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Decision-making method based on new entropy and refined single-valued neutrosophic sets and its application in typhoon disaster assessment

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Abstract
This study proposes a multi-attribute decision-making method for the decision-making problems with attributes and sub-attribute where the attribute weight is unknown, based on information entropy and the evaluation based on distance from average solution (EDAS) method under a refined single-valued neutrosophic set environment. First, the new distance measure, similarity measure, and neutrosophic entropy based on refined single-valued neutrosophic sets are defined. Further, the relationship between them is discussed and the attribute weights are determined based on the new neutrosophic entropy. Then, the EDAS method is used to rank and select the best alternative. Finally, two illustrative examples of typhoon disaster assessment (typhoon disaster assessment with multi-layer indicators and dynamic assessment of typhoon disaster) are presented to demonstrate the feasibility, effectiveness, and practicality of the proposed method. The advantages of the proposed method are illustrated by sensitive analysis and comparative analysis with other methods.

Keywords Multi-attribute decision-making. Refined single-valued neutrosophic sets (RSVNSs). Evaluation based on distance from average solution (EDAS) method. Neutrosophic entropy. Typhoon disaster assessment

1 Introduction

Decision-making is a common activity in various aspects of our daily lives. It is generally defined as the act of seeking the best alternative from a set of alternatives based on the judgments of one or several decision makers (DMs). Multi-attribute decision-making (MADM), an important branch of decision-making, involves the generation of decisions by DMs based on the information evaluated on a set of feasible alternatives, in which multiple attributes are used to find a common solution [1]. MADM has been widely used in many fields, such as pattern recognition [2, 3], medical diagnosis [3, 4], supplier selection [5], emergency decision [6], and disaster assessment [7]. However, the decision-making problem has become increasingly complicated owing to the growing amount of decision information and alternatives, the inherent uncertainty and complexity of decision problems, and the fuzzy nature of human thinking [8]. In some sudden, complicated situations in particular, some decision information is often not represented accurately; instead, it is often expressed as fuzzy, vague, hesitant, incomplete, indeterminate, and inconsistent. Thus, the fuzzy set (FS), hesitant fuzzy set, rough set, intuitionistic fuzzy set (IFS), Pythagorean fuzzy set (PFS), and neutrosophic set are used to model the decision information. Zadeh [9] proposed FSs, which are characterized by a membership degree, in 1965. Atanassov [10] proposed IFSs, which are characterized by a membership degree and non-membership degree, in 1986. Yager [11] proposed the PFS, which is the second type of IFS. In the past few decades, FSs, IFSs, and PFSs have become research hotspots [12, 13]. However, the incomplete, indeterminate, and inconsistent problems in real life cannot be explained using these theories because they cannot handle independent components and some dependent components. Therefore, the neutrosophic set [14], which is the latest theory in fuzzy fields, is proposed to deal with the above situations.

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The neutrosophic set (NS), which consists of the truth-membership degree (T), an indeterminacy-membership degree (I), and a falsity-membership degree (F), was proposed by Smarandache from a philosophical point of view in 1999. It is a generalization of sets such as crisp sets, FSs, and IFSs. In recent years, this novel concept has become a research hotspot and many scholars have proposed various forms of neutrosophic sets. For example, Wang et al. introduced single-valued neutrosophic sets (SVNSs) [15] for the practical application of NSs, along with interval neutrosophic sets (INSs) [16]. Ye [17] introduced simplified neutrosophic sets (SNSs) and applied them to MADM. Wang et al. defined the concept of multi-valued neutrosophic sets (MVNSs) [18] and multi-valued interval neutrosophic sets (MVINSs) [19]. Deli et al. introduced bipolar neutrosophic sets (BNSs) [20] and bipolar neutrosophic refined sets (BNRSs) [21]. Ali et al. [22] proposed the concept of bipolar neutrosophic soft sets (BNSSs). Tian et al. [23] defined simplified neutrosophic linguistic sets (SNLSs) and applied them to MADM. Biswas et al. [24], Ye [25], and Tan et al. [26] studied the trapezoidal fuzzy neutrosophic sets (TrFNSs). Broumi et al. [27], Tan et al. [28], and others combined the neutrosophic sets and graph theory to propose neutrosophic graphs (NGs), and used them to solve the shortest path problem. Liu et al. [29] studied linguistic neutrosophic sets (LNSs) and their application to multi-attribute group decision-making (MAGDM). Abdel-Basset et al. [30] defined the concept of type-2 neutrosophic number sets (T2NNSs). However, because there are both arguments and sub-arguments/refined arguments in the truth, indeterminacy, and falsity membership degrees of T, I, F in the neutrosophic set to express complex problems of the real world in detail, the truth, indeterminacy, and falsity information should be refined [31]. Hence, Smarandache [32] further extended the neutrosophic logic to n-valued refined neutrosophic logic by refining each neutrosophic component T, I, F into $T_1, T_2, \cdots, T_m$, $I_1, I_2, \cdots, I_P$, and $F_1, F_2, \cdots, F_r$, respectively. Broumi et al. [33] presented the cosine similarity measure of neutrosophic refined sets and applied it to medical diagnosis problems. Broumi et al. [34] proposed correlation measures for neutrosophic refined sets and applied them to medical diagnosis. Ye et al. [35] proposed the Dice similarity measure between single-valued neutrosophic multisets and applied it to medical diagnosis. Mondal et al. [36] proposed neutrosophic refined similarity measure based on a tangent function and applied it to MADM. In fact, the multi-valued neutrosophic sets/neutrosophic refined sets are neutrosophic multisets in their expressed forms [31, 37, 38]. Hence, these multi-valued neutrosophic sets/neutrosophic refined sets, i.e., neutrosophic multisets, and their decision-making methods cannot express and deal with decision-making problems with both attributes and sub-attributes [31]. Therefore, Ye and Smarandache [37] proposed a refined single-valued neutrosophic set, where the neutrosophic set $\{T, I, F\}$ is refined into RSVNS $\{(T_1, T_2, \cdots, T_r), (I_1, I_2, \cdots, I_r), (F_1, F_2, \cdots, F_r)\}$, and proposed the similarity measures based on RSVNSs to solve decision-making problems with both attributes and sub-attributes. Then, Fan and Ye [38] further presented the cosine measures of RSVNSs and refined interval neutrosophic sets (RINSs) based on the distance and cosine function and applied them to the decision-making problems with both attributes and sub-attributes under RSVNSs/RINSs environments. Chen et al. [31] proposed the vector similarity measures between refined simplified neutrosophic sets (RSNSs), which contain the RSVNS and RINS, and their MADM method. Throughout the existing literature, not many studies have been conducted on RSVNSs. Therefore, this study investigates the decision-making method with both attributes and sub-attributes, where the attribute weight is unknown, under the RSVNSs environment.

The objective determination of attribute weights is a key point in decision-making. For example, Chen et al. [39] established an objective programming model and used the Lagrange equation model to obtain the weights in a multi-granular hesitant fuzzy linguistic term environment. Harish [40] proposed linear programming based on these preferences and an improved score function to solve the MADM problems. Han et al. [41] used the deviation entropy weight method to determine the attribute weights based on the quantitative and qualitative indexes. Liu et al. [42] used the entropy weight method to determine the weights of attributes of the hybrid multiple attributes. Ye [43] proposed two weight models based on the improved similarity measures to derive the weights of the DMs and the attributes based on single-valued neutrosophic numbers (SVNNs). Tan et al. [44] used the entropy of neutrosophic linguistic sets (NLSs) to determine the attribute weights. Xiong et al. [45] presented a novel and simple nonlinear optimization model to determine the attribute weights by maximizing the total deviation in all attribute values based on SVNNs. Li et al. [46] constructed a single-objective programming model based on the smallest deviation degree between each alternative and linguistic neutrosophic positive ideal solution to determine attribute weights based on the linguistic neutrosophic numbers. However, there are few researches on the attribute weight determination methods based on RSVNSs. Therefore, this study examines the method of determining attribute weights based on the new distance measure and neutrosophic entropy in the RSVNSs environment.

Sorting methods are a research hotspot in decision making. Existing classic and commonly used methods include interactive multi-criteria decision making (which has the acronym TODIM in Portuguese) [47], technique for order preference by similarity to ideal solution (TOPSIS) [48], preference ranking organization method for enrichment evaluations (PROMETHEE) [49], elimination and choice expressing the reality (ELECTRE) [50], vlsekriterijumska optimizacija I kompromisno resenje (VIKOR) [51], grey relational analysis (GRA) [52], multi-objective optimization by ratio analysis (MULTIMOORA) [53], prospect theory (PT) [54], case-based reasoning (CBR) [55], Dempster–Shafer (D–S) theory, function method (score function, accuracy function, expectation function, etc.), and other correlation methods [56]. Here, the evaluation based on distance from average solution (EDAS) method, which can consider the conflicting attributes, is a relatively new method proposed by Keshavarz et al. [57] in 2015. It aims to find the best solution by using two distance measures, namely the positive distance from average (PDA) and the negative distance from average (NDA). The alternative with larger PDA and lower NDA is determined to be the best alternative [58]. Compared to the existing work, the EDAS model has the merit of only considering average solutions with respect to the intangibility of DMs and the uncertainty of the decision-making environment to obtain more accurate and effective aggregation results [59]. Zhang et al. [59] extended the EDAS method based on picture 2-tuple linguistic numbers. Liang et al. [60] proposed an extended EDAS method based on picture fuzzy information. Kahraman et al. [61] proposed an intuitionistic fuzzy EDAS method. Peng et al. [62] proposed algorithms for neutrosophic soft decision making based on EDAS. Although EDAS has been studied in recent years, the research is still limited. We expect to use the EDAS tool to manage the MADM problems and overcome the originality and simplicity of traditional compromise methods. Thus, we propose an extended EDAS-based MADM method whose evaluation information is expressed as refined single-valued neutrosophic sets.

A typhoon is a natural disaster with highly destructive power. Historically, the southeastern coastline of China has been prone to typhoons annually [63]. Typhoons are often accompanied by strong winds, rainstorms, and storm surges, which have the characteristics of high frequency, sudden occurrence, wide range of influence, and great intensity of disaster [64]. In the vulnerable areas of the geological environment, the rainstorms caused by typhoons can easily trigger geological disasters, such as collapses, landslides, and debris flow, and bring about heavy casualties and property losses. The destructive power of a typhoon typically exceeds that of an earthquake, making it one of the biggest disasters facing humanity; but to date no method has been developed to avoid it. Therefore, the assessment of typhoon disasters is a very important issue that can help disaster relief and management departments. However, the factors influencing typhoon disasters are difficult to describe accurately. For example, general economic loss includes many aspects, such as building collapse, number of deaths and missing persons, affected local economic conditions, degree of environmental damage, and degree of social instability. The assessment information is usually expressed as hesitant, ambiguous, incomplete, inconsistent, and uncertain. In an effort to mitigate these problems, FSs and IFSs have been used in typhoon disaster assessment (TDA) in recent years. Shi et al. [65] proposed a TDA method based on an MADM hybrid approach using analytic hierarchy process (AHP) and TOPSIS with fuzzy numbers. He et al. [66] proposed a TDA method based on Dombi aggregation operators with hesitant fuzzy information. Li et al. [67] proposed a TDA method based on TOPSIS with intuitionistic fuzzy numbers (IFNs). Yu [68] proposed a TDA method based on generalized intuitionistic fuzzy interactive aggregation operators. Tang et al. [69] studied the nature disaster risk evaluation based on incomplete hesitant fuzzy linguistic preference relations. Tan et al. [7] studied the TDA method based on the exponential aggregation operator of interval neutrosophic numbers (INNs). However, there are not many studies on TDA based on neutrosophic numbers. Thus, we propose an MADM method based on neutrosophic entropy and the EDAS method under the RSVNSs environment, and apply it to TDA.

The remainder of this paper is organized as follows. Section 2 briefly introduces some basic concepts, including neutrosophic sets and refined single-valued neutrosophic sets. Section 3 gives the definition of new distance measure, similarity measure, and entropy of RSVNSs, and discusses the relationship between them. Section 4 describes the problem and the attribute weights calculation method based on RSVNSs. Then, the MADM method based on the neutrosophic entropy and EDAS method with RSVNSs is presented. Section 5 gives two examples of typhoon disaster evaluation to verify the applicability of the proposed approach and its advantages by sensitive analysis and comparative analysis. Section 6 presents concluding remarks and suggestions for further research.

2 Preliminaries

In this section, we briefly outline various essential concepts, such as NSs, RSVNS, and operational rules of RSVNSs.

Definition 1 [14] Let X be a space of points (objects), with a generic element in X denoted by x. Then, a neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$, i.e., $T_A(x): X \to]0^-, 1^+[$, $I_A(x): X \to]0^-, 1^+[$, and $F_A(x): X \to]0^-, 1^+[$. Therefore, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition 2 [15] Let X be a universal set. A single-valued neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$.

Then, an SVNS A can be denoted by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}. \tag{1}$$

where $T_A(x), I_A(x), F_A(x) \in [0, 1]$ for each x in X. Then, the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$ satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$. For convenience, we can use $a = \langle T, I, F\rangle$ to represent a single-valued neutrosophic number (SVNN).

Definition 3 [37] Let $X = \{x_1, x_2, \cdots, x_n\}$ be a universe of discourse. Then, a refined single-valued neutrosophic set R in X can be expressed by the following form.

$$R = \{\langle x_i, (T_{1R}(x_i), T_{2R}(x_i), \cdots, T_{kR}(x_i)), (I_{1R}(x_i), I_{2R}(x_i), \cdots, I_{kR}(x_i)), (F_{1R}(x_i), F_{2R}(x_i), \cdots, F_{kR}(x_i))\rangle | x_i \in X\}. \tag{2}$$

where k is a positive integer, $T_{1R}(x_i)$, $T_{2R}(x_i)$, $\cdots$, $T_{kR}(x_i) \in [0, 1]$, $I_{1R}(x_i)$, $I_{2R}(x_i)$, $\cdots$, $I_{kR}(x_i) \in [0, 1]$, $F_{1R}(x_i)$, $F_{2R}(x_i)$, $\cdots$, $F_{kR}(x_i) \in [0, 1]$, and $0 \le T_{jR}(x_i) + I_{jR}(x_i) + F_{jR}(x_i) \le 3$ for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, k$.

Definition 4 [37] Let $X = \{x_1, x_2, \cdots, x_n\}$ be a universe of discourse, and M and N be two RSVNSs,

$$M = \{\langle x_i, (T_{1M}(x_i), T_{2M}(x_i), \cdots, T_{kM}(x_i)), (I_{1M}(x_i), I_{2M}(x_i), \cdots, I_{kM}(x_i)), (F_{1M}(x_i), F_{2M}(x_i), \cdots, F_{kM}(x_i))\rangle \; | x_i \in X\},$$

$$N = \{\langle x_i, (T_{1N}(x_i), T_{2N}(x_i), \cdots, T_{kN}(x_i)), (I_{1N}(x_i), I_{2N}(x_i), \cdots, I_{kN}(x_i)), (F_{1N}(x_i), F_{2N}(x_i), \cdots, F_{kN}(x_i))\rangle \; | x_i \in X\}.$$

Then, the relations between M and N are given as follows.

(1) Containment: $M \subseteq N$, if and only if $T_{jM}(x_i) \le T_{jN}(x_i)$, $I_{jM}(x_i) \ge I_{jN}(x_i)$, $F_{jM}(x_i) \ge F_{jN}(x_i)$ for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, k$.

(2) Equality: $M = N$, if and only if $T_{jM}(x_i) = T_{jN}(x_i)$, $I_{jM}(x_i) = I_{jN}(x_i)$, $F_{jM}(x_i) = F_{jN}(x_i)$ for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, k$.

(3) Union: $M \cup N = \left\{\left\langle \begin{array}{c} x_i, (T_{1M}(x_i) \vee T_{1N}(x_i), T_{2M}(x_i) \vee T_{2N}(x_i), \cdots, T_{kM}(x_i) \vee T_{kN}(x_i)), \\ (I_{1M}(x_i) \wedge I_{1N}(x_i), I_{2M}(x_i) \wedge I_{2N}(x_i), \cdots, I_{kM}(x_i) \wedge I_{kN}(x_i)), \\ (F_{1M}(x_i) \wedge F_{1N}(x_i), F_{2M}(x_i) \wedge F_{2N}(x_i), \cdots, F_{kM}(x_i) \wedge F_{kN}(x_i)) \end{array} \right\rangle | x_i \in X \right\}.$

(4) Intersection: $M \cap N = \left\{\left\langle \begin{array}{c} x_i, (T_{1M}(x_i) \wedge T_{1N}(x_i), T_{2M}(x_i) \wedge T_{2N}(x_i), \cdots, T_{kM}(x_i) \wedge T_{kN}(x_i)), \\ (I_{1M}(x_i) \vee I_{1N}(x_i), I_{2M}(x_i) \vee I_{2N}(x_i), \cdots, I_{kM}(x_i) \vee I_{kN}(x_i)), \\ (F_{1M}(x_i) \vee F_{1N}(x_i), F_{2M}(x_i) \vee F_{2N}(x_i), \cdots, F_{kM}(x_i) \vee F_{kN}(x_i)) \end{array} \right\rangle | x_i \in X \right\}.$

For convenient expression, a basic element $\{\langle x_i, (T_{1R}(x_i), T_{2R}(x_i), \cdots, T_{kR}(x_i)), (I_{1R}(x_i), I_{2R}(x_i), \cdots, I_{kR}(x_i)), (F_{1R}(x_i), F_{2R}(x_i), \cdots, F_{kR}(x_i))\rangle\}$ in R is simply denoted as $r = \langle (T_{1r}, T_{2r}, \cdots, T_{kr}), (I_{1r}, I_{2r}, \cdots, I_{kr}), (F_{1r}, F_{2r}, \cdots, F_{kr})\rangle$, which is called a refined single-valued neutrosophic number (RSNN).

