Applied Intelligence (2021) 51:7469–7483
https://doi.org/10.1007/s10489-021-02206-8

Stability and stabilization for uncertain fuzzy system with sampled-data control and state quantization

Liu Yang[1] · Jiayong Zhang[2] · Chao Ge[3] · Wei Li[4] · Zhiwei Zhao[5]

Accepted: 8 January 2021 / Published online: 11 March 2021

Abstract

This paper studies the stability and stabilization problems of the T-S fuzzy systems with uncertainty and state quantization. Considering that fuzzy membership functions(FMFs) are the main characteristic of T-S fuzzy model, if the information about the membership function is not added, it will be conservative. So, a novel Lyapunov-Krasovskii functional (LKF) which contains not only integral variables but also FMFs is constructed. To include more information about the sampling pattern, the states on both sides of the sampling interval are incorporated into the LKF. When taking the derivative of the LKF, the product terms which consist of derivative of FMFs and LKF coefficient are involved. Then, the product terms are discussed to ensure their negative definition. By further derivation, enough stability conditions are expressed in the form of linear matrix inequalities (LMIs). The sampling intervals and controller parameters for the T-S fuzzy system can be solved by MATLAB toolbox with the optimal parameters. Finally, two numerical examples are simulated to illustrate the effectiveness of the proposed method.

Keywords Sampled-data control · T-S fuzzy system · State quantization · Fuzzy membership function

1 Introduction

T-S fuzzy model has become the fundamental tool for studying nonlinear systems. With the help of the T-S fuzzy model, the nonlinear system can be expressed in a locally linear form, and then the fuzzy membership functions(FMFs) can be used to connect them smoothly. What's more, the original nonlinear system can be approximated with arbitrary precision [1–3]. Consequently, in recent years, the stability analysis and controller synthesis issues for T-S fuzzy systems have been deeply investigated and also gained a lot results [4–7]. On the other hand, an inevitable disadvantage that we should not ignore is the parameter uncertainties in T-S fuzzy systems, which may occur because of the model inaccuracies, unexpected exterior ambient perturbations or network-generated stochastic failures in the implementation of the model. These have caused many researchers to extensively study the stability problem for uncertain T-S fuzzy systems [8–11]. For example, H_∞ controller design problem for uncertain T-S fuzzy system via the parallel distributed compensation (PDC)has studied in [12], which includes event-triggered communication strategy. Based on the Lyapunov-Krasovskii functional (LKF) and combining delay-product-type functional method together with the state vector augmentation, less conservative delay-dependent stability conditions were obtained for T-S fuzzy systems [13]. Vadivel et al. [14] employed a new LKF together with linear matrix inequality (LMI) technique to investigate robust H_∞ stability for uncertain T-S fuzzy

✉ Jiayong Zhang
zjy815@163.com

✉ Chao Ge
gechao365@126.com

✉ Wei Li
heutliwei@163.com

1 The Institute of Electrical Engineering, North China University of Science and Technology, Hebei, Tangshan 063009, China

2 The Institute of Mining Engineering, North China University of Science and Technology, Hebei, Tangshan 063009, China

3 The Institute of Electrical Engineering, North China University of Science and Technology, Hebei, Tangshan 063009, China

4 The Institute of Artificial Intelligence, North China University of Science and Technology, Hebei, Tangshan 063009, China

5 Department of Computer Science and Technology, Tangshan University, Hebei, Tangshan 063009, China

systems which have distributed time delay and nonlinear disturbance.

Benefit from the advancement of digital technology and in-depth research on it, the digital controllers which provide many advantages including installation cost saving, lower communication channels occupancy and implementation simplicity are gradually applied to area of industry control, and resulted in the rapid development of sampling control. Sampling control scheme only uses signal value of the signal sampling instant, and stay unchanged between the sampling interval until the arrival of the next sampling time. In other words, the larger the sampling interval, the smaller the data quantity. Hence, the sampled-data control method has aroused great concern [15–19]. Liu et al. [20] utilized the sampling controller which has a constant signal transmission delay to tackle the stabilization problem for T-S fuzzy systems. Wu et al. [21] applied time-dependent Lyapunov functional way to study the sampled-data control problem of chaotic systems which represented by T-S fuzzy model. Unlike the above two approaches, the delay partitioning idea that delay interval is split into flexible terminals has been employed in [22]. In [23], the fuzzy sampled-data control of chaotic systems is presented by using a time-dependent Lyapunov function. The function is continuous at the time of sampling, but not necessarily positively defined within the sampling interval.

The advantage of signal quantization is that it can increase bandwidth efficiency, enhance anti-interference and reduce energy loss. Thus, quantization is widely considered in the design of controllers for T-S fuzzy systems [24–26]. The robust control problem of uncertain discrete time Takagi-Sugeno (T-S) fuzzy networked control systems (NCSs) which has state quantization had investigated in [27]. In [28], an improved LKF and a less conservative delay-dependent conditions are proposed for the stability analysis of closed-loop NCSs. The method of multiple Lyapunov functions are cited in both references [27] and [28]. Recently, The LKF that depends on membership functions has been widely studied. The effect of adding membership function to LKF in reducing conservatism has been verified in [29–34]. Inspired by the above articles, in the process of constructing a new LKF to ensure the stabilization of system and do controller design work, if the application of the FMFs and the actual sampling pattern is more sufficient, it will be less conservative. It is also necessary to incorporate uncertainty and quantification into the design of the controller. There is still much improvement about stability analysis and controller synthesis for uncertain sampled-data T-S fuzzy system. The above is the starting point of our writing.

As discussed above, a sampled-data feedback control scheme has been proposed for T-S fuzzy systems which have parameter uncertainties and state quantization. The following contributions can be summarized:

(1) In this paper, a novel Lyapunov function which depends on FMFs is established. In addition, to contain more information about the actual sampling pattern, the state information on both sides of the sampling interval is added.
(2) When deriving LKF, the positive and negative of the derivative of the FMFs are discussed, because results depend on not only the state variable, but also the product terms. And, these terms are product terms which consist of the derivative of the FMFs and some LKF coefficient.
(3) By using relaxed Free-matrix-based (FMB) integral inequality and reciprocally convex method, a series of new stability conditions are got with the LMIs form. Then, the gain K_j and the maximum sampling interval can be obtained by solving LMIs with the optimal parameters. In the end, the effectiveness of the proposed methodology is provided by two numerical examples.

Notation: In the process of constructing the stability conditions, some symbols are involved. In order to facilitate reading, the meanings of these symbols are annotated below:
$*$ represents the symmetric elements of the matrix, such as : $\Omega_1 = \begin{bmatrix} X & P \\ * & Y \end{bmatrix}$ is equivalent to $\Omega_1 = \begin{bmatrix} X & P \\ P^T & Y \end{bmatrix}$; $X > 0$ denotes X is the positive definite matrix ; X^T is the transpose of the matrix X; I denotes identity matrix with the appropriate dimensions; $\mathbb{R}^n$ n-dimensional vector Euclidean space; $\mathbb{R}^{m\times n}$ real matrices in $m \times n$ dimensions.

2 Problem statement and preliminary

Suppose that the model of the nonlinear system represented by the T-S fuzzy model is as follows:
Plant rule i:
IF $\zeta_1(t)$ is $\mu_{i1}, \ldots$,and $\zeta_p(t)$ is μ_{ip}
THEN

$$\dot{x}(t) = (A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))u(t), \tag{1}$$

where $i \in \mathfrak{R} \triangleq \{1, 2, ..., r\}$ represents the ith fuzzy rule; r represents the number of fuzzy rules; $\zeta_i(t)(i = 1, 2, ..., p)$ denote the premise variable ; $\mu_{ij}(j = 1, 2, ..., p)$ denote fuzzy sets; $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^n$ denote the system state variable and control input of the system respectively; A_i and B_i denote constant matrices with proper dimensions;

$\varDelta A_i(t)$ and $\varDelta B_i(t)$ denote time-varying matrices with compatible dimensions and are assumed to meet:

$$[\varDelta A_i(t), \varDelta B_i(t)] = H_i F_i(t)[E_{ai}, E_{bi}], \tag{2}$$

It is assumed that the uncertainty is energy bounded and the norm bounded conditions are satisfied as follows: H_i, E_{ai} and E_{bi} are known constant matrices with appropriate dimensions,and $F_i(t) \in R^{n\times n}$ is the unknown Lebesgue measurable time-varying matrix function, and $F_i(t)$ meets $F_i^T(t)F_i(t) \leq I$.

$$\dot{x}(t) = \sum_{i=1}^{r} \lambda_i(\zeta(t))[(A_i + \varDelta A_i(t))x(t) + (B_i + \varDelta B_i(t))u(t)], \tag{3}$$

where $\lambda_i(\zeta(t))$ denotes the normalized membership function and satisfies the following description:

$$\lambda_i(\zeta(t)) = \frac{\prod_{j=1}^{p} \mu_{ij}(\zeta_j(t))}{\sum_{i=1}^{r}\prod_{j=1}^{p} \mu_{ij}(\zeta_j(t))} \geq 0, \tag{4}$$

where $\zeta(t) = [\zeta_1(t), \zeta_2(t), ..., \zeta_p(t)]$ and $\sum_{i=1}^{r} \lambda_i(\zeta(t)) = 1$. For simplify, let $\lambda_i(t) \triangleq \lambda_i(\zeta(t))$ in the following.

Sampling is the timed measurement of analog signals. In other words, it is the process of converting time-varying analog signals into time-varying pulse signals. The sampled signal is stored temporarily until the next sampling. During interval, the sampled signal can be coded and quantified. Zero-order holder(ZOH) is an important tool to achieve the above operations. In this article, it is assumed that the ZOH generates $u(t)$. The ZOH is mathematically modeled. It includes two parts, one is using conventional digital-to-analog converter to complete practical signal reconstruction, the other is converting discrete signals into continuous pulse signals. ZOH keeps the sampled data unchanged during the sampling interval until the next sampling time with a battery of holding time. The holding time satisfies $0 = t_0 \leq t_1 \leq \cdots \leq t_k \leq \cdots \leq \lim_{k\to+\infty} t_k = +\infty$, i.e.. Only the measured discrete sampled data are used for control purpose. Through summarizing the above statement and combining knowledge of quantification, the following is the representation of controller.

