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ORIGINAL RESEARCH

# Fallacies and Their Place in the Foundations of Science

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## Abstract
It has been said that there is no scholarly consensus as to why Aristotle's logics of proof and refutation would have borne the title *Analytics.* But if we consulted Tarski's (Introduction to logic and the methodology of deductive sciences, Oxford University Press, New York, 1941) graduate-level primer, we would have the perfect title for them: *Introduction to logic and to the methodology of deductive sciences.* There are two strings to Aristotle's bow. The methodological string is the founding work on the epistemology of science, and the logical string sets down conditions on the proofs that bring this knowledge about. The logic of proof presents a difficulty whose solution exceeds its theoretical reach. The logic of refutation takes the problem on board, and advances a solution whose execution is framed by fallacy-avoidance at the beginning and fallacy-adoption at the end. But with a difference: the avoidance-fallacies are of Aristotle's own conception, whereas the adoption-fallacies, so judged in the modern tradition, aren't fallacies at all for Aristotle. The avoidance-fallacies are begging the question and *ignoratio elenchi*, and the adoption-fallacies, fallacies in name only, are the *ad hominem* and *ad ignorantiam*, an inductive turning in the first instance, and an abductive finish in the second.

**Keywords** Abduction · *Ad hominem* · *Ad ignorantiam* · Autoepistemic · Axiom · Begging the question · Deduction · Demonstration · Dialectic · Entailment · Fallacies · Foundations of science · *Ignoratio elenchi* · Induction · Inference · Proof · Syllogism · Validity

*In Memoriam*: I dedicate this essay to the memory of John Corcoran, March 8th 1937–January 8th, 2021.

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# 1 Part A: Proof

## 1.1 The Logic of Science is Born

It has been said[1] that there is no scholarly consensus as to why Aristotle's logics of proof and refutation would have borne the title *Analytics.* But if we consulted Tarski's (1941) graduate-level primer, we would have the perfect title for them: *Introduction to Logic and to the Methodology of Deductive Sciences* (Tarski 1941). There are two strings to Aristotle's bow. The methodological string is the founding work on the epistemology of science, and the logical string sets down conditions on the proofs that bring this knowledge about. The logic of proof presents a difficulty whose solution exceeds its theoretical reach. The logic of refutation takes the problem on board and advances a solution whose execution is framed by fallacy-avoidance at the beginning and fallacy-adoption at the end. But with a difference: the avoidance-fallacies are of Aristotle's own conception, whereas the adoption-fallacies, so judged in the modern tradition, aren't fallacies at all for Aristotle. The avoidance-fallacies are begging the question and *ignoratio elenchi*, and the adoption-fallacies, fallacies in name only, are the *ad hominem* and *ad ignorantiam*, an inductivist turning in the first instance, and an abductive finish in the second.

*Prior* and *Posterior Analytics* are two of the six volumes that make up the *Organon* of Aristotle's logical writings. The other four are *Categories, On Interpretation, Topics* and *On Sophistical Refutations.* Each is interesting and important in its own right, and each provides supporting links to the *Analytics* as, indeed, the *Prior* also does for the *Posterior.* Although an exaggeration, it would not be far wrong to say that it took a six-book effort to bring the logic of science to first flower. Among these supporting achievements is the near-perfect and now repaired proof in *Prior Analytics* of some near thing to a modern semi-decidability proof for a large class of valid arguments that qualify as syllogisms (for which latter, please see Sect. 1.4). In modern terms, a property is semi-decidable just when there exists an infallible algorithmic procedure for determining in finite time the presence of that property in anything that has it. When I say that the proof is "some near thing" to one of semi-decidability, I mean that its procedure is also inerrant, but quasi-mechanical, not algorithmic, and terminates in practicably low-finite time. Although the proof doesn't hold for all arguments expressible in Greek, it serves as a model of how one might be produced for them.[2]

Aristotle is not easy reading. His writing is ambiguous, and some of his most important points are implied rather than expressly declared. The logical works are "dense, elliptical, succinct, unpolished, convoluted, and technical, unnecessarily so

[1] By John Corcoran, for example, in his Logic School lectures on Aristotle's demonstrative logic in conjunction with UNILOG 2015, the 2015 conference on Universal Logic meeting at the University of Istanbul. In these same lectures, Corcoran reported that the first known axiomatization of plane geometry was made by Thales in the 6th century BC.

[2] See here Ebbinghaus (1964), Corcoran (1972) and Smiley (1973). Validity is decidable *tout court* if and only if the search algorithm spots it if present or spots its absence if not. Except where noted, the quotations from Aristotle are drawn from Aristotle (1984), the Barnes edition.

in the opinion of many". John Corcoran adds that it is his "opinion that Aristotle's prose in *Prior Analytics* is perversely 'reader unfriendly.'"[3] So, then it will take a good bit of figuring out to get to the bottom of what Aristotle was getting up to in his logic. In what follows I endeavour to steer clear of hermeneutical eddies, but in one respect I take a position on which there is vigorous disagreement in some quarters. It concerns Aristotle's definition of 'syllogism' and the word's inconsistent applications in various other passages of the *Organon*. I will say more of this when we get to Sect. 1.4.

Since my focus here is on the logics of proof and refutation or, equivalently, the demonstrative and dialectical logics, *Posterior Analytics* must be the primary textual source. We begin with its demonstrative logic, concerning which Corcoran is recommended reading.[4] In a nutshell, the logic of demonstration is the combination of an instruction manual supplemented by a blueprint for how to axiomatize any science that admits of it.[5] Before getting down to cases, we will go seriously amiss if we understand Aristotle's notion of axiom in the way that it is understood today. The core difference between then and now in matters axiomatic is the nature of the language in which axioms are formulated. In the present-day, thanks largely to Hilbert and the dominance of mathematical logic, if left to its own devices the language of a logicistic system is semantically lifeless. If falls to the theorist who has selected it for his own particular purposes to *assign* the notation-system's semantic properties. So interpreted formal language is a semantic artefact of the theorist's own making, in which there is no necessary commitment that axioms even be true.[6] In the older, indeed ancient conception, an axiom was a proposition selected from the working language of the science in question. In all cases the system's linguistic needs were tended by fully interpreted and semantically loaded mother tongues, supplemented, as needed, by specialized technical terms and neologisms.