Definition 5 Let $r = \langle (T_{1r}, T_{2r}, \cdots, T_{kr}), (I_{1r}, I_{2r}, \cdots, I_{kr}), (F_{1r}, F_{2r}, \cdots, F_{kr})\rangle$ and $l = \langle (T_{1l}, T_{2l}, \cdots, T_{kl}), (I_{1l}, I_{2l}, \cdots, I_{kl}), (F_{1l}, F_{2l}, \cdots, F_{kl})\rangle$ be two RSVNNs. Then, the following operational rules apply:

$$\lambda r = \left\langle \begin{array}{c} \left(1-(1-T_{1r})^\lambda, 1-(1-T_{2r})^\lambda, \cdots, 1-(1-T_{kr})^\lambda\right), \\ \left((I_{1r})^\lambda, (I_{2r})^\lambda, \cdots, (I_{kr})^\lambda\right), \left((F_{1r})^\lambda, (F_{2r})^\lambda, \cdots, (F_{kr})^\lambda\right) \end{array} \right\rangle, \lambda > 0.$$

$$r^\lambda = \left\langle \begin{array}{c} \left((T_{1r})^\lambda, (T_{2r})^\lambda, \cdots, (T_{kr})^\lambda\right), \left(1-(1-I_{1r})^\lambda, 1-(1-I_{2r})^\lambda, \cdots, 1-(1-I_{kr})^\lambda\right), \\ \left(1-(1-F_{1r})^\lambda, 1-(1-F_{2r})^\lambda, \cdots, 1-(1-F_{kr})^\lambda\right) \end{array} \right\rangle, \lambda > 0.$$

$$r + l = \left\langle \begin{array}{c} (T_{1r} + T_{1l} - T_{1r} \cdot T_{1l}, T_{2r} + T_{2l} - T_{2r} \cdot T_{2l}, \cdots, T_{kr} + T_{kl} - T_{kr} \cdot T_{kl}), \\ (I_{1r} \cdot I_{1l}, I_{2r} \cdot I_{2l}, \cdots, I_{kr} \cdot I_{kl}), (F_{1r} \cdot F_{1l}, F_{2r} \cdot F_{2l}, \cdots, F_{kr} \cdot F_{kl}) \end{array} \right\rangle.$$

$$r \cdot l = \left\langle \begin{array}{c} (T_{1r} \cdot T_{1l}, T_{2r} \cdot T_{2l}, \cdots, T_{kr} \cdot T_{kl}), (I_{1r} + I_{1l} - I_{1r} \cdot I_{1l}, I_{2r} + I_{2l} - I_{2r} \cdot I_{2l}, \cdots, I_{kr} + I_{kl} - I_{kr} \cdot I_{kl}), \\ (F_{1r} + F_{1l} - F_{1r} \cdot F_{1l}, F_{2r} + F_{2l} - F_{2r} \cdot F_{2l}, \cdots, F_{kr} + F_{kl} - F_{kr} \cdot F_{kl}) \end{array} \right\rangle.$$

3 New distance measure, similarity measure, and entropy of RSVNSs

3.1 Score function and accuracy function

Inexact numbers (such as fuzzy numbers, interval numbers, intuitionistic fuzzy numbers, and neutrosophic numbers) can represent the ambiguity, hesitation, uncertainty, and inconsistency of information. When solving some complex problems, the decision information expressed as inexact number will more fully reflect the problem attributes, but its calculation is more complicated and cannot be directly sorted. Therefore, most of the existing research work on inexact number processing convert it to exact numbers to complete the calculation and comparison of inexact numbers at the cost of losing some information. This conversion is best placed in the middle and late stages of the decision-making process. This practice can avoid the problem of inaccurate decision-making results caused by information loss. The score function and accuracy function are widely used in fuzzy decision methods because they are simple to calculate. Therefore, to deal with the problem of converting RSVNS into exact numbers, this study proposes the definition of score function and accuracy function based on RSVNS.

Because the size comparison of RSNN is used in the comparative analysis of the example, we define the score function and accuracy function of RSNN in this section. Inspired by the literature [70, 71], we defined the score function and accuracy function of the refined single-valued neutrosophic set. In the literature, there are several published studies on score function and accuracy function in different information environments [70–72]. However, their formula expression and calculation complexity differ. For some special cases, the calculation formula is more complicated. To simplify the calculation, we define simpler score and accuracy functions as follows.

Definition 6 Let $r=\langle(T_{1r}, T_{2r}, \cdots, T_{kr}), (I_{1r}, I_{2r}, \cdots, I_{kr}), (F_{1r}, F_{2r}, \cdots, F_{kr})\rangle$ be an RSVNN. Then, the score function of an RSVNN can be defined as

$$Sc(r)=\frac{1}{k}\sum_{j=1}^{k}\frac{1}{3}\left(2+T_{jr}-I_{jr}-F_{jr}\right), Sc(r)\in[0,1]. \tag{3}$$

where a larger value of $Sc(r)$ indicates a larger RSVNN r.

Definition 7 Let $r=\langle(T_{1r}, T_{2r}, \cdots, T_{kr}), (I_{1r}, I_{2r}, \cdots, I_{kr}), (F_{1r}, F_{2r}, \cdots, F_{kr})\rangle$ be an RSVNN. Then, the accuracy function of an RSVNN can be defined as

$$H(r)=\frac{1}{k}\sum_{j=1}^{k}\left(T_{jr}-F_{jr}\right), H(r)\in[-1,1]. \tag{4}$$

where a larger value of $H(r)$ indicates bigger the accuracy of RSVNN r.

Here, our accuracy function does not consider the indeterminacy-membership for three reasons. First, because the score function considers the indeterminacy membership and it is the main preferred sorting method, the accuracy function is only used when the score function cannot be distinguished. Therefore, the accuracy function can be simply defined to reduce the computational complexity. Second, the RSNN studied in this work has more data than the single-valued neutrosophic number. Thus, it is easier to distinguish based on the two functions. Finally, we study the TDA problem, which needs timeliness. Therefore, we need to reduce the calculation amount of the model. In summary, we define simpler score functions and accuracy functions that can solve the sorting problem. Even if they cannot solve some very special cases, the literature [72] can be referenced; that is, based on the literature [72], we can propose more complex accuracy functions to solve these special cases. The accuracy function definition is shown below.

$$H_2(r)=\frac{1}{k}\sum_{j=1}^{k}\left(T_{jr}-I_{jr}\cdot(1-T_{jr})-F_{jr}\cdot(1-I_{jr})\right), H_2(r)\in[-1,1].$$

Compared with the above two accuracy functions, $H(r)$ only considers the truth-membership function value and the falsity-membership function value of the evaluation information. Its purpose is to reduce the amount of calculation and speed up the evaluation process at the cost of losing some information. The advantage of $H_2(r)$ is that it considers the three membership values of the evaluation information, the information evaluation is more comprehensive, and the evaluation results will be more objective and reasonable. The choice of two functions can be based on the following principles: In the processing of inexact data, $Sc(\cdot)$ and $H(\cdot)$ are preferentially used to process information. If they cannot solve the problem, $H_2(\cdot)$ is selected, which is suitable for dealing with more complex problems and some special situations. For example, if $r=\langle(0,0.5),(0.5,1),(0,0.5)\rangle$, $l=\langle(0.5,1),(0.5,1),(0.5,1)\rangle$ are two RSVNNs, using the above functions to calculate $Sc(\cdot)$, we obtain $Sc(r)=\frac{1}{2}\left(\frac{1.5}{3}+\frac{1}{3}\right)=\frac{2.5}{6}$ and $Sc(l)=\frac{1}{2}\left(\frac{1.5}{3}+\frac{2}{3}\right)=\frac{2.5}{6}$. Then, when we calculate $H(\cdot)$, we obtain $H(r)=\frac{1}{2}(0+0)=0$ and $H(l)=\frac{1}{2}(0+0)=0$. Obviously, r and l cannot be distinguished by $Sc(\cdot)$ and $H(\cdot)$, so we calculate $H_2(\cdot)$, and obtain $H_2(r)=\frac{1}{2}(-0.5+0)=-0.25$ and $H_2(l)=\frac{1}{2}(0+1)=0.5$. Therefore, we can get $r<l$, which is also in line with people's subjective judgments.

On the basis of the score function $Sc(r)$ and accuracy function $H(r)$, we give the following ordered relation between two RSVNNs.

Definition 8 Let $r=\langle(T_{1r}, T_{2r}, \cdots, T_{kr}), (I_{1r}, I_{2r}, \cdots, I_{kr}), (F_{1r}, F_{2r}, \cdots, F_{kr})\rangle$ and $l=\langle(T_{1l}, T_{2l}, \cdots, T_{kl}), (I_{1l}, I_{2l}, \cdots, I_{kl}), (F_{1l}, F_{2l}, \cdots, F_{kl})\rangle$ be two RSVNNs. Thus, $Sc(r)$ and $Sc(l)$ are the scores of r and l, respectively, and $H(r)$ and $H(l)$ are the accuracy degree of r and l, respectively. Consequently, the ordered relation between two RSVNNs is defined as follows.

(1) If $Sc(r)>Sc(l)$, then $r>l$;
(2) If $Sc(r)<Sc(l)$, then $r<l$;
(3) If $Sc(r)=Sc(l)$, and

① $H(r)>H(l)$, then $r>l$;
② $H(r)<H(l)$, then $r<l$.

In particular, in some special cases, if the alternative cannot be sorted based on $Sc(\cdot)$ and $H(\cdot)$, $H(\cdot)$ can be replaced by $H_2(\cdot)$ in the sorting method of definition 7.

3.2 New distance measure, similarity measure of RSVNSs

Distance measure and similarity measure are widely researched and applied techniques in decision problems. The new distance measure of RSVNSs proposed in this study has several functions. On the one hand, the information entropy can be obtained based on the distance measure to calculate the attribute weight. On the other hand, it is used for comparative analysis in the example part, that is, one of the comparison methods with TOPSIS needs to calculate the distance between the solution and the positive/negative ideal solution. Distance measurement involves more than just that. The similarity measure is also a hotspot for many scholars. It can be used for scheme ranking. Based on the relationship between distance and similarity measure, this study can easily and conveniently obtain the similarity measure and use it for the comparative analysis of calculation examples. Therefore, we provide the following definitions for the new distance measure and similarity measure with RSVNSs in various forms.

Definition 9 Let M and N be two RSVNSs. Then, the distance measure $D(M, N)$ between M and N is defined as follows.

$$D(M,N) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|T_{jM}(x_i)-T_{jN}(x_i)\right|^{\lambda}+\left|I_{jM}(x_i)-I_{jN}(x_i)\right|^{\lambda}+\left|F_{jM}(x_i)-F_{jN}(x_i)\right|^{\lambda}\right)\right\}^{1/\lambda}, \lambda\geq 0. \tag{5}$$

If $\lambda = 1$, the distance Eq. (5) is reduced to the following Hamming distance.

$$D_H(M,N) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|T_{jM}(x_i)-T_{jN}(x_i)\right|+\left|I_{jM}(x_i)-I_{jN}(x_i)\right|+\left|F_{jM}(x_i)-F_{jN}(x_i)\right|\right)\right\}. \tag{6}$$

If $\lambda = 2$, the distance Eq. (5) is reduced to the following Euclidean distance.

$$D_E(M,N) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\sqrt{\frac{1}{3}\left(\left|T_{jM}(x_i)-T_{jN}(x_i)\right|^{2}+\left|I_{jM}(x_i)-I_{jN}(x_i)\right|^{2}+\left|F_{jM}(x_i)-F_{jN}(x_i)\right|^{2}\right)}. \tag{7}$$

Theorem 1 The above-defined distance $D(M, N)$ satisfies the following properties.

(P1) $0 \leq D(M, N) \leq 1$;
(P2) $D(M, N) = 0$ if and only if $M = N$;
(P3) $D(M, N) = D(N, M)$;

The proof process of Theorem 1 can be found in the Appendix.

Because $D_H(M, N)$ and $D_E(M, N)$ are special cases of $D(M, N)$, they also satisfy Theorem 1.

Here, we can get the similarity measures of RSVNSs based on the relationship between distance and similarity,

$$S(M,N) = 1-D(M,N) = 1-\frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|T_{jM}(x_i)-T_{jN}(x_i)\right|^{\lambda}+\left|I_{jM}(x_i)-I_{jN}(x_i)\right|^{\lambda}+\left|F_{jM}(x_i)-F_{jN}(x_i)\right|^{\lambda}\right)\right\}^{1/\lambda}. \tag{8}$$

If $\lambda = 1$, the similarity measure based on the Hamming distance is as follows.

$$S_H(M,N) = 1-D_H(M,N) = 1-\frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|T_{jM}(x_i)-T_{jN}(x_i)\right|+\left|I_{jM}(x_i)-I_{jN}(x_i)\right|+\left|F_{jM}(x_i)-F_{jN}(x_i)\right|\right)\right\}. \tag{9}$$

If $\lambda = 2$, the similarity measure based on the Euclidean distance is as follows.

$$S_E(M,N) = 1-D_E(M,N) = 1-\frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\sqrt{\frac{1}{3}\left(\left|T_{jM}(x_i)-T_{jN}(x_i)\right|^{2}+\left|I_{jM}(x_i)-I_{jN}(x_i)\right|^{2}+\left|F_{jM}(x_i)-F_{jN}(x_i)\right|^{2}\right)}. \tag{10}$$

Theorem 2 The similarity measures above evidently satisfy the following properties.

(P1) $0 \leq S(M, N) \leq 1$;
(P2) $S(M, N) = S(N, M)$;
(P3) $S(M, N) = 1$ for $M = N$, i.e., $T_{jM}(x_i) = T_{jN}(x_i)$, $I_{jM}(x_i) = I_{jN}(x_i)$, $F_{jM}(x_i) = F_{jN}(x_i)$.

Owing to the establishment of Theorem 1, Theorem 2 is also established, and no further proof is given here.

In general, we usually consider the weights of primary attributes. Assume that the weight of each primary attribute x_i is ω_i $(i = 1, 2, \cdots, n)$, with $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n}\omega_i = 1$. Then, we can introduce the weighted similarity measure formulas of RSVNS.

The generalized weighted similarity measure of RSVNS is as follows.

$$S(M,N) = 1-D(M,N) = 1-\sum_{i=1}^{n}\omega_i\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|T_{jM}(x_i)-T_{jN}(x_i)\right|^{\lambda}+\left|I_{jM}(x_i)-I_{jN}(x_i)\right|^{\lambda}+\left|F_{jM}(x_i)-F_{jN}(x_i)\right|^{\lambda}\right)\right\}^{1/\lambda}. \tag{11}$$

If $\lambda = 1$, the weighted similarity measure based on the Hamming distance is as follows.

$$S_H(M,N) = 1-D_H(M,N) = 1-\sum_{i=1}^{n}\omega_i\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|T_{jM}(x_i)-T_{jN}(x_i)\right|+\left|I_{jM}(x_i)-I_{jN}(x_i)\right|+\left|F_{jM}(x_i)-F_{jN}(x_i)\right|\right)\right\}. \tag{12}$$

If $\lambda = 2$, then the weighted similarity measure based on the Euclidean distance is as follows:

$$S_E(M,N) = 1-D_E(M,N) = 1-\sum_{i=1}^{n}\omega_i\frac{1}{k}\sum_{j=1}^{k}\sqrt{\frac{1}{3}\left(\left|T_{jM}(x_i)-T_{jN}(x_i)\right|^{2}+\left|I_{jM}(x_i)-I_{jN}(x_i)\right|^{2}+\left|F_{jM}(x_i)-F_{jN}(x_i)\right|^{2}\right)}. \tag{13}$$

3.3 New entropy of RSVNN

Determining attribute weights based on information entropy is a widely used method. Entropy is an important tool for measuring uncertain attribute information. A greater entropy value implies greater uncertainty in the information. Information entropy is a hot issue in the research of uncertain information [73, 74]. The neutrosophic set is mainly used to deal with uncertain, inconsistent information. Thus, using entropy to calculate weights is very suitable in the neutrosophic set environment. Simultaneously, we use the distance measure to define the neutrosophic entropy, which can simplify the overall decision-making model, reduce the calculation amount, and strike a balance between method objectivity, rationality, and complexity. Therefore, inspired by the literature [75, 76], this study proposes the extended entropy measures of RSVNN and gives the following definition of entropy for RSVNN in various forms.

