appropriated as a legal fact.

In oposition to that, the "institutional rules"-based approach (the approach based on "counts-as" rules) provides an *impersonal* frame of reference, which, if taken as exclusive, provides a picture that seems not to correspond to the *real practice* of legal reasoning and decision-making, as it has often been characterized [e.g., by Dworkin (1977), see Sect. 11 for more details].

Analogous argument applies, concerning the determination of which norms are *legally valid* in a legal system, when considering the notion proposed by Hart, of the "rule of recognition" (see also Sect. 11 for more on this): if the "rule of recognition" is, contrary to Hart's idea, taken to be an impersonal "count-as" rule, the account of the legal system misses the discretion inherent in the deliberation about what is, or is not, a *valid legal norm* (which, as Hart agrees, is a debatable issue, in general).

11 Confrontation with Dworkin, Hart, and Raz

To illustrate the usefulness of the formalization introduced in the paper for the critical analysis of the confrontation between *positivist* and *non-positivist* theories of law, we do include, in the present section, a brief discussion of an important and most influential non-positivist theory of law, highly antagonist to the positivist ones, namely, that by Dworkin (1977). In addition, we consider, also in a brief way, Raz's (1970) general meta-theoretical criteria for *theories of legal systems*, and his criticisms to the Kelsenian concept of legal system.

While analysing such rival theories and points of view, we further the presentation of the reading of Kelsen's theory of law that guided the elaboration of the operational semantical framework introduced in the present paper.

Clearly, however, the scholarship about Kelsen's approach to law, and the variety of readings that his works allow, is vast (see, e.g., Paulson and Paulson 2007), and as such its appraisal is beyond the limited purpose of the present work.

Thus, the choice we have made, to stick to Kelsen's first widely circulated presentation of the *Pure Theory of Law* (Kelsen 2009), is bound to a simple reason: we just needed a fix theoretical framework for the formalization we were working out.

But this choice does not imply that the operational semantical approach proposed here is limited to that particular presentation of the notion of legal system.

In fact, we claim that the approach is general enough to deal with any "positive" notion of law, amenable to acknowledge the above characterized *orthogonality* between the *formal* and *political* principles of legal systems (as is the case of H. Hart's concept of law Hart 2012, for instance, which we also consider in a critical way, in the following).

11.1 Hart

Hart's theory (Hart 2012) draws two main distinctions between *legal rules*:

1. a distinction between *two types* of legal rules, namely:

    – *primary rules*, which regulate the behaviors of the legal subjects;
    – and *secondary rules*, which regulate the way primary rules are created, applied, modified, and identified as pertaining to a given legal system.;

2. a distinction between *two ways* for legal rules to become *binding*, namely:

    – through their *acceptance* by those that are subject to them, but in such a way that the criteria to determine such acceptance need not be explicitly stated, but may just be implicit in the social practices of the legal subjects;
    – and by their becoming *valid*, by virtue of a *secondary rule*, specifically accepted for the purpose of determining such validity, the *rule of recognition*.

Hart (2012), then defines a *legal system* as a system of *primary* and *secondary* rules, where the belonging of a rule to the legal system is determined by the particular *rule of recognition* of that legal system.

Hart states that, since the *rule of recognition* can not be *valid*, for that (as stated in the above distinctions) would imply the need for another secondary rule, to recognize it, and so on (in an infinte regression), the *rule of recognition* can only be binding by virtue of its *acceptance* (which allows that the *rule of recognition* needs not be explictly and formally stated).

Thus, there is a clear *structural* difference between Hart's concept of law, and that of Kelsen, and such difference implies a crucial *foundational difference* between them, namely, that concerning the *source of validity* of legal norms.

Kelsen's conception of the source of validity of legal norms, the *basic norm*, supports the "pure" nature of his theory by clearly separating *factual issues* (like the social practices of the subjects of the legal norms) and *normative isues* (like that of the *validity* of legal norms), in a way that the *normative* features do *not* derive from the *factual* ones (even when the factual features are *external* to the legal system, like the social practices of the *legal subjects*).

That is, Kelsen carefully avoids to infringe the (undoubtedly, highly debated— see, e.g., Porter 1968) *Hume's law*, and to commit the related *naturalistic fallacy* (that is, the adoption of the idea that *normative elements* may be derived from *factual* ones), a precaution that Hart seems not to have cared about, when he took the factual, psycho-sociological disposition of *acceptance* of the *rule of recognition* by the law's subjects and legal organs as the source of the normative property of *validity* of the legal rules.

We remark, however, that Hart's "rule" of recognition is, in fact, a "procedure" of recognition, that is (as Hart himself states Hart 2012), a "complex practice" adopted by courts and legal subjects, in order to be able to determine which are, and which are not, the *valid legal norms* of a legal system.

Thus, it seems that Hart's notion of *rule of recognition* is not, in effect, totally incompatible with Kelsen's notion of *basic norm*, for the latter is assumed to exist in the *normative* realm of the legal system (i.e., in connection to its *legal order*) while the former is determined to exist in the *operational* realm of the system (i.e., in connection to the way of operation of the *legal organs*).

In fact, concerning *situated legal systems*, it seems that one can take Hart's notion of the *rule of recognition* as a conceptualization of the *procedural means* by which the *legal organs* determine the legal validity (or, non-validity) of a given legal norm, while having as a *formal basis* for such determination the assumed validity of the *basic norm*.[39]

Another crucial difference, distinguishing Hart's point of view from Kelsen's, is that concerning the importance of a specific *minimum content* for law.

