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Evaluating Arguments Based on Toulmin's Scheme

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ABSTRACT: Toulmin's scheme for the layout of arguments (1958, *The Uses of Argument*, Cambridge University Press, Cambridge) represents an influential tool for the analysis of arguments. The scheme enriches the traditional premises-conclusion model of arguments by distinguishing additional elements, like warrant, backing and rebuttal. The present paper contains a formal elaboration of Toulmin's scheme, and extends it with a treatment of the formal *evaluation* of Toulmin-style arguments, which Toulmin did not discuss at all. Arguments are evaluated in terms of a so-called dialectical interpretation of their assumptions. In such an interpretation, an argument's assumptions can be evaluated as defeated, e.g., when there is a defeating reason against the assumption. The present work builds on recent research on defeasible argumentation (cf. e.g. the work of Pollock, Reiter, Loui, Vreeswijk, Prakken, Hage and Dung). More specifically, the author's work on the dialectical logic DEFLOG and the argumentation tool ARGUMED serve as starting points.

KEY WORDS: argument evaluation, defeasible argumentation, Stephen E. Toulmin

1. INTRODUCTION

In his book *The Uses of Argument*, Stephen Toulmin (1958) has argued that arguments need to be analyzed using a richer format than the traditional one of formal logic in which only premises and conclusions are distinguished. He has proposed a scheme for the layout of arguments that in addition to *data* and *claim* distinguishes between *warrant*, *backing*, *rebuttal* and *qualifier*.

As an illustration, Toulmin discusses the claim that Harry is a British subject. The claim can be supported by the datum that Harry was born in Bermuda. That there is a connection at all between datum and claim is expressed by the warrant that a man born in Bermuda will generally be a British subject. In turn, the warrant can be supported by the backing that there are certain statutes and other legal provisions to that effect. The warrant does not have total justifying force, so the claim that Harry is a British subject must be qualified: it follows presumably. Moreover there are possible rebuttals, for instance when both his parents were aliens or he has become a naturalized American.

Schematically, the result is as in Figure 1 (Toulmin, 1958, p. 105). Toulmin et al., (1984) give many further examples.

Toulmin's scheme for the layout of arguments has had a continuing influence on argumentation researchers (cf., e.g., van Eemeren et al. (1996, pp. 129–160), Bench-Capon (1997)). Its general form is shown in Figure 2 (Toulmin, 1958, p. 104):

The Data consist of certain facts that support the Claim. The Warrant is an inference license according to which the Data support the Claim, while the Backing provides in turn support for the Warrant. A Rebuttal provides conditions of exception for the argument, and the Qualifier can express a degree of force that the Data give to the Claim by the Warrant.

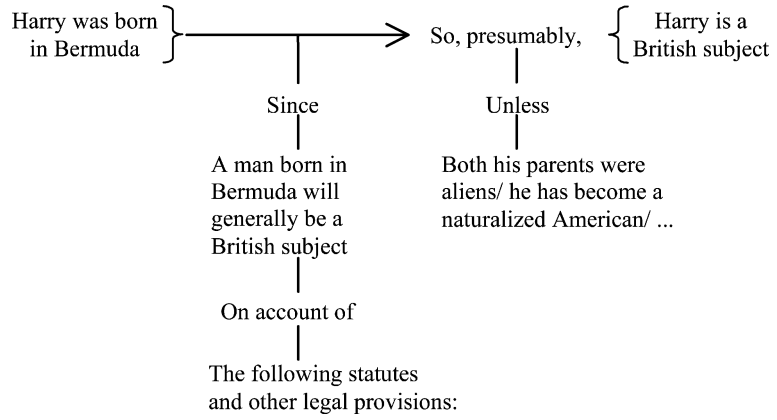

Figure 1. An example of Toulmin's scheme for the layout of arguments.

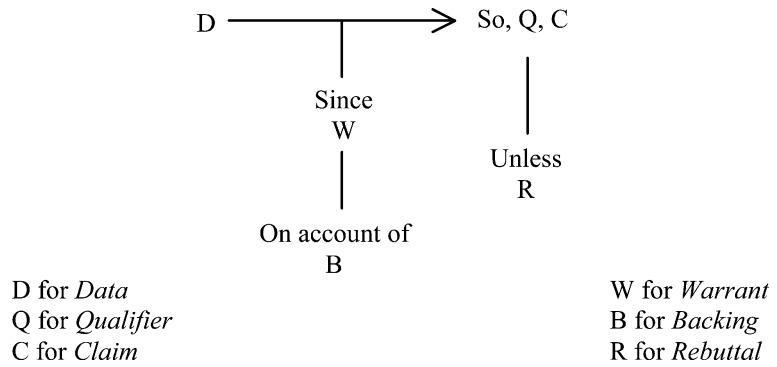

Figure 2. The general form of Toulmin's scheme for the layout of arguments.

Good points of Toulmin's work were his emphasis on the following:

- In argumentation, the warrants of arguments (in the sense of inference licenses) can be at issue and their backings can differ from domain to domain.
- Arguments can be subject to rebuttal in the sense that there can be conditions of exception.
- Arguments can have qualified conclusions.
- Other kinds of arguments than just those based on the standard logical quantifiers and connectives (for all x, for some x, not, and, or, etc.) need to be analyzed.
- Determining whether an argument is good or not involves substantive judgments and not only formal.

Since the appearance of Toulmin's book, all of these points have found increasing support in different research communities (under the direct influence of Toulmin or independently).

Notwithstanding Toulmin's critical stance towards formal logic,[1] in the present paper, a *formal* elaboration of Toulmin's central ideas will be given. It is shown that Toulmin's central ideas can well be set out in a formal way by the use of techniques of today's formal logic.

It goes without saying that the game of formalization is here not played for mathematical pleasure. The main reason for formalization is that it allows the repair of an omission in Toulmin's work. Toulmin's omission is that he has only discussed the *structure* of arguments (in terms of the roles of the different kinds of elements of arguments), but has not paid attention to the *evaluation* of arguments. In other words, he does not provide an analogue of logical validity as an evaluation criterion for arguments – perhaps because he does not believe there to be one.

This is a major omission, especially in view of the role of rebuttals in his scheme: clearly a rebuttal can influence the evaluation of an argument for a particular claim. Using Toulmin's example above, whether the datum that Harry is born in Bermuda justifies the claim that he is a British subject depends on whether Harry's parents were aliens. Assuming that Harry's parents were aliens, the datum does *not* justify the claim, while assuming that they weren't aliens, the datum will normally justify the claim (unless there is another rebuttal, such as that Harry has become a naturalized American). As stated previously, perhaps Toulmin did not believe there to be an evaluation criterion analogous to logical validity. Here it is shown that there is such a criterion by providing one: it is based on the idea of *dialectical interpretation* (cf. Verheij (2003a, b, 2005)). Briefly, the statements in an argument are evaluated with respect to the argument's assumptions. Statements can

be justified (e.g., when there is a justifying reason for them), defeated (e.g., when there is a defeating reason against them) or neither.