Definition 10 A real function $E_{RSVNN}: RSVNN \rightarrow [0, 1]$ is called an entropy measure for an RSVNN, and $r = \langle(T_{1r}, T_{2r}, \cdots, T_{kr}), (I_{1r}, I_{2r}, \cdots, I_{kr}), (F_{1r}, F_{2r}, \cdots, F_{kr})\rangle$ is an RSVNN. Then, the entropy measure is $E_{RSVNN}(r) = 1 - 2D(r, r')$, and $r' = \langle(0.5,0.5, \cdots, k\ \ times), (0.5,0.5, \cdots, k\ \ times), (0.5,0.5, \cdots, k\ \ times)\rangle$. The new entropy of RSVNN based on the distance measure is as follows.

$$E_{RSVNN}(r) = 1-2D\left(r, r'\right) = 1-2\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|T_{jr}(x_i)-T_{jr'}(x_i)\right|^{\lambda} + \left|I_{jr}(x_i)-I_{jr'}(x_i)\right|^{\lambda} + \left|F_{jr}(x_i)-F_{jr'}(x_i)\right|^{\lambda}\right)\right\}^{1/\lambda}. \tag{14}$$

Theorem 3 The defined entropy $E_{RSVNN}(r)$ of Definition 10 satisfies the following properties.

(P1) $E_{RSVNN}(r) = 0$if r is a crisp number;

(P2) $E_{RSVNN}(r) = 1$, if and only if $r = r' = \langle(0.5,0.5, \cdots, k\ \ times), (0.5,0.5, \cdots, k\ \ times), (0.5,0.5, \cdots, k\ \ times)\rangle$;

(P3) If $D(r, r') \geq D(l, r')$, then $E_{RSVNN}(r) \leq E_{RSVNN}(l)$ for r, l are RSVNNs, where D is the distance of two RSVNNs;

(P4) $E_{RSVNN}(r) = E_{RSVNN}(r^c)$, where r^c is the complement of r.

The proof process of Theorem 3 can be found in the Appendix.

Remark 1 In Theorem 3, some properties of entropy for RSVNN are introduced based on the above distance measures. These conditions in Theorem 3 imply the following properties.

(P1) A crisp set is not fuzzy, i.e., when an RSVNN reduces to a crisp number, it is not vague;

(P2) The RSVNN $r' = \langle(0.5,0.5, \cdots, k\ \ times), (0.5,0.5, \cdots, k\ \ times), (0.5,0.5, \cdots, k\ \ times)\rangle$ is the fuzziest one;

(P3) The closer an RSVNN is to r', the fuzzier it is;

(P4) An RSVNN has the same fuzziness as its complement;

Meanwhile, according to the relationship between similarity measure and distance measure $S(r, r') = 1 - D(r, r')$, we can get

$$E_{RSVNN}(r) = 1-2D\left(r, r'\right) = 2S\left(r, r'\right)-1 \tag{15}$$

When $\lambda = 1$, $D(r, r') = \frac{1}{k}\sum_{j=1}^{k}\{\frac{1}{3}(|T_{jr}(x_i)-0.5| + |I_{jr}(x_i)-0.5| + |F_{jr}(x_i)-0.5|)\}$ is a Hamming distance; then, the new entropy based on distance measure is given by

$$E_{RSVNN}\left(\tilde{n}\right) = 1-2\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|T_{jr}(x_i)-0.5\right| + \left|I_{jr}(x_i)-0.5\right| + \left|F_{jr}(x_i)-0.5\right|\right)\right\} \tag{16}$$

When $\lambda = 2$, $D(r, r') = \frac{1}{k}\sum_{j=1}^{k}\sqrt{\frac{1}{3}\left(|T_{jr}(x_i)-0.5|^2 + |I_{jr}(x_i)-0.5|^2 + |F_{jr}(x_i)-0.5|^2\right)}$ is a Euclidean distance; then, the new entropy based on distance measure is as follows.

$$E_{RSVNN}\left(\tilde{n}\right) = 1-2\frac{1}{k}\sum_{j=1}^{k}\sqrt{\frac{1}{3}\left(\left|T_{jr}(x_i)-0.5\right|^2 + \left|I_{jr}(x_i)-0.5\right|^2 + \left|F_{jr}(x_i)-0.5\right|^2\right)} \tag{17}$$

4 Multi-attribute decision making method based on neutrosophic entropy and EDAS method with RSVNSs

4.1 Problem description

RSVNSs can not only represent the primary attribute value and the sub-attribute value, but also represent the state value of the attribute at different times. To achieve our objectives, the problem description and the proposed method consist of the following several steps that are depicted graphically in Fig. 1.

In a decision-making problem with multiple attributes and sub-attributes, there is a set of alternatives $X = \{X_1, X_2, \cdots, X_m\}$ and a set of attributes $C = \{C_1, C_2, \cdots, C_n\}$, where $C_j (j = 1, 2, \cdots, n)$ may be split into a set of $k$sub-criteria $C_j = \{c_{1j}, c_{2j}, \cdots, c_{k_j j}\}$ $(j = 1, 2, \cdots, n)$. Let $\omega = \{\omega_1, \omega_2, \cdots, \omega_n\}$ be the potential weighting vector of the attribute, where $\omega_j \in [0, 1]$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n}\omega_j = 1$. If the DMs provide refined single-valued neutrosophic set to evaluate the alternative $X_i (i = 1, 2, \cdots, m)$ under the attribute $C_j (j = 1, 2, \cdots, n)$, it can be characterized by.

$SVNS_{X_i} = \left\{\left\langle \begin{matrix} C_j, (T_{X_i}(c_{1j}), T_{X_i}(c_{2j}), \cdots, T_{X_i}(c_{k_j j})), \\ (I_{X_i}(c_{1j}), I_{X_i}(c_{2j}), \cdots, I_{X_i}(c_{k_j j})), (F_{X_i}(c_{1j}), F_{X_i}(c_{2j}), \cdots, F_{X_i}(c_{k_j j})) \end{matrix} \right\rangle \middle| C_j \in C, c_{kj} \in C_j \right\}$.

Then, for convenience, each basic element in the RSVNS is represented by the RSVNN $x_{ij} = \langle(T_{i1j}, T_{i2j}, \cdots, T_{ik_j j}), (I_{i1j}, I_{i2j}, \cdots, I_{ik_j j}), (F_{i1j}, F_{i2j}, \cdots, F_{ik_j j})\rangle$ for $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$. Hence, we can construct the refined single-valued

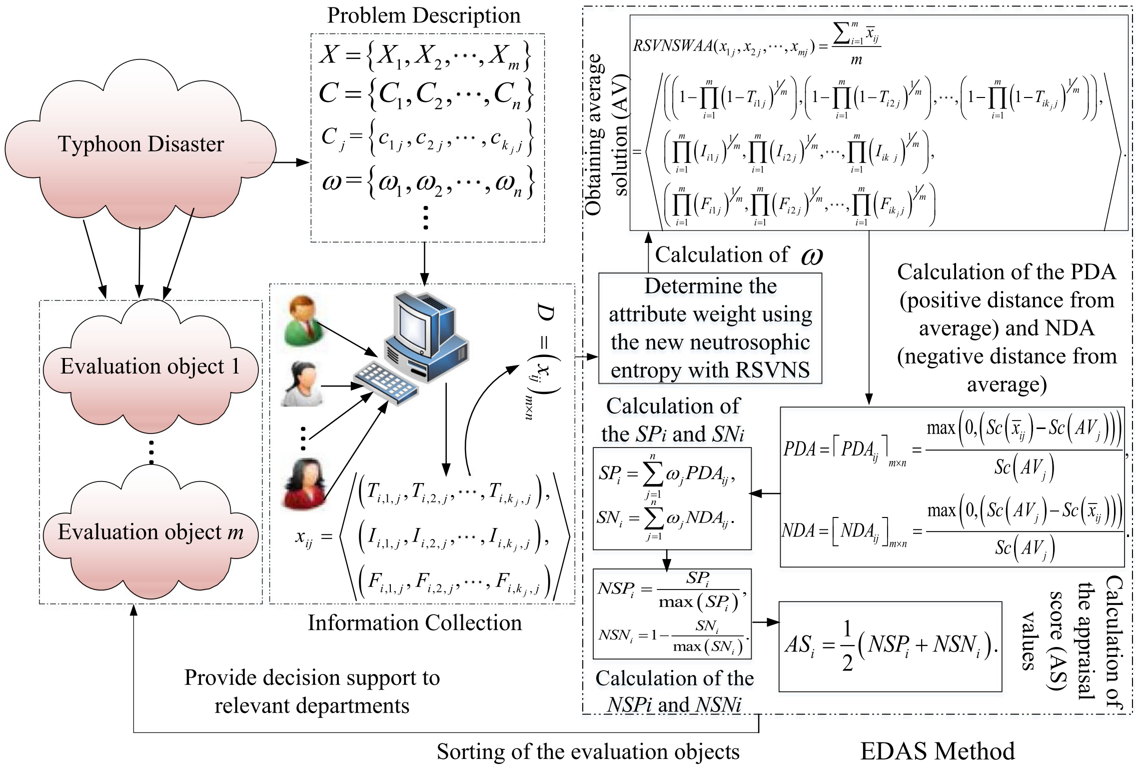


Fig. 1 Framework of the proposed MADM based on neutrosophic entropy and EDAS method with RSVNS

neutrosophic decision matrix $D = (x_{ij})_{m \times n}$, as shown in Table 1.

4.2 Determination of attribute weight-based entropy

For decision problems with unknown attribute weight, this study proposes a method based on neutrosophic entropy to determine the attribute weight. Information entropy represents the uncertainty in the attribute information. The greater the entropy value, the greater the uncertainty of the information. In this article, attribute indicators are divided into primary attributes and sub-attributes. Therefore, the calculation of attribute weight is a little complicated, which can be evaluated by the following formula.

$$\omega_j = \frac{1 - \frac{1}{m} \sum_{i=1}^{m} E_{RSVNN}(x_{ij})}{\sum_{j=1}^{n} \left(1 - \frac{1}{m} \sum_{i=1}^{m} E_{RSVNN}(x_{ij}) \right)}. \tag{18}$$

Of course, for the problem of unknown expert weight in group decision-making, information entropy can also be used to determine the expert weight.

Table 1 Refined single-valued neutrosophic decision matrix $D = (x_{ij})_{m \times n}$

Alternatives Attributes	C_1 $\{c_{11}, c_{21}, \cdots, c_{k_1 1}\}$	C_2 $\{c_{12}, c_{22}, \cdots, c_{k_2 2}\}$	$\cdots$ $\cdots$	C_n $\{c_{1n}, c_{2n}, \cdots, c_{k_n n}\}$
X_1	$\left\langle \begin{matrix} (T_{111}, T_{121}, \cdots, T_{1k_1 1}), \\ (I_{111}, I_{121}, \cdots, I_{1k_1 1}), \\ (F_{111}, F_{121}, \cdots, F_{1k_1 1}) \end{matrix} \right\rangle$	$\left\langle \begin{matrix} (T_{112}, T_{122}, \cdots, T_{1k_2 2}), \\ (I_{112}, I_{122}, \cdots, I_{1k_2 2}), \\ (F_{112}, F_{122}, \cdots, F_{1k_2 2}) \end{matrix} \right\rangle$	$\cdots$	$\left\langle \begin{matrix} (T_{11n}, T_{12n}, \cdots, T_{1k_n n}), \\ (I_{11n}, I_{12n}, \cdots, I_{1k_n n}), \\ (F_{11n}, F_{12n}, \cdots, F_{1k_n n}) \end{matrix} \right\rangle$
X_2	$\left\langle \begin{matrix} (T_{211}, T_{221}, \cdots, T_{2k_1 1}), \\ (I_{211}, I_{221}, \cdots, I_{2k_1 1}), \\ (F_{211}, F_{221}, \cdots, F_{2k_1 1}) \end{matrix} \right\rangle$	$\left\langle \begin{matrix} (T_{212}, T_{222}, \cdots, T_{2k_2 2}), \\ (I_{212}, I_{222}, \cdots, I_{2k_2 2}), \\ (F_{212}, F_{222}, \cdots, F_{2k_2 2}) \end{matrix} \right\rangle$	$\cdots$	$\left\langle \begin{matrix} (T_{21n}, T_{22n}, \cdots, T_{2k_n n}), \\ (I_{21n}, I_{22n}, \cdots, I_{2k_n n}), \\ (F_{21n}, F_{22n}, \cdots, F_{2k_n n}) \end{matrix} \right\rangle$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
X_m	$\left\langle \begin{matrix} (T_{m11}, T_{m21}, \cdots, T_{mk_1 1}), \\ (I_{m11}, I_{m21}, \cdots, I_{mk_1 1}), \\ (F_{m11}, F_{m21}, \cdots, F_{mk_1 1}) \end{matrix} \right\rangle$	$\left\langle \begin{matrix} (T_{m12}, T_{m22}, \cdots, T_{mk_2 2}), \\ (I_{m12}, I_{m22}, \cdots, I_{mk_2 2}), \\ (F_{m12}, F_{m22}, \cdots, F_{mk_2 2}) \end{matrix} \right\rangle$	$\cdots$	$\left\langle \begin{matrix} (T_{m1n}, T_{m2n}, \cdots, T_{mk_n n}), \\ (I_{m1n}, I_{m2n}, \cdots, I_{mk_n n}), \\ (F_{m1n}, F_{m2n}, \cdots, F_{mk_n n}) \end{matrix} \right\rangle$

$$e^f = \frac{\sum_{j=1}^{n}\omega_j\left(1-\frac{1}{m}\sum_{i=1}^{m}E_{RSVNN}\left(x_{ij}^f\right)\right)}{\sum_{f=1}^{g}\sum_{j=1}^{n}\omega_j\left(1-\frac{1}{m}\sum_{i=1}^{m}E_{RSVNN}\left(x_{ij}^f\right)\right)}. \quad (19)$$

where e^f represents the weight of the fth expert in the decision-making group composed of g experts and x_{ij}^f represents the decision value in the decision matrix $D^f = \left(x_{ij}^f\right)_{m\times n}$ of the f-th expert.

4.3 Steps in the proposed method

The current decision-making environment is increasingly complicated, where ω_j is unknown, and the evaluation information is expressed as RSVNSs. Thus, the steps in the proposed method are as follows.

Step 1: Give the decision matrix $D=(x_{ij})_{m\times n}$ provided by DMs in the form of RSVNS.

Step 2: Obtain the normalized decision matrix $\overline{D} = \left(\overline{x}_{ij}\right)_{m\times n}$. We need to standardize the decision information to ensure consistency in information. In general, attributes can be categorized into two types: benefit attributes and cost attributes. In this paper, all attribute values are converted to benefit-type attribute values, and if they are cost-type attribute values, they are changed using the following formula:

$$x_{ij}^c = \left\langle \left(F_{i1j}, F_{i2j}, \cdots, F_{ik_jj}\right), \left(1-I_{i1j}, 1-I_{i2j}, \cdots, 1-I_{ik_jj}\right), \left(T_{i1j}, T_{i2j}, \cdots, T_{ik_jj}\right) \right\rangle.$$

Step 3: Determine the attribute weights using the neutrosophic entropy based on Eq. (14) and Eq. (18).

Step 4: Aggregate the decision matrix to obtain the average solution (AV) for all attributes based on the RSVNS aggregation operator for aggregation operation. Here, because the essence of RSVNS is a set of single-valued neutrosophic numbers, we can define the RSVNS weighted arithmetic averaging (*RSVNSWAA*) operator such that the relevant calculation formula is as follows.

$$\begin{aligned} AV &= [AV_j]_{1\times n} = \left[\frac{\sum_{i=1}^{m}\overline{x}_{ij}}{m}\right]_{1\times n}, \\ RSVNSWAA&\left(x_{1j}, x_{2j}, \cdots, x_{mj}\right) = \frac{\sum_{i=1}^{m}\overline{x}_{ij}}{m} \\ &= \left\langle \begin{array}{c} \left(\left(1-\prod_{i=1}^{m}\left(1-T_{i1j}\right)^{1/m}\right), \left(1-\prod_{i=1}^{m}\left(1-T_{i2j}\right)^{1/m}\right), \cdots, \left(1-\prod_{i=1}^{m}\left(1-T_{ik_jj}\right)^{1/m}\right)\right), \\ \left(\prod_{i=1}^{m}\left(I_{i1j}\right)^{1/m}, \prod_{i=1}^{m}\left(I_{i2j}\right)^{1/m}, \cdots, \prod_{i=1}^{m}\left(I_{ik_jj}\right)^{1/m}\right), \\ \left(\prod_{i=1}^{m}\left(F_{i1j}\right)^{1/m}, \prod_{i=1}^{m}\left(F_{i2j}\right)^{1/m}, \cdots, \prod_{i=1}^{m}\left(F_{ik_jj}\right)^{1/m}\right) \end{array} \right\rangle. \end{aligned} \quad (20)$$

Step 5: Calculate the PDA and NDA based on AV, where

$$PDA = \left[PDA_{ij}\right]_{m\times n} = \frac{\max\left(0, \left(Sc\left(\overline{x}_{ij}\right) - Sc\left(AV_j\right)\right)\right)}{Sc\left(AV_j\right)}, \quad (21)$$

$$NDA = \left[NDA_{ij}\right]_{m\times n} = \frac{\max\left(0, \left(Sc\left(AV_j\right) - Sc\left(\overline{x}_{ij}\right)\right)\right)}{Sc\left(AV_j\right)}. \quad (22)$$

Here, for convenience, we can use the score function of RSVNS in Definition 6 to compare the two RSVNNs. However, if the PDA and NDA of the two schemes are the same, it is necessary to further recalculate the PDA and NDA based on the accuracy function to ensure that the alternatives can be fully sorted.