Again, in this respect, their points of view are not strictly *antagonic* to each other, but—on the contrary—*unrelated* to each other. For, while Hart's point of view stresses the indispensability, for any system of norms, to have a *minimum content*

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[39] An idea that, in fact, can be thought to be implicitly assumed by Hart, in his notion of the *ultimate rule of recognition* (Hart 2012).

(protection for persons, property, and promises), for the system to merit to be called a "legal system," for Kelsen the issue is not about what content (minimum or not) a system of norms should have, in order to merit to be called a "legal system", but simply that such content is not a relevant matter for the *theory of law* (which should, thus, be kept "free" of non-formal issues).

In other words, while Hart is concerned with looking for the set of "essential" set of coercive *functions* that any legal system should perform for the society where it operates (Hart 2012), Kelsen is just concerned with the coercive function of law in its most general sense (thus, not with any specific type of coercive functions, such as those included by Hart in his notion of *minimum content* of law).

In the same sense, on the other hand, one sees that Kelsen's point of view *does not fall* under what Hart tersely classified as "positivism", namely, the point of view that "law may have any content" (Hart 2012).

For, Kelsen's idea (Kelsen 2009) (see also Kelsen 2007) is not that "law may have any content" (in the sense that it is enough for a legal system to be correctly structured from the formal point of view, to be considered a "good" legal system, whatever its content), but just that the problem of what specific content a legal system "must have", in each particular society, at each particular moment, is not an issue to be decided by the *theory of law*, but by other theories (political, moral, ideological, etc.), if any.[40]

Thus, here also, one sees that no incompatibility prevents the idea of a situated legal system whose legal order satisfies Hart's requirements for a *minimum content*.

11.2 Dworkin

Dworkin's theory of law has been highly influential since the late 1970s, having presented itself as an alternative to "positivism and utilitarianism" (Dworkin 1977).

A direct confrontation between Dworkin's and Kelsen's theories of law is somewhat problematic to establish, however, since Dworkin never mentioned Kelsen explicitly, in any of his major books (Dworkin 1977, 1985, 1986) (probably, because Dworkin took Hart's theory Hart (2012), not Kelsen's, as the most "powerful" version of "legal positivism").

Notwithstanding that, one may consider the following analysis as one possible way to relate Dworkin's theory of law to Kelsen's, and to find a possible place, in *situated legal systems*, for at least an important part of Dworkin's conceptions.

In Dworkin (1977), Dworkin presents his concept of a *general theory of law* as a theory that has to be "normative as well as conceptual", the conceptual part determining "what the law is", the normative part determining "what the law should be" .

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[40] Interestingly enough, that characterization of "positivism" would put Hart himself outside the field of "positivism", due to his strong concern with the definite minimal content that he thinks every legal system should have (Dworkin 1977, e.g., says Hart is a "moral philosopher"). On the other hand, Hart explictly classifies himself as "positivist", in the usual sense of someone who admits the possibility of independence between *descriptive* and *evaluative* (or, *justificatory*) points of view toward an object of study (Hart 2012).

Thus, in opposition to Kelsen, which considers that the determination of the type of "content" of the legal systems is not a matter of concern for the *theory of law* (but of concern to political, moral, ideological,, etc. theories),

Dworkin states that the normative part of the *theory of law* should be "embedded in a more general political and moral philosophy". In the particular case of the *theory of law* that he proposes, the contents required from legal systems are to have a politically "liberal" orientation, having at their core certain "pre-legal" (thus, political and moral) types of *individual rights* (Dworkin 1977).

Three particular *normative* theories should be encompassed by the normative part of such *general theory of law*, namely, a theory of *legislation*, a theory of *adjudication*, and a theory of *compliance*.

In Dworkin's characterization of those particular normative theories, one may identify *two types of goals* that each should meet: to morally and politically guide the *deliberation processes* of the legal organs involved in the operation of law, and to define the *operational conditions* under which such deliberation processes should be realized.

This is shown in Table 1, which indicates the two main aspects of each one of the particular normative theories identified by Dworkin.

One clearly notices, from Dworkin's explanation of those particular theories (Dworkin 1977), that the "operative parts" of the theories may be reduced to what Kelsen called *authorizations*, because authorizations involve determinations not only of the *legal actions* that they authorize to be performed, and the determination of their corresponding *legal performers*, but also the spatial, temporal, and instrumental *conditions* for their realization.

On the other hand, the "deliberative" parts of the theories, which purport to morally and politically guide the legal organs, concern precisely what Kelsen's theory assigns to the *discretion* of those legal organs. That is, apparently only the *deliberative* parts of the particular *normative theories* encompassed in Dworkin's conception of *general theory of law* are not reducible to elements of Kelsen's theory of law.

Thus, as we see it, Dworkin's conception of law seems to relate to that of Kelsen in a way similar to that with which Hart's conception relates to Kelsen's, that is, more as a possible operational *complement* (dealing with principles for the determination of the contents of legal norms, and of methods for *legal reasoning* and *decision-making*) than as a full-fledged theoretical *alternative* to it.