This would be the place to say what Aristotle is not doing here. He was not laying down any axiom of *any* science. That would be for the scientific experts to do. Nor was he setting out his *own* contributions in axiomatizable form. His objective was to provide a model-answer to any question of the form, as might be put to him by a scientist of the day.[7]

- "What must be done to axiomatize my science: What would it look like when so structured, and what would be the good of it?"

[3] John Corcoran (2003, p. 261).

[4] Corcoran (2009). See also Angioni and Zuppolini (2019).

[5] It could be that, irrespective of its subject-matter, Aristotle took a science to be axiomatizable if it has well-developed proof methods and/or some widely-shared unproved postulates framed in a way that gives rise to lawlike generalizations. In a word, if it is *mature*.

[6] To accommodate formalist preferences, for example. For more on these differences in how axioms have been conceived of, see Blanchette (2019).

[7] Contrary to Łukasiewicz (195). Ebbinghaus, Corcoran and Smiley frame the syllogistic in modern natural deduction terms. Clearly an improvement, Aristotle sees logic as philosophy, and philosophy in turn not as a science, but rather as a Speculative Art.

Here is Aristotle's answer. The basic structure of a science's axiomatization can be schematized as an ordered pair $\langle \mathcal{A}, \mathcal{P} \rangle$ of its axioms $\mathcal{A}$, and its proof-rules $\mathcal{P}$.[8] Consider now the class $\mathcal{C}$ of all theorems provable from $\langle \mathcal{A}, \mathcal{P} \rangle$. $\mathcal{C}$ is the *demonstrative closure* of the science's axioms. It is the epistemic consummation of the $\langle \mathcal{A}, \mathcal{P} \rangle$ pair. $\mathcal{C}$ not only contains all and only the true propositions of a given science but does so in a way that induces a knowledge of their truth in anyone who understands the proofs. Given that axioms are always true, that proof is truth-preserving and that demonstrative closure is exhaustive, the axiomatization of any science is both sound and complete. If we thought of a science's axiomatization as its deconstruction to the foundational elements from which the whole of its knowledge inerrantly flows, Aristotle's model plan for how this is done could quite plausibly be called *logical analysis*.[9]

The central task for the logic of demonstration is to expose the conditions under which the $\langle \mathcal{A}, \mathcal{P} \rangle$ structure brings its $\mathcal{C}$ to pass. And, for Aristotle, the critical heart of that task is unearthing the conditions under which a true scientific proposition qualifies as one of its axioms. But before attending to that, it would repay us to work our way up to it more gradually. So, we begin our considerations with the second member of the $\mathcal{A}, \mathcal{P}$, pair, the concept of demonstrative proof.

### 1.2 Validity

A first requirement on demonstrative proof is that its conclusion must follow of necessity (*ex anakês subainein*) from its prior lines. When that condition is met, the proof is valid or, put another way, its conclusion is entailed by the prior lines. It might therefore strike us as odd, even to the point of negligence, that Aristotle provides no theoretical analysis of these linked notions. Since he is careful to define the terms he thinks need it,[10] the mere fact that Aristotle left entailment and validity to speak for themselves is a fair indication that he thought that any reader of the *Analytics* would have a clear (and correct) understanding of them. Although it is not essential for our purposes that we settle the matter conclusively, my own view is that anyone with the language of a capable 10 year old has a nontrivial grasp of the relevant meanings. Once he has developed the capacity to know that his beautiful Auntie Bea could not possibly be a hockey puck or that, if someone had three apples, then they would absolutely have to be fewer than twelve, he has cottoned on to the modal basics and the conditional, which is enough to grasp the meaning of the NLE inset just a few lines below. Think too of the contradiction-by-negation phenomenon, as in "No! It was Billy who spilled the milk, not Suzie", and truth-attributions such

[8] In Aristotle's case, it is taken as given that all relevant terms are clear without express definition or have been well-defined in the run-up to axiomatic remodelling. Frege, whose *Grundgesetze* bears a striking structural similarity to Aristotle's framework, gave definitions an express (and *creative*) role in the axiomatization process. But we needn't take that further here.

[9] The scholarly consensus is that the word 'logic' was first applied to Aristotle's writings by Alexander of Aphrodisias (2nd-3rd century), in Wallies (1881–1883).

[10] Notably, the concept of syllogism.

as in: "Yes, mother, what Suzie told you was true". Then there are the irritations excited by parental behaviour which sometimes discomports with firm instructions to the young. ("But you smoke, Dad!) So, our young friend has a decent command of negation and of what has come to be known as pragmatic inconsistency. Moreover, in English, the meaning of 'entails' is at least this clear:

- *The natural-language meaning of entailment* (NLE): If S entails S′, then it is in no sense possible for S to be true and S′ concurrently not true.

Why should it be any different for what is meant by "*ex anakês subainein*" in ancient Greek? Aristotle takes it as given, rightly in my opinion, that anyone competent in Greek would understand the meaning of "*ex anakês subainein*" if he had an interest in it and could do so without formal tuition.

Of course, if read as a *biconditional*, controversies certainly arise.[11] When read from left to right, there is nothing to ruffle the semantic intuitions of competent speakers. But when read in reverse, that is, from right to left, spots of bother appear. One is that every logical truth is entailed by any proposition at all, even by its own negation.[12] An even better-known irritation is the entailment of every proposition whatever by any logical contradiction. So, from "The Tiber flows through Rome and it is not the case that the Tiber flows through Rome" it follows of necessity that the Toronto Maple Leafs will win the Stanley Cup sometime before the heat-death of the universe.[13] It is all rather interesting in its way, but we needn't tarry with it here. Nothing I will say in this paper hinges on the biconditional reading.