Step 6: Calculate the weighted PDA_{ij} and weighted NDA_{ij} to obtain SP_i and SN_i, where

$$SP_i = \sum_{j=1}^{n}\omega_j PDA_{ij}, \quad (23)$$

$$SN_i = \sum_{j=1}^{n}\omega_j NDA_{ij}. \quad (24)$$

Step 7: Normalize SP_i and SN_i to obtain NSP_i and NSN_i, where

$$NSP_i = \frac{SP_i}{\max(SP_i)}, \quad (25)$$

$$NSN_i = 1 - \frac{SN_i}{\max(SN_i)}. \quad (26)$$

Step 8: Calculate the appraisal score (AS) values of every alternative based on NSP_i and NSN_i, where

$$AS_i = \frac{1}{2}\left(NSP_i + NSN_i\right). \quad (27)$$

Step 9: Rank the alternatives according to the AS. Thus, we can choose the best alternative (alternatives) or rank alternatives. In general, the bigger the value of U_i is, the better the selected alternative will be.

5 Case study and comparative analysis

5.1 Case study

Fujian Province is located in the southeastern coastal area of China and is one of the most economically developed areas in China. However, the region has been devastated by typhoons on an annual basis, affecting the economic development of the region and causing huge losses in terms of people's lives and property. Effectively estimating the disaster losses caused by typhoons is a matter of great concern to the decision-making departments and the people. Thus, we evaluate the post-stroke situation and apply the theory of neutrosophic set to the disaster losses due to typhoons to provide auxiliary decision-making for the disasters in relevant departments and provide assistance for keeping the people informed and for social stability. Taking the strong typhoon "Maria" that occurred in Fujian Province in July 2018 as an example, after the disaster, we quickly obtained data from multiple sources for disaster assessment. Inspired by the literature [68, 77], we constructed an evaluation indicator system. The assessment targets $X=\{X_1, X_2, \cdots, X_9\}$ were nine counties and cities in Fujian Province; specifically, Nanping (X_1), Ningde (X_2), Sanming (X_3), Fuzhou (X_4), Putian (X_5), Longyan (X_6), Quanzhou (X_7), Xiamen (X_8), and Zhangzhou (X_9). The primary indicators $C=\{C_1, C_2, C_3, C_4\}$ in the assessment included People's safety (C_1), Economic loss (C_2), Environmental damage (C_3), and Social impact (C_4). The sub-indicators were $C_1=\{c_{11}, c_{2_11}\}$ with Population death (c_{11}) and Population affected (c_{2_11}), $C_2=\{c_{12}, c_{2_22}\}$ with Housing damage (c_{12}) and Economic damage (c_{2_22}), $C_3=\{c_{13}, c_{2_33}\}$ with Environmental impact (c_{13}) and Agricultural damage (c_{2_33}), and $C_4=\{c_{14}, c_{2_44}\}$ with Social stability (c_{14}) and Other impact (c_{2_44}). The nine assessment objects were evaluated using the refined single-valued neutrosophic sets by DMs or experts under two-level indicators. Thus, the assessment matrix $D=(x_{ij})_{m\times n}$ was given in the form of RSVNSs. In addition, in this section, we study the disaster assessment problem in two cases from practical problems, where the evaluation indicators are divided into first-level indicators and sub-indicators. One case is multi-level indicator evaluation; that is, the evaluation indicators are divided into first and second-level indicators. Another case is dynamic evaluation, that is, the value of the evaluation matrix is not static; it has multiple evaluation values under different states in different periods.

5.1.1 Case 1: Typhoon disaster assessment with multi-layer indicators

It is suitable to use RSVNS to evaluate the multi-layer indicators of typhoon disasters. It can not only express the uncertainty of the evaluation, but also express the evaluation data simply and concisely, and it is convenient to calculate. According to Section 4, TDA using the proposed MADM model contains the following steps.

Step 1: Obtain evaluation data. First, we invited several experts to give the assessment data expressed by RSVNSs in Table 2.

Step 2: Obtain the normalized evaluation matrix $\overline{D}$. Because all the attributes in this article are of the same type, they do not need to be standardized.

Step 3: Determine the attribute weights using the neutrosophic entropy based on Eq. (14) and Eq. (18). First, the entropy values of evaluation information based on Eq. (14) are presented in Table 3.

Then, the attribute weights are determined according to Eq. (18). $\omega_1=0.2283$, $\omega_2=0.2554$, $\omega_3=0.2659$, $\omega_4=0.2503$.

Step 4: Obtain average solution (AV) for all attributes using the *RSVNSWAA* operator defined in Eq. (20).

$$AV_1=\langle(0.6768, 0.6505), (0.3209, 0.3427), (0.3541, 0.3898)\rangle,$$

$$AV_2=\langle(0.5923, 0.6202), (0.3998, 0.3725), (0.4404, 0.3430)\rangle,$$

$$AV_3=\langle(0.5210, 0.5357), (0.4666, 0.4531), (0.5342, 0.5102)\rangle,$$

$$AV_4=\langle(0.3522, 0.7087), (0.6319, 0.2601), (0.6037, 0.2319)\rangle.$$

Step 5: Calculate the PDA_{ij} and NDA_{ij} based on Eq. (21) and Eq. (22), respectively. The calculation results are presented in Table 4.

Step 6: Obtain SP_i and SN_i based on the weighted PDA_{ij} and NDA_{ij} in Eq. (23) and Eq. (24), respectively.

$SP_1=0.0000$, $SP_2=0.2987$, $SP_3=0.0000$, $SP_4=0.1207$, $SP_5=0.2509$, $SP_6=0.0000$, $SP_7=0.0388$, $SP_8=0.0000$, $SP_9=0.2071$.

$SN_1=0.7615$, $SN_2=0.000$, $SN_3=0.8054$, $SN_4=0.0000$, $SN_5=0.0000$, $SN_6=0.4809$, $SN_7=0.2738$, $SN_8=0.2907$, $SN_9=0.0075$.

Step 7: Obtain NSP_i and NSN_i by normalizing SP_i and SN_i based on Eq. (25) and Eq. (26), respectively .

$NSP_1=0.0000$, $NSP_2=1.000$, $NSP_3=0.0000$, $NSP_4=0.4040$, $NSP_5=0.8400$, $NSP_6=0.0000$, $NSP_7=0.1299$, $NSP_8=0.0000$, $NSP_9=0.6935$.

$NSN_1=0.0546$, $NSN_2=1.000$, $NSN_3=0.0000$, $NSN_4=1.0000$, $NSN_5=1.0000$, $NSN_6=0.4030$, $NSN_7=0.6600$, $NSN_8=0.6391$, $NSN_9=0.9907$.

Step 8: Calculate the AS_i values using NSP_i and NSN_i based on Eq. (27).

$AS_1=0.0273$, $AS_2=1.0000$, $AS_3=0.0000$, $AS_4=0.7020$, $AS_5=0.9200$, $AS_6=0.2015$, $AS_7=0.3950$, $AS_8=0.3195$, $AS_9=0.8421$.

Step 9: Rank the alternatives based on AS_i. From the calculation results in step 8 and Fig. 2, it is evident that $U_{ND}\succ U_{PT}\succ U_{ZZ}\succ U_{FZ}\succ U_{QZ}\succ U_{XM}\succ U_{LY}\succ U_{NP}\succ U_{SM}$. Thus, the ranking results of the impact of typhoons in nine cities are $ND\succ PT\succ ZZ\succ FZ\succ QZ\succ XM\succ LY\succ NP\succ SM$. According to the assessment results, the relevant management departments can effectively respond to disasters while rationally distributing relief materials and resources, thereby reducing the bad impact of typhoon disasters on people's lives.

Table 2 Multi-layer indicators assessment matrix $D=(x_{ij})_{m \times n}$

Index Cities	C_1 $\{c_{11}, c_{2_1 1}\}$	C_2 $\{c_{12}, c_{2_2 2}\}$	C_3 $\{c_{13}, c_{2_3 3}\}$	C_4 $\{c_{14}, c_{2_4 4}\}$
Nanping (NP)	$\langle (0.35, 0.10), (0.65, 0.85), (0.60, 0.90) \rangle$	$\langle (0.05, 0.05), (0.90, 0.90), (0.95, 0.95) \rangle$	$\langle (0.10, 0.05), (0.85, 0.90), (0.90, 0.95) \rangle$	$\langle (0.05, 0.20), (0.90, 0.60), (0.95, 0.75) \rangle$
Ningde (ND)	$\langle (0.95, 0.65), (0.05, 0.35), (0.05, 0.30) \rangle$	$\langle (0.50, 0.95), (0.50, 0.05), (0.45, 0.05) \rangle$	$\langle (0.90, 0.80), (0.10, 0.20), (0.50, 0.15) \rangle$	$\langle (0.80, 0.50), (0.20, 0.50), (0.15, 0.45) \rangle$
Sanming (SM)	$\langle (0.20, 0.05), (0.75, 0.90), (0.80, 0.95) \rangle$	$\langle (0.10, 0.05), (0.85, 0.90), (0.90, 0.95) \rangle$	$\langle (0.10, 0.05), (0.85, 0.90), (0.90, 0.95) \rangle$	$\langle (0.05, 0.10), (0.90, 0.66), (0.95, 0.85) \rangle$
Fuzhou (FZ)	$\langle (0.90, 0.80), (0.10, 0.20), (0.50, 0.15) \rangle$	$\langle (0.65, 0.80), (0.35, 0.20), (0.30, 0.15) \rangle$	$\langle (0.50, 0.50), (0.50, 0.50), (0.45, 0.45) \rangle$	$\langle (0.35, 0.80), (0.65, 0.20), (0.60, 0.15) \rangle$
Putian (PT)	$\langle (0.65, 0.95), (0.35, 0.05), (0.30, 0.05) \rangle$	$\langle (0.90, 0.65), (0.10, 0.35), (0.50, 0.30) \rangle$	$\langle (0.65, 0.80), (0.35, 0.20), (0.30, 0.15) \rangle$	$\langle (0.50, 0.70), (0.50, 0.25), (0.45, 0.20) \rangle$
Longyan (LY)	$\langle (0.35, 0.35), (0.65, 0.65), (0.60, 0.60) \rangle$	$\langle (0.20, 0.50), (0.75, 0.50), (0.80, 0.45) \rangle$	$\langle (0.35, 0.10), (0.65, 0.85), (0.60, 0.90) \rangle$	$\langle (0.10, 0.30), (0.85, 0.55), (0.90, 0.60) \rangle$
Quanzhou (QZ)	$\langle (0.50, 0.50), (0.50, 0.50), (0.45, 0.45) \rangle$	$\langle (0.80, 0.35), (0.20, 0.65), (0.15, 0.60) \rangle$	$\langle (0.10, 0.05), (0.85, 0.90), (0.90, 0.95) \rangle$	$\langle (0.35, 0.90), (0.65, 0.10), (0.60, 0.05) \rangle$
Xiamen (XM)	$\langle (0.35, 0.20), (0.65, 0.75), (0.60, 0.80) \rangle$	$\langle (0.80, 0.20), (0.20, 0.75), (0.15, 0.80) \rangle$	$\langle (0.20, 0.35), (0.75, 0.65), (0.80, 0.60) \rangle$	$\langle (0.10, 0.95), (0.85, 0.05), (0.90, 0.05) \rangle$
Zhangzhou (ZZ)	$\langle (0.80, 0.90), (0.20, 0.10), (0.15, 0.50) \rangle$	$\langle (0.35, 0.80), (0.65, 0.20), (0.60, 0.15) \rangle$	$\langle (0.80, 0.90), (0.20, 0.10), (0.15, 0.50) \rangle$	$\langle (0.35, 0.80), (0.65, 0.20), (0.60, 0.15) \rangle$

5.1.2 Case 2: Dynamic assessment of typhoon disaster

The dynamic decision algorithm differs from the traditional static decision method. It takes multiple values at different time points. The decision result of the dynamic decision method is more scientific and effective, and it can better provide decision support for relevant departments. Assume the assessment targets $X=\{X_1, X_2, \cdots, X_9\}$ are nine counties and the indicator system is $C=\{C_1, C_2, C_3, C_4\}$ in the assessment, including People's life safety (C_1), Economic loss (C_2), Environmental damage (C_3), and Social impact (C_4). The evaluation values are sampled at three time points t_1, t_2, t_3 after the typhoon disaster occurs. Thus, it is appropriate that the nine assessment objects are to be evaluated using the refined single-valued neutrosophic sets by DMs or experts at multiple points in time. Hence, the assessment matrix $D=(x_{ij})_{m \times n}$ with $x_{ij}=\langle (T_{i1j}, T_{i2j}, \cdots, T_{ik_j j}), (I_{i1j}, I_{i2j}, \cdots, I_{ik_j j}), (F_{i1j}, F_{i2j}, \cdots, F_{ik_j j}) \rangle$ is given in the form of RSVNS. According to Section 4, the dynamic assessment of typhoon disasters using the proposed MADM model contains the following steps:

Table 3 Entropy values of evaluation information

Index Cities	C_1	C_2	C_3	C_4
Nanping (NP)	0.4833	0.1333	0.1833	0.3500
Ningde (ND)	0.3833	0.5333	0.4167	0.6667
Sanming (SM)	0.2833	0.1833	0.1833	0.2633
Fuzhou (FZ)	0.4167	0.5167	0.9667	0.5500
Putian (PT)	0.3833	0.5667	0.5167	0.7333
Longyan (LY)	0.7333	0.7000	0.4836	0.5000
Quanzhou (QZ)	0.9667	0.5500	0.1836	0.4500
Xiamen (XM)	0.5833	0.4000	0.5836	0.1667
Zhangzhou (ZZ)	0.4167	0.5500	0.4167	0.5500

Step 1: Obtain evaluation data. First, we invited several experts to give the assessment data sampled at three time points expressed by RSVNSs in Table 5.

Step 2: Because all attributes are of the same type, the assessment matrix $D=(x_{ij})_{m \times n}$ does not need to be standardized.

Step 3: Determine the attribute weights for the dynamic assessment matrix based on Eq. (14) and Eq. (18), as follows.

$$\omega_1=0.2333,\ \omega_2=0.2372,\ \omega_3=0.2863,\ \omega_4=0.2431.$$

Step 4: Obtain the AV for all attributes using the *RSVNSWAA* operator defined in Eq. (20).

$$AV_1=\langle (0.5893, 0.5906, 0.7055), (0.4081, 0.3983, 0.2830), (0.4435, 0.3454, 0.3298) \rangle,$$

$$AV_2=\langle (0.5225, 0.5081, 0.6202), (0.4681, 0.4757, 0.3725), (0.5342, 0.4296, 0.3430) \rangle,$$

$$AV_3=\langle (0.4967, 0.5456, 0.5425), (0.4864, 0.4423, 0.4465), (0.5695, 0.5040, 0.4267) \rangle,$$

$$AV_4=\langle (0.5562, 0.5529, 0.5618), (0.4269, 0.4106, 0.3972), (0.4054, 0.3753, 0.3654) \rangle.$$

Table 4 The values of PDA_{ij} and NDA_{ij}

Index	PDA_{ij}				NDA_{ij}			
Cities	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4
Nanping (NP)	0.0000	0.0000	0.0000	0.0000	0.6301	0.8906	0.8222	0.6850
Ningde (ND)	0.2373	0.2033	0.5359	0.2000	0.0000	0.0000	0.0000	0.0000
Sanming (SM)	0.0000	0.0000	0.0000	0.0000	0.7832	0.8496	0.8222	0.7630
Fuzhou (FZ)	0.2118	0.2169	0.0024	0.0650	0.0000	0.0000	0.0000	0.0000
Putian(PT)	0.2373	0.1759	0.4389	0.1400	0.0000	0.0000	0.0000	0.0000
Longyan (LY)	0.0000	0.0000	0.0000	0.0000	0.4388	0.3984	0.5311	0.5500
Quanzhou (QZ)	0.0000	0.0000	0.0000	0.1550	0.2092	0.0292	0.8222	0.0000
Xiamen (XM)	0.0000	0.0000	0.0000	0.0000	0.5536	0.1523	0.4341	0.0400
Zhangzhou (ZZ)	0.2118	0.0000	0.5359	0.0650	0.0000	0.0292	0.0000	0.0000

Step 5: Calculate PDA_{ij} and NDA_{ij} based on Eq. (21) and Eq. (22), respectively; the calculation results are presented in Table 6.

Step 6: Obtain SP_i and SN_i based on the weighted PDA_{ij} and weighted NDA_{ij} in Eq. (23) and Eq. (24), respectively.

$SP_1 = 0.0000$, $SP_2 = 0.2881$, $SP_3 = 0.0000$, $SP_4 = 0.1570$, $SP_5 = 0.2416$, $SP_6 = 0.0000$, $SP_7 = 0.0257$, $SP_8 = 0.0418$, $SP_9 = 0.2167$.