Thhis point of view seems to be neatly in consonance with the role that Kelsen's theory of law reserves for the (political, moral, ideological, etc.) points of view in the construction and evolution of legal systems, namely, the driving of the legal system toward certain directions, regarding its "content".

Table 1 Normative part of a "general theory of law", according to Dworkin (Dworkin 1977)

Normative theory	"Deliberative" part	"Operativel" part
Theory of legislation	Theory of legislative justice	Theory of legitimacy
Theory of adjudication	Theory of controversy	Theory of jurisdiction
Theory of compliance	Theory of deference	Theory of enforcement

Moreover, this point of view seems also to be in neat consonance with Hart's own point of view about Dworkin's theory, which Hart exposed in the *Postscript* to Hart (2012), stating that it was "not obvious why there should be or indeed could be any significant conflict" between his and Dowrkin's theories (Hart 2012).[41]

On the other hand, one should acknowledge the *foundational difference* between the two theories being confronted here, namely, the difference in the way each of them determines the *source of the validity* of legal norms.

For, while Kelsen plainly puts that source of validity of legal norms *inside* the *legal order* proper, in its *basic norm* (so that his notion of *validity* of legal norms is kept "pure", i.e., exempt of components *external* to the legal order), Dworkin puts the source of validity of legal norms *outside* the *legal order*, in the *principles* (Dworkin's terms are: "principles, policies, and other sorts of standards" Dworkin 1977, Chap. 2) that determine the *practice* of the legal organs of the legal system.

In other words, while Kelsen purports to present a *pure* foundation for law, in an almost *transcendental* kantian sense (Paulson and Paulson 2007), Dworkin seems to aim at a more concrete and specific *social*, or *cultural*, foundation, seeking support for the validity of legal norms in certain "liberal" moral, political, and economical principles (having the *individual rights* as their core): such *principles* would be responsible for determing both certain *standards of behaviors* for the *legal organs*, and certain *standards of content* for the legal norms.[42]

However, the crucial *conceptual* difference between the two approaches to law concerns Dworkin's idea that *individual rights* can "pre-exist any form of legislation" (Dworkin 1977).

Dworkin states in the following way his basic conception of *individual right*, taking the notion of a *collective social goal* as a reference (Dworkin 1977):

> Individuals have rights when, for some reason, a collective goal is not a sufficient justification for denying them what they wish, as individuals, to have or to do, or not a sufficient justification for imposing some loss or injury upon them.

We focus here on the first type of right, relative to wishes to do or have something. We use the following expression, in standard logical notation and with an intuitive reading, as our formalization of Dworkin's conception of such type of right (*ag* is an agent, *r* is a right, and *g, g'* are goals):

$$\forall ag, r :$$
$$hasIndividualRight(ag, r) \Leftrightarrow$$
$$\exists g(hasGoal(ag, g) \land \neg\exists g'(collectiveGoal(g') \land justifDenial(g', g)))$$

Notice, firstly, that Dworkin acknowledges that only a *legal system* can bind all the legal subjects of a society to the predicate *hasIndividualRight*, given the possible

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[41] In fact, there seems to be a strong similarity, if not plain identity, between what Dworkin calls *individual rights* and what Hart requires as the *minimum content* of law, when taken in *concrete* terms.

[42] At the end, Dworkin effectively rules out *legal orders* as the main components of legal systems, and plainly treats "principles as law" (Dworkin 1977, Chap. 2), that is, construes legal systems essentially as *systems of legal principles and legal policies*.

variety of opinions of the legal subjects about the relative strengths of the different *collective* and *individual goals* that may be taken into consideration.

For, Dworkin's conception is not that *hasIndividualRight* would be binding previously to the existence of any *legal system*, as would be the case if his notion of *individual right* were a variation of the classical notion of *natural right*. His conception is, rather, that *hasIndividualRight* can be binding previously to the existence of any *pertinent legislation* concerning the object g (in confrontation with g') of the presumed right.

That is, the issue concerns not the way the predicate *hasIndividualRight* can become binding in general, but the way an individual instance of it, $hasIndividualRight(ag, r)$, can be determined to be binding in a given *hard case*, for, then, the *legal validity* of that $hasIndividualRight(ag, r)$, so determined, may serve as a *principled guidance* for solving the hard case being tried.

For Dworkin, the determination of the validity of an *individual right*, in a given *hard case*, should result from a judge "discovering" the "pre-existence" of that *individual right*, in that case (Dworkin 1977).

Given the non-applicability of a *pertinent legislation*, that "discovery" has, then, to be guided by certain *moral* and *political* principles (that Dworkin exposes Dworkin 1977), which would serve also the purpose of supporting the argument for the validity of the thus "discovered", "pre-existent" *individual right*.

However, that seems to imply that for Dworkin (in opposition to Kelsen's notion of *authentic intepretation* Kelsen 2009[43]) there is no space for *innovations* in the interpretation of law, in the sense that *legal organs* are never allowed to provide solutions to legal cases on bases that are *new* to the legal system and to its social context, but should always look for, and make use of, *standards* that are already *established* in the culture of the society.[44]

Thus, Kelsen's understanding of the solutions to *hard cases* seems to be not in complete opposition to that of Dworkin: the difference seems to be just that, for Kelsen, the *theory of law* should not be concerned with guiding the conduct of the legal organs, when dealing with such *hard cases*, while for Dworkin, that guidance should be taken as a legitimate concern for that theory.[45]

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[43] Kelsen calls "authentic" (i.e., *creative of new valid legal norms*), not only an interpretation that instantiates for the concrete case being tried (as a new, *individual* norm) one of the possibilities within the frame of possibilities determined by the legal norm appropriate for the case, but also, in the trial of a *hard case*, an interpretation that solves the case by *creating* a new norm, specific for the case, if it happens that the legal organ responsible for the trial is *legally authorized* to create such a norm, and this even if it happens that there is no valid legal norm that can determine the scope of content of the new norm, which means that this content is wholly determined by the *discretion* of the legal organ that creates it.