This would be an appropriate place to take note of something else that the axiomatization of a science doesn't do besides not delving into validity. It doesn't specify the proof rules of either general or particular applicability. There is nothing in this that merits censure. It was not Aristotle's task to specify the proof rules of any given science or to write up any of its respective axioms. *Posterior Analytics* is a how-to-do manual and was not itself laid out in axiomatic form.[14] Although validity is a necessary condition of a valid rule's proof-worthiness, it falls far short of being sufficient. In the modern era alone, it has been known for a half-century and more that the standard transformation rules of modern systems of logic—*modus ponens*,[15]

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[11] And, with them, substantial and profitable careers. See, for example, Etchemendy (1990) and Field (2008). A well-received anthology is Caret and Hjortland (2015).

[12] Since it is in no sense possible for a logical truth S* to be false, it is in no sense possible for a proposition S to be true and S* not true. On the biconditional definition, this difficulty cannot be avoided.

[13] By parity of argument, if it is in no sense possible for 'S and not-S' to be true, there is no sense in which arbitrary S′ is true and "S and not-S" is not true.

[14] It is true that Aristotle provides selective proofs of his own, as for example, the proof of the reducibility of syllogistic schemata in all figures to one or other of the first-figure schemata. For example, at *Prior Analytics*, $27^a$ 9–12, Aristotle reduces a second figure syllogism by propositional conversion, and at $27^a$ 36–$27^b$ he reduces a third figure syllogism "through the impossible." I will come back to this in Sect. 1.4.

[15] Harman (1970). Harman's main objective was to blow the whistle on the inadequacy of the rules of the probability calculus for inductive inference. The deductive detour was his *amuse-bouche* to get the critical juices flowing.

*modus tollens*, and so on—are valid but not proof-worthy and, contrary to what is widely put about, belief actually isn't closed under consequence either. In the well-known example, suppose that Sally currently believes statement S but hasn't taken any notice of S′. Yet when new information awakens her to the fact that S actually implies S′, she also awakens to the belief that S′ is false. Rather that adding it to her belief-stock, Sally rejects S′, and along with it S as well. Harman's example generalizes to a powerful insight. Proving things is a highly context-sensitive thing to do. Whether a valid rule is proof-worthy in any given case depends on the subject-matter of the science in question, the nature of the enquiry, what the prover already knows, what theorems are already to hand, the prover's background information, his skill, the particular target of the present proof, and the proof's design. Considerations such as these simply don't generalize, and because that is so it is not possible to cite the conditions on proof-worthiness for the general case even within the domain of enquiry of a particular science.[16]

This is far from saying that since a prover cannot state the criterion of proof-worthiness, his own demonstrative efforts are mere shots in the dark. The deductive sciences brim with proof-making success and owe their large prosperity to formidable proof-making skills. What this tells us is not that proof's success-conditions aren't known, but only that they are known *implicitly* by those who have the training and the "nose" for them.

### 1.3 What Proof Does Not Preserve

It is not in dispute that a demonstrative proof is truth-preserving. This is provable from NLE's characterization of the entailment relation. A true proposition S cannot entail a false one S′; for it if did it would be possible for it to be impossible for S to be true and S′ concurrently not true, yet also for S′ concurrently to be true. Truth-preservation is one of several properties intrinsic to entailment, provable from the very condition that characterizes it in NLE. Other well-known properties follow suit—e.g. reflexivity, nonsymmetry and transitivity. But no proof-seeking scientist would want proofs to be reflexive, for no one seriously believes that a proposition in need of proof is proved by itself. Other intrinsic properties are somewhat less well known, monotonicity for one. It is slightly more reader-friendly to explain monotonicity for valid arguments. Let $V = \langle\{S_1, \ldots, S_n\}, S'\rangle$ be any argument, and let $V^+$ arise from V by supplementing its premisses with any and as many propositions one chooses and in any number of occurrences.[17] If validity is monotonic, then $V^+$ is valid if and only if V is. Consider now the simplified case $\langle\{S\}, S\rangle$, valid by the reflexivity of entailment. It follows by monotony that $\langle\{S, \text{not-}S\}, S\rangle$ is valid. Generalizing, every entailed proposition is entailed by itself combined with its own negation. Since every proposition is self-entailed, it is also entailed by the totality of

[16] Consider writing the advance manual that would regulate the proof of the binomial theorem and also as the unsolvability proof for Hilbert's tenth problem.

[17] The angles flank an ordered pair of elements. The curly brackets enclose the arguments premiss(es). The rightmost element is the argument's conclusion.

propositions expressible in any language that expresses it. More unwelcome results pop up, but enough is now at hand to persuade us that demonstrative proof cannot preserve monotonicity.

Please keep in mind that these proof-unworthy results are *not* products of the biconditional definition of entailment, but solely of the intuitive left-to-right reading, as in NLE. Although they are properties intrinsic to the entailment relation, they cannot be considered normative for proof in the general case. But this is no reason to give up on the entailment relation. True, *one* of its intrinsic properties is dispositive for valid proof, but that is only to say that if you want your proof to be valid, there is no hope of its being so unless the tie that binds premisses to conclusion is truth-preserving. As we now begin to see more sure-footedly, validity is necessary for valid proof-worthiness, but well short of sufficient.

Another property intrinsic to entailment is topical irrelevance. Consider Peano arithmetic as an example, and the theorem that addition is commutative. Then on the principle R that if some S is true, so too is at least one proposition of any set of propositions containing S, it follows that addition operator is commutative or the beautiful Auntie Bea went swimming today in Monaco.[18] True though it is, it is neither a truth nor a theorem of Peano arithmetic. More generally,

- *Intrinsic property limitation*: Of all entailment's intrinsic properties, only truth is preserved by well-made proofs.
- *Corollary*: Besides, most of what is true isn't worth knowing, and comparatively little is worth the effort to prove it.