$SN_1 = 0.7040$, $SN_2 = 0.000$, $SN_3 = 0.7149$, $SN_4 = 0.0087$, $SN_5 = 0.0000$, $SN_6 = 0.4545$, $SN_7 = 0.3157$, $SN_8 = 0.3447$, $SN_9 = 0.0000$.

Step 7: Obtain NSP_i and NSN_i by normalizing SP_i and SN_i based on Eq. (25) and Eq. (26), respectively.

$NSP_1 = 0.0000$, $NSP_2 = 1.000$, $NSP_3 = 0.0000$, $NSP_4 = 0.5449$, $NSP_5 = 0.8336$, $NSP_6 = 0.0000$, $NSP_7 = 0.0893$, $NSP_8 = 0.1452$, $NSP_9 = 0.7521$.

$NSN_1 = 0.0152$, $NSN_2 = 1.000$, $NSN_3 = 0.0000$, $NSN_4 = 0.9878$, $NSN_5 = 1.0000$, $NSN_6 = 0.3643$, $NSN_7 = 0.5582$, $NSN_8 = 0.5178$, $NSN_9 = 1.000$.

Step 8: Calculate the AS_i values using NSP_i and NSN_i based on Eq. (27).

$AS_1 = 0.0076$, $AS_2 = 1.0000$, $AS_3 = 0.0000$, $AS_4 = 0.7664$, $AS_5 = 0.9193$, $AS_6 = 0.1821$, $AS_7 = 0.3238$, $AS_8 = 0.3315$, $AS_9 = 0.8760$.

Step 9: Rank the alternatives based on AS_i. From the calculation results in the previous step and Fig. 3, it is evident that $U_{ND} \succ U_{PT} \succ U_{ZZ} \succ U_{FZ} \succ U_{XM} \succ U_{QZ} \succ U_{LY} \succ U_{NP} \succ U_{SM}$. Thus, the ranking results of the impact of typhoons in the nine cities are $ND \succ PT \succ ZZ \succ FZ \succ XM \succ QZ \succ LY \succ NP \succ SM$. The results of dynamic assessment are basically consistent with the results of multi-level indicator evaluation. Only a slight difference was observed because dynamic assessment takes data at different time points and the data is large. Simultaneously, the severity of the typhoon disaster is different at different time points. In particular, in the short time immediately after the typhoon, the data monitored by the public sentiment are uncertain and not sufficiently accurate. Therefore, the average evaluation value after multiple sampling can eliminate the adverse effects.

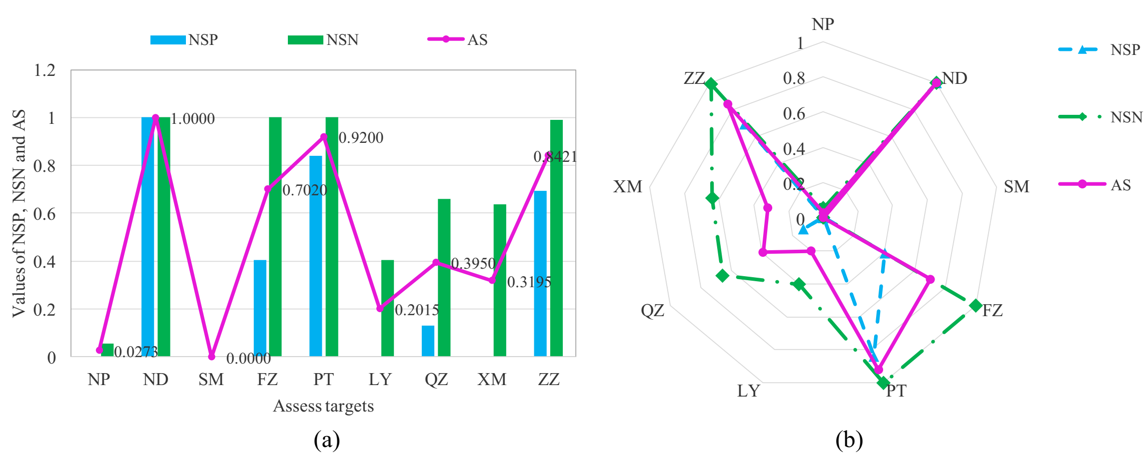


Fig. 2 Ranking results of the evaluation cities

Table 5 Dynamic assessment matrix $D=(x_{ij})_{m\times n}$

Index Cities	C_1 $\{t_1,t_2,t_3\}$	C_2 $\{t_1,t_2,t_3\}$	C_3 $\{t_1,t_2,t_3\}$	C_4 $\{t_1,t_2,t_3\}$
Nanping (NP)	$\langle(0.50,0.35,0.10),(0.50,0.65,0.85),(0.45,0.60,0.90)\rangle$	$\langle(0.10,0.05,0.05),(0.85,0.90,0.90),(0.90,0.95,0.95)\rangle$	$\langle(0.20,0.10,0.05),(0.75,0.85,0.90),(0.80,0.90,0.95)\rangle$	$\langle(0.05,0.10,0.20),(0.90,0.85,0.60),(0.95,0.90,0.75)\rangle$
Ningde (ND)	$\langle(0.80,0.65,0.95),(0.20,0.35,0.05),(0.15,0.30,0.05)\rangle$	$\langle(0.50,0.65,0.95),(0.50,0.35,0.05),(0.45,0.25,0.00)\rangle$	$\langle(0.90,0.80,0.65),(0.10,0.20,0.35),(0.50,0.15,0.30)\rangle$	$\langle(0.50,0.80,0.65),(0.50,0.20,0.35),(0.45,0.15,0.30)\rangle$
Sanming (SM)	$\langle(0.35,0.20,0.05),(0.65,0.75,0.90),(0.60,0.80,0.95)\rangle$	$\langle(0.10,0.20,0.05),(0.85,0.75,0.90),(0.90,0.80,0.95)\rangle$	$\langle(0.20,0.10,0.05),(0.75,0.85,0.90),(0.80,0.90,0.95)\rangle$	$\langle(0.05,0.20,0.10),(0.90,0.60,0.66),(0.95,0.75,0.85)\rangle$
Fuzhou (FZ)	$\langle(0.90,0.80,0.70),(0.65,0.35,0.30),(0.50,0.15,0.20)\rangle$	$\langle(0.65,0.70,0.80),(0.35,0.25,0.20),(0.30,0.20,0.15)\rangle$	$\langle(0.50,0.65,0.50),(0.50,0.35,0.50),(0.45,0.30,0.45)\rangle$	$\langle(0.80,0.50,0.35),(0.20,0.50,0.65),(0.15,0.45,0.60)\rangle$
Putian(PT)	$\langle(0.65,0.70,0.95),(0.35,0.25,0.05),(0.30,0.20,0.05)\rangle$	$\langle(0.90,0.80,0.65),(0.10,0.20,0.35),(0.50,0.15,0.30)\rangle$	$\langle(0.50,0.65,0.80),(0.50,0.35,0.20),(0.45,0.30,0.15)\rangle$	$\langle(0.50,0.70,0.65),(0.50,0.25,0.35),(0.45,0.20,0.30)\rangle$
Longyan (LY)	$\langle(0.50,0.35,0.35),(0.50,0.65,0.65),(0.45,0.60,0.60)\rangle$	$\langle(0.20,0.35,0.50),(0.75,0.65,0.50),(0.80,0.60,0.45)\rangle$	$\langle(0.20,0.35,0.10),(0.75,0.65,0.85),(0.80,0.60,0.90)\rangle$	$\langle(0.30,0.10,0.20),(0.55,0.85,0.60),(0.60,0.90,0.75)\rangle$
Quanzhou (QZ)	$\langle(0.35,0.65,0.50),(0.65,0.35,0.50),(0.60,0.30,0.45)\rangle$	$\langle(0.50,0.35,0.35),(0.50,0.65,0.65),(0.45,0.60,0.60)\rangle$	$\langle(0.10,0.20,0.05),(0.85,0.75,0.90),(0.90,0.80,0.95)\rangle$	$\langle(0.35,0.65,0.90),(0.65,0.35,0.10),(0.60,0.30,0.05)\rangle$
Xiamen (XM)	$\langle(0.10,0.35,0.20),(0.85,0.65,0.75),(0.90,0.60,0.80)\rangle$	$\langle(0.65,0.50,0.20),(0.35,0.50,0.75),(0.30,0.45,0.80)\rangle$	$\langle(0.10,0.20,0.35),(0.85,0.75,0.65),(0.90,0.80,0.60)\rangle$	$\langle(0.95,0.70,0.35),(0.05,0.25,0.65),(0.05,0.20,0.60)\rangle$
Zhangzhou (ZZ)	$\langle(0.50,0.80,0.90),(0.50,0.20,0.10),(0.45,0.15,0.50)\rangle$	$\langle(0.35,0.50,0.80),(0.65,0.50,0.20),(0.60,0.45,0.15)\rangle$	$\langle(0.80,0.90,0.95),(0.20,0.10,0.05),(0.15,0.50,0.05)\rangle$	$\langle(0.35,0.65,0.80),(0.65,0.35,0.20),(0.60,0.30,0.15)\rangle$

5.2 Validity test for the proposed method

Inspired by the literature [78, 79], this study ascertained the validity of the proposed method. The preference relationship between the alternatives meets the following three criteria:

(1) **Non-substitutability:** An effective MADM method should not change the indication of the best alternative on replacing a non-optimal alternative by another worse alternative without changing the relative importance of each decision criterion [79].

(2) **Transitivity:** An effective method should follow the transitive property. That is, if $X_i \succ X_j$ and $X_j \succ X_k$, then $X_i \succ X_k$, $(i,j,k=1,2,\cdots,m)$.

(3) **Consistency:** If all alternatives are considered as a set and each subset of the set is ranked by the same

Table 6 The values of PDA_{ij} and NDA_{ij} for dynamic assessment

Index	PDA_{ij}				NDA_{ij}			
Cities	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4
Nanping (NP)	0.000	0.000	0.000	0.000	0.472	0.851	0.745	0.735
Ningde (ND)	0.286	0.273	0.433	0.134	0.000	0.000	0.000	0.000
Sanming (SM)	0.000	0.000	0.000	0.000	0.657	0.761	0.745	0.690
Fuzhou (FZ)	0.233	0.333	0.083	0.000	0.000	0.000	0.000	0.036
Putian(PT)	0.251	0.343	0.274	0.096	0.000	0.000	0.000	0.000
Longyan (LY)	0.000	0.000	0.000	0.000	0.339	0.344	0.554	0.556
Quanzhou (QZ)	0.000	0.000	0.000	0.106	0.181	0.254	0.745	0.000
Xiamen (XM)	0.000	0.000	0.000	0.172	0.630	0.165	0.554	0.000
Zhangzhou (ZZ)	0.110	0.014	0.614	0.049	0.000	0.000	0.000	0.000

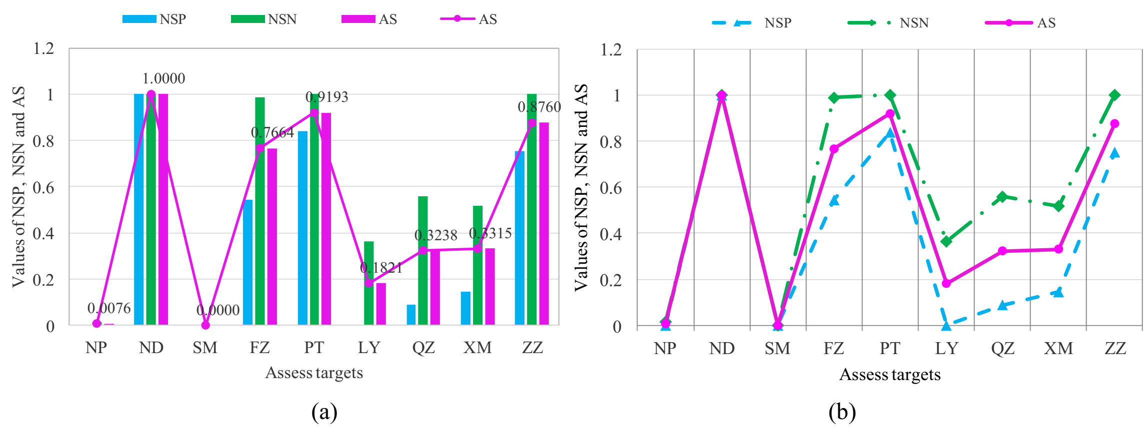


Fig. 3 Ranking results of the evaluation cities

evaluation method, the preference relations of the alternatives of the subset and those of the alternatives in the set are consistent. That is, there is a set of alternatives $X=\{X_i, X_j, X_k, X_l, X_m\}$, and the sorting result based on our method is $X_i \succ X_j \succ X_k \succ X_l \succ X_m$. If $X^{'}=\{X_j, X_l, X_m\}$ is a subset of X, the sorting result is $X_j \succ X_l \succ X_m$.

5.2.1 Validity test of the assessment methods using criterion 1

(1) For the validity test of the multi-layer indicator assessment method, first, we exchange the truth-membership degree (T) with the falsity-membership degree (F) for any non-optimal and worse alternative, and then apply the proposed method to evaluate them. If we choose X_1 (NP) as the non-optimal alternative and X_3 (SM) as the worse alternative, then the transformed evaluation matrix is $D^c=(x_{ijc})_{m\times n}$:

$$D_1^c=\begin{array}{c} NP \\ ND \\ SM \\ FZ \\ PT \\ LY \\ QZ \\ XM \\ ZZ \end{array}\begin{bmatrix} \left\langle\begin{array}{c}(0.60,0.90),\\(0.65,0.85),\\(0.35,0.10)\end{array}\right\rangle, & \left\langle\begin{array}{c}(0.95,0.95),\\(0.90,0.90),\\(0.05,0.05)\end{array}\right\rangle, & \left\langle\begin{array}{c}(0.90,0.95),\\(0.85,0.90),\\(0.10,0.05)\end{array}\right\rangle, & \left\langle\begin{array}{c}(0.95,0.75),\\(0.90,0.60),\\(0.05,0.20)\end{array}\right\rangle \\ \cdots & \cdots & \cdots & \cdots \\ \left\langle\begin{array}{c}(0.80,0.95),\\(0.75,0.90),\\(0.20,0.05)\end{array}\right\rangle, & \left\langle\begin{array}{c}(0.90,0.95),\\(0.85,0.90),\\(0.10,0.05)\end{array}\right\rangle, & \left\langle\begin{array}{c}(0.90,0.95),\\(0.85,0.90),\\(0.10,0.05)\end{array}\right\rangle, & \left\langle\begin{array}{c}(0.95,0.85),\\(0.90,0.66),\\(0.05,0.10)\end{array}\right\rangle \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

Based on the transformed matrix, the new AS_i for each alternative can be obtained as $AS_1=0.4354$, $AS_2=0.9952$, $AS_3=0.4542$, $AS_4=0.5718$, $AS_5=0.8451$, $AS_6=0.0000$, $AS_7=0.1844$, $AS_8=0.1351$, $AS_9=0.8861$. It is evident that $U_{ND}\succ U_{ZZ}\succ U_{PT}\succ U_{FZ}\succ U_{SM}\succ U_{NP}\succ U_{QZ}\succ U_{XM}\succ U_{LY}$. and the ranking results of the impact of typhoons in nine cities are $ND\succ ZZ\succ PT\succ FZ\succ SM\succ NP\succ QZ\succ XM\succ LY$. Because the best alternative is again ND, which is the same as that of the evaluation problem before exchanging data, it is confirmed that the proposed method does not change the indication of the best alternative when a non-optimal alternative is replaced by another worst alternative. Hence the proposed multi-layer indicator assessment method is valid for the non-substitutability of criterion 1 [78, 79].

(2) For the validity test of the dynamic assessment method, we use X_1 (NP) and X_3 (SM) as the transformation objects. The transformed X_1^c (NP) and X_3^c (SM) are as follows.

$$X_1^c=\left\{\left\langle\begin{array}{c}(0.45,0.60,0.90),\\(0.50,0.65,0.85),\\(0.50,0.35,0.10)\end{array}\right\rangle,\left\langle\begin{array}{c}(0.90,0.95,0.95),\\(0.85,0.90,0.90),\\(0.10,0.05,0.05)\end{array}\right\rangle,\left\langle\begin{array}{c}(0.80,0.90,0.95),\\(0.75,0.85,0.90),\\(0.20,0.10,0.05)\end{array}\right\rangle,\left\langle\begin{array}{c}(0.95,0.90,0.75),\\(0.90,0.85,0.60),\\(0.05,0.10,0.20)\end{array}\right\rangle\right\},$$

$$X_3^c=\left\{\left\langle\begin{array}{c}(0.60,0.80,0.95),\\(0.65,0.75,0.90),\\(0.35,0.20,0.05)\end{array}\right\rangle,\left\langle\begin{array}{c}(0.90,0.80,0.95),\\(0.85,0.75,0.90),\\(0.10,0.20,0.05)\end{array}\right\rangle,\left\langle\begin{array}{c}(0.80,0.90,0.95),\\(0.75,0.85,0.90),\\(0.20,0.10,0.05)\end{array}\right\rangle,\left\langle\begin{array}{c}(0.95,0.75,0.85),\\(0.90,0.60,0.66),\\(0.05,0.20,0.10)\end{array}\right\rangle\right\}.$$

Based on the transformed matrix, the new AS_i for each alternative can be obtained as $AS_1=0.4453$, $AS_2=0.9927$, $AS_3=0.4611$, $AS_4=0.6979$, $AS_5=0.8342$, $AS_6=0.0000$, $AS_7=0.1355$, $AS_8=0.1158$, $AS_9=0.9053$. Thus, $U_{ND}\succ U_{ZZ}\succ U_{PT}\succ U_{FZ}\succ U_{SM}\succ U_{NP}\succ U_{QZ}\succ U_{XM}\succ U_{LY}$, and the ranking results of the impact of typhoons in the nine cities are $ND\succ ZZ\succ PT\succ FZ\succ SM\succ NP\succ QZ\succ XM\succ LY$. Hence, the proposed dynamic assessment method is valid for the non-substitutability of criterion 1.