[44] In Dworkin's terms Dworkin (1977), the solution of *hard cases* is achieved by "reaching beyond the law for some other sort of *standard* to guide him [the judge] in manufacturing a fresh legal rule or supplementing an old one." (our emphasis).

[45] Kelsen's expression about this problem is that it is not a "problem of legal theory but of legal politics" (Kelsen 2009).

11.3 Raz's meta-theoretical requirements for theories of legal systems

Joseph Raz's thesis (Raz 1970), written under the supervision of Hart, presents an interesting attempt to produce a systematic and encompassing *analytical framework* for theories of legal systems, including *criteria* for the analysis of existant theories and for the development of new ones.

Raz construes the idea of a *theory of legal systems* as any theory aiming to solve four main problems:

1. The problem of the *existence* of legal systems: *What are the criteria to determine if a legal system exists, or not, in a given location and time?*
2. The problem of the *identity* of legal systems (and the correlate problem of the *membership* to legal systems): *What are the criteria to determine to what legal system belongs a given norm, and the set of laws that belong to a given legal system?*
3. The problem of the *structure* of legal systems: *Is there a structure that is common to all legal systems? Are there different types of legal systems?*
4. The problem of the *content* of legal systems: *Is there a content that is common to all legal systems? Are there different sets of contents, characteristic of different types of legal systems?*

Concerning the problem of the *existence* of legal systems,[46] Raz states a distinction between theories that are based on *material* and on *formal* criteria for determining the identity of legal systems (Raz 1971): the first ones try to determine the identity of legal systems on the basis of their content, the latter on the basis of their form. One may say, in that sense, that Dworkin's theory is of the first kind, while Kelsen's and Hart's are of the second kind.

Concerning the problem of the *identity* of (*membership* to) legal systems, one may say that Dworkin seeks to determine it on the basis of *principles and policies*, Kelsen on the basis of the *basic norm*, and Hart on the basis of the *rule of recognition*.

Concerning the problem of the *structure* of legal systems, one may say that all the three theories that we have examined adopt the idea that there is a general structure *common* to all legal systems, but that those general structures are *different* for each theory, although both Hart's and Dworkin's theories refer to law as it presents itself in its "surface" structure, while only Kelsen's theory aims to explicitly reach its "deep" structure. Raz's theory attempts that only implicitly, regarding what he calls the *problem of the individuation of norms*, see below.

Concerning the problem of the *content* of legal systems, it becomes clear that Dworkin is explicitly concerned with such issue (even though he provides an answer restricted to just one particular type of content, the *politically liberal* one), while Kelsen states that such issue does not concern explicitly the *pure* theory of legal

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[46] Notice that the problem of the *validity* of the legal norms, and of the legal system as a whole, is not made explicit by that list of four problems. One may conceive that it is *implict* in the combination of the problems of *existence* and *identity*. That, however, is in direct opposition to Kelsen's approach, which emphasizes the importance of the *explicit* treatment of the problem of the *validity* of legal norms.

systems (the only one he provides), and Hart provides important remarks about the separation (and systematic connections) between *law and morals* in general, and determines a *minimum content* for law (protection for persons, property, and promises).

Raz's own theory of legal systems, in the final part of Raz (1970), aims at an "improvement" of Kelsen's theory of legal systems, through the incorporation of contributions found in Bentham and Hart, with the core "improvement" being an alternative formulation of the *source of the validity* of legal norms, allowing the suppression of the "doctrine of the *basic norm*". The detailed analysis of such theory, however, falls out of the scope of the present paper.

We remark, however, an important part of Raz's work, namely, his methodological effort to establish principles for solving what he calls the *problem of the individuation* of legal norms. That is, an effort to establish a *method* for determining the *deep structure* that underlies the *surface structure* of the set of legal texts of a *legal system*, a methodological concern to which Kelsen gave no explicit attention.

12 Complementary topics

We treat in this section, in a very brief way, three complementary topics, which are relevant for the subject of *situated legal systems*.

12.1 Formalization of history-savvy situated legal systems

The concept of *legal system* adopted in the present paper, being based on Kelsen (2009), concerns Kelsen's account of *statutory law*, not *customary law*, which he considered only later, in Kelsen (2007).

From the *operational* point of view, the formalization of *customary law*, with its *creation* and *derogation* (i.e., *desuetude*) of norms on the basis of the evolution of *social customs*, introduces the particular difficulty of providing a formal account of such type of system evolution.

That difficulty is similar to the one that should be overcome in the treatment of an issue that we have not treated in our formalization, namely, the issue of the *efficacy* of legal systems.