Of course it is impossible to retain all of Toulmin's central ideas in their exact original form – some ideas will be adapted and extended. The task set here is one of reconstruction with a contemporary eye, and thus by necessity involves an *interpretation* of Toulmin's ideas. Still the goal is that the elements of Toulmin's scheme will find a recognizable place in the present elaboration.

The starting point for the reconstruction of Toulmin's scheme is a particular theory of dialectical argumentation, called DefLog. It is related to my work on argument assistance software (see, e.g., Verheij (2003a, 2005) on the ArguMed system). In the present paper, the relevant parts of DefLog are introduced alongside the reconstruction of Toulmin's scheme. For a more extensive account of DefLog, the reader may want to consult Verheij (2003b, c). The present work builds on research on defeasible and dialectical argumentation (cf. e.g. the work of Pollock (1987), Reiter (1980), Loui (1998), Vreeswijk (1997), Prakken (1997), Hage (1997) and Dung (1995)). An earlier version of this paper was presented at the OSSA 2001 conference (Verheij, 2002).

There are many relations of the points discussed in this paper with work in the field of argumentation theory. For instance, the treatment of warrants in this paper is related to argumentation schemes (see, e.g., Walton (1996), cf. also Verheij (2003c)) and Hitchcock's use of covering generalizations (1998). Such cross-connections between formal and informal approaches to argumentation deserve further attention, but fall outside the scope of this paper. The book *Argumentation Machines. New Frontiers in Argument and Computation* (2003; edited by Reed and Norman) is a useful resource in this respect.

2. ELABORATING ON TOULMIN'S SCHEME FOR THE LAYOUT OF ARGUMENTS

In the following, the elements of Toulmin's scheme for the layout of arguments will be reconstructed, in a way that is formally explicit. A step-by-step strategy will be followed. Simple arguments will be analyzed in terms of primitive notions like statements, justifying reasons etc.

A graphical representation will be used that arranges the elements of Toulmin's scheme in a different way than in Toulmin's original. The reason for this is a pragmatic one: in this way it becomes possible to use the ArguMed software described by Verheij (2003a, 2005). ArguMed is argument assistance software that can graphically represent arguments and evaluate the status of the statements involved. It can be downloaded at http://www.ai.rug.nl/~verheij/aaa/.

Figure 3 shows Toulmin's original arrangement on the right and the new arrangement on the left. For reasons that will become clear below (in section 3, notably Figure 10), the element of rebuttal has been left out here. Note also that in Figure 3 some of Toulmin's lines (e.g., between Backing and Warrant) have been replaced by arrows. The reason for this adaptation is that in the formal treatment below each of the arrows corresponds to a conditional sentence.

The main difference between the original arrangement and the new one is that reasons appear below their conclusions. For instance, the data D appear below the qualified claim QC (why the qualifier and the claim are taken together is explained below) and the backing B appears below the warrant W. Other differences will be explained when they become relevant.

2.1. *Data and claim*

The first step in the present reconstruction is to consider Data and Claim. In Toulmin's (1958) own words, we have

> 'one distinction to start with: between the *claim* or conclusion whose merits we are seeking to establish (C) and the facts we appeal to as a foundation for the claim what I shall refer to as our *data* (D)' (p. 97).

Let's look at Toulmin's example: if we seek to establish the claim that Harry is a British subject, we could for instance appeal to the datum that Harry was born in Bermuda. In an ordinary language argument this could be expressed thus:

Harry was born in Bermuda. So he is a British subject.

Already at this early stage, it is convenient to adopt some conventions that elaborate on Toulmin's. The first is the convention that arguments are constructed from one or more *statements* expressed by *sentences*.[2] In the example we find the two statements that Harry is a British subject and that Harry was born in Bermuda, expressed by the

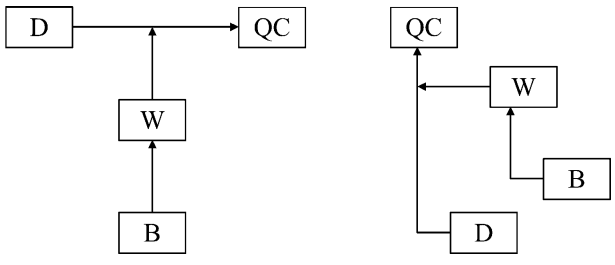

Figure 3. Toulmin's original arrangement (without rebuttal; on the left) presented differently (on the right).

sentences 'Harry is a British subject' and 'Harry was born in Bermuda', respectively.

The second convention is to distinguish between two different *roles* of statements in arguments: some are the argument's *assumptions*, others its *issues*. An argument's assumptions are the foundation on which the argument is built, whereas an argument's issues can be regarded as the topics or themes of the argument. A peculiarity of this convention is that an argument can have the effect that an issue of the argument can be settled by the argument (and hence is not 'at issue'). This occurs when the evaluation of the argument shows that an issue of the argument is justified or defeated.

In the example, it is assumed that Harry was born in Bermuda, and an issue that Harry is a British subject. If it were assumed that Harry is a British subject, there would be no need to provide a reason for it. If the datum were itself not assumed, i.e., if it were an issue that Harry was born in Bermuda, that issue would have to be established by further argument.

The third convention concerns the *evaluation* of the statements in an argument. The idea is that the statements in an argument can be evaluated, depending on the information expressed in the argument. For instance, the assumption that Harry was born in Bermuda is – since it is an assumption – taken to be justified. Also the argument's issue that Harry is a British subject is evaluated as justified, but only since there is a reason justifying it, viz. that Harry was born in Bermuda. At present, two evaluation statuses are distinguished: statements can be justified or they can be unevaluated. If for instance the statement that Harry was born in Bermuda were itself an issue, that statement and the statement that Harry is a British subject would not be justified. (Later we will also distinguish statements that are defeated. See section 3.)

This brings us to the fourth convention, which concerns *support by reasons*. One point of an argument like 'Harry was born in Bermuda. So he is a British subject' is that it expresses that the statement that Harry was born in Bermuda *supports* the statement that he is a British subject. It not only expresses the assumption that Harry was born in Bermuda and the issue that Harry is a British subject, but also that there is a support relation between the two statements. In other words, the argument implies that there is a conditional relation between the two statements, that can – for instance – be expressed by the compound sentence 'If Harry was born in Bermuda, he is a British subject'.[3]

The conditional 'If ..., (then) ...' is here not construed as the notorious material conditional of standard logic. One important reason for this is that a material conditional is truth-functional: its truth value is determined by the truth values of the conditional's antecedent and consequent. Cf. Haack (1978), Hage (1997), Prakken (1997), Verheij

(1996). The conditional 'If D, then C' implied by an argument 'D. So C' should however intuitively reflect some relation between D and C that is not captured by the truth values of D and C alone.