5.2.2 Validity test for the assessment methods using criterion 2 and criterion 3

(1) We first decompose this large evaluation problem into a small problem set, and then solve the small problem separately using the proposed method. The results are listed in Table 7:

If we merge the sorting results of small problems, we can get the overall sorting results of case 1 and case 2 as $ND\succ PT\succ ZZ\succ FZ\succ QZ\succ XM\succ LY\succ NP\succ SM$ and $ND\succ PT\succ ZZ\succ FZ\succ XM\succ QZ\succ LY\succ NP\succ SM$, respectively. These results are consistent with the ranking results of the undecomposed evaluation

Table 7 Small problems of decomposition and their sorting results

Small problems of decomposition	Sorting results for small problems	
	Case 1	Case 2
$\{X_2, X_3, \cdots, X_9\}$	ND≻PT≻ZZ≻FZ≻QZ≻XM≻LY≻SM	ND≻PT≻ZZ≻FZ≻XM≻QZ≻LY≻SM
$\{X_1, X_3, \cdots, X_9\}$	PT≻ZZ≻FZ≻QZ≻XM≻LY≻NP≻SM	PT≻ZZ≻FZ≻XM≻QZ≻LY≻NP≻SM
$\{X_1, X_2, X_4, \cdots, X_9\}$	ND≻PT≻ZZ≻FZ≻QZ≻XM≻LY≻NP	ND≻PT≻ZZ≻FZ≻XM≻QZ≻LY≻NP
$\{X_1, \cdots, X_3, X_5, \cdots, X_9\}$	ND≻PT≻ZZ≻QZ≻XM≻LY≻NP≻SM	ND≻PT≻ZZ≻XM≻QZ≻LY≻NP≻SM
$\{X_1, \cdots, X_4, X_6, \cdots, X_9\}$	ND≻ZZ≻FZ≻QZ≻XM≻LY≻NP≻SM	ND≻ZZ≻FZ≻XM≻QZ≻LY≻NP≻SM
$\{X_1, \cdots, X_5, X_7, \cdots, X_9\}$	ND≻PT≻ZZ≻FZ≻QZ≻XM≻NP≻SM	ND≻PT≻ZZ≻FZ≻XM≻QZ≻NP≻SM
$\{X_1, \cdots, X_6, X_8, X_9\}$	ND≻PT≻ZZ≻FZ≻XM≻LY≻NP≻SM	ND≻PT≻ZZ≻FZ≻XM≻LY≻NP≻SM
$\{X_1, \cdots, X_7, X_9\}$	ND≻PT≻ZZ≻FZ≻QZ≻LY≻NP≻SM	ND≻PT≻ZZ≻FZ≻QZ≻LY≻NP≻SM
$\{X_1, \cdots, X_7, X_8\}$	ND≻PT≻FZ≻QZ≻XM≻LY≻NP≻SM	ND≻PT≻FZ≻XM≻QZ≻LY≻NP≻SM

questions. Thus, the proposed method is valid for the transitivity of criterion 2 and consistency of criterion 3.

5.3 Comparative analysis

5.3.1 Sensitivity analysis of parameter in distance measure

To illustrate the robustness of the proposed evaluation method, sensitivity analysis was conducted in this study on the parameters λ in the proposed new distance formula. The results are shown in Table 8 and Fig. 4.

It is evident from Fig. 4 that when the parameter λ in distance measurement takes different values, the ranking results of the evaluated objects are exactly the same, which shows that the parameters have little or no influence on the evaluation results, thus indicating that the proposed method has certain compatibility and robustness.

5.3.2 Comparative analysis of evaluation methods based on different distance measures

In this section, inspired by the literature [80], we compare our decision-making methods with several existing approaches. First, this study modified the distance measures proposed in the literature [31, 37, 38, 81–84] to adapt to the refined single-valued neutrosophic sets environment, and then compared them with the distance measures proposed in this study. The corresponding comparison results of the two cases (case 1 is an evaluation

Table 8 Ranking result R and attribute weight ω for different λ

Result λ	Attribute weight ω	Ranking result R
Case 1: Multi-layer indicator assessment		
$\lambda = 1$	$\omega_1 = 0.2283$, $\omega_2 = 0.2554$, $\omega_3 = 0.2659$, $\omega_4 = 0.2503$	ND ≻ PT ≻ ZZ ≻ FZ ≻ QZ ≻ XM ≻ LY ≻ NP ≻ SM
$\lambda = 2$	$\omega_1 = 0.2197$, $\omega_2 = 0.2550$, $\omega_3 = 0.2757$, $\omega_4 = 0.2496$	ND ≻ PT ≻ ZZ ≻ FZ ≻ QZ ≻ XM ≻ LY ≻ NP ≻ SM
$\lambda = 3$	$\omega_1 = 0.2174$, $\omega_2 = 0.2537$, $\omega_3 = 0.2788$,$\omega_4 = 0.2500$	ND ≻ PT ≻ ZZ ≻ FZ ≻ QZ ≻ XM ≻ LY ≻ NP ≻ SM
$\lambda = 4$	$\omega_1 = 0.2183$, $\omega_2 = 0.2531$, $\omega_3 = 0.2781$, $\omega_4 = 0.2506$	ND ≻ PT ≻ ZZ ≻ FZ ≻ QZ ≻ XM ≻ LY ≻ NP ≻ SM
$\lambda = 5$	$\omega_1 = 0.2204$, $\omega_2 = 0.2532$, $\omega_3 = 0.2753$, $\omega_4 = 0.2511$	ND ≻ PT ≻ ZZ ≻ FZ ≻ QZ ≻ XM ≻ LY ≻ NP ≻ SM
$\lambda = 7$	$\omega_1 = 0.2260$, $\omega_2 = 0.2554$, $\omega_3 = 0.2666$,$\omega_4 = 0.2520$	ND ≻ PT ≻ ZZ ≻ FZ ≻ QZ ≻ XM ≻ LY ≻ NP ≻ SM
$\lambda = 10$	$\omega_1 = 0.2340$, $\omega_2 = 0.2614$, $\omega_3 = 0.2515$,$\omega_4 = 0.2531$	ND ≻ PT ≻ ZZ ≻ FZ ≻ QZ ≻ XM ≻ LY ≻ NP ≻ SM
$\lambda = 15$	$\omega_1 = 0.2439$, $\omega_2 = 0.2716$, $\omega_3 = 0.2297$,$\omega_4 = 0.2547$	ND ≻ PT ≻ ZZ ≻ FZ ≻ QZ ≻ XM ≻ LY ≻ NP ≻ SM
$\lambda = 20$	$\omega_1 = 0.2499$, $\omega_2 = 0.2791$, $\omega_3 = 0.2150$,$\omega_4 = 0.2560$	ND ≻ PT ≻ ZZ ≻ FZ ≻ QZ ≻ XM ≻ LY ≻ NP ≻ SM
Case 2: Dynamic assessment		
$\lambda = 1$	$\omega_1 = 0.2333$, $\omega_2 = 0.2372$, $\omega_3 = 0.2863$, $\omega_4 = 0.2431$	ND ≻ PT ≻ ZZ ≻ FZ ≻ XM ≻ QZ ≻ LY ≻ NP ≻ SM
$\lambda = 2$	$\omega_1 = 0.2263$, $\omega_2 = 0.2355$, $\omega_3 = 0.3049$, $\omega_4 = 0.2333$	ND ≻ PT ≻ ZZ ≻ FZ ≻ XM ≻ QZ ≻ LY ≻ NP ≻ SM
$\lambda = 3$	$\omega_1 = 0.2233$, $\omega_2 = 0.2354$, $\omega_3 = 0.3129$, $\omega_4 = 0.2285$	ND ≻ PT ≻ ZZ ≻ FZ ≻ XM ≻ QZ ≻ LY ≻ NP ≻ SM
$\lambda = 4$	$\omega_1 = 0.2223$, $\omega_2 = 0.2362$, $\omega_3 = 0.3152$, $\omega_4 = 0.2263$	ND ≻ PT ≻ ZZ ≻ FZ ≻ XM ≻ QZ ≻ LY ≻ NP ≻ SM
$\lambda = 5$	$\omega_1 = 0.2223$, $\omega_2 = 0.2375$, $\omega_3 = 0.3146$, $\omega_4 = 0.2255$	ND ≻ PT ≻ ZZ ≻ FZ ≻ XM ≻ QZ ≻ LY ≻ NP ≻ SM
$\lambda = 7$	$\omega_1 = 0.2235$, $\omega_2 = 0.2408$, $\omega_3 = 0.3100$, $\omega_4 = 0.2257$	ND ≻ PT ≻ ZZ ≻ FZ ≻ XM ≻ QZ ≻ LY ≻ NP ≻ SM
$\lambda = 10$	$\omega_1 = 0.2259$, $\omega_2 = 0.2461$, $\omega_3 = 0.3008$, $\omega_4 = 0.2272$	ND ≻ PT ≻ ZZ ≻ FZ ≻ XM ≻ QZ ≻ LY ≻ NP ≻ SM
$\lambda = 15$	$\omega_1 = 0.2294$, $\omega_2 = 0.2532$, $\omega_3 = 0.2874$, $\omega_4 = 0.2299$	ND ≻ PT ≻ ZZ ≻ FZ ≻ XM ≻ QZ ≻ LY ≻ NP ≻ SM
$\lambda = 20$	$\omega_1 = 0.2318$, $\omega_2 = 0.2579$, $\omega_3 = 0.2782$, $\omega_4 = 0.2320$	ND ≻ PT ≻ ZZ ≻ FZ ≻ XM ≻ QZ ≻ LY ≻ NP ≻ SM

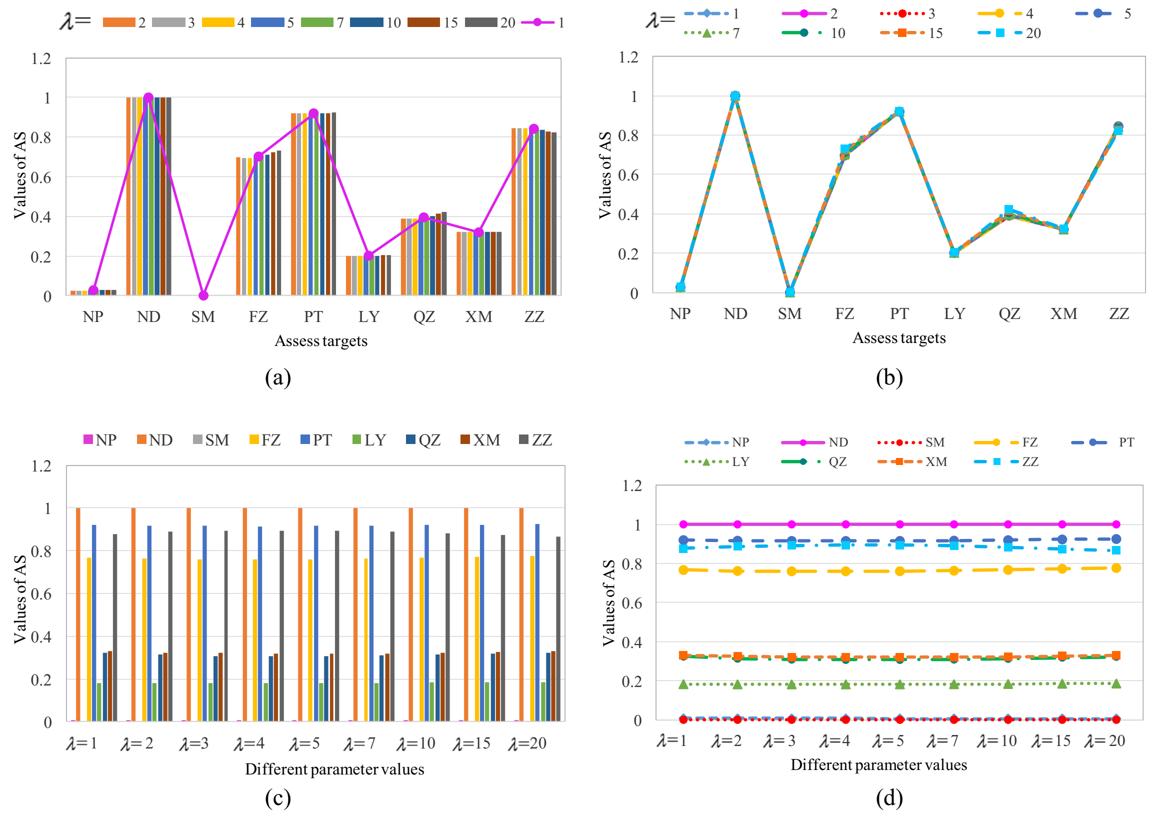


Fig. 4 Ranking results of the evaluation cities for different λ in distance measure for case 1

Table 9 Ranking results based on different distance measures for Case 1

Evaluation results Different distance measures	Appraisal scores (AS) of the evaluation cities NP	ND	SM	FZ	PT	LY	QZ	XM	ZZ	Ranking results
Distance measure based on Jaccard similarity [31, 81]	0.0276	1.0000	0.0000	0.7046	0.9200	0.2015	0.3996	0.3207	0.8397	*ND≻PT≻ZZ≻FZ≻QZ ≻XM≻LY≻NP≻SM*
Distance measure based on Dice similarity [31, 81]	0.0279	1.0000	0.0000	0.7061	0.9200	0.2014	0.4023	0.3211	0.8385	*ND≻PT≻ZZ≻FZ≻QZ ≻XM≻LY≻NP≻SM*
Distance measure based on Cosine similarity [31, 81]	0.0290	1.0000	0.0000	0.7132	0.9206	0.2016	0.4113	0.3218	0.8341	*ND≻PT≻ZZ≻FZ≻QZ ≻XM≻LY≻NP≻SM*
Distance measure based on Cosine similarity [38]	0.0262	1.0000	0.0000	0.6973	0.9189	0.2010	0.3933	0.3217	0.8423	*ND≻PT≻ZZ≻FZ≻QZ ≻XM≻LY≻NP≻SM*
Modified distance measure based on Cosine similarity [82]	0.0268	1.0000	0.0000	0.6961	0.9191	0.2006	0.3917	0.3202	0.8446	*ND≻PT≻ZZ≻FZ≻QZ ≻XM≻LY≻NP≻SM*
Modified distance measure-1 based on Tangent similarity [83]	0.0277	1.0000	0.0000	0.7014	0.9200	0.2012	0.3950	0.3190	0.8431	*ND≻PT≻ZZ≻FZ≻QZ ≻XM≻LY≻NP≻SM*
Modified distance measure-2 based on Tangent similarity [83]	0.0273	1.0000	0.0000	0.7019	0.9199	0.2015	0.3950	0.3196	0.8421	*ND≻PT≻ZZ≻FZ≻QZ ≻XM≻LY≻NP≻SM*
Modified distance measure based on Majumdar and Samanta's similarity [37]	0.0286	1.0000	0.0000	0.7102	0.9211	0.2021	0.4017	0.3187	0.8388	*ND≻PT≻ZZ≻FZ≻QZ ≻XM≻LY≻NP≻SM*
Modified distance measure based on Hybrid vector similarity [84]	0.0284	1.0000	0.0000	0.7094	0.9203	0.2015	0.4064	0.3214	0.8365	*ND≻PT≻ZZ≻FZ≻QZ ≻XM≻LY≻NP≻SM*
Proposed Hamming distance	0.0273	1.0000	0.0000	0.7020	0.9200	0.2015	0.3950	0.3195	0.8421	*ND≻PT≻ZZ≻FZ≻QZ ≻XM≻LY≻NP≻SM*
Proposed Euclidean distance	0.0265	1.0000	0.0000	0.6968	0.9193	0.2011	0.3903	0.3200	0.8443	*ND≻PT≻ZZ≻FZ≻QZ ≻XM≻LY≻NP≻SM*

example with multi-level indicators and case 2 is an example for dynamic evaluation) are given in Table 9, Table 10, and Fig. 5.