It is difficult to see this as problematic or even puzzling. Entailment is an alethic relation. Inference and proof are epistemic relations. Entailing is what propositions "do", whereas proving is what people do. These are differences in kind. The requisite conditions are therefore bound to be different.[19]

A complaint that can fairly be lodged against the modern logical theorist, including its early and later greats—Frege, Peirce, Russell, Tarski—is that when they listed their proof-rules, not one of them was proof-worthy in its own right. All the leading textbooks repeat the omission. It would be interesting to know whether any of the earlier greats had made a better fist of it, Aristotle, say? The answer, I think, is no and yes; no in the sense that he makes no express mention of the characteristics that make a valid rule a proof-worthy one; but yes in the sense that he builds structures in which these very properties are within any proof-maker's reach. To see how this comes to be, we must turn to the logic of syllogisms and, as we do, we should also keep in mind a second reason to examine them. There is a troubling

[18] It is even more promiscuous than that. From the commutativity of addition we have it that addition is commutative or beautiful Auntie Bea went swimming today in Monaco or the Blue Jays whipped the Orioles on Wednesday or the real numbers are uncountably many or … or …. We should note that principle R gives the truth conditions for inclusive alternation. That's why we can simplify the discussion by switching to "or".

[19] The quotation marks are advisory. In no literal sense do propositions do anything. Propositions lack agency.

problem with axioms. In Aristotle's meaning of the term, axioms do not effectively present themselves as such. Axioms currently in use might not be the real thing, and we might miss some of those that are indeed the right thing. Before turning to the details, it would be helpful to have some grasp of the tools of repair provided by the syllogistic. They are to be found in *Topics* and *On Sophistical Refutations.*

### 1.4 Syllogisity

A syllogism is a valid argument meeting certain limiting conditions. They can be found in *Sophistical Refutations* 1, $165^{a}$ 1–3, *Topics* 1, $100^{a}$ 25–27, and *Prior Analytics* A $24^{b}$ 19–24. Here is the passage from *Sophistical Refutations*:

> A *sullogismos* rests on certain propositions such that they involve necessarily the assertion of something other than what has been stated, through what has been stated.

Aristotle adds the by "because these things are so" he means "resulting through them", and by "resulting through them" he means "needing no further term from the outside in order for the necessity to come about." (20–24). From what has already been said about validity, it is clear that syllogisity isn't it, but rather a special case of it:

- Indeed, "… that which is necessary is wider than *sullogismos;* for every *sullogismos* is necessary, but not everything which is necessary is a *sullogismos.* (Barnes, *Pr. An.* A 32, $47^{a}$ 34–35)
- "Of the present enquiry [= syllogisms], on the other hand, it was not the case that part of the work had been thoroughly don before, while part had not. Nothing existed at all." (*Soph. Ref.* 34, $183^{b}$ 34–36).

I read this to say that the validity part had not been worked out in advance (since it hadn't been anywhere "worked up" in Aristotle's logic) and that the part that comes into play now is newly-made.

In large measure, then, the overall importance of syllogisity is that it is a property of Aristotle's own creation, and made-up for the good that it can do for the demonstrative proof rules. A further constraint, and heaviest of them all, is that all propositions constitutive of a syllogism must be framed as categorical statements, that is, as propositions in one or other of the following schematic forms: 'Every A is B'; 'No A is B'; 'Not every A is B', and 'Some A is B'. It is a striking limitation. No science could approximate to an axiomatizable maturity if its working language were exhaustively categorical and its proofs limited to three lines. Aristotle also requires that in their syllogistic occurrences its categorical propositions be *assertively* expressed as, "affirming an denying something of something" (*Pr. An.* $24^{a}$ 17–22.) In Charles Morris' much later trichotomy of the dimensions of language – the syntactic, the semantic and the pragmatic – proof is a bred-in-the-bone pragmatic enterprise. In non-syllogistic contexts, the assertive requirement is relaxed as, for example, in the case of proofs by assumption ($24^{a}$ 22-$24^{b}$ 15), but the context remains fixedly pragmatic. The thing to note here is that the meaning of '*sullogismos*' can shift depending on context of use.

I parse these passages as follows:

- *Syllogisity defined*: In its core sense, a syllogism is a three line-deductively valid argument that displays exactly one more nonlogical term than its two premisses. It is structured in such a way that each line displays two different terms, no single line entails the conclusion, and each term of the conclusion must have exactly one occurrence in exactly one premisss, a different premiss for each of the conclusion's terms.

If we now examined the constraints imposed on valid proof rules, various attractive characteristics of proof-worthiness would be evident. Besides truth-preservation, the following wouldn't be difficult to spot.

- Nonmonotonicity[20]
- Subject-matter preservation.
- Off-topic preclusion.
- Theorem-generation[21]
- Premissory economy[22]
- Knowledge-inducement.[23]

It is not my place to say which of these an expert physicist would opt for if he were proceeding in Aristotle's axiomatic ways, but I daresay the choice would be most at least. Among the further characteristics not on this list, the first to be dropped by a working scientist would be the categoricality feature. Since Aristotle's mission was to show the exact sciences the route to axiomatic reconstitution, why would he have constructed these things with such unrealistic tightness? The shortest correct answer is captured by the old phrase, "if he coulda he woulda" Had Aristotle the engineering wherewithal to implement his demonstrative designs in a language as expressly rich and complex as those of the sciences, it would take some explaining as to why he wouldn't have followed suit. So, he did the next best thing. His purpose was to *model* the way for demonstration to proceed in the real sciences, and, to that end, it sufficed to show how it works for a model-language, and a greatly simplified one.

It is also helpful to keep it in mind that Aristotle's philosophy of science is a two-logic affair. The logic of demonstration can only go so far and, at the point at which it comes under stress, must defer to the logic that will provide the desired

[20] This, the arbitrary promissory expansion of valid arguments is disallowed.