As is well know in *legal theory*, and Kelsen stresses it adequately (Kelsen 2009), a necessary condition of the *validity* of the legal norms of a legal system is that of the *efficacy* of the system's norms: legal norms whose enforcement can not have a *minimal level* of efficacy in the social system where the legal system operates, should be considered *non-valid*.

In both cases (the formalization of *customs* and the formalization of the *efficacy* of legal norms), an explicit *formal model* should be provided for *history-savvy situated legal systems*, something that is only implicit in the formalization we gave here.

12.2 Normativity in multiagent systems

Normativity is by now an established subject of research in the Multiagent Systems area (Boella et al. 2006), with the usual approach being to tackle legal issues through formalizations based on *deontic logic*, or on other specific types of logics (defeasible, input-output, etc.), as surveyed, e.g., in Grossi and Rotolo (2011).

However, most of the work carried out following this direction seems to have been concerned only with the logical aspects of the *legal reasoning* and *decision-making processes* of legal organs (see, e.g., Governatori and Sartor 2011, or Grossi and Rotolo 2011), not with legal systems as *functional systems* situated in given agent societies, as we have aimed here.

12.3 Process-based models of legal systems

In this paper, we have introduced a formalization of Kelsen's concept of *legal systems* in terms of an *operational semantical framework* based on the model of *transition systems*, whose general principles were first introduced in Plotkin (1981).

Since, from an operational point of view, an *imputation* is a relation between two *events* (that is, two *occurrences* of actions), an alternative formalization may possibily be given (perhaps in a more direct, simpler, and abstract way than we have done here), in terms of *process-based* models, *à la*Hoare (1985) or Milner (1980).

As a first step toward the formal capturing of the *structural and operational features* of *legal systems*, however, we believe that the *transistion systems* approach that we have adopted is more convenient, due to its being more concrete, and apparently more intuitive.[47]

13 An illustrative example

In this section, we illustrate the use of the proposed *operational semantical framework* to derive the validity of legal decisions in a sample *situated legal system*, viewed from the perspective of agent-based *social simulation* studies (Gilbert and Trotzsch 2005). The example concerns the application of a fine to a delict, regarding a public policy that regulates fishing activity in Brazil. In the example, one can see the use of the StAgSoc component of the legal configuration of situated legal systems as a source of *social facts* for the semantical *derivation rules*.

13.1 Situated legal systems and the modeling of public policy processes

For the application of legal systems and their operational semantics to the *agent-tebased modeling and simulation* of the execution phase of public policy processes, the framework depicted in Fig. 4 should be useful (Costa and Santos 2012).

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[47] At the price of having to make crucial use of an auxiliary set, the *record of legal facts* RLFact, which is an *ad-hoc* addition to Kelsen's theory.

The central feature of the framework is the structure of the *public policies* themselves, which are cast as *sets of norms and plans*.

Public policies are assumed to be emitted by an *issuer agent* (the *Government*, in Fig. 4), and *government agencies* are taken to act as *detectors* and *effectors* (for both norms and plans), following the architecture for public policy systems proposed in Hood (1983) (see also the location of *legal systems* in agent societies, shown in Fig. 1).

The framework is not only *agent-based*, but also *artifact-based*, meaning that the concept of *artifact* (Ricci et al. 2006) is used to computationally model the *structural components* of the legal systems (in particular, the *plans* and *norms* that constitute the public policies), which are cast as the so-called *legal artifacts* (Costa and Santos 2012).

Besides *design patterns* for legal artifacts, the framework also defines design patterns for the *behaviors* that the agents, both *social* and *legal*, may have toward the public policies norms and plans (both to comply and to not comply with those norms and plans).

13.2 The public policy for the *Defeso*

In Brazil, the activity of *fishing* is regulated by IBAMA (Brazilian Institute for the Environment and Renewable Natural Resources),[48] which annually decrees specific regulations for each region in the country, with special attention to the restrictions on fishing activities during the spawning period, called the *defeso*.[49]

The typical content of a regulation decree for a *defeso*, in a given area, can be summarized as follows:

– This year, the *defeso* period in the area runs from October 15th to February 28th.
– During the *defeso*:

1. Fishing in the area is allowed only for the subsistence of native people (max. of 03 kg per fisher per day, for local consumption) and for scientific research.
2. Fishing boats are strictly forbidden in the rivers of the area.
3. Fishing can only be performed with hand-held fishing sticks or hand-held fishing lines, with natural or artificial baits.
4. It is strictly forbidden to use baits that are exotic to the area.
5. Lines with multiple hooks are only allowed with artificial baits.
6. Native fishers are required to obtain a special license before they start their activity.

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[48] http://www.ibama.gov.br.