1. Jaccard, Dice, and cosine distance measures proposed by Chen et al. [31, 81].

$$D_{Jaccard}(M,N) = 1-R_{Jaccard}(M,N) = 1-\frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\frac{T_{jM}(x_i)T_{jN}(x_i)+I_{jM}(x_i)I_{jN}(x_i)+F_{jM}(x_i)F_{jN}(x_i)}{\left[\left(T_{jM}^2(x_i)+I_{jM}^2(x_i)+F_{jM}^2(x_i)\right)+\left(T_{jN}^2(x_i)+I_{jN}^2(x_i)+F_{jN}^2(x_i)\right)-\left(T_{jM}(x_i)T_{jN}(x_i)+I_{jM}(x_i)I_{jN}(x_i)+F_{jM}(x_i)F_{jN}(x_i)\right)\right]}. \quad (28)$$

$$D_{Dice}(M,N) = 1-R_{Dice}(M,N) = 1-\frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\frac{2\left(T_{jM}(x_i)T_{jN}(x_i)+I_{jM}(x_i)I_{jN}(x_i)+F_{jM}(x_i)F_{jN}(x_i)\right)}{\left(T_{jM}^2(x_i)+I_{jM}^2(x_i)+F_{jM}^2(x_i)\right)+\left(T_{jN}^2(x_i)+I_{jN}^2(x_i)+F_{jN}^2(x_i)\right)}. \quad (29)$$

$$D_{cosine}(M,N) = 1-R_{cosine}(M,N) = 1-\frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\frac{T_{jM}(x_i)T_{jN}(x_i)+I_{jM}(x_i)I_{jN}(x_i)+F_{jM}(x_i)F_{jN}(x_i)}{\sqrt{T_{jM}^2(x_i)+I_{jM}^2(x_i)+F_{jM}^2(x_i)}\sqrt{T_{jN}^2(x_i)+I_{jN}^2(x_i)+F_{jN}^2(x_i)}}. \quad (30)$$

2. Distance measure proposed by Fan and Ye [38].

$$D(M,N) = 1-\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{k}\cos\left\{\frac{\pi}{6}\left(\left|T_{jM}(x_i)-T_{jN}(x_i)\right|+\left|I_{jM}(x_i)-I_{jN}(x_i)\right|+\left|F_{jM}(x_i)-F_{jN}(x_i)\right|\right)\right\}/k. \quad (31)$$

3. Modified distance measure based on the cosine similarity measure proposed by Ye [82].

$$D(M,N) = 1-S(M,N) = 1-\frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\cos\left\{\frac{\pi}{2}\left(\left|T_{jM}(x_i)-T_{jN}(x_i)\right|\vee\left|I_{jM}(x_i)-I_{jN}(x_i)\right|\vee\left|F_{jM}(x_i)-F_{jN}(x_i)\right|\right)\right\}. \quad (32)$$

4. Modified distance measure-1 based on the tangent similarity measure proposed by Ye [83].

$$D(M,N) = 1-ST_1(M,N) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\tan\left(\frac{\pi}{4}\max\left(\left|T_{jM}(x_i)-T_{jN}(x_i)\right|,\left|I_{jM}(x_i)-I_{jN}(x_i)\right|,\left|F_{jM}(x_i)-F_{jN}(x_i)\right|\right)\right). \quad (33)$$

5. Modified distance measure-2 based on tangent similarity measure proposed by Ye [83].

$$D(M,N) = 1-ST_2(M,N) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\tan\left(\frac{\pi}{12}\left(\left|T_{jM}(x_i)-T_{jN}(x_i)\right|+\left|I_{jM}(x_i)-I_{jN}(x_i)\right|+\left|F_{jM}(x_i)-F_{jN}(x_i)\right|\right)\right). \quad (34)$$

6. Modified distance measure based on Majumdar and Samanta's similarity measure proposed by Ye and Smarandache [37].

$$D(M,N) = 1-M(M,N) = 1-\frac{\sum_{i=1}^{n}\sum_{j=1}^{k}\left\{\min\left(T_{jM}(x_i),T_{jN}(x_i)\right)+\min\left(I_{jM}(x_i),I_{jN}(x_i)\right),\min\left(F_{jM}(x_i),F_{jN}(x_i)\right)\right\}}{\sum_{i=1}^{n}\sum_{j=1}^{k}\left\{\max\left(T_{jM}(x_i),T_{jN}(x_i)\right)+\max\left(I_{jM}(x_i),I_{jN}(x_i)\right),\max\left(F_{jM}(x_i),F_{jN}(x_i)\right)\right\}}. \quad (35)$$

7. Modified distance measure based on the hybrid vector similarity measure proposed by Pramanik et al. [84].

$$D(M,N) = 1-Hyb_{SVRNS}(M,N) = 1-\frac{1}{k}\sum_{j=1}^{k}\left\langle\frac{1}{n}\left[\begin{array}{l}\alpha\sum_{i=1}^{n}\frac{2\left(T_{jM}(x_i)\cdot T_{jN}(x_i)+I_{jM}(x_i)\cdot I_{jN}(x_i)+F_{jM}(x_i)\cdot F_{jN}(x_i)\right)}{\left[\left(T_{jM}(x_i)\right)^2+\left(I_{jM}(x_i)\right)^2+\left(F_{jM}(x_i)\right)^2+\left(T_{jN}(x_i)\right)^2+\left(I_{jN}(x_i)\right)^2+\left(F_{jN}(x_i)\right)^2\right]}\\+(1-\alpha)\sum_{i=1}^{n}\frac{\left(T_{jM}(x_i)\cdot T_{jN}(x_i)+I_{jM}(x_i)\cdot I_{jN}(x_i)+F_{jM}(x_i)\cdot F_{jN}(x_i)\right)}{\left[\sqrt{\left(T_{jM}(x_i)\right)^2+\left(I_{jM}(x_i)\right)^2+\left(F_{jM}(x_i)\right)^2}\cdot\sqrt{\left(T_{jN}(x_i)\right)^2+\left(I_{jN}(x_i)\right)^2+\left(F_{jN}(x_i)\right)^2}\right]}\end{array}\right]\right\rangle. \quad (36)$$

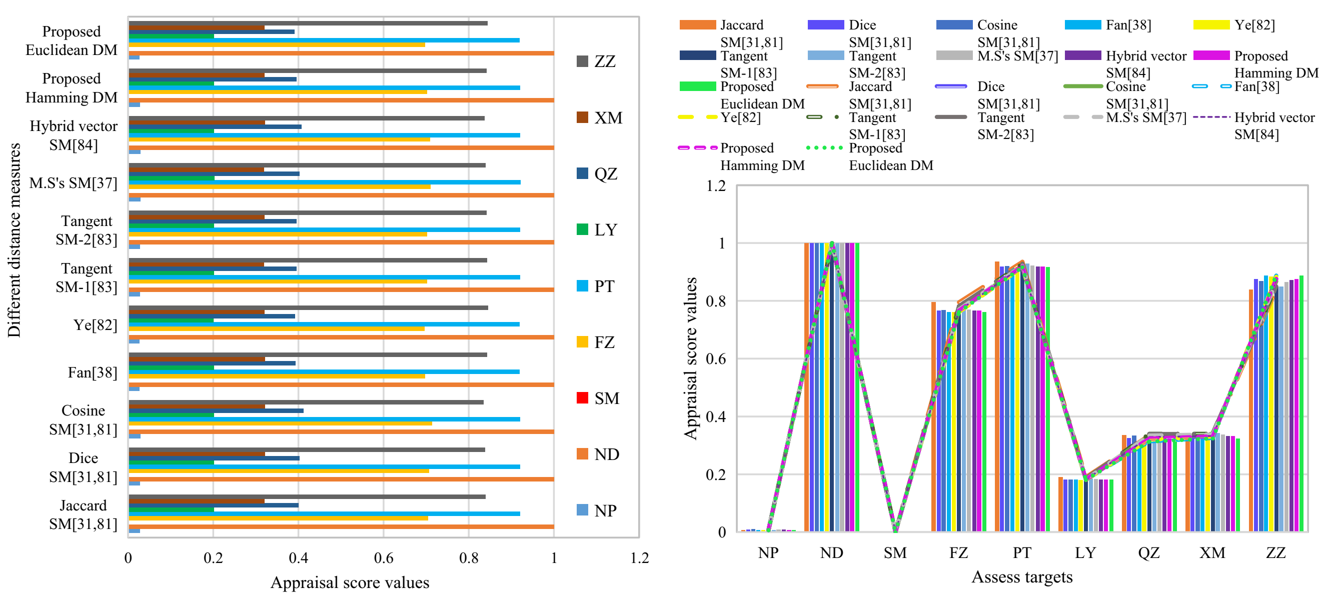
Table 10 Ranking results based on different distance measures for Case 2

Evaluation results Different distance measures	Appraisal scores (AS) of the evaluation cities NP	ND	SM	FZ	PT	LY	QZ	XM	ZZ	Ranking results
Distance measure based on Jaccard similarity [31, 81]	0.0067	1.0000	0.0000	0.7952	0.9352	0.1904	0.3349	0.3354	0.8393	*ND≻PT≻ZZ≻FZ≻XM≻QZ≻LY≻NP≻SM*
Distance measure based on Dice similarity [31, 81]	0.0087	1.0000	0.0000	0.7655	0.9185	0.1818	0.3250	0.3305	0.8759	*ND≻PT≻ZZ≻FZ≻XM≻QZ≻LY≻NP≻SM*
Distance measure based on Cosine similarity [31, 81]	0.0099	1.0000	0.0000	0.7679	0.9198	0.1821	0.3333	0.3351	0.8679	*ND≻PT≻ZZ≻FZ≻XM≻QZ≻LY≻NP≻SM*
Distance measure based on Cosine similarity [38]	0.0066	1.0000	0.0000	0.7615	0.9166	0.1812	0.3140	0.3259	0.8875	*ND≻PT≻ZZ≻FZ≻XM≻QZ≻LY≻NP≻SM*
Modified distance measure based on Cosine similarity [82]	0.0072	1.0000	0.0000	0.7610	0.9165	0.1806	0.3204	0.3309	0.8837	*ND≻PT≻ZZ≻FZ≻XM≻QZ≻LY≻NP≻SM*
Modified distance measure-1 based on Tangent similarity [83]	0.0075	1.0000	0.0000	0.7813	0.9282	0.1856	0.3400	0.3428	0.8500	*ND≻PT≻ZZ≻FZ≻XM≻QZ≻LY≻NP≻SM*
Modified distance measure-2 based on Tangent similarity [83]	0.0075	1.0000	0.0000	0.7814	0.9282	0.1856	0.3396	0.3423	0.8501	*ND≻PT≻ZZ≻FZ≻XM≻QZ≻LY≻NP≻SM*
Modified distance measure based on Majumdar and Samanta's similarity [37]	0.0087	1.0000	0.0000	0.7702	0.9216	0.1826	0.3344	0.3381	0.8650	*ND≻PT≻ZZ≻FZ≻XM≻QZ≻LY≻NP≻SM*
Modified distance measure based on Hybrid vector similarity [84]	0.0093	1.0000	0.0000	0.7666	0.9191	0.1819	0.3289	0.3326	0.8722	*ND≻PT≻ZZ≻FZ≻XM≻QZ≻LY≻NP≻SM*
Proposed Hamming distance	0.0076	1.0000	0.0000	0.7664	0.9193	0.1821	0.3238	0.3315	0.8760	*ND≻PT≻ZZ≻FZ≻XM≻QZ≻LY≻NP≻SM*
Proposed Euclidean distance	0.0071	1.0000	0.0000	0.7615	0.9163	0.1813	0.3132	0.3240	0.8878	*ND≻PT≻ZZ≻FZ≻XM≻QZ≻LY≻NP≻SM*

From Table 9, Table 10, and Fig. 5, it is evident that the results of the proposed method and the existing methods are completely consistent, which illustrates the rationality and effectiveness of the proposed method.

5.3.3 Comparative analysis of different methods

In this section, we consider the proposed method based on the EDAS approach and nine other methods (specifically, the



(a) (b)

Fig. 5 Comparison results of evaluation methods based on different distance measures

Table 11 Comparative analysis of different methods

Different Methods Cities	Case 1 Traditional function-based method		TOPSIS-based method		Our similarity-based method		Method proposed by Fan et al. [38]		Method-1 proposed by Chen et al. [31, 81]	
	Score function values (SFV)	Ranking results	Closeness coefficients (CC)	Ranking results	Our Similarity (OS)	Ranking results	Weighted cosine similarity (WCS)	Ranking results	Weighted jaccard similarity (WJS)	Ranking results
Nanping (NP)	0.1404	8	0.5514	8	0.3070	8	0.3533	8	0.1420	8
Ningde (ND)	0.7493	1	0.8850	1	0.9763	1	0.9374	2	0.8300	2
Sanming (SM)	0.1131	9	0.5493	9	0.2742	9	0.3173	9	0.1182	9
Fuzhou (FZ)	0.6557	4	0.6756	4	0.8533	4	0.9057	4	0.7517	4
Putian(PT)	0.7234	2	0.7453	2	0.9175	2	0.9462	1	0.8310	1
Longyan (LY)	0.3042	7	0.5657	7	0.4752	7	0.5761	7	0.2802	7
Quanzhou (QZ)	0.4541	5	0.6121	5	0.7251	5	0.6848	5	0.4901	5
Xiamen (XM)	0.4096	6	0.6070	6	0.7082	6	0.6366	6	0.4213	6
Zhangzhou (ZZ)	0.6905	3	0.7150	3	0.8948	3	0.9141	3	0.7839	3
	Case 2									
Nanping (NP)	0.1735	8	0.0452	8	0.6501	8	0.4656	8	0.2021	8
Ningde (ND)	0.7348	1	0.9605	1	0.9855	1	0.9547	1	0.8568	1
Sanming (SM)	0.1647	9	0.0302	9	0.6447	9	0.4609	9	0.1814	9
Fuzhou (FZ)	0.6581	4	0.9425	2	0.9789	4	0.9276	3	0.7812	4
Putian(PT)	0.7095	2	0.9148	3	0.9688	2	0.9483	2	0.8406	2
Longyan (LY)	0.3145	7	0.2443	7	0.7231	7	0.6453	7	0.3263	7
Quanzhou (QZ)	0.4156	5	0.4327	6	0.7921	5	0.6969	5	0.4741	5
Xiamen (XM)	0.3994	6	0.6966	5	0.8888	6	0.6931	6	0.4453	6
Zhangzhou (ZZ)	0.6895	3	0.8143	4	0.9319	3	0.9204	4	0.7846	3
Different Methods Cities	Case 1 Method-2 proposed by Chen et al. and Karaaslan [31, 81]		Method-3 proposed by Chen et al. and Karaaslan [31, 81]		Method proposed by Pramanik et al. [84]		Method proposed by Ye and Smarandache [37]		Our proposed method based on EDAS	
	Weighted dice similarity (WDS)	Ranking results	Weighted cosine similarity (WCS)	Ranking results	Weighted hybrid vector similarity (WHVS)	Ranking results	Weighted extended similarity (WES)	Ranking results	Appraisal score (AS)	Ranking results
Nanping (NP)	0.2484	8	0.2594	8	0.2539	8	0.1185	8	0.0273	8
Ningde (ND)	0.9101	2	0.9135	2	0.9118	2	0.6687	1	1.0000	1
Sanming (SM)	0.2115	9	0.2228	9	0.2171	9	0.1041	9	0.0000	9
Fuzhou (FZ)	0.8687	4	0.8730	4	0.8708	4	0.5417	4	0.7020	4
Putian(PT)	0.9271	1	0.9328	1	0.9300	1	0.6271	2	0.9200	2
Longyan (LY)	0.4400	7	0.4462	7	0.4431	7	0.2067	7	0.2015	7
Quanzhou (QZ)	0.6185	5	0.6248	5	0.6217	5	0.3526	5	0.3950	5

Table 11 (continued)

Xiamen (XM)	0.5417	6	0.5487	6	0.5452	6	0.2943	6	0.3195	6
Zhangzhou (ZZ)	0.8811	3	0.8850	3	0.8831	3	0.5937	3	0.8421	3
	Case 2									
Nanping (NP)	0.3122	8	0.3224	8	0.3173	8	0.1557	8	0.0076	8
Ningde (ND)	0.9143	1	0.9184	1	0.9164	1	0.6859	1	1.0000	1
Sanming (SM)	0.2928	9	0.3012	9	0.2970	9	0.1458	9	0.0000	9
Fuzhou (FZ)	0.8600	3	0.8668	3	0.8634	3	0.5959	4	0.7615	4
Putian(PT)	0.9040	2	0.9095	2	0.8634	2	0.6545	2	0.9163	2
Longyan (LY)	0.4752	7	0.4806	7	0.4779	7	0.2414	7	0.1813	7
Quanzhou (QZ)	0.5891	5	0.5928	5	0.5909	5	0.3440	5	0.3132	6
Xiamen (XM)	0.5522	6	0.5574	6	0.5548	6	0.3288	6	0.3240	5
Zhangzhou (ZZ)	0.8578	4	0.8603	4	0.8591	4	0.6127	3	0.8878	3

function-based method, TOPSIS-based method, our similarity-based method, method proposed by Fan et al. [38], method of weighted Jaccard similarity proposed by Chen et al. [31, 81], method of weighted dice similarity proposed by Chen et al. [31, 81], method of weighted cosine similarity proposed by Chen et al. [31, 81], method of hybrid vector similarity proposed by Pramanik et al. [84], and the method of extended similarity proposed by Ye and Smarandache [37]) to illustrate the advantages of the proposed method. The comparison results are presented in Table 11 and Fig. 6.