[21] To be a theorem of a given science, a proposition's ancestral line must originate with an axiom. But no axiom is itself a theorem since no axiom is its own ancestor.

[22] The fewer the axioms that get the job done, the less likely that an axiom will turn out to have been provable, hence no axiom at all.

[23] Which is the whole point of a theory of scientific knowledge.

foundational stability. And as I keep saying, the switch cannot succeed with the tools provided by the syllogistic – properties that make a deduction a genuinely proof-worthy demonstration. Since we, too, are at that point now, we must make the move to dialectic, but not before a brief pause to take up a point raised by John Corcoran.

Writing in "Aristotle's demonstrative logic", Corcoran turns his mind to the idea that demonstrations must not contain redundant premisses:

- "There is no justification for attributing to Aristotle, or any other accomplished logician, the absurd view that no demonstration has a 'redundant' premiss—one that is not needed for the deduction of the conclusion." (Corcoran 2009; p. 4)[24]

One sees Corcoran's point. Some scientific demonstrations are more efficiently rendered than others, but apparently equal in their knowledge-inducing virtue. It bears on the wording of Corcoran's remark that 6 years earlier he administered to Aristotle's writings, that deserved dressing-down: dense, elliptical, succinct, unpolished, convoluted, unnecessarily technical and perversely user-unfriendly. (Corcoran 2003, p. 262). One of the respects in which the ellipticality complaint is justly levied concerns the use of the term '*sullogismos*'. In the texts I have cited, which may well have antedated *Posterior Analytics*, the anti-redundancy reading is nothing close to absurd. Yet in other places, *sullogismoi* are plainly birds of another feather, as in the aforementioned reduction proofs at *Prior Analytics*, 27$^a$ 9–12 and 27$^a$ 36-27$^b$. Especially striking is the make-up of *ecthetic* syllogisms that depend on the introduction of "terms from the outside".[25]

The source of the difficulty is Aristotle's ambiguous use of "*sullogismos"*, combined with a translator's interest in finding a term for *sullogismoi* that might appear to fit all cases without obvious ambiguity in the translating language. To this end, some translators opt for 'deduction' as the one-size-fits-all term. In his Aristotle and Boole paper (2003), Corcoran allows for the possibility that key aspects of *Prior Analytics* had been anticipated in earlier writings, perhaps all four volumes of the *Organon* apart from the *Analytics*.[26] At the beginning of Chap. 4 of *Prior Analytics*, Aristotle distinguishes between syllogisms and demonstrations, pointing out that while syllogisity is the prior and more general notion, no demonstration can proceed without incorporating requisite features of syllogisity. For this to be so, the proof-rules must be a *mix* of the common rules, e.g. *reductio absurdum, modus ponens* and the syllogistic rules, e.g., the rule corresponding to a syllogism in *Barbara.* This feature alone guarantees the subject-matter preservation of demonstrative proof and marks the only respect in which "every demonstration is a syllogism." My own position is

[24] Hans Hansen points out in correspondence, it is difficult to square this dismissal with Aristotle's non-cause as cause fallacy. See here John Woods and Hans V. Hansen, "The subtleties of Aristotle on Non-Cause", *Logique et Analyse*, 44 (2001), 325–415.

[25] *Pr. An.* 25$^a$ 15–17; 28$^a$ 24–26; 28$^b$ 14–15; 28$^b$ 20–21; and 30$^a$ 9–10.

[26] Corcoran writes: "In his recent book *Aristotle's Earlier Logic*, John Woods 2001 presents further evidence that Aristotle addressed these issues in earlier works. (p. 263, *n.* 5)".

- *The thricewise ambituity of 'sullogismos'*: It would better serve Aristotle's purposes to reserve the word 'syllogism' for the tightly constrained validities under present consideration, the word 'deduction' for a valid argument of any propositional construction, and the word 'demonstration' for valid knowledge-producing proofs, guided by proof rules carrying properties also carried by syllogisms under certain of the syllogistic constraints.

We now have a chance to correct one of the most enduring misconceptions in logic's long history and a blight on the subject's pedagogy. To put it in a nutshell, Aristotle's logic is not to be found in his provisions for syllogisity. In its basic meaning, a fallacy is a common misconception, which is precisely what we have at hand here. Here is how and why.

Trifling as their make-up is, there is enough complexity in syllogisms to make them theoretically interesting. One notable claim is that in all of Greek, there are just fourteen ways for a valid categorical assertoric argument to be a syllogisms, made so in each case by instantiation of syllogistic schema.[27] Embedded in the notion of schema is a workable distinction between a system's logical and nonlogical terms, and with it comes a serviceable substitutivity principle under which uniform substitution of different nonlogical terms for the nonlogical terms of a syllogism is syllogisity-preserving. Another feature of interest is that corresponding to all fourteen syllogism is a valid syllogistic rule licensing the derivation of a syllogism's conclusion from its premisses. An important distinction is that between "perfect" an "imperfect" syllogism. A perfect syllogism is a syllogism whose validity is evident to anyone who understands the argument and is imperfect when it can be understood without recognizing its validity.

The proofs by which an imperfect syllogism's validity could be made apparent draws on features of perfect syllogisms's premisses. However, neither the semi-decidability proof nor the reduction proofs can succeed by relying on syllogistic rules alone. If we considered the theory of syllogisms as a logic then we would have it that syllogisms cannot provide their own proof rules. Among the nonsyllogistic rules that retain propositional categoricality, there are those whose input-lines are not premisses but assumptions. And among the rules that do not honour the propositional categoricality constraint are the "common rules" such as *modus ponens.* It would be churlish to deny to the systematic study of such features the name of logic. It is the logic which, with some embellishment by the Schoolmen that occupied the logic curricula of our universities until ≈ 1925 AD. But it is not, and never could be *Aristotle*'s logic, that is, the logic he built for scientific knowledge. One cannot do science under categorical constraint. And now, since the logic of refutation impatiently awaits, it is past time to give it proper notice.