[49] http://www.ibama.gov.br/servicos-recursos-pesqueiros/defeso-aguas-continentais.
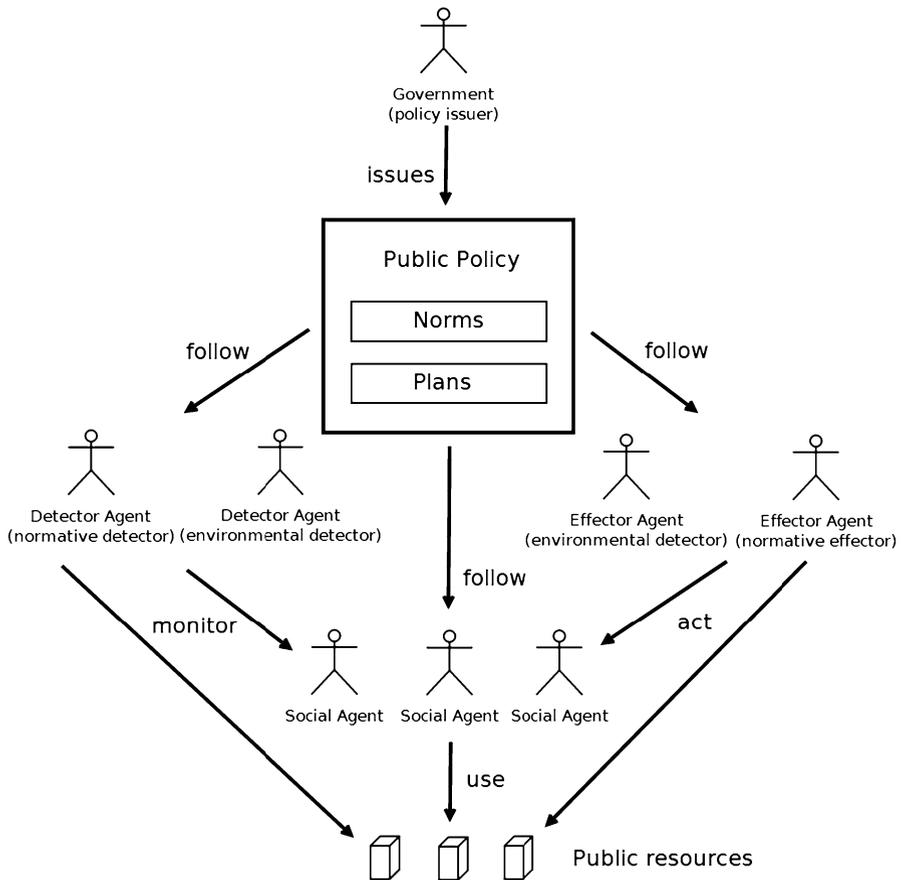
Fig. 4 A framework for the agent-based modeling of the execution phase of public policy processes (Costa and Santos 2012)

7. Industrialization and transportation of industrially processed fish is prohibited in the area, except in the case of farm raised fishes, in which case the farm should be specifically licensed.

8. A certificate of controlled origin is required for transportation of industrially processed fish, in the latter case.

9. Incompliance with this regulation implies liability to fines, retention of fishing equipment and boats, and administrative penalties by the IBAMA inspectors.

10. Surveillance of rivers and roads will be active in the area, and inspectors are formally authorized to inspect fishing equipment, boats, and road vehicles.

13.3 Formal presentation of the public policy

13.3.1 Some of the basic concepts

- **Agent types:**
  - *RegulationAgency*
  - *Inspector*
  - *Fisher*, with *Fisher* = *NativeFisher* ∪ $\overline{NativeFisher}$

- **Action types:**
  - *fine*—impose a fine
  - *fish3kg*—fish more than 3 kg of fishes in a given day
  - *boat*—use fishing boat
  - *boatret*—retention of fishing boat
  - *license*—have a license

- **Corporate Agents:**
  - *IBAMA* ∈ *RegulationAgency*

- **Legal norms:** (concerning regulations 1, 2, 6, and 9)
  - (*NativeFisher*, *fish3kg*) ⇓ (*Inspector*, *fine*)
  - ($\overline{NativeFisher}$, *fish*) ⇓ (*Inspector*, *fine*)
  - (*NativeFisher*, $\overline{license}$) ⇓ (*Inspector*, *fine*)
  - (*Fisher*, *boat*) ⇓ (*Inspector*, *boatret*)

- **Authorizations:**
  - *Auth*(*IBAMA*, *createlnrm*(*fnrm*)), for any fishing norm *fnrm*
  - *Auth*(*Inspector*, *fine*)
  - *Auth*(*Inspector*, *boatret*)

13.3.2 Some of the derivation rules

- **Registration of legal fact about fishing more than 3 kg of fishes:**
  For the *defeso* regulation, the rule R_1

$$\frac{did(nf,fish3kg) \in \text{StAgSoc} \qquad Aut(Inspector, reg(fish3kg)) \in LOrd}{\langle \text{LOrd, LOrg, RLFact, StAgSoc} \rangle \xrightarrow{insp \,\triangleright\, recordlfct(did(nf,fish3kg))} \langle \text{LOrd, LOrg, RLFact}', \text{StAgSoc} \rangle} R_1$$

where:

- $did(nf,fish3kg) \in \text{StAgSoc}$ means that $did(nf,fish3kg)$ is a social fact in StAgSoc
- $\text{RLFact}' = \text{RLFact} \cup \{did(nf,fish3kg)\}$

holds for any native fisher $nf \in NativeFisher$ and any inspector $insp \in Inspector$.

- **Imposition of fine for fishing more than 3 kg of fishes:**
  For the *defeso* regulation, the rule R_2

$$\frac{did(nf,fish3kg) \in \text{RLFact} \qquad Auth(Inspector, createlauth(fine(NativeFisher))) \in LOrd}{\langle \text{LOrd, LOrg, RLFact, StAgSoc} \rangle \xrightarrow{insp \,\triangleright\, createlauth(fine(nf))} \langle \text{LOrd, LOrg, RLFact}', \text{StAgSoc} \rangle} R_2$$

where:

- $\text{RLFact}' = \text{RLFact} \cup \{fine(nf)\}$ where, for the sake of space, $fine(nf)$ encodes an authorization $Auth(lo, charge(nf,fv3kg))$ for lo to charge the fine value $fv3kg$ from nf.

holds for any native fisher $nf \in NativeFisher$, any legal organ $lo \in \text{LOrg}$, and any inspector $insp \in Inspector$.