In order to prevent confusion with the material conditional, a dedicated arrow $\sim>$ is used here. A conditional D $\sim>$ C only validates Modus ponens. There is no way of deriving a conditional D $\sim>$ C on the basis of logic alone; deriving D $\sim>$ C will always be based on premises. For instance, D $\sim>$ C does not follow from a deduction of C from D.[4] Still D $\sim>$ C can follow, namely when it is the consequent of another conditional, e.g., W $\sim>$ (D $\sim>$ C). See section 2.2.

Statements expressed by sentences of the form 'If ..., (then) ...' can themselves be assumed or an issue, just like all other statements. For instance, an argument like 'Harry was born in Bermuda. So he is a British subject' can be considered to imply the assumption that if Harry was born in Bermuda, he is a British subject. A conditional assumption of this form can be referred to as the *associated conditional* of the argument (cf. Hitchcock, (1985)).[5]

According to the present conventions, the argument 'Harry was born in Bermuda. So he is a British subject' implies the following:

It is assumed that Harry was born in Bermuda.
It is an issue that Harry is a British subject.
It is assumed that if Harry was born in Bermuda, he is a British subject.
It is justified that Harry was born in Bermuda.
It is justified that Harry is a British subject.
It is justified that if Harry was born in Bermuda, he is a British subject.

The issue that Harry is a British subject is justified, since the statement that Harry was born in Bermuda is a justifying reason for it. In general, a statement D is said to be a *justifying reason* for another statement C if the statements that D and that if D, then C are both justified.

The above can be summarized in a graphical representation of the argument (cf. Verheij (2003a, b)) (Figure 4).

The exclamation mark indicates an assumed statement, the question mark a statement that is an issue. The associated conditional (that if Harry was born in Bermuda, he is a British subject) is depicted by the

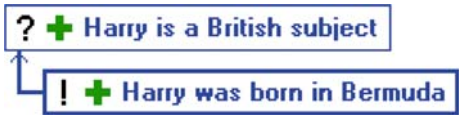

Figure 4. An elementary argument.

arrow. That the two elementary statements are justified is indicated by the plus signs and the dark bold font.

Figure 5 shows what happens if it were an issue that Harry was born in Bermuda.

Now both elementary statements are unevaluated (as is indicated by the light italic font and the zero sign) since for neither is there a justifying reason. The dark colour of the arrow indicates that the corresponding conditional statement is a justified assumption.

By the present conventions an argument like 'Harry was born in Bermuda. So he is a British subject' is closely linked to the following formal derivation based on Modus ponens (From φ and $\varphi \sim\!> \Psi$, conclude Ψ):

$$\frac{\text{Harry was born in Bermuda.} \quad \text{Harry was born in Bermuda} \sim\!> \text{Harry is a British subject.}}{\text{Harry is a British subject.}}$$

The two premises of the derivation correspond to the two assumptions of the datum-claim argument (one of them the associated conditional), the conclusion of the derivation to the issue. The Modus ponens form of the derivation makes explicit how the evaluation status is transferred from the assumptions to the issue: when the statements that φ and that $\varphi \sim\!> \Psi$ are both justified, then the statement that Ψ is also justified.

It is tempting to read the Modus ponens-derivation itself as the following informal argument:

(*)Harry was born in Bermuda. If Harry was born in Bermuda, then he is a British subject. So he is a British subject.[6]

However, this informal argument is clearly *another* argument than the example 'Harry was born in Bermuda. So he is a British subject' (one relevant difference being that while the shorter example could occur in an ordinary argumentative text, the extended version (*) only occurs in logic-oriented texts). (See also Carroll's dialogue 'What the tortoise said to Achilles', reprinted in Hofstadter (1980), that shows how adding the 'If..., then ...'-assumption underlying an argument leads to an infinite regress.)

Until now, we have for convenience omitted the qualifier in Toulmin's example. In the example, the datum 'Harry was born in Bermuda' does not support the claim 'Harry is a British subject' in an

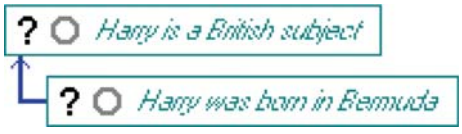

Figure 5. If the datum were an issue.

unqualified manner, but only *presumably*. Concerning such *qualifiers* as Toulmin distinguishes in his scheme, we will here be very brief. A qualifier is simply thought of as some kind of modal operator on statements. As a result, Toulmin's qualifier will be considered as being a *part* of the sentence that expresses the claim supported by the data. In Toulmin's example, we will take it that the datum supports the claim 'Presumably, Harry was born in Bermuda'. As a result, the relevant conditional is 'If Harry was born in Bermuda, then presumably he is a British subject' instead of the one used above. No semantic assumptions about the qualifiers are assumed here, in style with Toulmin's loose treatment. For instance, he seems to use the qualifiers 'presumably' and 'generally' more or less interchangeably (even though the former suggests an epistemological and the latter an ontological point of view).

There is much more to say about qualifiers (for instance, Peter McBurney has suggested that a proper treatment of qualifiers will lead to a richer set of rebuttals than discussed in section 3), but this falls outside of the scope of this paper.

2.2. *Warrant*

The next step is to consider the *warrants* of arguments. Toulmin (1958) describes warrants as 'general, hypothetical statements, which can act as bridges [between data and claim, BV], and authorize the sort of step to which our particular argument commits us' (p. 98). In other words, warrants are inference licenses that express that certain claims follow from certain data. The warrant of Toulmin's example on British subjecthood is the statement 'A man born in Bermuda will generally be a British subject'. Toulmin thinks of warrants as *general* inference licenses: not only if *Harry* was born in Bermuda, he is a British subject, but also if *John* was born in Bermuda, he is a British subject, etc.[7] It seems that for Toulmin the statement 'A man born in Bermuda will generally be a British subject' is equivalent to a scheme like the following, where *Person* stands for any person:

> If *Person* was born in Bermuda, then generally *Person* is a British subject.

Here '*Person* was born in Bermuda' and '*Person* is a British subject' can be taken as a *generic* datum and claim, respectively. The apparent equivalence for Toulmin between rule statements like 'A man born in Bermuda will generally be a British subject' and conditional schemes like the one above is for instance suggested by his emphasis on the bridge-like character of warrants (e.g., p. 98 as cited above, and p. 105, where he discusses the difference between warrants and backings).