Controller Rule j:

IF $\zeta_1(t)$ is $\mu_{j1}, \ldots$, and $\zeta_p(t)$ is μ_{jp}
THEN

$$u(t) = K_j L(t_k), \quad t_k \leq t < t_{k+1}, \quad j = 1, 2, \cdots, r, \tag{5}$$

where K_j are local gain matrix with compatible dimension. Then, consider the logarithmic quantizer as follows:

$$L(\cdot) = [L_1(\cdot), L_2(\cdot), \ldots, L_n(\cdot)]^T$$

and the m-level sub-quantizer $L_m(\cdot)$ with symmetric properties:

$$L_m(x_m(t_k)) = -L_m(-x_m(t_k))$$

At the same time, the quantization level of L_m is described by:

$$\{\pm U_m^r | U_m^r = (\rho_m)^r U_m^{(0)}, r = \pm 1, \pm 2, \ldots\} \cup 0,$$
$$0 \leq \rho_m < 1, U_m^{(0)} \geq 0.$$

where ρ_m and $U_m^{(0)}$ denote the quantizer density and intinal quantization value, respectively. The following is the rigorous definition of the quantization $L_m(\cdot)$:

$$L_m(x_m(t_k)) = \begin{cases} U_m^{(r)}, \quad \frac{U_m^{(r)}}{1+l_m} < x_m(t_k) \leq \frac{U_m^{(r)}}{1-l_m}, \text{if } x_m(t_k) > 0, \\ 0, \quad \text{if } x_m(t_k) = 0 \\ -L_m(-x_m(t_k)), \quad \text{if } x_m(t_k) < 0 \end{cases}$$

where the $l_m = \frac{1-\rho_m}{1+\rho_m} (m = 1, 2, \ldots, n)$ is the quantier density. Hence, the quantizer is the characteristic of

$$L(x(t_k)) = x(t_k) + g(x(t_k))$$

where

$$g(x(t_k)) = [g_1(x_1(t_k)), g_2(x_2(t_k)) \ldots, g_n(x_n(t_k))]^T$$

with

$$-l_m[x_m(t_k)]^2 \leq x_m(t_k)g_m(x_m(t_k)) \leq l_m[x_m(t_k)]^2 \tag{6}$$

Thus, the overall state feedback controller is inferred by :

$$u(t_k) = \sum_{j=1}^{r} \lambda_j(t_k)K_j\,[x(t_k) + g(x(t_k))] \tag{7}$$

This article does not require sampling to be periodic. It only assumes that the distance between any two consecutive sampling instants belongs to an interval. In particular, suppose:

$$t_{k+1} - t_k = h_k \leq h \tag{8}$$

after substituting the (7) into the system (3), the following closed-loop model of fuzzy system can be got:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(t)\lambda_j(t_k)\big[(A_i + \Delta A_i(t))x(t) + (B_i + \Delta B_i(t))K_j(x(t_k) + g(x(t_k)))\big], \quad (9)$$

Next are the lemmas and assumption that will be used in the subsequent proof process:

Assumption 1 *Due to* $0 \le \lambda_j(t), \lambda_j(t_k) \le 1$*, assume* $|\lambda_j(t) - \lambda_j(t_k)| \le \eta_j, \forall k \in N$ *and* $j \in \Re$*, with* $0 \le \eta_j \le 1$.

Lemma 1 [35] *Let x be a differentiable function:* $[t_1, t_2] \to \mathbb{R}^n$*. For symmetric matrices* $R(\in \mathbb{R}^{n\times n}) > 0$ *,*$\xi \in \mathbb{R}^m$ *and* $N_1, N_2 \in \mathbb{R}^{n\times m}$*, the inequality is established:*

$$\begin{aligned} &-\int_{t_1}^{t_2}\dot{x}^T(s)R\dot{x}(s)ds \\ &\le (t_2 - t_1)\xi^T[N_1^T R^{-1}N_1 + \frac{(t_2 - t_1)^2}{3}N_2^T R^{-1}N_2]\xi \\ &\quad + 2\xi^T\left[N_1^T(x(t_2) - x(t_1)) - 2N_2^T\int_{t_1}^{t_2}x^T(s)ds\right] \\ &\quad + 2(t_2 - t_1)\xi^T N_2^T[x(t_2) + x(t_1)]. \end{aligned}$$

Lemma 2 [36] *For given positive integers* n, m*, a scalar* $\alpha\epsilon(0, 1)$*, a given matrix G in* $\mathbb{R}^{n\times n} > 0$*, two matrices* M_1 *and* M_2 *in* $\mathbb{R}^{n\times m}$*, for all vector* ζ *in* $\mathbb{R}^m$*, the function* $\Theta(\alpha, G)$ *given by:*

$$\Theta(\alpha, G) = \frac{1}{\alpha}\zeta^T M_1^T G M_1\zeta + \frac{1}{1-\alpha}\zeta^T M_2^T G M_2\zeta$$

if there exists a matrix X in $\mathbb{R}^{n\times n}$ *such that* $\begin{bmatrix} G & X \\ * & G\end{bmatrix} > 0$*, then the following inequality holds:*

$$\min_{\alpha\in(0,1)}\Theta(\alpha, R) \ge \begin{bmatrix} M_1\zeta \\ M_2\zeta\end{bmatrix}^T\begin{bmatrix} G & X \\ * & G\end{bmatrix}\begin{bmatrix} M_1\zeta \\ M_2\zeta\end{bmatrix}.$$

Lemma 3 [37] *For given* $b_1 > 0$ *and* $b_2 > 0$*, if there exists an LKF* $\mathcal{W}(t, x(t))$ *which depends on FMFs and satisfies the conditions as follows,* $\forall t \in [t_k, t_{k+1})$*:*

- (i) $b_1|x(t_k)|^2 \le \mathcal{W}(t_k, x(t_k)) \le b_2|x(t_k)|^2$;
- (ii) $\mathcal{W}(t_k, x(t_k)) \le \mathcal{W}^-(t_k, x(t_k))$;
- (iii) $\mathfrak{L}\mathcal{W}(t, x(t)) \triangleq \lim_{\Delta\to 0^+}\frac{1}{\Delta}\{\mathbb{E}\{\mathcal{W}(t+\Delta, x(t+\Delta))|t\} - \mathcal{W}(t, x(t))\} < 0$;

where $\mathfrak{L}\mathcal{W}(t, x(t))$ *is the infinitesimal operator along fuzzy system* (10) *and* $\mathcal{W}^-(t, x(t)) \triangleq \lim_{s\uparrow t}\mathcal{W}(s, s(t))$*. Then, the fuzzy system* (9) *has the stochastic stability.*

Lemma 4 [38] *Given matrices* $W_1 = W_1^T, W_2, W_3$ *with compatible dimensions:*

$$W_1 + W_2F(t)W_3 + W_3^T F^T(t)W_2^T < 0,$$

for all $F(t)$ *satisfying* $F^T(t)F(t) \le I$*, it is equivalent to the existence of a scalar* $\sigma > 0$*, thus satisfying:*

$$W_1 + \sigma W_2W_2^T + \sigma^{-1}W_3^T W_3 < 0.$$

3 Main results

In the following part, we obtain the stability constraint conditions for the T-S fuzzy system which has uncertainty and state quantization. To simplify the matrix expression, the symbols involved are shown as following:

$e_i = [0_{n\times(i-1)n}, I_n, 0_{n\times(7-i)n}](i = 1, 2, 3, 4, 5, 6, 7)$,
$h_1(t) = t - t_k,\ \ h_2(t) = t_{k+1} - t$,
$\eta_1^T(t) = \left[x^T(t), x^T(t_k), x^T(t_{k+1}), g^T(x(t_k))\right]$, $\eta_2^T(t) = \left[\eta_1^T(t), \dot{x}^T(t)\right]$,
$\eta_3^T(t) = \left[\eta_1^T(t), \int_{t_k}^t x^T(s)ds, \int_t^{t_{k+1}}x^T(s)ds\right]$, $\eta_4^T(t) = \left[x^T(t) - x^T(t_k), 0, \int_{t_k}^t x^T(s)ds, 0\right]$,
$\eta_5^T(t) = \left[0, x^T(t) - x^T(t_{k+1}), 0, \int_t^{t_{k+1}}x^T(s)ds\right]$, $\eta^T(t) = \left[\eta_3^T(t), \dot{x}^T(t)\right]$.

Theorem 1 *for given scalars* $h > 0, \delta_1 > 0, \delta_2 > 0$ *and control gain matrices* K_j*, the fuzzy system* (9) *is asymptotically stable, if there exist positive matrices* $P_i, \mathcal{R}_i = \begin{bmatrix} R_{11i} & R_{12i} & R_{13i} & R_{14i} & R_{15i} \\ * & R_{22i} & R_{23i} & R_{24i} & R_{25i} \\ * & * & R_{33i} & R_{34i} & R_{35i} \\ * & * & * & R_{44i} & R_{45i} \\ * & * & * & * & R_{55i}\end{bmatrix}, \mathcal{S}_i = \begin{bmatrix} S_{11i} & S_{12i} & S_{13i} & S_{14i} & S_{15i} \\ * & S_{22i} & S_{23i} & S_{24i} & S_{25i} \\ * & * & S_{33i} & S_{34i} & S_{35i} \\ * & * & * & S_{44i} & S_{45i} \\ * & * & * & * & S_{55i}\end{bmatrix}$*, with* $R_{15i}^T = R_{15i}$ *and* $S_{15i}^T = S_{15i}$*, any appropriate dimensional matrix* $\mathcal{Q}, \mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3, \mathcal{N}_4, G_1, G_2, G_3$ *and X, and any appropriate dimensional diagonal matrix* Z_{ij}*, such that for all* $i, j \in \Re$*, the following LMIs yield:*

$$\begin{cases} \text{if } \dot{\lambda}_i < 0, \text{then } P_i - P_r > 0, R_i - R_r > 0, S_i - S_r > 0 \\ \text{if } \dot{\lambda}_i > 0, \text{then } P_i - P_r < 0, R_i - R_r < 0, S_i - S_r < 0 \end{cases},$$
$$i = 1, 2, \cdots, r-1. \quad (10)$$

$$\Upsilon = \begin{bmatrix} R_{11i} & X \\ * & S_{11i}\end{bmatrix} > 0, \quad (11)$$

$$\begin{bmatrix} \Omega_1 & \Xi_6 & h_k\bar{\mathcal{N}}_3 & h_k^2\bar{\mathcal{N}}_4 \\ * & -Z_{ij} & 0 & 0 \\ * & * & -h_k S_{55i} & 0 \\ * & * & * & -3h_k S_{55i} \end{bmatrix} < 0, \tag{12}$$

$$\begin{bmatrix} \Omega_2 & \Xi_6 & h_k\bar{\mathcal{N}}_1 & h_k^2\bar{\mathcal{N}}_2 \\ * & -Z_{ij} & 0 & 0 \\ * & * & -h_k R_{55i} & 0 \\ * & * & * & -3h_k R_{55i} \end{bmatrix} < 0, \tag{13}$$

where

$$\Omega_1 = \begin{bmatrix} \Xi_1 & \Xi_4 & \Xi_5 \\ * & -\delta_1 I & 0 \\ * & * & -\delta_2 I \end{bmatrix} + diag\{\Xi_2, 0, 0\} + T_{ij}^T Z_{ij} T_{ij}^T,$$

$$\Omega_2 = \begin{bmatrix} \Xi_1 & \Xi_4 & \Xi_5 \\ * & -\delta_1 I & 0 \\ * & * & -\delta_2 I \end{bmatrix} + diag\{\Xi_3, 0, 0\} + T_{ij} Z_{ij} T_{ij}^T,$$