13.4 Derivation of a legally valid authorization to sanction

To illustrate the use of the proposed operational semantical framework in determining the validity of a legal authorization to sanction, we provide here an example derivation of a legally valid authorization to fine a fisherman for irregular fishing.

This can be obtained, in the context of the formal presentation of the public policy for the *defeso*, by a simple *sequential application* of the two rules, R_1 and R_2:

$$\langle \text{LOrd, LOrg, RLFact, StAgSoc} \rangle$$
$$\xrightarrow{R_1}$$
$$\langle \text{LOrd, LOrg, RLFact}', \text{StAgSoc} \rangle$$
$$\xrightarrow{R_2}$$
$$\langle \text{LOrd, LOrg, RLFact}'', \text{StAgSoc} \rangle$$

where:

- $\text{RLFact}' = \text{RLFact} \cup \{did(nffish3kg)\}$
- $\text{RLFact}'' = \text{RLFact}' \cup \{Auth(lo, charge(nf, fv3kg))\}$

This composition formally realizes one of the *operational intents* of the joint regulations one and nine of the public policy for the *defeso*, namely: to *enable the charging of a fine* from any native fisher that fishes more than 3 kg of fishes in a given day.

At the same time, the composition formally guarantees the *legal validity* of that *enabling*, in the context of the given legislation for the *defeso*, and the concrete situation found in $\langle \text{LOrd}, \text{LOrg}, \text{RLFact}, \text{StAgSoc} \rangle$.[50]

14 Conclusion, and the way ahead

From the perspective of our work, the main idea we derived from Kelsen's theory of legal systems, and from his conception of *law* as a *social technology* Kelsen (2009), is that (in the terms of the architectural style of *agent societies* presented in Sect. 2) legal systems are *open, dynamic social systems* that *normatively* perform *coercive regulation functions* in the agent societies where they are put to operate.

Such idea stimulated us to show, in the present paper, that the *deep structure* of law can be cast in *operational* terms, through an *action-based* semantical framework whose essential actions are *legal acts* performed by *legal organs*.[51]

Even though we haved focused, here, almost exclusively on the *internal operation* of legal systems (as Kelsen himself did), the operational semantics that we have introduced is also capable to support the formal modeling of the way legal systems operate *externally*, that is, is capable to model the way the dynamics of a situated legal system *couples with* the dynamics of its underlying agent society (cf., e.g., (Costa and Dimuro 2009)), a subject that we should tackle in a future work.

In doing that, we will be able to relate our approach, of *situating* legal systems in agent societies, to two other *systemic* approaches to legal systems, namely, the *institutional* approach, by MacCormick and Weinberger (2010), which presents a renewed *positivist* conception of law, and the *autopoietic* approach, by Luhmann (2008), where the notion of "structural coupling" between legal systems and societies is of central importance.

Similarly to the present work, those two approaches also aim to present legal systems as *operational structures* that are simultaneously *normative* and *functional*, in the societies where they are situated.

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[50] On the basis of this type of formalization, an *agent-based model* for this public policy for fishing activity was developed, and preliminary simulation studies relating costs of norm enforcing and positive results for the sustainability of fishing activities were presented in Santos et al. (2012).

[51] That this *action-based* view of legal systems is compatible with Kelsen's own view of law is shown by his general conception of *norms as meanings of acts of will directed toward the conducts of others*, and of *legal norms* as the meanings of such acts, when *legally authorized* (Kelsen 2009).

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References

Alchourrón CE (1969) Logic of norms and logic of normative propositions. Log Anal 12(47):242–268

Alchourrón CE, Bulygin E (1971) Normative systems. Springer, Berlin

Austin J (1995) The province of jurisprudence determined. Cambridge University Press, Cambridge

Bentham J (1970a) The limits of jursiprudence defined. Greenwood Publishing Group, Westport

Bentham J (1970b) Of laws in general. Athlone Press, London

Boella G, van der Torre L, Verhagen H (2006) Introduction to normative multiagent systems. Comput Math Organ Theory 12:71–79

Burrell G, Morgan G (1979) Sociological paradigms and organisational analysis: elements of the sociology of corporate life. Ashgate Publishing, Surrey

Chomsky N (2002) Syntatic structures. De Gruyter Mouton, Berlin

Costa ACR (2014a) Proposal for a notion of modularity in multiagent systems. In: Birna van Riemskijk M, Dalpiaz F, Dix J (eds) Informal proceedings of EMAS 2014. AAMAS@Paris

Costa ACR (2014b) On the legal aspects of agent societies. Open publication on www.ResearchGate.net. doi:10.13140/2.1.4345.7923

Costa ACR (2014c) On the bases of an architectural style for agent societies: concept and core operational structure. Open publication on www.ResearchGate.net. doi:10.13140/2.1.4583.8720