[27] *Pr. An.* I 4–6, but slightly adjusted upward in I7. We need not determine the exact number here. The most famous schema is *Barbara*: ⟨{"Every A is B", "Every B is C'}, "Every A is C"⟩.

## 2 Part Two: Refutation

### 2.1 The Elusiveness of Axioms

Perhaps the first thing to do is to emphasize that how Aristotle conceives of axioms is so out of touch with today's understanding of them.[28] In Aristotle's approach, for a proposition to qualify as an axiom (*axioma*) it must be a first principle (*archê*),

- "true, primary, immediate, better known than, prior to, and causative of the conclusion." (*Post An*, I. 2 $72^b$ 20–23).

Moreover, it

- "must be primary and indemonstrable …". ($71^b$ 27[29]).

By the last quarter of the 19th century, only two mathematicians of note shared the old-fashioned concept, Frege in the logic of arithmetic, and Moritz Pasch in projective geometry.[30] Here is Frege, writing in 1899:

- "I call axioms propositions that are true but are not proved because our knowledge of them flows from a source very different from the logical source …. From the truth of the axioms it follows that they do not contradict one another. There is no need for a further proof." [31]

Writing later Frege adds:

- "No thought that is held to be false can be accepted as an axiom. Furthermore, it is part of the concept of an axiom that it can be recognized as true independently of other truths."[32]

Frege's characterization lacks the detailed heft of Aristotle's own. But there is no doubting Frege's steadfastness, as his correspondence with Hilbert makes clear. He

[28] According to one modern critic, Frege's approach to axioms – hence Aristotle's too –was "a dinosaur" Blanchette (2014, p. 201.)

[29] For a further discussion of why a first principle must itself be an unprovable truth, readers could consult Woods (2019; p. 31.) The Corcoran reference is to that book's 2001 edition, which contains material on the turn to dialectic (Chap. 8) which, as I now regret, wasn't retained in the second edition. The translation is Tredennick's in Aristotle (1960).

[30] Pasch, (1882).

[31] Frege (1980), p. 37, The remark cited here is from the Frege-Hilbert correspondence in the interval 1895–1903.

[32] Frege (1979), p. 168. Further details about the similarities between *Grundgesetze* and *Analytics* can be found in Woods (2021).

repeatedly champions the Euclidean notion of axiom as the uniquely correct one, apparently unaware of Aristotle's prior employment of it.[33]

What I want to touch on now is the problem posed by the first-principles requirement, for it is there that the link to fallacy theory takes on a large material relevancy. It is a problem engendered by the fact is that axioms do not announce or self-certify their axiomaticity.

- *The first-principles problem*: The first principles of any given science are not effectively presentable. So, neither are their demonstrative closures self-announcing.
- *Aristotle's corollary*: It cannot therefore be determined with inerrant accuracy that any given proposition meets all the criteria for first principlehood. From which it follows in turn that the soundness and completeness of the logic of demonstration lies in jeopardy of collapse.

For axioms to deliver the completeness and soundness result in a way that is causative of knowledge, they must themselves be known to have the characteristics that make them so. Since they cannot themselves be in the demonstrative closure of any science, they must be made known to their users by some nondeductive means. Aristotle made no pretence of being surprised by this, whereas Frege could never quite come to terms with it. Frege abandoned the philosophy of arithmetic, and Aristotle switched his logic from the demonstrative to the dialectical, the logic of trial by combat.

As with the logic of syllogisms, the logic of dialectic introduces a simplified model of the actual slings and arrows of attack-and-defend arguments. In the first instance, he borrowed a common word and gave it an entirely different stipulated meaning. In the present case, he does the same with the common word 'refutation' which means a falsifying argument and for which he stipulates a different and purpose-built meaning. If we recall Aristotle's assurances that the theoretical meaning of 'syllogism' was entirely new to logical theory, we should be prompted to regard Aristotle's use of 'refutation' in the same way.

As already mentioned, the rudiments of dialectic are laid out in stylized form in *Topics* and *Sophistical Refutations*. Their chief accomplishment is a well-polished accounting of *ad hominem* proofs, which not only are *not* fallacies, but mark a major theoretical advance in the study of logical inconsistency. To see how this works, we begin with the distinction between *endoxon* and *elenchus*, each word being used in a special-purpose way, not in the ways of ordinary Greek. In ordinary speech, *endoxa* are beliefs—for example, that the next-door neighbours have visiting relatives at present. But here they are beliefs with wide and dominan*t* market-share. *Elenchi* in the everyday sense are arguments that establish the falsity of some targeted proposition. That is not the meaning here. In present-day English, notably in journalism, 'refute' is frequently misused as a synonym of 'deny'. Another common misusage is taking the expression 'begs the question of whether' to mean 'invites or raises the

[33] Aristotle's axiomatics preceded Euclid's *Elements* by two generations.

question of whether ….' In Aristotle's sense a question is begged when a proposition is used as a premiss in a refutation-argument which has not been conceded by one's opponent. (See just ahead). So, then, an *endoxon* is an opinion had by all ('Water is wet'), or by the many (Water's chemical structure is $H_2O$'), or by the wise ('The shortest line between two points is curved'). Among the wise are the leading experts of the mature sciences, and an *elenchus* here is a form of proof to the contrary of some stated *endoxon. Sophistical Refutations* tells the story of how these notions interact, set out by Aristotle in a highly simplified context.