The discussion of the formal validity of arguments of the type 'D. W. So C' (p. 118f., especially p. 119), where D stands for the data, W for the warrant and C for the claim, strengthens this suggestion. The formal structure of such an argument could be made explicit in the following way:

$$D(t).\ D(x) \sim\!> C(x).\ \text{So}\ C(t).$$

Here x is a variable instantiated by t. (In general, there can be several variables.) Toulmin mentions a variant of this formal structure in the style of classical syllogisms:

X is an A. All A′s are B′s. So X is a B.

Somewhat confusingly, Toulmin is not fully consistent in his discussion of warrants: occasionally, he seems to refer to an *instance* of a conditional scheme (like the one above) as a warrant, for instance when he mentions the short form 'If D, then C', where D and C are particular (and not a generic) data and claim (p. 98). However, as stated previously, at other places Toulmin unambiguously emphasizes the generality of warrants (e.g., in the description of warrants cited above, p. 98, but also on p. 100, where he states that warrants certify the soundness of all arguments of the appropriate type).

Summarizing, it is convenient to distinguish the following three:

(1) A man born in Bermuda will generally be a British subject.
(2) If *Person* was born in Bermuda, then generally *Person* is a British subject.
(3) If Harry was born in Bermuda, then generally he is a British subject.

The first is the ordinary language expression of a warrant. It is a rule statement that connects certain kinds of data to certain kinds of claims. It is these that in the present paper will be referred to as warrants (which is in agreement with Toulmin's conception of warrants and with most of Toulmin's examples).

The second is the conditional scheme that is the formal explication of the bridge-like connection resulting from warrants. In a sense (and, as stated previously, apparently according to Toulmin) an ordinary language warrant like (1) above is equivalent to a conditional scheme such as (2): it may be taken that each implies the other. Moreover, it seems straightforward to write algorithms that translate the one to the other (at least for significant subsets of ordinary language examples of warrants, e.g., for simple generic sentences like 'Bermudans are British subjects' and 'Thieves are punishable'). It should be noted however that in ordinary language argument, the conditional schemes will themselves never occur. They are merely convenient constructs in

order to summarize the range of particular argument steps as they are licensed by the generic warrant that corresponds to the scheme.

The third, 'If Harry was born in Bermuda, then generally he is a British subject', is the associated conditional that expresses that a particular datum implies a particular claim. Conditionals such as (3) are the instances of the conditional schemes like (2) that correspond to ordinary language warrants (1).[8]

The only formal connection between the three that will be used below is that between the warrant (in the sense of an ordinary language rule statement) and the associated conditional that forms the bridge from datum to claim. It can be expressed as the nested conditional 'If W, then if D, then C', or formally as W $\sim>$ (D $\sim>$ C).[9] The nested conditional expresses that it follows from the warrant that the claim follows from the datum. This nested conditional is normally left implicit in an argument, just like the conditional 'If D, then C'.

The result is graphically represented in Figure 6.

That the statement that a man born in Bermuda will generally be a British subject, is a warrant is visualized by an arrow pointing to an arrow: here the arrow from the warrant statement points to the arrow representing the associated conditional that if Harry was born in Bermuda, he is a British subject. The warrant justifies the connection between datum and claim.

The transfer of evaluation status from assumptions to issues can be made explicit as the two-step Modus ponens derivation shown in Figure 7. D, W and W $\sim>$ (D $\sim>$ C) are assumptions of the data-claim-warrant argument. In Figure 6, W $\sim>$ (D $\sim>$ C) is shown using a thicker line than D $\sim>$ C in order to show that the former is an assumption of the argument instead of an issue.

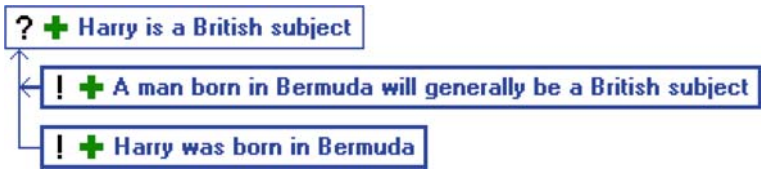

Figure 6. Datum, claim and warrant.

$$\frac{\dfrac{}{\text{W. } \text{W} \sim> (\text{D} \sim> \text{C}).} \qquad \text{D.}}{\text{D} \sim> \text{C.}} \qquad \text{D.}$$

$$\text{C.}$$

Figure 7. A two-step Modus ponens derivation.

2.3. *Backing*

The next important element of arguments distinguished by Toulmin is that of the *backing* of warrants. Backings provide support for warrants. They become relevant when a warrant is challenged. This occurs when the legitimacy of the range of arguments as licensed by a warrant is challenged. In Toulmin's example, the warrant that a man born in Bermuda will generally be a British subject, is supported by appeal to particular statutes and legal provisions (which are not explicitly mentioned by Toulmin). Formally, the relation between backing and warrant is the same as the relation between datum and claim,[10] and requires nothing new. An argument 'B. So W' (where B is a backing of some warrant B) can be analyzed just like an argument 'D. So C'. In the former case, the associated conditional has the form 'If B, then W'. In the example, it becomes 'If the statutes and other legal provisions so-and-so obtain, then a man born in Bermuda will generally be a British subject'.

Even though datum and backing formally play a related role, Toulmin notes a relevant difference between them: there are arguments containing data but without explicit backing (as in 'D. So C'), while there are no arguments containing backing but lacking data. According to Toulmin, the occurrence of a backing *presupposes* the occurrence of data (and claim).[11] Toulmin also emphasizes the difference between backing and warrant: backings can be categorical statements of fact just like data, while warrants always are general bridge-like statements (cf. p. 105). A central point in Toulmin's book is that different kinds of backings occur in different fields of argument. Among Toulmin's examples of backings are statutes and acts of Parliament, statistical reports, appeals to the results of experiments and references to taxonomical systems. All can provide the backing that warrant the arguments as they are acceptable in particular fields.

The graphical representation of an argument involving a backing is shown in Figure 8.

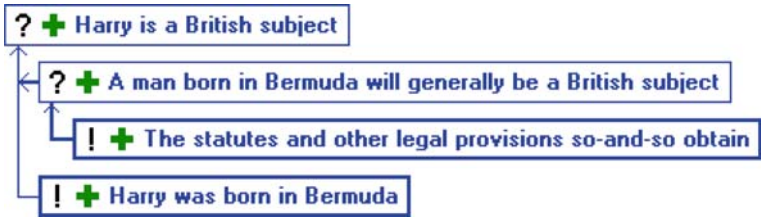

Figure 8. Adding the backing.