$$\begin{aligned}
\Xi_1 &= 2e_1^T P_i e_7 + 2\Pi_4^T \mathcal{Q}\Pi_5 - 2\Pi_8^T \mathcal{R}_i \Pi_9 - 2\Pi_8^T \mathcal{S}_i \Pi_{10} + e_2^T R_{15i} e_2 - e_1^T R_{15i} e_1 + 2\Gamma_1^T \Gamma_{2ij} \\
&\quad + e_1^T S_{15i} e_1 - e_3^T S_{15i} e_3 + 2\Pi_{14}^T \left[\mathcal{N}_1^T, \mathcal{N}_2^T, \mathcal{N}_3^T, \mathcal{N}_4^T\right] \Pi_{11} - \frac{1}{h}[e_5^T, e_6^T]\Upsilon[e_5^T, e_6^T]^T \\
\Xi_2 &= 2\Pi_1^T \mathcal{Q}\Pi_2 + 2\Pi_4^T \mathcal{Q}\Pi_6 + \Pi_{12}^T \mathcal{R}_i \Pi_{12} - \Pi_{13}^T \mathcal{S}_i \Pi_{13} + 2\Pi_{14}^T \mathcal{N}_4^T (e_1 + e_3), \\
\Xi_3 &= 2\Pi_1^T \mathcal{Q}\Pi_3 + 2\Pi_4^T \mathcal{Q}\Pi_7 + \Pi_{12}^T \mathcal{S}_i \Pi_{12} - \Pi_{13}^T \mathcal{R}_i \Pi_{13} + 2\Pi_{14}^T \mathcal{N}_2^T (e_1 + e_2), \\
\Xi_4 &= [\delta_1 \Gamma_1^T H_i, e_1^T E_{ai}^T], \ \Xi_5 = [\delta_2 \Gamma_1^T H_i, (e_2^T + e_4^T) K_j^T E_{bi}^T] \\
\Xi_6 &= col\{(e_2 + e_4)^T [\eta_1 (K_1 + L_{ij})^T, \eta_2 (K_2 + L_{ij})^T, \cdots, \eta_r (K_r + L_{ij})^T), 0, 0\} \\
T_{ij} &= col\{\Gamma_1^T B_i \underbrace{[I, I, \cdots, I]}_{r}, 0, \underbrace{[I, I, \cdots, I]}_{r}\}
\end{aligned}$$

$$\bar{\mathcal{N}}_l = col[\mathcal{N}_l, 0, 0], \ l = 1, 2, 3, 4$$

$$\Pi_1^T = \left[e_7^T, 0, 0, 0, e_1^T, -e_1^T\right], \ \Pi_2^T = \left[e_1^T - e_2^T, 0, e_5^T, 0\right], \ \Pi_3^T = \left[0, e_1^T - e_3^T, 0, e_6^T\right], \ \Pi_4^T = \left[e_1^T, e_2^T, e_3^T, e_4^T, e_5^T, e_6^T\right],$$

$$\Pi_5^T = \left[e_2^T - e_1^T, e_1^T - e_3^T, -e_5^T, e_6^T\right], \ \Pi_6^T = \left[e_7^T, 0, e_1^T, 0\right], \ \Pi_7^T = \left[0, e_7^T, 0, -e_1^T\right], \ \Pi_8^T = \left[0, e_2^T, e_3^T, e_4^T, 0\right],$$

$$\Pi_9^T = \left[e_5^T, 0, 0, 0, e_1^T - e_2^T\right], \ \Pi_{10}^T = \left[e_6^T, 0, 0, 0, e_3^T - e_1^T\right], \ \Pi_{11}^T = \left[e_1^T - e_2^T, -2e_5^T, e_3^T - e_1^T, -2e_6^T\right],$$

$$\Pi_{12}^T = \left[e_1^T, e_2^T, e_3^T, e_4^T, e_7^T\right], \Pi_{13}^T = \left[0, e_2^T, e_3^T, e_4^T, 0\right], \Pi_{14}^T = \left[e_1^T, e_2^T, e_3^T, e_4^T, e_5^T, e_6^T, e_7^T\right], \Gamma_1 = \left[G_1^T, G_2^T, 0, 0, 0, 0, G_3^T\right],$$

$$\Gamma_{2ij} = \left[A_i, B_i K_j, 0, B_i K_j, 0, 0, -I\right], \ E_{ij} = \left[E_{ai}, E_{bi} K_j, 0, E_{bi} K_j, 0, 0, 0\right].$$

Proof The following LKF are Chosen:

$$\mathcal{W}(t, x(t)) = \sum_{i=1}^{4} V_i(t, x(t)), \quad t \in \left[t_k, t_{k+1}\right), \tag{14}$$

with

$$V_1(t, x(t)) = x^T(t)\mathcal{P}(t)x(t),$$

$$V_2(t, x(t)) = h_2(t) \int_{t_k}^{t} \eta_2^T(s)\mathcal{R}(t)\eta_2(s)ds,$$

$$V_3(t, x(t)) = -h_1(t) \int_{t}^{t_{k+1}} \eta_2^T(s)\mathcal{S}(t)\eta_2(s)ds,$$

$$V_4(t, x(t)) = 2\eta_3^T(t)\mathcal{Q}(h_2(t)\eta_4(t) + h_1(t)\eta_5(t)),$$

where $\mathcal{P}(t) = \sum_{i=1}^{r} \lambda_i(t) P_i, \mathcal{R}(t) = \sum_{i=1}^{r} \lambda_i(t)\mathcal{R}_i, \mathcal{S}(t) = \sum_{i=1}^{r} \lambda_i(t)\mathcal{S}_i$. □

Let $b_1 = \min\{\lambda_{min}(P_i)|(i \in \mathfrak{R})\}$ and $b_2 = \max\{\lambda_{max}(P_i)|(i \in \mathfrak{R})\}$. Moreover, $\mathcal{W}(t_k, x(t_k)) = \mathcal{W}^-(t_k, x(t_k))$. Then, $\mathcal{W}(t, x(t))$ meet the (i),(ii) of Lemma 3 .

Calculating the time derivative of $\mathcal{W}(t, x(t))$ along the trajectories of the fuzzy system (9) and the result is shown as follows:

$$\dot{\mathcal{W}}(t, x(t)) = \sum_{i=1}^{4} \dot{V}_i(t, x(t)), t \in [t_k, t_{k+1}),$$

with

$$\dot{V}_1(t, x(t)) = 2x^T(t)\mathcal{P}(t)\dot{x}(t) + x^T(t)\dot{\mathcal{P}}(t)x(t), \tag{15}$$

$$\begin{aligned}
\dot{V}_2(t, x(t)) &= h_2(t)\eta_2^T(t)\mathcal{R}(t)\eta_2(t) \\
&\quad + h_2(t) \int_{t_k}^{t} \eta_2^T(s)\dot{\mathcal{R}}(t)\eta_2(s)ds \\
&\quad - \int_{t_k}^{t} \eta_2^T(s)\mathcal{R}(t)\eta_2(s)ds,
\end{aligned} \tag{16}$$

$$
\begin{aligned}
&-\int_{t_k}^{t}\eta_2^T(s)\mathcal{R}(t)\eta_2(s)ds\\
=&\sum_{i=1}^{r}\lambda_i(t)\Big[-\int_{t_k}^{t}x^T(s)R_{11i}x(s)ds-\int_{t_k}^{t}\dot{x}^T(s)R_{55i}\dot{x}(s)ds-2\int_{t_k}^{t}x^T(s)dsR_{12i}x(t_k)\\
&-2\int_{t_k}^{t}x^T(s)dsR_{13i}x(t_{k+1})-2\int_{t_k}^{t}x^T(s)dsR_{14i}g(x(t_k))-2\int_{t_k}^{t}x^T(s)R_{15i}\dot{x}(s)ds\Big]\\
&-2\sum_{i=1}^{r}\lambda_i(t)\Big[x^T(t_k)R_{25i}+x^T(t_{k+1})R_{35i}+g^T(x(t_k))R_{45i}\Big][x(t)-x(t_k)]\\
&-h_1(t)\sum_{i=1}^{r}\lambda_i(t)\Big[x^T(t_k)R_{22i}x(t_k)+2x^T(t_k)R_{23i}x(t_{k+1})+2x^T(t_k)R_{24i}g(x(t_k))\\
&+x^T(t_{k+1})R_{33i}x(t_{k+1})+2x^T(t_{k+1})R_{34i}g(x(t_k))+g^T(x(t_k))R_{44i}g(x(t_k))\Big],
\end{aligned}\tag{17}
$$

$$
\dot{V}_3(t,x(t))=h_1(t)\eta_2^T(t)\mathcal{S}(t)\eta_2(t)-h_1(t)\int_t^{t_{k+1}}\eta_2^T(s)\dot{\mathcal{S}}(t)\eta_2(s)ds-\int_t^{t_{k+1}}\eta_2^T(s)\mathcal{S}(t)\eta_2(s)ds,\tag{18}
$$

$$
\begin{aligned}
&-\int_{t}^{t_{k+1}}\eta_2^T(s)\mathcal{S}(t)\eta_2(s)ds\\
=&\sum_{i=1}^{r}\lambda_i(t)\Big[-\int_{t}^{t_{k+1}}x^T(s)S_{11i}x(s)ds-\int_{t}^{t_{k+1}}\dot{x}^T(s)S_{55i}\dot{x}(s)ds-2\int_{t}^{t_{k+1}}x^T(s)dsS_{12i}x(t_k)\\
&-2\int_{t}^{t_{k+1}}x^T(s)dsS_{13i}x(t_{k+1})-2\int_{t}^{t_{k+1}}x^T(s)dsS_{14i}g(x(t_k))-2\int_{t}^{t_{k+1}}x^T(s)S_{15i}\dot{x}(s)ds\Big]\\
&-2\sum_{i=1}^{r}\lambda_i(t)\Big[x^T(t_k)S_{25i}+x^T(t_{k+1})S_{35i}+g^T(x(t_k))S_{45i}\Big]\Big[x(t_{k+1})-x(t)\Big]\\
&-h_2(t)\sum_{i=1}^{r}\lambda_i(t)\Big[x^T(t_k)S_{22i}x(t_k)+2x^T(t_k)S_{23i}x(t_{k+1})+2x^T(t_k)S_{24i}g(x(t_k))+x^T(t_{k+1})S_{33i}x(t_{k+1})\\
&+2x^T(t_{k+1})S_{34i}g(x(t_k))+g^T(x(t_k))S_{44i}g(x(t_k))\Big],
\end{aligned}\tag{19}
$$

$$
\begin{aligned}
\dot{V}_4(t,x(t))=&\ 2\dot{\eta}_3^T(t)\mathcal{Q}[h_2(t)\eta_4(t)+h_1(t)\eta_5(t)]+2\eta_3^T(t)\mathcal{Q}[h_2(t)\dot{\eta}_4(t)-\eta_4(t)+\eta_5(t)+h_1(t)\dot{\eta}_5(t)]\\
=&\ 2\eta^T(t)\big[h_2(t)\Pi_1^T\mathcal{Q}\Pi_2+h_1(t)\Pi_1^T\mathcal{Q}\Pi_3+\Pi_4^T\mathcal{Q}\Pi_5+h_2(t)\Pi_4^T\mathcal{Q}\Pi_6+h_1(t)\Pi_4^T\mathcal{Q}\Pi_7\big]\eta(t),
\end{aligned}\tag{20}
$$