Aristotle knows as well as anyone the large frequency with which humans hold conflicting beliefs, often about some of life's nastiest problems. He also knew that left to our own untutored devices, we are not especially adept at resolving our differences. That the ancient schools and academies would give instruction in the arts of argument and persuasion makes at least as much sense as the teaching of geometry to the young. Aristotle's Lyceum was a seat of high learning in such matters. One of its central objectives was to teach the ways and means of evading raw argument's constant scourge, the fallacy (as Aristotle called it) of begging one another's questions (*petitio principii*), especially in matters that are considered important enough by philosophers to take principled positions upon. Of course, begging the other person's question can be embarrassing, but Aristotle's point is that, in n-party scientific engagements, unbridled question-begging crashes the system. Had he known of it, he would have agreed that Frank Ramsey's famous line was seriously intended: "How can a philosophical enquiry be conducted without a perpetual *petitio principii*?"[34]

One of the more interesting features of the Lyceum's mock contests is that the matters up for disputatious engagement must in all cases be opinions that are generally agreed upon. This places the attacker in a difficult position and accords a correspondingly lighter one to the proposition's defender, for the proposition in question is very likely to be believed by each of them. Another respect in which dialectical duties were unevenly imposed is that, once the thesis to be debated has been declared, its defender has no role to play in its defence apart from answering each question put to him by his adversary with a simple yes-or-no answer. Correspondingly, the adversary is given only two roles. First and foremost, he must put to the defender clear questions, one at a time, each fully and truthfully answerable by a yes-or-no response. His further requirement is to make an inventory of the propositions advanced by yes- and no answers and if the conditions to and fro permit, construct a syllogism whose premisses are drawn from the inventory of his opponent's concessions, and whose conclusion is the logical contradictory of the thesis in question. When these conditions have been met, the refutation has succeeded, made so by a perfectly constructed *ad hominem* proof. And in its construction, no question has been begged. The closest thing we have in present-day life to Aristotle's refutations can be found in common law trials at the cross-examination stage. These, too, are highly structured affairs, but not at all up to the artificial standards imposed by

[34] *Philosophical Papers*, p. 2. Aristotle took pains with question-begging. For details, readers could consult Woods and Walton (1982).

*Topics* and *Sophistical Refutations.* The gap between them is very wide. Criminal defendants have no obligation in law to answer questions. Another difference is that criminal trials are the real thing, and Aristotle's dialectical exchanges are a model of the real thing. Even in the training sessions in the Lyceum, it is virtually impossible to see how students could have met these burdens with categorical propositions alone. It is more reasonable to suppose that what Aristotle means here by '*sullogismos*' is what I am calling 'deduction' – a valid argument from concessions already at hand.

The winning proof will not prove that the *endoxon* is false (will not be a proof in the full sense, as Aristotle says), but rather a proof against the man who holds it. What has been shown by it is that the *endoxon*'s supporter has made a logically inconsistent defence of it. He cannot without contradiction hold the opinion he supports together with his attestations under cross-examination. Although not a proof in the demonstrative sense, a well-made *ad hominem* is a truth-preserving proof, but not a falsifying one. Centuries later, the point was picked up by Locke, who rightly observed that when an arguer has made a successful *ad hominem* against a dialectical opponent he has pressed him with "consequences drawn from his own principles or concessions". Locke's *ad hominem* argument is not a fallacy either, and Locke never suggested that it was.[35] Neither did Aristotle.

This is an important development, attested to in Aristotle's distinction between proofs against the man and proofs in the full sense; in other words, *demonstrative.* Come back now to the properties carried by the conditions that define syllogisms which render a rule proof-worthy. Apart from truth-preservation, none of the others is needed for *ad hominem* proof. Accordingly, the syllogistic has no role to play in the logic of refutation. We should note that the concept of dialectic has a large and varied presence in Greek philosophy, but in its employment here, it is solely that part of it that pertains to the logic of refutation by *ad hominem* proof.

It is also in the dialectical writings that Aristotle launches his doctrine of *paralogismos*, which is the kind of fallacy committed in mistaking a non-refutation for a refutation. One way in which to fall into this trap is by advancing as a refutation a deduction whose conclusion is something other than the contradictory of the thesis under attack. It is a case of what Aristotle calls the *ignoratio elenchi* fallacy, that is, the fallacy of misconceiving what it takes for a validly derived argument to be a refutation (*Soph. Ref.*5, 167$^{b}$ 21–36; 6, 168$^{b}$ 17–21; 7, 169$^{b}$ 9–13). It is also the mistake of reasoning correctly but to topically irrelevant effect: If the attacker's conclusion isn't the logical contradictory of the thesis under challenge the refutation fails because it is off-topic.[36]

[35] Locke (1962). Locke's purpose was to direct them to others so as "to prevail on their assent, or at least so awe them as to silence their opposition. For Aristotle, see *Soph. Ref.* 22 178$^{b}$ 17: 10, 170$^{a}$ 13, 17–18, 20;20. 177$^{b}$ 33–38; 33 183$^{a}$ 22, 24, 8; *Top.* 161$^{a}$.

[36] It is worth noting that a perfectly made syllogism can be a failed type of argument. This necessitates a distinction between a syllogism-as such and a syllogism-in use. Arguments satisfying the definition of syllogisms are, just so, syllogism-as such. Further conditions bind syllogism-in use, and these vary with the argument's objectives and the procedural means of bringing them to heel. Discussion of the distinction pervades Woods (2014).

Lying at the heart of the logic of demonstration is how a science's axioms are to be sorted out from its other unquestioned truths. At the heart of this heart are the basic facts about a theory's first principles. True, necessary, primary and best-understood, and neither needful of nor susceptible to proof, they are not however inerrantly self-announcing as self-certified. The question is not, however, whether such propositions exist, but rather how they are to be spotted as *first principles*. And once spotted, would the process that brought them to notice have been a knowledge-producing one? In Aristotle's model-based telling, the process unfolds in the following way he tells it in an idealized abstraction description of the dynamically complex cognitive economies in which real science—indeed all human enquiry—is actually practised. Related is what we might call *cognitive economics*, conceived of as a descriptive behavioural science of what actually happens in workaday science.[37] In this abstract model, candidates for axiomatic consideration are assembled for test by the theory's leading experts. At the start, each candidate's proposition is already an *endoxon*, an opinion held by the wise, a scientific "given" with a very large market-share. Each candidate, in turn, is put on trial, with some experts assigned the task of supporting it against all the efforts by the others to bring a successful *ad hominem* proof against it. The trial continues until such time as the dialectic exhausts itself, with no case for inconsistent defence having found merit. At that point that *dialektikos* runs dry, *epagôgê* takes over. In lieu of successful *ad hominem* prosecution, reasoning *ad ignorantiam* now takes precedence.[38] In later writings, including those of present-day fallacy theory, what we have here is one fallacy superceding another. In fact, and in logic, neither is fallacious at all.