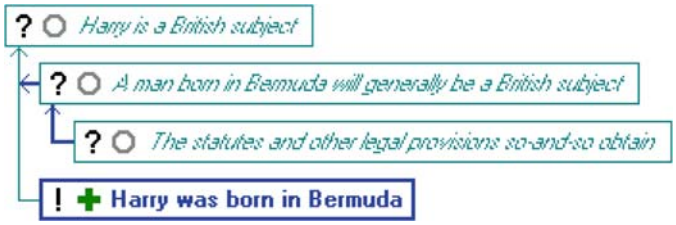

Figure 9. If the backing were an issue.

If the backing were itself an issue, we would get what is shown in Figure 9.

Neither the statement that Harry is a British subject nor the warrant statement that a man born in Bermuda will generally be a British subject, is now justified since there is no justifying reason for them. Note however that the arrows from backing to warrant and from warrant to the arrow from datum to claim are justified assumptions. The arrow from datum to claim is now an unevaluated issue (while it was a justified issue in Figure 8).

It should be noted that the reconstruction of Toulmin's analysis of arguments with data, (qualified) claim, warrant and backing as above is hardly a deviation from standard logical notions, though perhaps a different use of them. This is in contrast with Toulmin's claims. As has been noted by others, the distinction of data, claim, qualifier, warrant and backing can be dealt with in a way that is not too far from standard logic, while retaining the differences between them. In the present reconstruction, the technical tools used are essentially a Modus ponens-validating conditional and variables (plus the unelaborated suggestion to consider modal operators for the qualifiers). The main differences between Toulmin's treatment and standard logic arise from differences in emphasis with respect to philosophical and argumentation-theoretical starting points (such as the mentioned field-dependency of the backings of warrants and the distinction of different roles of statements in an argument).

3. REBUTTAL

A genuine and radical deviation from standard logic is required by Toulmin's notion of *rebuttals*. However, Toulmin hardly elaborates on the nature of rebuttals.

3.1. *Kinds of rebuttals*

As Toulmin puts it, rebuttals involve conditions of exception for the argument (p. 101). Apparently, for Toulmin, rebuttals can have several

functions. For instance, rebuttals can 'indicate circumstances in which the general authority of the warrant would have to be set aside' (p. 101), but can also be (and for Toulmin apparently equivalently) 'exceptional circumstances which might be capable of defeating or rebutting the warranted conclusion' (p. 101). On p. 102, he also speaks about the applicability of a warrant in connection with rebuttals. In other words, Toulmin speaks of the defeat (or rebutting) of the conclusion, of the applicability of the warrant and of the authority of the warrant, in a rather loose manner, without further distinction. Toulmin is unclear about the relation of these seemingly different situations. Here the three will be distinguished, in a way that naturally fits the reconstruction of the other elements of Toulmin's scheme above, as follows. If we look at the warrant-data-claim part of Toulmin's scheme (which in our reconstruction includes the qualifier), there are five statements that can be argued against:

1. The data D
2. The claim C
3. The warrant W
4. The associated conditional 'If D, then C' that expresses the bridge from datum to claim.
5. The associated conditional 'If W, then if D, then C' that expresses the bridge between warrant and the previous associated conditional.[12]

Reasons against any of these statements can be seen as a *kind of rebuttal* of an argument that consists of warrant, data and claim (Figure 10).

The first three speak for themselves, and are clearly all different. An argument against the datum that Harry was born in Bermuda (for instance by claiming that Harry was born in London) differs from an argument against the claim that Harry is a British subject (for instance by claiming that Harry has become a naturalized American) and from an argument against the warrant that a man born in Bermuda will

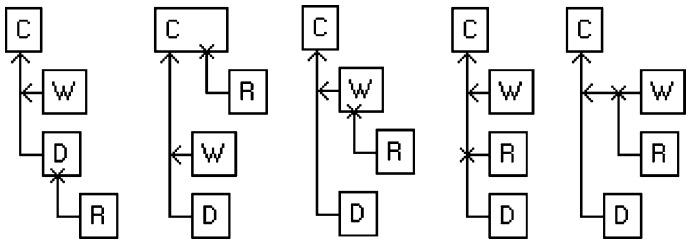

Figure 10. Five kinds of rebuttal.

generally be a British subject (for instance by claiming that those born in Bermuda are normally French).

An argument against the fourth kind of statement (the first associated conditional), can be regarded as an attack on the connection between data and claim. Such attacks have been dubbed *undercutting defeaters* by Pollock (1987).[13] Harry having become a naturalized American could be an argument against the connection between Harry being born in Bermuda and Harry being a British subject.

An argument against the fifth kind of statement can be regarded as an attack against the warrant's applicability: normally the warrant can justify the conditional that connects data and claim, but since there is a rebuttal, the warrant does not apply. In other words, when the associated conditional if W, then if D, then C, is not justified, the warrant, which normally gives rise to a bridge between data of type D and claim of type C, does *not* give rise to such a bridge for the actual data D and claim C at hand. For instance, Harry's parents both being aliens could well be an argument against the applicability of the warrant that a man born in Bermuda will generally be a British subject.[14]

The three situations to which Toulmin attaches the term rebuttal (defeat of the conclusion, of the applicability of the warrant and of the authority of the warrant) are among these five kinds of rebuttals, viz. the second, fifth and third, respectively. The other two kinds of rebuttals of a warrant-data-claim argument, viz. that of the first and fourth kind, are apparently not mentioned by Toulmin.

3.2. *The effect of rebuttal on argument evaluation*

We now turn to a major omission of Toulmin's discussion: a topic he does not address is the *effect* of rebuttal on the evaluation status of the statements in an argument. Interestingly, this has turned out to be a notorious, but rewarding topic for logical study (cf. the work on formalizing defeasible and dialectical argument; see e.g. Chesñevar et al. (2000), Prakken and Vreeswijk (2002)).

In the present approach, a third evaluation status is introduced in order to analyze the effect of rebuttal on the evaluation status: statements can not only be justified or unevaluated, but also *defeated*.[15] The latter status applies when there is a defeating reason against the statement. A defeated statement is 'contra-justified'. For instance, in the example, it can be defeated that Harry is a British subject in light of the reason against it that he has become a naturalized American, even though Harry was born in Bermuda. (See Verheij (2003a, b, 2005) for a discussion of technical difficulties of the evaluation of arguments.)

One difficulty is the fundamental *nonmonotonicity* of the resulting logic: when assumptions are added it can occur that previously justified

statements are no longer justified (cf. in a more general logical setting Gabbay et al. (1994)). In standard logic, when sentences are added to an initial set of sentences the truth of which is assumed, the set of implied truths never gets smaller. Additional assumptions normally allow more conclusions to be drawn, and never fewer. In contrast, in a context of reasoning with pros and cons, as for instance in Toulmin's analysis of argument, where there is not only support but also attack, statements that are initially justified with respect to a set of assumptions can become defeated (or unevaluated) in light of additional assumptions.