By using Jensen's inequality and Lemma 2, for any compatible matrix X and $\Upsilon=\begin{bmatrix}R_{11i} & X\\ * & S_{11i}\end{bmatrix}>0$, the following inequality is obtained:

$$
-\sum_{i=1}^{r}\lambda_i(t)\Big[\int_{t_k}^{t}x^T(s)R_{11i}x(s)ds+\int_{t}^{t_{k+1}}x^T(s)S_{11i}x(s)ds\Big]\le-\frac{1}{h}\sum_{i=1}^{r}\lambda_i(t)\eta^T(t)[e_5^T,e_6^T]\Upsilon[e_5^T,e_6^T]^T\eta(t).\tag{21}
$$

By using Lemma 1, we can get

$$
\begin{aligned}
-\sum_{i=1}^{r}\lambda_i(t)\int_{t_k}^{t}\dot{x}^T(s)R_{55i}\dot{x}(s)ds\ \le\ & h_1(t)\sum_{i=1}^{r}\lambda_i(t)\eta^T(t)\Big[\mathcal{N}_1^TR_{55i}^{-1}\mathcal{N}_1+\frac{h_k^2}{3}\mathcal{N}_2^TR_{55i}^{-1}\mathcal{N}_2\Big]\eta(t)\\
&+2h_1(t)\eta^T(t)\mathcal{N}_2^T[x(t)+x(t_k)]\\
&+2\eta^T(t)\Big[\mathcal{N}_1^T(x(t)-x(t_k))-2\mathcal{N}_2^T\int_{t_k}^{t}x(s)ds\Big],
\end{aligned}\tag{22}
$$

$$
\begin{aligned}
&-\sum_{i=1}^{r}\lambda_i(t)\int_t^{t_{k+1}}\dot{x}^T(s)S_{55i}\dot{x}(s)ds\\
&\le h_2(t)\sum_{i=1}^{r}\lambda_i(t)\eta^T(t)\left[\mathcal{N}_3^T S_{55i}^{-1}\mathcal{N}_3+\frac{h_k^2}{3}\mathcal{N}_4^T S_{55i}^{-1}\mathcal{N}_4\right]\\
&\quad+2h_2(t)\eta^T(t)\mathcal{N}_4^T\left[x(t_{k+1})+x(t)\right]\\
&\quad+2\eta^T(t)\left[\mathcal{N}_3^T(x(t_{k+1})-x(t))-2\mathcal{N}_4^T\int_t^{t_k+1}x(s)ds\right].
\end{aligned}\tag{23}
$$

In addition, given that the $\sum_{i=1}^{r}\dot{\lambda}_i(t)=0$, we can do the following rewrite: $\dot{\mathcal{P}}(t)=\sum_{i=1}^{r}\dot{\lambda}_i(t)P_i=\sum_{i=1}^{r-1}\dot{\lambda}_i(P_i-P_r)$, $\dot{\mathcal{R}}(t)=\sum_{i=1}^{r}\dot{\lambda}_i(t)\mathcal{R}_i=\sum_{i=1}^{r-1}\dot{\lambda}_i(\mathcal{R}_i-\mathcal{R}_r)$, $\dot{\mathcal{S}}(t)=\sum_{i=1}^{r}\dot{\lambda}_i(t)\mathcal{S}_i=\sum_{i=1}^{r-1}\dot{\lambda}_i(\mathcal{S}_i-\mathcal{S}_r)$. then discuss how to ensure the $\dot{\mathcal{P}}(t)<0, \dot{\mathcal{R}}(t)<0, \dot{\mathcal{S}}(t)<0$,

$$
\begin{cases}
\text{if } \dot{\lambda}_i<0, \text{ then } P_i-P_r>0, \mathcal{R}_i-\mathcal{R}_r>0, \mathcal{S}_i-\mathcal{S}_r>0\\
\text{if } \dot{\lambda}_i>0, \text{ then } P_i-P_r<0, \mathcal{R}_i-\mathcal{R}_r<0, \mathcal{S}_i-\mathcal{S}_r<0
\end{cases}\tag{24}
$$

where $i=1,2,\cdots,r-1$

What's more, for any matrix G_1, G_2 and G_3 with appropriate dimensions, the formula (25) can be established.

$$
\begin{aligned}
0&=2\left[x^T(t)G_1+x^T(t_k)G_2+\dot{x}^T(t)G_3\right]\\
&\quad\times\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(t)\lambda_j(t_k)\left[(A_i+\Delta A_i(t))x(t)\right.\\
&\quad\left.+(B_i+\Delta B_i(t))K_j(x(t_k)+g(x(t_k))-\dot{x}(t)\right]\\
&=2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(t)\lambda_j(t_k)\eta^T(t)\Gamma_1^T\left[(A_i+\Delta A_i(t))x(t)\right.\\
&\quad\left.+(B_i+\Delta B_i(t))K_j(x(t_k)+g(x(t_k))-\dot{x}(t)\right]\eta(t).
\end{aligned}\tag{25}
$$

Then, combing (15)–(25), we can obtain that for $t\in[t_k, t_{k+1}]$

$$
\dot{\mathcal{W}}(t,x(t))\le\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(t)\lambda_j(t_k)\eta^T(t)\Xi_{ij}\eta(t),\tag{26}
$$

where

$$
\begin{aligned}
\Xi_{ij}&=\Xi_1+\Gamma_1^T[H_iF(t)E_{ai}e_1+H_iF(t)E_{bi}K_j(e_2+e_4)]\\
&\quad+h_2(t)\left(\Xi_2+\mathcal{N}_3^T S_{55i}^{-1}\mathcal{N}_3+\frac{h_k^2}{3}\mathcal{N}_4^T S_{55i}^{-1}\mathcal{N}_4\right)\\
&\quad+h_1(t)\left(\Xi_3+\mathcal{N}_1^T R_{55i}^{-1}\mathcal{N}_1+\frac{h_k^2}{3}\mathcal{N}_2^T R_{55i}^{-1}\mathcal{N}_2\right).
\end{aligned}
$$

Denoting $\gamma_j(t)=\lambda_j(t_k)-\lambda_j(t)$, $j\in\Re$ and $\zeta(t)=diag\{\gamma_1(t)I,\gamma_2(t)I,\cdots,\gamma_r(t)I\}$, because of $\sum_{j=1}^{r}\gamma_j(t)=0$ similar to [39], we have

$$
\begin{aligned}
\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(t)\lambda_j(t_k)\Xi_{ij}&=\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(t)\lambda_j(t)\frac{\Xi_{ij}+\Xi_{ji}}{2}\\
&\quad+\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(t)\gamma_j(t)\Xi_{ij}\\
&=\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(t)\lambda_j(t)\\
&\quad\times\left(\frac{\Xi_{ij}+\Xi_{ji}}{2}+\tilde{\Xi}_{ij}\right),
\end{aligned}\tag{27}
$$

where $\tilde{\Xi}_{ij}=G(B_i+H_iF(t)E_{bi})\underbrace{[I,...,I]}_{r}\zeta(\gamma(t))col\{K_1,\cdots,K_r\}(e_2+e_4)$.

In the following, some degree of freedom L_{ij} can be introduced into $\tilde{\Xi}_{ij}$. And then, it holds $\tilde{\Xi}_{ij}=G(B_i+H_iF(t)E_{bi})\underbrace{[I,...,I]}_{r}\zeta(\gamma(t))col\{K_1+L_{ij},\cdots,K_r+L_{ij}\}(e_2+e_4)$.

For $t\in\{t_k,t_{k+1}\}$, $\Xi_{ij}<0$ is equivalent as follows:

$$
\begin{aligned}
&\Xi_1+\tilde{\Xi}_{ij}+\Gamma_1^T[H_iF(t)E_{ai}e_1+H_iF(t)E_{bi}K_j(e_2+e_4)]\\
&\quad+h_k\left(\Xi_2+\mathcal{N}_3^T S_{55i}^{-1}\mathcal{N}_3+\frac{h_k^2}{3}\mathcal{N}_4^T S_{55i}^{-1}\mathcal{N}_4\right)<0,
\end{aligned}\tag{28}
$$

$$
\begin{aligned}
&\Xi_1+\tilde{\Xi}_{ij}+\Gamma_1^T[H_iF(t)E_{ai}e_1+H_iF(t)E_{bi}K_j(e_2+e_4)]\\
&\quad+h_k\left(\Xi_3+\mathcal{N}_1^T R_{55i}^{-1}\mathcal{N}_1+\frac{h_k^2}{3}\mathcal{N}_2^T R_{55i}^{-1}\mathcal{N}_2\right)<0.
\end{aligned}\tag{29}
$$

Combining the Schur complement and Lemma 4, (28) and (29) can be further expressed as (12) and (13). Then, if (10), (11), (12) and (13) are satisfied, we can get $\dot{\mathcal{W}}(t,x(t))<0$. So far, (i)-(iii) of Lemma 3 can be satisfied, which can guarantee the asymptotic stability of system (9).

Remark 1 In Theorem 1, based on the derivative of the FMFs, constraints are carried out in the stability conditions to ensure the terms $\dot{P}(t)<0$, $\dot{R}(t)<0$, $\dot{S}(t)<0$. If fuzzy rule number is r, there will be 2^{r-1} possible situations. The possible cases are named Case γ ($\gamma=1,2,\cdots,2^{r-1}$). For each Case γ, the stability conditions constructed should be satisfied.

Remark 2 A novel LKF which depends on FMFs is established in (16). It does not require positive characterization of all terms. Unlike the LKFs constructed in [6, 20, 39], in this paper, $x(t_{k+1})$ was obtained in terms $V_2(t,x(t))$ and $V_4(t,x(t))$. Therefore, less conservatism is expected to be achieved.

Remark 3 In [35], an improved FMB integral inequality is introduced. The upper bound provided by the improved FMB is closer than the upper bound taken under Jensen inequality. Therefore, inequality in lemma 1 is chosen to handle $-\int_{t_k}^{t}\dot{x}^T(s)S_{55i}\dot{x}(s)ds$ and $-\int_{t}^{t_{k+1}}\dot{x}^T(s)R_{55i}\dot{x}(s)ds$.

4 Controller design

In Theorem 1, by means of Lyapunov function as well as free-matrix-based(FMB) inequalities and other lemmas, stability conditions of the fuzzy sampling system are obtained. In this part, the controller design and parameters piteration methods are given. If the controller parameters K_j are unknown, the matrix inequalities obtained are nonlinear. Then, the matrix inequalities will be linearized, then stability conditions that expressed in the form of LMI can be obtained. Also, the controller gain K_j can be solved from LMIs stability conditions.