As constructed here, the *ad ignorantiam* is epagogically stimulated but framed autoepistemically[39] and with *apagogic* intent:

> *The ad Ignorantiam rule*: Were the winning candidate not in fact an axiom, then given the dialectical pressure that has been brought to bear upon it here, that fact would have come to light by now. That not in fact being so, the reasonable inference—though not a truth-preserving one—is that the winning candidate actually is the real article, a *bona fide* first principle.

'*Epagôgê*' is Aristotle's word for *induction*, and the message of Book A of *Posterior Analytics.* is that while all the action *within* an axiomatized science is truth-preserving, its foundations rest inductively on a battlefield of dialectical exhaustion. '*Apagôgê*' is Aristotle's word for *abduction.* If Charles Peirce could have been on hand, he might have observed that the dialectical exhaustion that follows a science's trial by combat raises an obvious question. "What would it take to account for the fact that systematic communal pressure against the surviving candidate has been so unavailing?" Peirce's answer, and Aristotle's too, would be that it is best accounted

[37] Some reflections on how cognitive economies work can be found in Woods (2022). I try for the full account in Woods (2023).

[38] Named so for Locke (1962), but not as a fallacy.

[39] In some literatures, autoepistemic inference is inference from negation-as-failure. My own first contact with the word was Moore (1983). See also Moore (1988).

for by the assumption that had there been defeaters of them, they would have made themselves known by now, especially in light of science's inbuilt combativeness. So, the undefeated propositions in current axiomatic employment can reasonably be seen as the real article. It is a nice story in the telling, but the point to emphasize now is the manifest impossibility of implementing Aristotle's measures in the circumstances of real-life. For one thing, refutational exchanges are time-consuming and structurally complex. In too many cases, they take too long to unfold and they are too complicated to be done as well as they should.

There is a brief discussion of *apagôgê*'s structural differences from *epagôgê* at *Prior Anaytics* 2. 25. The differences are laid out syllogistically. In my reading of his logic of science, the extent of his syllogistic influence on the foundations of science end with the contribution to the proof-worthiness of a science's rules of demonstrative inference, gleaned from select properties of syllogism's defining conditions. If, as I am supposing, this exhausts the relevance of the syllogistic for the axiomatization of science, the syllogistic turn to *epagôgê* and *apagôgê* are irrelevant to that end.[40]

## 2.2 And so?

In one of his more insightful *aperçues* it fell to Quine to observe that all enquiry begins in *media res*. He might well have added—and possibly left it as an inference—that one also leaves enquiry *in medias res.* It is a fundamental datum for cognitive economics. This alone should alert us to the great liberties taken by Aristotle in founding the logic of science. The very idea that everything whatever to be learned from a science can be found compacted *in potentia* in some finite list of single propositions is preposterous, and Aristotle knew it. Why, then, did he take such liberties and devote to his elucidations the labour of six substantial volumes of analytical effort? The right answer will elude us unless we give some thought to what Aristotle's liberties were liberties from. If I am not mistaken, we now have at hand a way of answering this question. It is simply beyond the powers of any finite being to give an exhaustive description of the state of play in any mature science, still less of all of them at once. So, he took the considerable pains of constructing highly idealized abstract models of what he saw as the basic structural features of scientific enquiry. What I have proposed in this essay is that fallacy theory was founded to further the structural modelling of scientific practice. No science can survive the rot of pervasive question-begging; so, the model will have to reflect that fact. Then, too, no science, not even nonmathematical ones, can survive without some truth-preserving nourishment; so that, too, must be picked up by the model. No less obvious is that, if only on pain of infinite regress, no science can prove its every finding. Part of the

[40] Still, it must be noted, that substantial efforts by scholars of high standing have been made to bring the notions of *Pr. An.* 2.25 into fuller theoretical bloom. It might well be that, in due course, those efforts will achieve fruitful landing in the dialectical logic of refutation. But for the present, I think that we must wait to see. See for example Magnani (2017), especially Chap. 5, and Bellucci (2019), and a reply by Woods at pp. 565–570. Closer to the mark among recent writings, I see Park (2021).

reason is the sheer sprawl of a science's complexities. Another is the extent to which those very proofs are backed and filled by the implicities of background information. This, too, will have to be prefigured in the model. Perhaps Aristotle's greatest insight into these matters is that no mature science can foundationalize itself by generalizing from particular cases. There is no need to fault such inferences in principle, but they cannot achieve the foundational target until question-begging is brought down to a sustainable level. Thus, the prior need is for *petitio*-containment, and the most efficient way of attaining it is by submitting the whole operational wherewithal of the sciences to the aggressions of competitive free-market trial and error. And when patches of subduance present themselves by surviving all *ad hominem* efforts against them, the market must have the resources to make a coherent response to it. It is here that "our" (not his) fallacies come into play. Knowing no case against the consensus, a decision *ad ignorantiam* is made. It is decided by autoepistemic inference that the consensus renders foundational support. If there were reason to doubt the patch of consensus, we experts would surely know it by now. But we don't; so, there isn't. Thus does the dialectical exhaustion of inductive *ad hominem*-striving set the stage for abductive finish. And, in so saying, no question has been begged and no refutation misconceived.

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