Toulmin's example can be used as an illustration: when it is not assumed or otherwise justified that Harry's parents were aliens or that he has become a naturalized American, etc., i.e., when the possible rebuttals are not effective, it is justified that Harry is a British subject on the assumption that he was born in Bermuda and that a man born in Bermuda will generally be a British subject. However, when one of the rebuttals is effective, e.g., when it is assumed that Harry's parents were aliens, Harry being a British subject no longer follows as a justified statement, but is unevaluated (given that there are no *other* reasons for or against it).

Just as justifying reasons for a statement can make an issue justified, defeating reasons against a statement can make it defeated. Formally, it is convenient to distinguish statements of the form 'It is defeated that ...', where the dots indicate a sentence expressing another statement. In this way, *attack by reasons* can be dealt with by analogy with support by reasons (cf. section 2.1). Whereas the support relation between data D and claim C is expressed by the associated conditional 'If D, then C', the attack relation between a rebuttal R and a claim C is expressed by the statement 'If R, then it is defeated that C' (for a rebuttal of the second kind above). The latter will formally be denoted as R xC (where xC expresses that it is defeated that C and is the conditional used to express support)[16] or more briefly as R $\sim>$ xC. In general, a statement R is said to be a *defeating reason* against another statement C if the statement that R, and the statement that if R, then it is defeated that C, are both justified (cf. the notion of a justifying reason in section 2.1).

It can now be defined which statements are supported by a set of assumptions (using the simple logical language with connectives $\sim>$ and x as used above) and which attacked. A statement S is *supported* by the assumptions if the statement S is itself an assumption or follows from the assumptions by the repeated application of Modus ponens (From ϕ and $\phi \sim> \psi$, conclude ψ). A statement S is *attacked* by assumptions if the statement that the statement is defeated (i.e. the statement that xS) is one of the assumptions or follows from the

assumptions by the repeated application of Modus ponens. When a set of assumptions is *conflict free*, i.e., when there are no statements that are both supported and attacked by the assumptions, it is now easy to define the *evaluation status* of statements with respect to the assumptions, as follows. A statement is *justified* (with respect to the conflict-free set of assumptions) when it is supported by the assumptions. A statement is *defeated* (with respect to the conflict-free set of assumptions) when it is attacked by the assumptions. Any other statement is *unevaluated* (with respect to the conflict-free set of assumptions).

Evaluation in the more important general case of possibly conflicting sets of assumptions is subtle (and those who are not formally inclined may want to pass over this). The definition for conflict-free sets does not work since when there is a conflict there are statements that are both supported and attacked by the assumptions. Therefore in the general case, the assumptions are considered to be *defeasible* in the sense that not all statements assumed to be justified need also turn out to be evaluated as justified when the whole set of assumptions is evaluated. The idea is that some of the assumptions are themselves defeated since they are attacked by other assumptions. As a result, a set of prima facie justified assumptions Δ is divided into two parts: the justified part and the defeated part. In words, the justified part is a maximal conflict-free subset of Δ that attacks all elements of the defeated part. Such a justified part is said to *dialectically interpret* Δ.

The formal definition is as follows:

Let Δ be a set of sentences defeasibly assumed to be justified, and let J be a subset of Δ (possibly equal to Δ). Then J *dialectically interprets* Δ when the following hold:

1. J is conflict free.
2. Any statement in Δ that is not in J is attacked by J.

A *dialectical interpretation* of Δ is a pair of sets (Supp(J), Att(J)), where J dialectically interprets Δ, Supp(J) consists of the statements supported by J, and Att(J) consists of the statements attacked by J. The statements supported by J are said to be *justified* in the dialectical interpretation of Δ corresponding to J, while those attacked by J are said to be *defeated*.

There is a close formal connection between this definition of dialectical interpretation and Dung's argumentation frameworks (1995). Other related work has for instance been done by Pollock (1987), Reiter (1980), Loui (1998), Vreeswijk (1997), Prakken (1997), Hage (1997).

There is a lot to say about this definition of dialectical interpretation. For instance, while some theories have a unique dialectical interpretation, others have none or several. For further information, the reader is referred to Verheij (2003a, b, 2005) and the software that can

be downloaded at http://www.ai.rug.nl/~verheij/aaa/. Here we will confine ourselves to examples related to Toulmin's scheme.

3.3. *Toulmin's example*

Let's again consider Toulmin's example concerning Harry. Assume as datum that Harry was born in Bermuda (D) and as rebuttal that Harry has become a naturalized American (R). It is an issue that Harry is a British subject (the claim C). For simplicity we first forget about warrant and backing. The relation between datum and claim is implicitly assumed, and can formally be expressed as D $\sim>$ C. The relation of the rebuttal with datum and claim is assumed to be of the *fourth* kind above, i.e., as an attack against the associated conditional that If D, then C. Therefore the relation is expressible as R $\sim>$ x(D $\sim>$ C). Formally, the set of assumptions Δ consists of four sentences, viz. D, R, D $\sim>$ C and R $\sim>$ x(D $\sim>$ C). The set contains a conflict: D $\sim>$ C is both supported (since it is an assumption) and attacked (by applying Modus ponens on R and R $\sim>$ x(D $\sim>$ C)). The set of assumptions has a dialectical interpretation however: in it, the assumption D $\sim>$ C is defeated, while the other three are justified. The claim C is unevaluated since it is neither justified nor defeated in the dialectical interpretation. This is in accordance with the intuition that there is neither a justifying reason for C, nor a defeating reason against it. It can be checked that this is the *only* dialectical interpretation.[17]

Graphically, we get Figure 11 for the case that the rebuttal is assumed.

The arrow ending in a cross indicates that the rebuttal attacks the associated conditional that if Harry was born in Bermuda, he is a British subject. It is now not justified that Harry is a British subject since it is an issue for which there is no justifying reason. (Nor is it defeated.) The associated conditional is defeated (indicated by the dotted line) since there is a defeating reason against it, viz. the rebuttal.

If it were an issue that Harry has become a naturalized American, Figure 12 would be the result.

Now it is justified that Harry is a British subject, since the rebuttal does not have effect and the associated conditional is not defeated, but

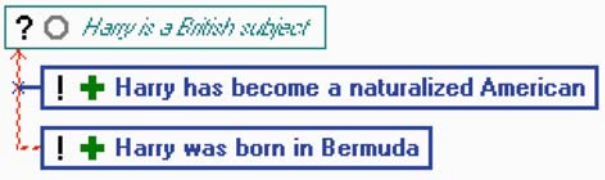

Figure 11. A rebuttal.