Theorem 2 *For given scalars $h > 0, \delta_1 > 0, \delta_2 > 0, \varepsilon_1$ and ε_2, the fuzzy system (9) is asymptotically stable, if there exists positive matrices $\bar{P}_i, \bar{\mathcal{S}}_i, \bar{\mathcal{R}}_i$ with $\bar{S}_{15i}^T = \bar{S}_{15i}$ and $\bar{R}_{15i}^T = \bar{R}_{15i}$, any appropriate dimensional matrix $\bar{\mathcal{Q}}, \bar{\mathcal{N}}_1, \bar{\mathcal{N}}_2, \bar{\mathcal{N}}_3, \bar{\mathcal{N}}_4, \bar{G}$ and $\bar{X}$ and appropriate dimensional diagnal matrix Z_{ij} such that for all $i, j \in \mathfrak{R}$, the following LMIs yield:*

$$\begin{cases} \text{if } \dot{\lambda}_i < 0, \ \textit{then } \ \bar{P}_i - \bar{P}_r > 0, \bar{R}_i - \bar{R}_r > 0, \bar{S}_i - \bar{S}_r > 0 \\ \text{if } \dot{\lambda}_i > 0, \ \textit{then } \ \bar{P}_i - \bar{P}_r < 0, \bar{R}_i - \bar{R}_r < 0, \bar{S}_i - \bar{S}_r < 0 \end{cases}, \quad i = 1, 2, \cdots, r-1. \tag{30}$$

$$\bar{\Upsilon} = \begin{bmatrix} \bar{R}_{11i} & \bar{X} \\ * & \bar{S}_{11i} \end{bmatrix} > 0, \tag{31}$$

$$\begin{bmatrix} \bar{\Omega}_1 & \bar{\Xi}_6 & h_k\bar{\bar{\mathcal{N}}}_3 & h_k^2\bar{\bar{\mathcal{N}}}_4 \\ * & -Z_{ij} & 0 & 0 \\ * & * & -h_k\bar{S}_{55i} & 0 \\ * & * & * & -3h_k\bar{S}_{55i} \end{bmatrix} < 0, \tag{32}$$

$$\begin{bmatrix} \bar{\Omega}_2 & \bar{\Xi}_6 & h_k\bar{\bar{\mathcal{N}}}_1 & h_k^2\bar{\bar{\mathcal{N}}}_2 \\ * & -Z_{ij} & 0 & 0 \\ * & * & -2h_k\bar{R}_{55i} & 0 \\ * & * & * & -2h_k\bar{R}_{55i} \end{bmatrix} < 0, \tag{33}$$

where

$$\bar{\Omega}_{ij1} = \begin{bmatrix} \bar{\Xi}_1 & \bar{\Xi}_4 & \bar{\Xi}_5 \\ * & -\delta_1 I & 0 \\ * & * & -\delta_2 I \end{bmatrix} + diag\{\bar{\Xi}_2, 0, 0\} + \bar{T}_{ij}Z_{ij}\bar{T}_{ij}^T, \tag{34}$$

$$\bar{\Omega}_{ij2} = \begin{bmatrix} \bar{\Xi}_1 & \bar{\Xi}_4 & \bar{\Xi}_5 \\ * & -\delta_1 I & 0 \\ * & * & -\delta_2 I \end{bmatrix} + diag\{\bar{\Xi}_3, 0, 0\} + \bar{T}_{ij}Z_{ij}\bar{T}_{ij}^T, \tag{35}$$

$$\begin{aligned}
\bar{\Xi}_1 &= 2e_1^T\bar{P}_ie_7 + 2\Pi_4^T\bar{\mathcal{Q}}\Pi_5 - 2\Pi_8^T\bar{\mathcal{R}}_i\Pi_9 - 2\Pi_8^T\bar{\mathcal{S}}_i\Pi_{10} + e_2^T\bar{R}_{15i}e_2 - e_1^T\bar{R}_{15i}e_1 + 2\Gamma_1^T\Gamma_{2ij} \\
&\quad + e_1^T\bar{S}_{15i}e_1 - e_3^T\bar{S}_{15i}e_3 + 2\Pi_{14}^T\left[\bar{\mathcal{N}}_1^T, \bar{\mathcal{N}}_2^T, \bar{\mathcal{N}}_3^T, \bar{\mathcal{N}}_4^T\right]\Pi_{11} - \frac{1}{h}[e_5^T, e_6^T]\Upsilon[e_5^T, e_6^T]^T \\
\bar{\Xi}_2 &= 2\Pi_1^T\bar{\mathcal{Q}}\Pi_2 + 2\Pi_4^T\bar{\mathcal{Q}}\Pi_6 + \Pi_{12}^T\bar{\mathcal{R}}_i\Pi_{12} - \Pi_{13}^T\bar{\mathcal{S}}_i\Pi_{13} + 2\Pi_{14}^T\mathcal{N}_4^T(e_1 + e_3), \\
\bar{\Xi}_3 &= 2\Pi_1^T\bar{\mathcal{Q}}\Pi_3 + 2\Pi_4^T\bar{\mathcal{Q}}\Pi_7 + \Pi_{12}^T\bar{\mathcal{S}}_i\Pi_{12} - \Pi_{13}^T\bar{\mathcal{R}}_i\Pi_{13} + 2\Pi_{14}^T\mathcal{N}_2^T(e_1 + e_2), \\
\bar{\Xi}_4 &= [\delta_1(\epsilon_1e_1^T + \epsilon_2e_2^T + e_7^T)H_i, e_1^TE_{ai}^T], \ \bar{\Xi}_5 = [\delta_2(\epsilon_1e_1^T + \epsilon_2e_2^T + e_7^T)H_i, (e_2^T + e_4^T)\bar{K}_j^TE_{bi}^T] \\
\bar{\Xi}_6 &= col\{(e_2 + e_4)^T[\eta_1(\bar{K}_1 + \bar{L}_{ij})^T, \eta_2(\bar{K}_2 + \bar{L}_{ij})^T, \cdots, \eta_r(\bar{K}_r + \bar{L}_{ij})^T), 0, 0\} \\
T_{ij} &= col\{(\epsilon_1e_1^T + \epsilon_2e_2^T + e_7^T)B_i\underbrace{[I, I, \cdots, I]}_{r}, 0, \underbrace{[I, I, \cdots, I]}_{r}\} \\
\bar{\bar{\mathcal{N}}}_l &= [\bar{\mathcal{N}}_l, 0, 0], l = 1, 2, 3, 4
\end{aligned}$$

The controller parameters K_j can be given through the following demonstration:

$$K_j = \bar{K}_j\bar{G}^{-T}(i = 1, 2, \ldots, r). \tag{36}$$

Proof For simplicity of expression, set $w = \epsilon_1e_1^TG + \epsilon_2e_2^TG + e_6^T$, In the formula, ϵ_1, ϵ_2 denote the scalar parameters and G is a nonsingular matrix. In the following proof, $\bar{G} \triangleq G^{-1}$

$$\begin{cases} \text{if } \dot{\lambda_p} < 0, \text{then } \ \bar{G}(P_i - P_r)\bar{G}^T \geq 0, \quad \bar{G}(R_i - R_r)\bar{G}^T \geq 0, \quad \bar{G}(S_i - S_r)\bar{G}^T \geq 0 \\ \text{if } \dot{\lambda_p} > 0, \text{then } \ \bar{G}(P_i - P_r)\bar{G}^T \leq 0, \quad \bar{G}(R_i - R_r)\bar{G}^T \leq 0, \quad \bar{G}(S_i - S_r)\bar{G}^T \leq 0 \end{cases} \quad i = 1, 2, \cdots, r-1 \tag{37}$$

Set $G_2 = \epsilon_1 G_1, G_3 = \epsilon_2 G_1, G_1 = \bar{G}^{-1}, \bar{P}_i = \bar{G} P_i \bar{G}^T$, $\bar{X}_{ij} = \bar{G} X_{ij} \bar{G}^T, \bar{K}_j = K_j \bar{G}^T, \Lambda_1 = diag\left\{\bar{G}, \bar{G}\right\}, \Lambda_2 = diag\left\{\Lambda_1, \Lambda_1\right\}, \Lambda_3 = diag\left\{\Lambda_2, \bar{G}\right\}, \Lambda_4 = diag\left\{\Lambda_3, \bar{G}\right\}$, $\Lambda_5 = diag\left\{\Lambda_4, \bar{G}\right\}, \Lambda_6 = diag\left\{\Lambda_5, \Lambda_1, I, \Lambda_1\right\}, \Lambda_7 = diag\left\{\Lambda_5, I, \bar{G}, I, \Lambda_1\right\}, \bar{\mathcal{Q}} = \Lambda_4 \mathcal{Q} \Lambda_2^T, \bar{\mathcal{R}} = \Lambda_3 \mathcal{R} \Lambda_3^T$, $\bar{\mathcal{S}} = \Lambda_3 \mathcal{S} \Lambda_3^T, \bar{\mathcal{N}}_l^T = \Lambda_5 \mathcal{N}_l^T \bar{G}^T (l = 1, 2, 3, 4)$.

Pre and postmultiplying (11) by Λ_2 and Λ_2^T, get (31); Pre and postmultiplying (12) and (13) by Λ_6 and Λ_7^T, correspondingly get (32) and (33). The above is the proof process. □

It is notable that the conditions (32) and (33) become linear matrix inequalities form under the only premise that ε_1 and ε_2 are known.

By the algorithm which based on the method that appears in [41] that is shown below, then optimal solution parameters ε_1 and ε_2 can be obtained.

Algorithm 1

Step 1: *for each Case γ, specify range $[l_\gamma, L_\gamma]$ for $\epsilon_{1,\gamma}$ and $\epsilon_{2,\gamma}$, and increment $\triangle h$ for h, $\triangle \epsilon_i$ for $\epsilon_{i,\gamma}$, where $\epsilon_{1,\gamma}, \epsilon_{2,\gamma} \leq L_\gamma - l_\gamma$. let $\epsilon_{1,\gamma,0} = 0$ and $\epsilon_{2,\gamma,0} = 0$ and $h_{\gamma,0} = h_0$, where h_0 is a small enough positive scalar.*

Step 2: *Use the LMIs toolbox in matlab to solve the stability conditions (30), (31), (32) with specified $\epsilon_{1,\gamma}, \epsilon_{2,\gamma}, h_\gamma$ If there is a set of feasible solutions, jump to **Step 3**; otherwise jump to **Step 4**.*

Step 3: *Set $\epsilon_{1,\gamma,0} = \epsilon_{1,\gamma}, \epsilon_{2,\gamma,0} = \epsilon_{2,\gamma}$, and $h_{\gamma,0} = h_\gamma$, and then jump to **Step 2** with $h_\gamma = h_\gamma + \triangle h$*

Step 4: $\epsilon_{1,\gamma} = \epsilon_{1,\gamma} + \triangle \epsilon_1$.

If $\epsilon_{1,\gamma} \leq L_\gamma$, go to step 2; else, let $\epsilon_{1,\gamma} = l_\gamma$ and $\epsilon_{2,\gamma} = \epsilon_{2,\gamma} + \triangle \epsilon_2$.