Costa ACR (2014d) Toward a formal reconstruction of Kelsen's theory of legal systems. In: WEIT 2013 - 2nd workshop-school on theoretical computer science, IEEE, New York

Costa ACR, Dimuro GP (2009) A minimal dynamical organization model. In: Dignum V (ed) Hanbook of multi-agent systems: semantics and dynamics of organizational models. IGI Global, pp 419–445

Costa ACR, Santos IAS (2012) Toward a framework for simulating agent-based models of public policy processes on the Jason-CArtAgO platform. In: AMPLE@ECAI 2012 - 2nd international workshop on agent-based modeling for policy engineering. ECAI, Montpellier

Costa ACR, Hübner JF, Bordini RH (1994) On entering an open society. In: Proceedings of the XI Brazilian symposium on artificial intelligence. Sociedade Brasileira de Computação, Fortaleza, pp 535–546

Davis KC (1976) Discretionary justice: a preliminary inquiry. University of Illinois Press, Champaign

Demazeau Y, Costa ACR (1996) Populations and organizations in open multi-agent systems. In: 1st national symposium on parallel and distributed artificial intelligence (PDAI'96). Hyderabad, India

Dworkin R (1977) Taking rights seriously. Harvard University Press, Cambridge

Dworkin R (1985) A matter of principle. Harvard University Press, Cambridge

Dworkin R (1986) Law's empire. Harvard University Press, Cambridge

Fernández M (2014) Programming languages and operational semantics: a concise overview. Springer, Berlin

Fox MS (1979) Organization structuring: designing large complex software. Technical report, Carnegie-Mellon University (Technical Report CMU-CS-79-155), Pittsburgh

Gilbert N, Troitzsch K (2005) Simulation for the social scientist. Open University Press, London

Governatori G, Rotolo A (2008) A computational framework for institutional agency. AI Law 16(1):25–52

Governatori G, Rotolo A (2010) Changing legal systems: legal abrogations and annulments in Defeasible Logic. J IGPL 18(1):157–194

Governatori G, Sartor G (eds) (2011) Special issue: deontic logic and normative systems. AI Law 19(2–3)

Grossi D, Rotolo A (2011) Logic in the law: a concise overview. In: Gupta A, van Benthem J (eds) Logic and philosophy today. College Publications, London, pp 251–274

Hart HLA (1958) Positivism and the separation of law and morals. Harv Law Rev 71(4):593–629

Hart HLA (1962–1963) Kelsen visited. UCLA Law Rev 10:709–728

Hart HLA (2012) The concept of law. Oxford University Press, Oxford

Hoare CAR (1985) Communicating sequential processes. Prentice-Hall, Englewood Cliffs

Hood C (1983) The tools of government. Macmillan, London

Hübner JF, Boissier O, Kitio R, Ricci A (2010) Instrumenting multi-agent organisations with organisational artifacts and agents: giving the organisational power back to the agents. J Auton Agents Multi-Agent Syst 20(3):369–400

Kelsen H (1991) General theory of norms. Oxford University Press, Oxford

Kelsen H (2007) General theory of law and state. The Law Book Exchange, New Jersey

Kelsen H (2009) Pure theory of law. The Law Book Exchange, New Jersey

Kelsen H, Klug U (1981) Rechtsnormen und Logishe Analyse. Ein Briefwechsel 1959 bis 1965. Deuticke (Brazilian translation: Normas Jurídicas e Análise Lógica - Correspondência 1959–1965. Editora Forense, 1984)

Ladyman J (2001) Understanding philosophy of science. Routledge, London

Lehmann J, Breuker J, Brouwer B (2004) Causation in AI and law. AI Law 12:279–315

Luhmann N (2008) Law as a social system. Oxford University Press, Oxford

MacCormick N, Weinberger O (2010) An institutional theory of law: new approaches to legal positivism. Springer, Berlin

Milner R (1980) A calculus of communicating systems. Springer, Berlin

Paulson SL, Paulson BL (eds) (2007) Normativity and norms—critical perspectives on Kelsenian themes. Claredon Press, Oxford

Plotkin G (1981) A structural approach to operational semantics. Technical report, University of Aarhus

Porter BF (1968) Deity and morality—with regard to the naturalistic fallacy. Routledge, London

Raz J (1970) Concept of a legal system: an introduction to the theory of legal system. Oxford University Press, Oxford

Raz J (1971) The identity of legal systems. Calif Law Rev 59(3):795–815

Ricci A, Viroli M, Omicini A (2006) Programming MAS with artifacts. In: Bordini RP, Dastani M, Dix J, El Fallah Seghrouchni A (eds) Programming multi-agent systems, vol 3862., LNAI Springer, pp 206–221. Springer, 2006. 3rd International workshop (PROMAS 2005), AAMAS 2005, Utrecht, The Netherlands, 26/jul/2005. Revised and Invited Papers

Santos IAS, Mota FP, Costa ACR, Dimuro GP (2012) Um framework para simulação de políticas públicas aplicado ao caso da piracema, sob o olhar da teoria dos jogos. In Encontro Nacional de Inteligência Artificial - ENIA 2012, Sociedade Brasileira de Computação, Porto Alegre

Searle JR (1995) The construction of social reality. The Free Press, New York

Turner JH (2010) Theoretical principles of sociology. Springer, Berlin

Wegner P (1968) Programming languages, information structures and machine organization. McGraw-Hill, New York