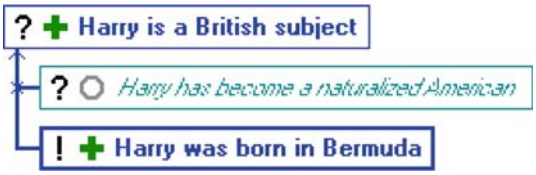

Figure 12. If the rebuttal were an issue.

justified. The graphical representation clearly demonstrates that rebuttals can have a decisive effect on the evaluation of arguments and the statements in them: when the rebuttal is an issue, the claim is (prima facie) justified, but when the rebuttal is assumed, the claim is not justified.

Recall that Toulmin did not distinguish between kinds of rebuttals. In the previous paragraphs, the rebuttal R that Harry has become a naturalized American was considered to be a rebuttal of the fourth kind. Let's now analyze the example considering R as a rebuttal of the *fifth* kind, i.e., it attacks the applicability of the warrant in Harry's case. Formally, this is denoted thus: R $\sim>$ x(W $\sim>$ (D $\sim>$ C)). In other words, if the rebuttal R is justified, then it is defeated that the conditional that if D then C, follows from the warrant W. As a result, in case of R, W does not imply that D can support C. The following are assumed:

D: Harry was born in Bermuda
W: A man born in Bermuda will generally be a British subject
R: Harry has become a naturalized American
B: The statutes and other legal provisions so-and-so obtain

It is an issue that Harry is a British subject (C). The assumed logical connections (that are normally left implicit) are as follows:

B $\sim>$ W
W $\sim>$ (D $\sim>$ C)
R $\sim>$ x(W $\sim>$ (D $\sim>$ C))

The set of assumptions contains a conflict: the nested conditional W $\sim>$ (D $\sim>$ C) that expresses that W implies that D can support C, is both supported (as an assumption) and attacked (since its defeat follows from R and R $\sim>$ x(W $\sim>$ (D $\sim>$ C))). In the unique dialectical interpretation, only W $\sim>$ (D $\sim>$ C) is defeated (by the defeating reason R against it), while the other six assumptions are justified. As a result, the statements that D $\sim>$ C and that C are unevaluated since there is no justifying reason for them.

Assuming that the rebuttal is of the fifth kind, we get the graphical representation shown in Figure 13.

The arrow ending in a cross indicates that the rebuttal that Harry has become a naturalized American attacks that the conditional that if Harry was born in Bermuda, he is a British subject, follows from the warrant statement. It is now neither justified nor defeated that Harry is a British subject since there is neither a justifying reason for it nor a defeating reason against it.

If the rebuttal were itself an issue, the evaluation would be different, and it would be justified that Harry is a British subject (Figure 14).

3.4. *The reinstatement of a claim*

An important phenomenon related to the evaluation of arguments as discussed above is the *reinstatement* of a claim. This occurs when a claim is at first justified, for instance in the light of an argument without rebuttal, subsequently unjustified, for instance when a rebuttal is added, and finally again justified, when the rebuttal is in turn shown

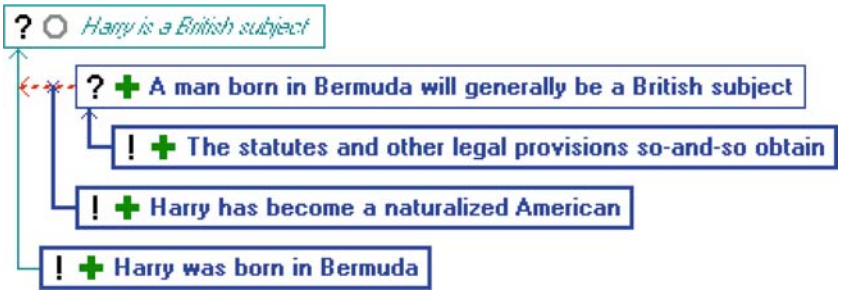

Figure 13. Rebuttal of the fifth kind.

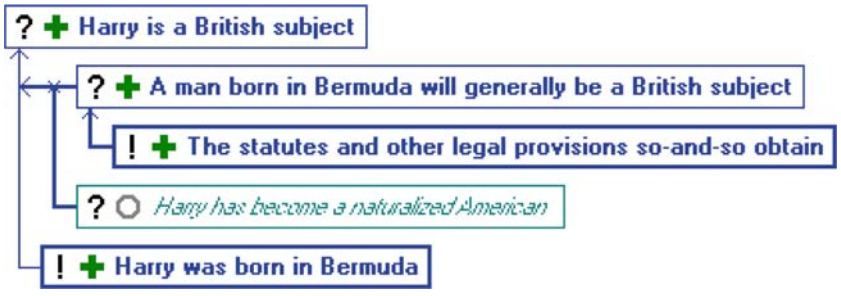

Figure 14. If the rebuttal were an issue.

to be unjustified. The latter might occur when the rebuttal is itself the claim of a scheme that is extended by a rebuttal.

Toulmin does not discuss this phenomenon of reinstatement (simply because he does not discuss the evaluation of arguments at all). An example based on Dutch tort law is the following. It may be claimed that someone has committed an unlawful act on the basis of the datum that he has violated a property right. An example of the violation of a property right is the breaking of someone else's window. In Dutch tort law, a warrant could be that violations of property rights are unlawful, which can be backed by article 6:162 of the Civil Code. If the argument is limited to the present information, the claim can be thought of as justified (on the basis of the argument). It can subsequently be argued that there is a rebuttal: there is a ground of justification for the act since the person obeyed an official's command. (Note that the rebuttal is here the claim of a second scheme with the obeying of the official's command as datum.) At this stage, the claim that an unlawful act has been committed is no longer justified on the basis of the argument involved. The argument can further be extended by adding a rebuttal to the second scheme: the official giving the command was unauthorized to do so. As a result the claim of the second scheme (viz. the existence of a ground of justification) is no longer justified. Since it is at the same time the rebuttal of the first scheme, it loses its rebutting effect in that scheme, and the claim that an unlawful act has been committed is again justified.

Verheij (2003a, b) gives more examples of the evaluation of arguments and the reinstatement of statements. A full theory of dialectical arguments is presented in which statements (including conditional statements corresponding to support and attack) can be supported or attacked by any number of other statements and in which support and attack can be chained (thus for instance allowing the attack of an attacking statement).

4. CONCLUSION

In the present paper, Toulmin's scheme for the layout of arguments has been formally reconstructed. An important omission of Toulmin's treatment has been repaired by providing an account of the evaluation of Toulmin-styled arguments. It has been discussed how the evaluation status of the statements in an argument is determined by a dialectical interpretation of the assumptions of the argument. In such an interpretation, assumptions are taken to be defeasible, so that they need not all be evaluated as justified in an interpretation: some can be defeated, viz. when they are attacked by a defeating reason against them. In order to achieve this, Toulmin's scheme for the layout of arguments

has been interpreted in terms of a theory of dialectical arguments (Verheij 2003a, b).