*If $\epsilon_{2,\gamma} > L_\gamma$, $\epsilon_{1,\gamma,0}$ and $\epsilon_{2,\gamma,0}$ and $h_{\gamma,0}$ are the desire values of $\epsilon_{1,\gamma}$ and $\epsilon_{2,\gamma}$ and h_γ, respectively; else, go to **Step 2**.*

Remark 4 At the time of adding (24), parameters $\varepsilon_1, \varepsilon_2$ are introduced to add more degrees of freedom. The parameters ε_1, ε_2 should be given in advance such that the stability conditions in the form of the linear matrix inequalities (LMIs) in Theorem 2 can be got. The above algorithm is a search process of the optimal ε_1, ε_2 for each Case γ, and after getting the optimal parameters, the maximum sampling interval and the controller gain matrices can be solved from the LMIs. It should be noted that $\varepsilon_{1,\gamma}$ and $\varepsilon_{2,\gamma}$ $(\gamma = 1, 2 \cdots, r-1)$ were expressed as ε_1 and ε_2 in each Case γ for simplicity.

Remark 5 By discussing the derivative of the membership function, the conservativeness in the process of designing the controller and calculating the maximum sampling interval is reduced. For each case γ, the controller parameters and maximum sampling interval to be solved are represented by $K_{j,\gamma}$ and h_γ, $(\gamma = 1, 2, \cdots, 2^{r-1}; j = 1, \cdots, r)$. For simplicity, $K_{j,\gamma}$ can be expressed as K_j. For each cases, the stabilituy conditions should be satisfied and each possible Case γ exists separately.

In Theorem 2, all the stability conditions are expressed in the form of LMI. Later, the LMI toolbox can be used to solve the controller parameters and the maximum sampling interval. The following are stability conditions that remove the uncertainties in Theorem 2:

Corollary 1 *For the given scalars $h > 0, \delta_1 > 0, \delta_2 > 0, \varepsilon_1$ and ε_2, if there exist positive matrices $\bar{P}_i, \bar{\mathcal{R}}_i, \bar{\mathcal{S}}_i$ with $\bar{S}_{15i}^T = \bar{S}_{15i}$ and $\bar{R}_{15i}^T = \bar{R}_{15i}$, any appropriate dimensional matrix $\bar{\mathcal{Q}}, \bar{\mathcal{N}}_1, \bar{\mathcal{N}}_2, \bar{\mathcal{N}}_3, \bar{\mathcal{N}}_4$ and $\bar{X}$ and any appropriate dimensional diagonal matrix Z_{ij} such that for all $i, j \in \Re$, the T-S fuzzy system which removes uncertainty is asymptotically stable.*

$$\bar{\Upsilon} = \begin{bmatrix} \bar{R}_{11i} & \bar{X} \\ * & \bar{S}_{11i} \end{bmatrix} > 0, \tag{38}$$

$$\begin{bmatrix} \hat{\Omega}_1 & \hat{\Xi}_6 & h_k \bar{\mathcal{N}}_3 & h_k^2 \bar{\mathcal{N}}_4 \\ * & -Z_{ij} & 0 & 0 \\ * & * & -h_k \bar{S}_{55i} & 0 \\ * & * & * & -3h_k \bar{S}_{55i} \end{bmatrix} < 0, \tag{39}$$

$$\begin{bmatrix} \hat{\Omega}_2 & \hat{\Xi}_6 & h_k \bar{\mathcal{N}}_1 & h_k^2 \bar{\mathcal{N}}_2 \\ * & -Z_{ij} & 0 & 0 \\ * & * & -2h_k \bar{R}_{55i} & 0 \\ * & * & * & -2h_k \bar{R}_{55i} \end{bmatrix} < 0, \tag{40}$$

where

$$\hat{\Omega}_{1ij} = \bar{\Xi}_{ij} + \Xi_{2ij} + \hat{T}_{ij} Z_{ij} \hat{T}_{ij} \tag{41}$$

$$\hat{\Omega}_{2ij} = \bar{\Xi}_{ij} + \Xi_{3ij} + \hat{T}_{ij} Z_{ij} \hat{T}_{ij} \tag{42}$$

$$\hat{\Xi}_{6ij} = (e_2^T + e_4^T)[\eta_1(\bar{K}_1 + \bar{L}_{ij})^T, \cdots, \eta_r(\bar{K}_r + \bar{L}_{ij})^T] \tag{43}$$

$$\hat{T}_{ij} = (\epsilon_1 e_1^T + \epsilon_2 e_2^T + e_7^T) B_i \underbrace{[I, I, \cdots, I]}_{r} \tag{44}$$

The other notations appearing above are consistent with Theorem 2. Similarly, K_j can be obtained from the following expression:

$$K_j = \bar{K}_j \bar{G}^{-T}. \tag{45}$$

5 Numerical example

In order to verify the effectiveness of the proposed method, two examples are simulated in this part.

Example 1 The following example is the simulation analysis of the inverted pendulum. Through controlling the inverted pendulum system, the control ability of the designed controller rule to the nonlinear system can be detected. And the inverted pendulum can be applied in many aspects, such as military, aerospace, robotics and general industrial process. And some specific applications can also be list, such as balance control during robot walking, verticality control during rocket launch and attitude control during satellite flight. The inverted pendulum model is shown in Fig. 1. The following are the state equations of the inverted pendulum system:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_1(t) = \dfrac{gsin(x_1(t)) - am_1lx_2^2(t)sin(2x_1(t))/2 - acos(x_1(t))u(t)}{4l/3 - am_1lcos^2(x_1(t))} \end{cases},$$

Where $x_1(t)$ represents the angle of the pendulum from vertical and $x_2(t)$ denotes the angular velocity. Set $M = 8kg, m_1 = 2kg, l = 0.5m, a = \frac{1}{m_1+M}, g = 9.8m/s^2$, and $\beta = cos(88°)$. According to the subsystem division in reference [40], the following subsystem model expression can be got:

Rule 1:

$$IF\ x_1(t)\ is\ about\ 0, THEN$$
$$\dot{x}_1(t) = A_1x(t) + B_1u(t),$$

Rule 2:

$$IF\ x_1(t)\ is\ about\ \pm\frac{\pi}{2}, THEN$$
$$\dot{x}_1(t) = A_2x(t) + B_2u(t).$$

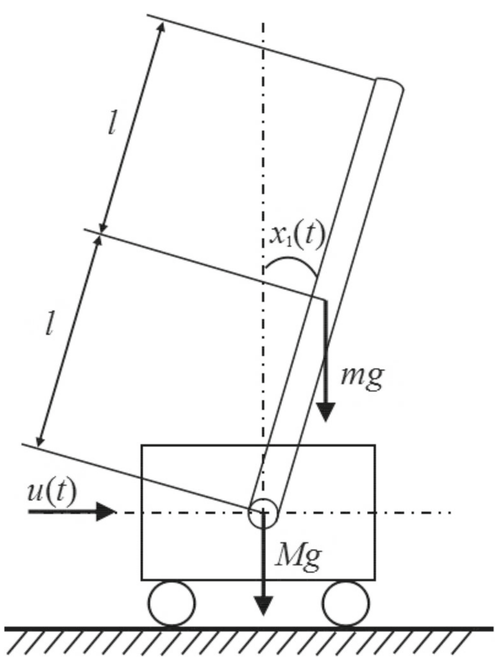

Fig. 1 Inverted pendulum control system model of Example 1

The state space matrices of the inverted pendulum system are shown below:

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3-am_1l} & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ -\frac{a}{4l/3-am_1l} \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3-am_1l\beta^2)} & 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ -\frac{a\beta}{4l/3-am_1l\beta^2} \end{bmatrix}.$$

The fuzzy membership functions are

$$\lambda_1(t) = \begin{cases} 1 - \dfrac{2}{\pi}x_1(t), & \text{if}\ \ 0 \le x_1(t) < \dfrac{\pi}{2} \\ 1 + \dfrac{2}{\pi}x_1(t), & \text{if}\ \ -\dfrac{\pi}{2} < x_1(t) < 0 \end{cases},$$

and $\lambda_2(t) = 1 - \lambda_1(t)$. When $|x_2(t)| \le 10\ rad/s$,in addition, from the relation of the membership function, we can get $\eta_1 = \eta_2 = (20/\pi) \times h^*$. In Example 1, r=2, there are two possible cases, named Case 1 and Case 2:

$$Case1:\ \ \bar{P}_1 - \bar{P}_2 > 0, \bar{R}_1 - \bar{R}_2 > 0, \bar{S}_1 - \bar{S}_2 > 0;$$
$$Case2:\ \ \bar{P}_1 - \bar{P}_2 < 0, \bar{R}_1 - \bar{R}_2 < 0, \bar{S}_1 - \bar{S}_2 < 0.$$

By means of the Algorithm 1 in Theorem 2, the optimal parameters are searched in a given range to conduct a iterative optimization process. Firstly, specify the search interval of parameters $\varepsilon_i(i = 1, 2)$ and set the increment $\triangle\ h$ for h and $\triangle\ \varepsilon_i(i = 1, 2)$ for ε which satisfy $\varepsilon_i \le L - l(i = 1, 2)$. The initial parameters value were taken into the stability conditions obtained in corollary 1 to find a solution. Then the value of the optimal parameter can be obtained by combining the specific steps in the Algorithm 1.

When the system has achieved the maximum sampling interval, the optimal parameters have been obtained at same time. The controller gain parameters and the sampling interval of the system are obtained under the optimal parameters. For case 1, $\varepsilon^*_{1,1} = 1.6, \varepsilon^*_{2,1} = 27.8$; for case 2, $\varepsilon^*_{1,2} = 2.5, \varepsilon^*_{2,2} = 27.8$. And the maximum sampling

Table 1 Comparison of the maximums h_{max} obtained by various methods

Method	Wu et al. [21]	Wang et al. [29]	Zhao et al. [41]	Corollary 1
h_{max}	0.0692	0.1107	0.1292	0.1528

interval $h_1^* = h_2^* = 0.040s$ can be obtained. Here are the parameters of the controller, correspondingly:

$$K_{1,1} = [563.8033, 182.5967];$$
$$K_{2,1} = [1950.0910, 627.9089].$$
$$K_{1,2} = [602.0910, 189.9089];$$
$$K_{2,2} = [1979.7786, 624.4515].$$

The maximum sampling intervals obtained by different methods [39–41] are listed in Table 1. It can be seen that the result in corollary 1 are improved. From this point, the propose control method has less conservative.

The state space matrix of the system and the gain matrices of the controller obtained above are brought into the fuzzy system (9). The initial value $x_0(t) = [-2, 3, 4]$ was given, then the controller input curve and the state response curve of the system are drawn with the help of Matlab toolbox. The state response curve has been shown in Fig. 2. The control input curve $u(t)$ has shown in Fig. 3. It is observed that T-S fuzzy system is asymptotically stable under the control strategy designed in this paper.

Example 2 The following is a simulation analysis of the chaotic Lurenz system. The chaotic system reacts when the initial conditions change slightly, but after continuous changes, the future state will be greatly different. By controlling the behavior of chaos, the control method can be applied to some practical application process, such as meteorology, aerospace and other fields. And the state equations of the system are shown below:

$$\begin{cases} \dot{x}_1(t) = -x_2(t) - x_3(t), \\ \dot{x}_2(t) = x_1(t) + m x_3(t), \\ \dot{x}_3(t) = n x_1(t) - (p - x_1(t)) x_3(t) + u_(t). \end{cases} .$$

By using the T-S fuzzy system model, the above Lurenz nonlinear system was represent. When $x_1(t) \in [p-q, p+q]$, the state space matrix of the system are obtained by referring to the reference [21, 41], The state transition matrices are shown as follow:

$$A_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & m & 0 \\ n & 0 & -q \end{bmatrix},\ A_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & m & 0 \\ n & 0 & q \end{bmatrix},\ B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

The FMFs of this Example are $\lambda_1(x_1(t)) = \frac{1}{2}(1 + \frac{p - x_{1(t)}}{q})$ and $\lambda_2(x_1(t)) = \frac{1}{2} - \frac{p - x_1(t)}{2q}$. The parameters values in the above matrix are $m = 0.3$, $n = 0.5$, $p = 5$ and $q = 10$, respectively.

In this example, r=2, so there are two possible cases and they are called $Case1$ and $Case2$:

$$Case1: \quad \bar{P}_1 - \bar{P}_2 > 0,\ \bar{R}_1 - \bar{R}_2 > 0,\ \bar{S}_1 - \bar{S}_2 > 0;$$
$$Case2: \quad \bar{P}_1 - \bar{P}_2 < 0,\ \bar{R}_1 - \bar{R}_2 < 0,\ \bar{S}_1 - \bar{S}_2 < 0.$$

When $|x_1(t)| \leq 10\ rad/s$, it can be calculated that $\eta_1 = \eta_2 = min\{1, \frac{7.2}{2*q} * h_\gamma^*\}$. After getting the optimal parameters of the algorithm in Theorem 2, the stability conditions in the form of LMIs in corollary 1 were solved, then the maximum sampling interval and controller parameters got.

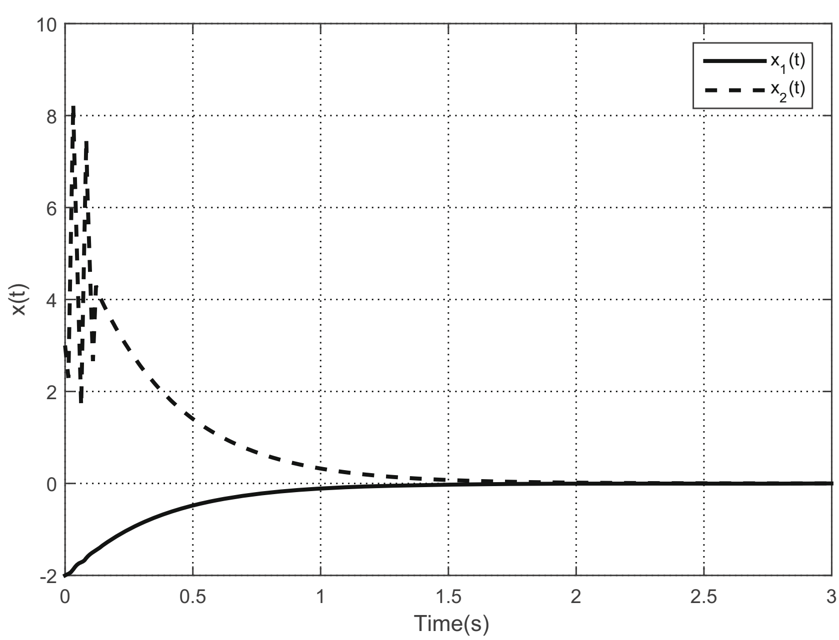

Fig. 2 State response curve of Example 1

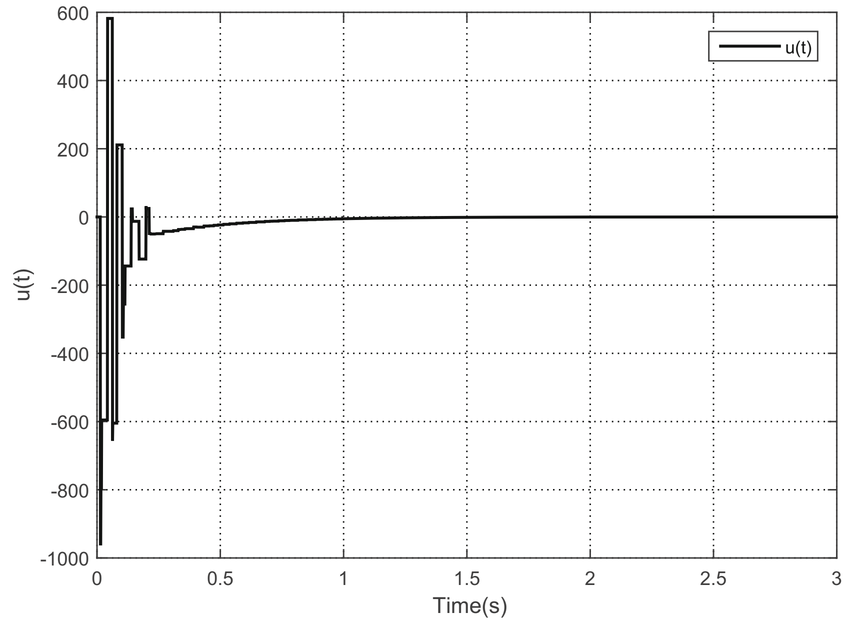

Fig. 3 Control input curve of Example 1

The maximum sampling interval obtained is $h_1^* = 0.1752s$ in Case 1, when $\varepsilon_{1,1}^* = 1.43, \varepsilon_{2,1}^* = 1.96$. And for Case 2, $h_2^* = 0.1528s$, when $\varepsilon_{1,2}^* = 2.21, \varepsilon_{2,2}^* = 2.01$. The maximum sampling interval is determined by the minimum of them: $h_{max} = min_{1 \leq \gamma \leq 2^{r-1}} h_\gamma^* = h_2^* = 0.1528s$.

Then, the value of the controller gain matrix K_j are correspondingly given as follow:

$$K_{1,1} = [3.4179, -1.5759, 5.0301];$$
$$K_{2,1} = [-2.8793, 0.8127, -9.8471];$$
$$K_{1,2} = [3.3571, -1.6107, 4.1251];$$
$$K_{2,2} = [-2.7399, 0.7931, -9.9243].$$

Comparing h_{max} with [21, 29, 41], the comparison of the maximum sampling interval is gathered in Table 2. It can be seen that the maximum sampling interval is improved. In this sense, conservatism has been reduced. The state space matrix of the system and the parameter K_j of the controller obtained above are brought into the fuzzy system (9). Given the initial value of $x_0(t) = [-2, 3, 4]$, the controller input curve and the state response curve are drawn with the help of Matlab. The state response curve has been shown in Fig. 4, and control input curve $u(t)$ has been shown in Fig. 5. It can be seen from the curves of Figs. 4 and 5 that the system (9) is asymptotically stable under the control strategy designed in this paper.

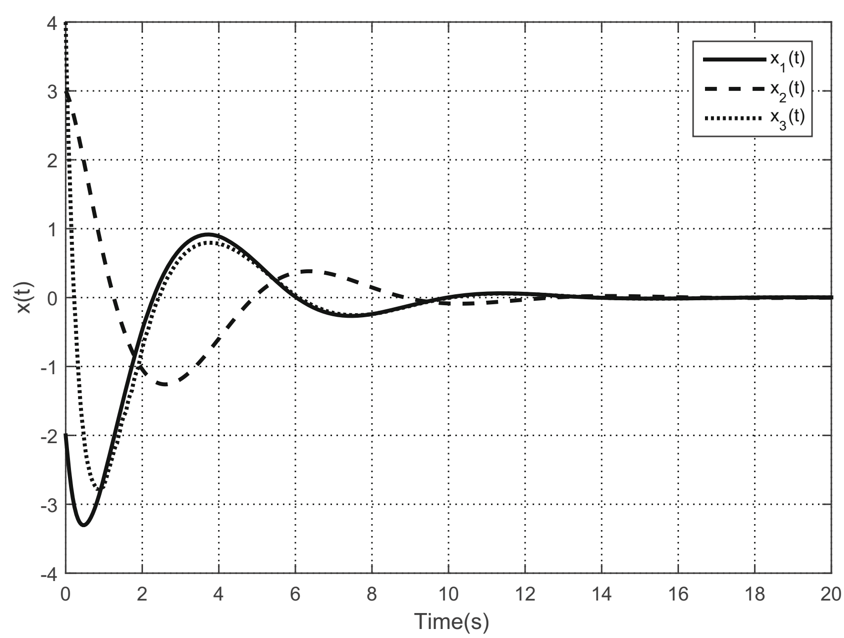

Fig. 4 State response curve of Example 2

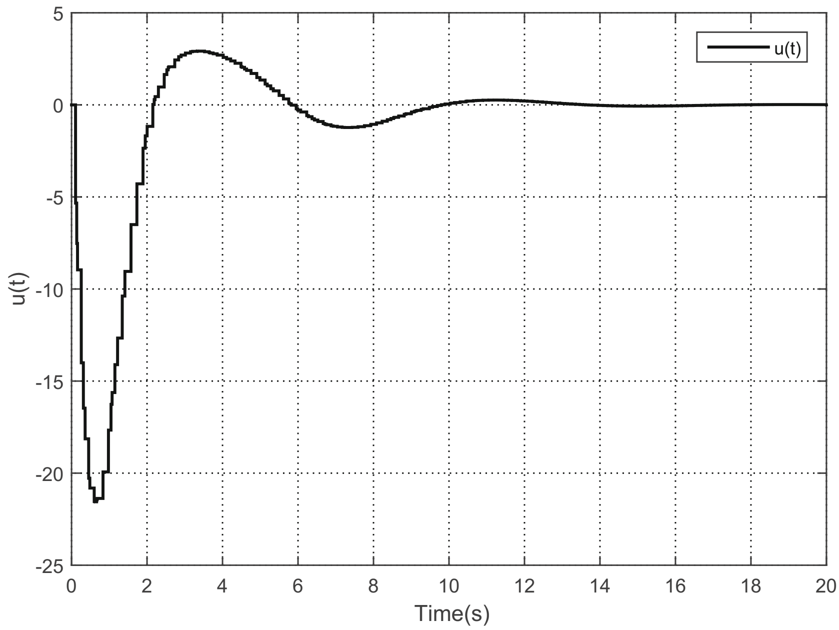

Fig. 5 Control input curve of Example 2

6 Conclusion

The stability and stabilization problems have been analyzed for the T-S fuzzy systems which have parameter uncertainty and state quantization. A novel LKF which depends on FMFs is constructed. In addition, both sampling states $x(t_k)$ and $x(t_{k+1})$ are added. When deriving of LKF, the terms which consist of the derivative of the FMFs and some LKF coefficients appear. Through discussing the terms, constrains are added to the stability conditions. Then, by using relaxed Free-matrix-based (FMB) integral inequality and reciprocally convex method, the stability conditions expressed in the form of LMIs are obtained. Later, maximum sampling interval and gain K_j are solved by LMI toolbox. At last, simulation examples are given to illustrate the effectiveness of the proposed method. In the future work, the dissipation control for the system with uncertainty can be carried out. And the content of network attack could also be considered. The rapid development of network control has led to its wide range of applications, but at the same time, the openness of the network may bring vulnerable attacks, leading to serious consequences. In addition, the ability of learning evolution is one of the important manifestations of the highly intelligent autonomous control system. It refers to the ability to improve the relevant performance of the system through autonomous learning, modification and continuous evolution. The combination of fuzzy control and the above content can be regarded as the future development direction.