It can be concluded from the present work that the main deviation from standard logic of Toulmin's scheme is the element of rebuttal. Since Toulmin, the logic of rebuttal has deservedly received widespread attention (under the name of defeasible argumentation). The present paper is a continuation of that line of research.

Another important point of Toulmin's work is the field-dependency of the backings of the warrants underlying arguments. Toulmin has stressed that the rules of argument can vary from domain to domain. This raises a fundamental and a practical question. The fundamental question is what remains of logic when the rules of argument are variable. The practical question is to determine the rules of argument in particular domains. Both questions still require substantial research. Beginnings of answers to these important questions can be found in the works of Girle et al. (2003), Hage (1997), McBurney and Parsons (2000), Verheij (1999, 2003c) and Walton (1996).

NOTES

[1] Toulmin's recent book *Return to Reason* (Toulmin, 2001) can be regarded as an elaboration of this critical stance. He speaks of a centuries-old imbalance in our pursuit of knowledge and defends reasonable judgment (grounded on practical experience) against the dominance of rationality (based on theory).

[2] It is more precise to say that *the propositional content* of statements is expressed by sentences. Statements can be regarded as a kind of speech act (cf. Austin (1962), Searle (1969)).

[3] David Hitchcock has informed me that the claim that an argument 'Data, so claim' implies the conditional statement 'If data, then claim' can be given historical support by reference to the Stoic interpretation of conditional propositions, as reported in Diogenes Laertius, *Lives and Opinions of the Eminent Philosophers* 7.71: '... the conditional connective 'if' ... declares that the second [i.e., the consequent] follows from the first [i.e., the antecedent].'

[4] In terms of the jargon associated with the rules of inference of natural deduction: $\sim>$ has the standard elimination rule for conditionals (viz. *Modus ponens*), but lacks the standard introduction rule. In fact, there is no rule of inference introducing $\sim>$. I have introduced the term 'primitive implication' for such a conditional (Verheij, 2003b).

[5] The associated conditional of a data-claim argument is a 'minimal' commitment implicit in the argument, viz. simply the conditional that connects data and claim. In argumentation theory, the generalization of such a conditional to a rule statement (in our example the rule that those born in Bermuda are British subjects) is also often referred as the implicit assumption of an argument (van Eemeren et al., 1996, p. 14 and elsewhere). Those generalizations are here referred to as the warrants of an argument. See section 2.2 below and the last paragraph of section 1 of this paper. Hitchcock's associated conditional of an argument (1985) is construed as a material conditional. There are other types of implicit assumptions (see, e.g., Ennis, 1982).

[6] Such a reading of logical derivations as informal arguments is the basis of many textbooks that connect logic with informal argument. Here logical derivations play a related, but more modest role, viz. as explications of the transfer of evaluation status from assumptions to issues.

[7] It has been argued that a given data-claim argument can have different warrants, dependent on the scope of the generalization (cf. Hitchcock, 1998).

[8] It may be noted that our treatment has the effect that warrants, conditional schemes and their instances all use the same qualifier (unlike in Toulmin's example where the qualifier in the warrant is 'generally' and the qualifier of the claim is 'presumably'). The 'generally' in 'If Harry was born in Bermuda, then generally he is a British subject' refers to the class of (real or hypothetical) cases or situations in which Harry was born in Bermuda. Cf. also the remarks on qualifiers as modal operators in section 2.1.

[9] We use these conditionals here since DefLog and ArguMed (as described by Verheij (2003a, b, 2005)) do not have variables and variable instantiation. If these would be added to DefLog, the present treatment using conditionals W $\sim>$ (D $\sim>$ C) could be replaced by stipulating equivalences between warrants W and conditional generalizations D(x) $\sim>$ C(x) and then deriving instances D(t) $\sim>$ C(t) of D(x) $\sim>$ C(x).

[10] As a result, the warrant of an argument 'B. So W' (where B is a backing of a warrant W) can again be challenged and require further backing. When the warrant W* of the argument 'B. So W' is backed by B*, the warrant of the corresponding argument 'B*. So W*' can in turn be challenged, and so on, *ad infinitum*. As Toulmin remarks, *some* warrants must be left unchallenged (p. 106), just as some data for that matter. See e.g. Hage (2000) for a recent discussion of (amongst other topics) the resulting problem of justification (with an emphasis on the context of legal reasoning).

[11] Whether Toulmin is right here is arguable. For instance, in the law it is common to argue about the validity of legal rules, even outside the context of a concrete case to apply the rule to. This would correspond to an argument concerning the backing of a warrant (the legal rule), without having concrete data (the case facts) and claim (the legal consequence) in mind.

[12] Including variables and variable instantiation as suggested in note 9 would affect this fifth statement.

[13] The second kind of rebuttal is related to what Pollock (1987) calls rebutting defeaters. We do not elaborate on these here because that would involve technical detail that is not relevant here. Cf. e.g. Hage (1997), Prakken (1997), Verheij (1996) on priorities and weighing reasons.

[14] Especially in the law, the applicability of rules (which play a role similar to Toulmin's warrants) is often at issue. Cf. Hage (1997), Prakken (1997), Verheij (1996). In Hage's and Verheij's work, it is also discussed how the *weighing* of reasons for and against a conclusion can be formally treated.

[15] It depends on one's taste whether this approach is regarded as a three-valued interpretation of the whole of a logical language (in terms of the values *j*, *d* and *u* for justified, defeated and unevaluated) or as a two-valued interpretation of a part of the language (in terms of the values *j* and *d*, with possibly some sentences of the language left uninterpreted).

[16] The sentence 'It is defeated that C' expresses the *dialectical negation* of the statement that C. Dialectical negation expresses the denial that a prima facie justified assumption is justified. See Verheij (2003a, b, 2005). An omission that I hope to fill in future work is that I did not yet provide a good explanation of the relation between ordinary negation and dialectical negation.

[17] In this case, a straightforward, but tedious approach would be to check *all* 16 subsets of the set of four assumptions. When Δ gets larger, this quickly becomes impossible in practice. A shorter argument for the present example goes as follows. Let Δ consist of the four assumptions D, R, D $\sim>$ C and R $\sim>$ x(D $\sim>$ C). Δ attacks only one sentence, namely D $\sim>$ C since only the dialectical negation x(D $\sim>$ C) of that sentence follows by *Modus ponens* from Δ. Hence only D $\sim>$ C is a candidate for being defeated in a dialectical interpretation of Δ. As a result, there are only two subsets that might dialectically interpret Δ, namely Δ itself or Δ minus D $\sim>$ C. Δ itself cannot be a dialectically interpreting set since it is not conflict free. Δ minus D $\sim>$ C is a dialectically interpreting set since it is conflict free and attacks D $\sim>$ C.

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