Acknowledgements This work was funded by Natural Science Foundation-Steel and Iron Foundation of Hebei Province under Grant E2019105123, and Science and Technology Project of Hebei Education Department under Grant ZD2019311.

References

1. Gao Q, Feng G, Wang Y, Qiu JB (2012) Universal fuzzy models and universal fuzzy controllers for stochastic nonaffine nonlinear systems. IEEE Trans Fuzzy Syst 21(2):328–341
2. Gao Q, Feng G, Dong D, Liu L (2014) Universal fuzzy models and universal fuzzy controllers for discrete-time nonlinear systems. IEEE Trans Fuzzy Syst 45(5):880–887
3. Shen H, Chen MS, Wu ZG, Cao JD, Park JH (2019) Reliable event-triggered asynchronous extended passive control for semi-Markov jump fuzzy systems and its application. IEEE Trans Fuzzy Syst 28(8):1708–1722
4. Kwon OM, Park MJ, Park Ju H, Lee SM (2016) Stability and stabilization of T-S fuzzy systems with time-varying delays via augmented Lyapunov-Krasovskii functionals. Inf Sci 372:1–15
5. Lian Z, He Y, Zhang CK, Wu M (2019) Stability and stabilization of T-S fuzzy systems with time-varying delays via delay-product-type functional method. IEEE Trans Cybern 50(6):2580–2589
6. Wang LK, Lam HK (2017) A new approach to stability and stabilization analysis for continuous-time Takagi-Sugeno fuzzy systems with time delay. IEEE Trans Fuzzy Syst 26(4):2460–2465
7. Zhao X, Lin C, Chen B, Wang QG (2018) A novel Lyapunov-Krasovskii functional approach to stability and stabilization for T-S fuzzy systems with time delay. Neurocomputing 313:288–294
8. Chang XH, Zhang L, Park JH (2015) Robust static output feedback h_∞ control for uncertain fuzzy systems. Fuzzy Sets Syst 273:87–104
9. Kwon OM, Park MJ, Lee SM, Park JH (2012) Augmented Lyapunov-Krasovskii functional approaches to robust stability criteria for uncertain Takagi-Sugeno fuzzy systems with time-varying delays. Fuzzy Sets Syst 201:1–19

10. Peng C, Fei MR (2013) An improved result on the stability of uncertain T-S fuzzy systems with interval time-varying delay. Fuzzy Sets Syst 212:97–109
11. Wang YY, Shen H, Hamid RK, Duan DP (2017) Dissipativity-based fuzzy integral sliding mode control of continuous-time T-S fuzzy systems. IEEE Trans Fuzzy Syst 26(3):1164–1176
12. Yan HC, Wang TT, Zhang H, Shi HB (2015) Event-triggered h_∞ control for uncertain networked T-S fuzzy systems with time delay. Neurocomputing 157:273–279
13. Lian Z, He Y, Zhang CK, Wu M (2017) Further robust stability analysis for uncertain Takagi-Sugeno fuzzy systems with time-varying delay via relaxed integral inequality. Inf Sci 409:139–150
14. Vadivel P, Sakthivel R, Mathiyalagan K, Thangaraj P (2012) Robust stabilization of nonlinear uncertain Takagi-Sugeno fuzzy systems by h_∞ control. IET Control Theory Appl 6(16):2556–2566
15. Du HB, Qian CJ, Li SH, Chu ZB (2019) Global sampled-data output feedback stabilization for a class of uncertain nonlinear systems. Automatica 99:403–411
16. Li HY, Sun XJ, Shi P, Lam HK (2015) Control design of interval type-2 fuzzy systems with actuator fault: sampled-data control approach. Inf Sci 302:1–13
17. Luo JN, Liu XZ, Tian WH, Zhong SM, Shi KB (2019) Nonfragile sampled-data filtering of uncertain fuzzy systems with time-varying delays. IEEE Trans Syst Man Cybern. https://doi.org/10.1109/TSMC.2019.2946189,2019
18. Peng C, Han QL, Yue D, Ti EG (2011) Sampled-data robust h_∞ control for T-S fuzzy systems with time delay and uncertainties. Fuzzy Sets Syst 179(1):20–33
19. Wen S, Huang T, Yu X, Chen MZQ, Zeng ZG (2015) Aperiodic sampled-data sliding-mode control of fuzzy systems with communication delays via the event-triggered method. IEEE Trans Fuzzy Syst 24(5):1048–1057
20. Liu YJ, Park JH, Guo BZ, Shu YJ (2017) Further results on stabilization of chaotic systems based on fuzzy memory sampled-data control. IEEE Trans Fuzzy Syst 26:1040–1045
21. Wu ZG, Shi P, Su HY, Chu J (2013) Sampled-data fuzzy control of chaotic systems based on a T-S fuzzy model. IEEE Trans Fuzzy Syst 22(1):153–163
22. Wang B, Cheng J, Al-Barakai A, Fardoun HM (2017) A mismatched membership function approach to sampled-data stabilization for T-S fuzzy systems with time-varying delayed signals. Signal Process 140:161–170
23. Wang ZP, Wu HN (2014) On fuzzy sampled-data control of chaotic systems via a time-dependent Lyapunov functional approach. IEEE Trans Cybern 45(4):819–829
24. Dong SL, Wu ZG, Shi P, Su HY, Lu RQ (2017) Reliable control of fuzzy systems with quantization and switched actuator failures. Neurocomputing 47:2198–2208
25. Liu YJ, Lee SM (2015) Stability and stabilization of Takagi-Sugeno fuzzy systems via sampled-data and state quantized controller. IEEE Trans Fuzzy Syst 24:635–644
26. Mahmoud MS, Saif AA (2012) Robust quantized approach to fuzzy networked control systems. IEEE J Emerg Sel Topic Circuits Syst 2:71–81
27. Lu RQ, Cheng HL, Bai JJ (2015) Fuzzy-model-based quantized guaranteed cost control of nonlinear networked systems. IEEE Trans Fuzzy Syst 23:567–575
28. Shi WX (2014) Adaptive fuzzy control for MIMO nonlinear systems with nonsymmetric control gain matrix and unknown control direction. IEEE Trans Fuzzy Syst 22:1288–1300
29. Wang X, Park JH, She K, Zhong S, Shi L (2019) Stabilization of chaotic systems with T-S fuzzy model and nonuniform sampling: a switched fuzzy control approach. IEEE Trans Fuzzy Syst 27(6):1263–1271
30. Sofianos N, Boutalis YS (2013) Stable indirect adaptive switching control for fuzzy dynamical systems based on T-S multiple models. Int J Syst Sci 44(8):1546–1565
31. Sofianos NA, Boutalis YS (2016) Robust adaptive multiple models based fuzzy control of nonlinear systems. Neurocomputing 173:1733–1742
32. Wang LK, Lam HK (2018) New stability criterion for continuous-time Takagi-Sugeno fuzzy systems with time-varying delay. IEEE Trnas Cybern 49:1551–1556
33. Wang YY, Xia YQ, Zhou PF (2017) Fuzzy-model-based sampled-data control of chaotic systems: a fuzzy time-dependent Lyapunov-Krasovskii functional approach. IEEE Trans Fuzzy Syst 25(6):1672–1684
34. Wang ZP, Wu HN (2014) On fuzzy sampled-data control of chaotic systems via a time-dependent Lyapunov functional approach. IEEE Trans Cybern 45(4):819–829
35. Zeng HB, Lay TK (2017) He Y,others Sampled-data synchronization control for chaotic neural networks subject to actuator saturation. Neurocomputing 260(18):25–31
36. Park PG, Ko JW, Jeong C (2011) Reciprocally convex approach to stability of systems with time-varying delays. Automatica 47(1):235–238
37. Shao HY, Han QL, Zhang ZQ, Zhu XL (2014) Sampling-interval-dependent stability for sampled-data systems with state quantization. Int J Robust Nonlinear Control 24(17):2995–3008
38. Ge C, Shi YP, Park JH, Hua CC (2019) Robust h_∞ stabilization for T-S fuzzy systems with time-varying delays and memory sampled-data control. Appl Math Comput 346:500–512
39. Wang M, Qiu JB, Chadli M, Wang M (2015) A switched system approach to exponential stabilization of sampled-data T-S fuzzy systems with packet dropouts. IEEE Trans Cybern 46(12):3145–3156
40. Zhu XL, C B YueD, Wang YY (2012) An improved input delay approach to stabilization of fuzzy systems under variable sampling. IEEE Trans Fuzzy Syst 20(2):330–341
41. Zhao JR, Xu SY, Park JH (2019) Improved criteria for the stabilization of T-S fuzzy systems with actuator failures via a sampled-data fuzzy controller. Fuzzy Sets Syst 392(1):154–169

Liu Yang received the B.S. degree in electronic information science and technology from Tangshan Normal University, Tangshan, China, in 2018. Now she is currently pursing the M.S. Degree in control engineering, North China University of Science and Technology, Tangshan, China.

Jiayong Zhang received the Ph.D. degree in mining engineering from the China University of Mining and Technology, Xuzhou, China, in 2014. He is currently a Full Professor with the Institution of Mining Engineering, North China University of Science and Technology, Tangshan, China. His research interests include multiagent systems, neural networks, fuzzy systems, and networked control systems.

Chao Ge received the Ph.D. degree in electrical engineering from Yanshan University, Qinhuangdao, China, in 2015. He is currently a Full Professor with the North China University of Science and Technology, Tangshan, China. His research interests include time-delay systems, neural networks, fuzzy systems, and networked control systems.

Wei Li received his MS degree in Computer Applications Technology from North China University of Science and Technology, Tangshan, China, in 2010. Currently, he is an associate professor at the College of Artificial Intelligence, North China University of Science and Technology, Tangshan, China. His research interests include reliability consideration and networked control systems.

Zhiwei Zhao received the B.S. and Ph.D. degrees from Yanshan University, Qinhuangdao, China, in 2003 and 2016, respectively. He received the M.S. degree from University of Science and Technology, Beijing, China, in 2009. He is currently a Full Professor with the Department of Computer Science and Technology, Tangshan University, Tangshan, China. His research interests include evolutionary computation, multi-objective optimization, wireless network, and resource allocation optimization.