In this section, the traditional function-based method is used to calculate the weighted score function value of each scheme based on Eq. (3) proposed in this study, and obtain the sorting result. The TOPSIS-based method calculates the distance between each solution and the positive ideal solution $x_{ij}^{+} = \langle (\max T_{i1j}, \max T_{i2j}, \cdots, \max T_{ik_j j})$, $(\min I_{i1j}, \min I_{i2j}, \cdots$, $\min I_{ik_j j}), (\min F_{i1j}, \min F_{i2j}, \cdots, \min F_{ik_j j})\rangle$ and the distance between each solution and the negative ideal solution.

$x_{ij}^{-} = \langle (\min T_{i1j}, \min T_{i2j}, \cdots, \min T_{ik_j j}), (\max I_{i1j}, \max I_{i2j}, \cdots,$ $\max I_{ik_j j}), (\max F_{i1j}$, $\max F_{i2j}, \cdots, \max F_{ik_j j})\rangle$, to obtain the values of closeness coefficients, and finally, the sorting result. Our similarity-based method calculates the weighted similarity value (OS) for each alternative. The positive ideal solution based on the proposed similarity measure is calculated to be x_{ij}^{+}. The method proposed by Fan et al. [38] first introduced a cosine similarity measure, then calculated the weighted cosine similarity (WCS) measure value for each alternative and the positive ideal solution x_{ij}^{+}, and finally ranked the alternatives according to WCS. The method-1 proposed by Chen et al. and Karaaslan [31, 81] is similar to the method proposed by Fan and Ye [38], which calculates the weighted Jaccard similarity (WJS) measure value for each alternative and the positive ideal solution x_{ij}^{+}, and finally ranks the alternatives according to WJS. Method-2 proposed by Chen et al. and Karaaslan [31, 81] calculates the weighted dice similarity (WDS) measure value for each alternative and the positive ideal solution x_{ij}^{+}, and finally ranks the alternatives according to WDS. Method-3 proposed by Chen et al. and Karaaslan [31, 81] calculates the weighted cosine similarity (WCS) measure value for each alternative and the positive ideal solution x_{ij}^{+}, and finally ranks the alternatives according to WCS. The method proposed by Pramanik et al. [84] calculates the weighted hybrid vector similarity (WHVS) measure value for each alternative and the positive ideal solution x_{ij}^{+}, and finally ranks the alternatives according to WHVS. The method proposed by Ye and Smarandache [37] calculates the weighted extended similarity (WES) measure value for each alternative and the positive ideal solution x_{ij}^{+}, and finally ranks the alternatives according to WES. It is evident from Table 11 and Fig. 6 that the sorting results for the five methods are basically the same, and the optimal alternative is identical, except for the methods proposed by

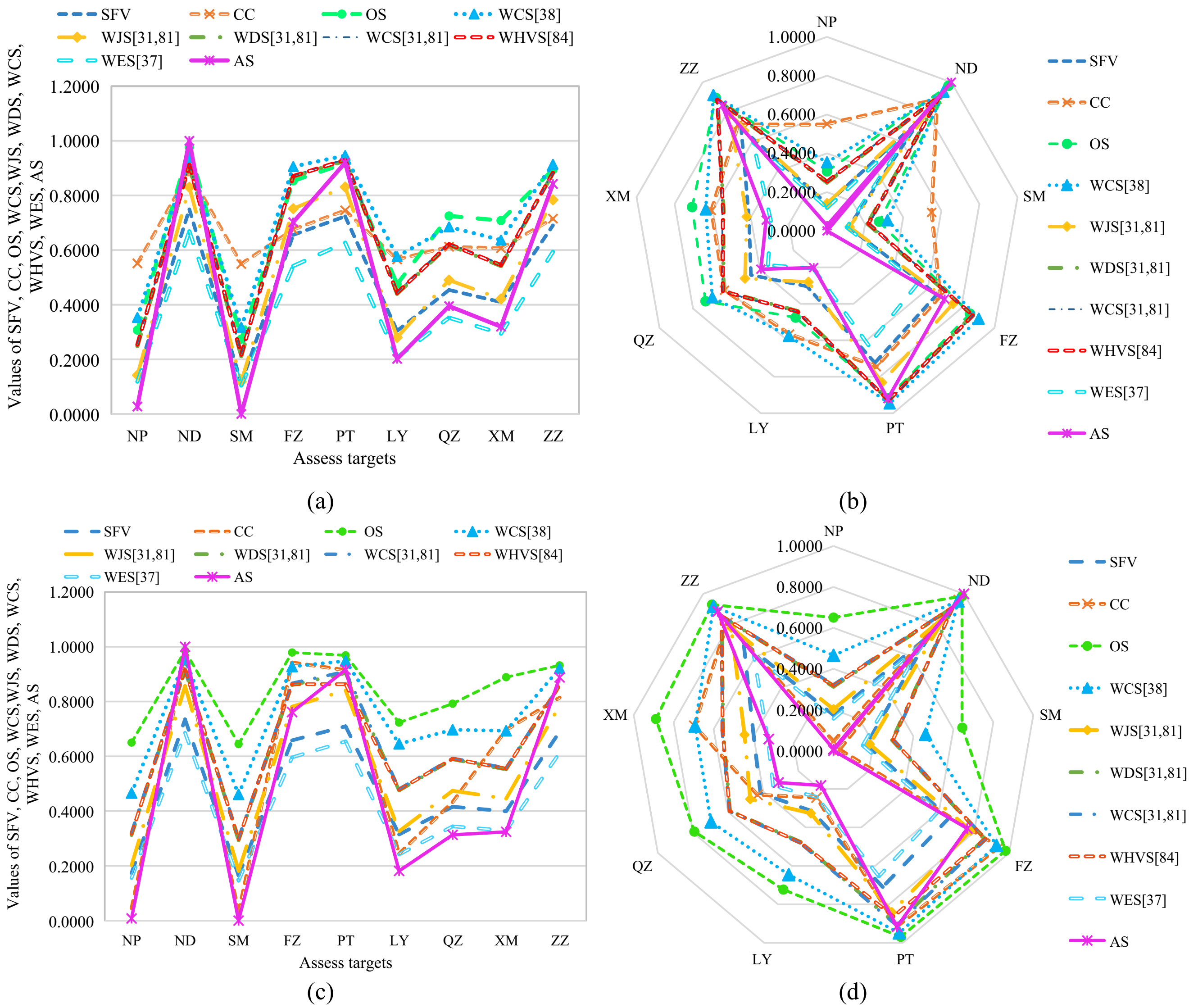


Fig. 6 Comparative analysis of different methods

Chen et al. [31, 81], literature [38], and Pramanik et al. [84] for case 1. We believe that this happened because the weights of the attributes used in the calculation are uniformly obtained using the information entropy method in this study; the other reason is that as compared to the proposed EDAS and TOPSIS methods, these methods only consider the relationship between the scheme and ideal solution, instead of that between the scheme and negative ideal solution. Although the methods proposed by Chen et al. [31, 81], Fan et al. [38], and Pramanik et al. [84] are simple, they only calculate the weighted similarity between each alternative and the ideal solution, which has certain irrationality. However, the EDAS method in this study considers the solution more comprehensively. In addition, the relative value of the evaluation object based on the EDAS method is more evident and easy to distinguish.

5.4 Advantages of the proposed method

In this section, we analyze and summarize the advantages of the proposed method as follows.

(1) We proposed a multi-criteria decision-making method under refined single-valued neutrosophic sets environment, for which, to the best of our knowledge, not many related studies have been conducted. Then, we applied the method to the typhoon disaster assessment problem and studied multi-level index evaluation and dynamic evaluation, which have not been seen in related research.
(2) The EDAS method adopted here aims to find the best solution through two distance measures, namely, the PDA and the NDA. In comparison to previously proposed methods [31–38], which only considered the weighted similarity between the alternatives and the

positive ideal solution, our method is more comprehensive and reasonable.

(3) In this study, the generalized distance measure of refined single-valued neutrosophic sets is proposed, and the definition of refined single-valued neutrosophic entropy is introduced based on this distance measure which helps in determining the attribute weight. Such distance and information entropy balance definition makes our evaluation method simple and effective, and ensures that the evaluation results are more objective and effective in comparison with the direct assumptions of other methods [31–38].

6 Conclusion

This paper proposed a multi-attribute decision-making method based on information entropy and the EDAS method under a refined single-valued neutrosophic sets environment to solve the decision-making problem that has both attributes and sub-attributes, in which the attribute weight is unknown. The new distance measure, similarity measure, and neutrosophic entropy were defined based on the refined single-valued neutrosophic sets. The relationship between these attributes was also discussed and the attribute weights obtained using the new entropy. The extended EDAS method was used to rank and select the best alternatives. Two illustrative examples for TDA (TDA with multi-layer indicators and dynamic assessment of typhoon disaster) were presented to demonstrate the feasibility, effectiveness, and practicality of the proposed method. The advantages of the algorithm were demonstrated by sensitivity analysis and comparative analysis with other methods. In the future, we will further enhance the proposed method considering the following aspects: more accurate network data acquisition, practical application and improvement of the proposed method, and extension of the proposed method to other data environments, such as Pythagorean fuzzy set and picture fuzzy set. We will also continue to study the theory of neutrosophic sets, their decision-making methods, and their applications in TDA.

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Compliance with ethical standards

Conflict of interest The authors declare that there is no conflict of interest regarding the publication of this paper.

Appendix

Here, we prove the properties of Theorem 1 and Theorem 3.

(1) Proof of the three properties of Theorem 1:

Proof.

(P1) $0 \le T_{jM}(x_i),\ I_{jM}(x_i),\ F_{jM}(x_i),\ T_{jN}(x_i),\ I_{jN}(x_i),\ F_{jN}(x_i) \le 1$, then $0 \le |T_{jM}(x_i) - T_{jN}(x_i)| \le 1$, $0 \le |I_{jM}(x_i) - I_{jN}(x_i)| \le 1$, $0 \le |F_{jM}(x_i) - F_{jN}(x_i)| \le 1$, and $0 \le |T_{jM}(x_i) - T_{jN}(x_i)|^{\lambda} \le 1$, $0 \le |I_{jM}(x_i) - I_{jN}(x_i)|^{\lambda} \le 1$, $0 \le |F_{jM}(x_i) - F_{jN}(x_i)|^{\lambda} \le 1$, so $0 \le \frac{1}{3}\left(\left|T_{jM}(x_i) - T_{jN}(x_i)\right|^{\lambda} + \left|I_{jM}(x_i) - I_{jN}(x_i)\right|^{\lambda} + \left|F_{jM}(x_i) - F_{jN}(x_i)\right|^{\lambda}\right) \le 1$,

$$0 \le \left\{\frac{1}{3}\left(\left|T_{jM}(x_i) - T_{jN}(x_i)\right|^{\lambda} + \left|I_{jM}(x_i) - I_{jN}(x_i)\right|^{\lambda} + \left|F_{jM}(x_i) - F_{jN}(x_i)\right|^{\lambda}\right)\right\}^{1/\lambda} \le 1,$$

$$0 \le \frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|T_{jM}(x_i) - T_{jN}(x_i)\right|^{\lambda} + \left|I_{jM}(x_i) - I_{jN}(x_i)\right|^{\lambda} + \left|F_{jM}(x_i) - F_{jN}(x_i)\right|^{\lambda}\right)\right\}^{1/\lambda} \le 1.$$

Thus, $0 \le D(M, N) \le 1$ is established.

(P2) If $M = N$, then $T_{jM}(x_i) = T_{jN}(x_i)$, $I_{jM}(x_i) = I_{jN}(x_i)$, $F_{jM}(x_i) = F_{jN}(x_i)$, $T_{jM}(x_i) - T_{jN}(x_i) = 0$,

$I_{jM}(x_i) - I_{jN}(x_i) = 0$, $F_{jM}(x_i) - F_{jN}(x_i) = 0$,

so $|T_{jM}(x_i) - T_{jN}(x_i)|^{\lambda} + |I_{jM}(x_i) - I_{jN}(x_i)|^{\lambda} + |F_{jM}(x_i) - F_{jN}(x_i)|^{\lambda} = 0$,

$$\frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|T_{jM}(x_i) - T_{jN}(x_i)\right|^{\lambda} + \left|I_{jM}(x_i) - I_{jN}(x_i)\right|^{\lambda} + \left|F_{jM}(x_i) - F_{jN}(x_i)\right|^{\lambda}\right)\right\}^{1/\lambda} = 0,$$

that is, $D(M, N) = 0$.

Conversely, if $D(M, N) = 0$, then $|T_{jM}(x_i) - T_{jN}(x_i)|^{\lambda} + |I_{jM}(x_i) - I_{jN}(x_i)|^{\lambda} + |F_{jM}(x_i) - F_{jN}(x_i)|^{\lambda} = 0$,

$T_{jM}(x_i) - T_{jN}(x_i) = 0$, $I_{jM}(x_i) - I_{jN}(x_i) = 0$, $F_{jM}(x_i) - F_{jN}(x_i) = 0$, then $T_{jM}(x_i) = T_{jN}(x_i)$, $I_{jM}(x_i) = I_{jN}(x_i)$, $F_{jM}(x_i) = F_{jN}(x_i)$, so $M = N$. Thus, we can get $D(M, N) = 0$ if and only if $M = N$.

(P3) $|T_{jM}(x_i) - T_{jN}(x_i)| = |T_{jN}(x_i) - T_{jM}(x_i)|$, $|I_{jM}(x_i) - I_{jN}(x_i)| = |I_{jN}(x_i) - I_{jM}(x_i)|$, $|F_{jM}(x_i) - F_{jN}(x_i)| = |F_{jN}(x_i) - F_{jM}(x_i)|$, so $|T_{jM}(x_i) - T_{jN}(x_i)|^{\lambda} = |T_{jN}(x_i) - T_{jM}(x_i)|^{\lambda}$, $|I_{jM}(x_i) - I_{jN}(x_i)|^{\lambda} = |I_{jN}(x_i) - I_{jM}(x_i)|^{\lambda}$, $|F_{jM}(x_i) - F_{jN}(x_i)|^{\lambda} = |F_{jN}(x_i) - F_{jM}(x_i)|^{\lambda}$,

$$\left(\left|T_{jM}(x_i) - T_{jN}(x_i)\right|^{\lambda} + \left|I_{jM}(x_i) - I_{jN}(x_i)\right|^{\lambda} + \left|F_{jM}(x_i) - F_{jN}(x_i)\right|^{\lambda}\right) = \left(\left|T_{jN}(x_i) - T_{jM}(x_i)\right|^{\lambda} + \left|I_{jN}(x_i) - I_{jM}(x_i)\right|^{\lambda} + \left|F_{jN}(x_i) - F_{jM}(x_i)\right|^{\lambda}\right),$$

$$\frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|T_{jM}(x_i) - T_{jN}(x_i)\right|^{\lambda} + \left|I_{jM}(x_i) - I_{jN}(x_i)\right|^{\lambda} + \left|F_{jM}(x_i) - F_{jN}(x_i)\right|^{\lambda}\right)\right\}^{1/\lambda} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|T_{jN}(x_i) - T_{jM}(x_i)\right|^{\lambda} + \left|I_{jN}(x_i) - I_{jM}(x_i)\right|^{\lambda} + \left|F_{jN}(x_i) - F_{jM}(x_i)\right|^{\lambda}\right)\right\}^{1/\lambda},$$

Thus, $D(M, N) = D(N, M)$ is established.

(2) Proof of the four properties of Theorem 3:

Proof.

(P1) If r is a crisp number, then r is not fuzzy; thus, its entropy is zero. In this case, for example $r = \langle(1, 1, \cdots, k$

times), (0, 0, ⋯, k *times*), (0, 0, ⋯, k *times*)⟩, or $r = \langle (0, 0, \cdots, k\ times), (0, 0, \cdots, k\ times), (1, 1, \cdots, k\ times)\rangle$,

So $E_{RSVNN}(r) == 1-2D(r, r') = 1-2\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}(0.5^{\lambda}+0.5^{\lambda}+0.5^{\lambda})\right\}^{1/\lambda} = 0.$

Thus, we can get $E_{RSVNN}(r) = 0$if r is a crisp number.

(P2) Because $E_{RSVNN}(r) = 1$, that is $1 - 2D(r, r') = 1$, then $D(r, r') = 0$. Based on the Theorem 1 of Section 3.2, we can get $r = r' = \langle (0.5, 0.5, \cdots, k\ times), (0.5, 0.5, \cdots, k\ times),$- $(0.5, 0.5, \cdots, k\ times)\rangle$.

If $r = r' = \langle (0.5, 0.5, \cdots, k\ times), (0.5, 0.5, \cdots, k\ times),$- $(0.5, 0.5, \cdots, k\ times)\rangle$, then $D(r, r') = 0$, $1 - 2D(r, r') = 1$, so $E_{RSVNN}(r) = 1$. Thus, we can get $E_{RSVNN}(r) = 1$, if and only if $r = r' = \langle (0.5, 0.5, \cdots, k\ times), (0.5, 0.5, \cdots, k\ times), (0.5, 0.5, \cdots, k\ times)\rangle$;

(P3) If $D(r, r') \geq D(l, r')$, then $2D(r, r') \geq 2D(l, r')$, $1 - 2D(r, r') \leq 1 - 2D(l, r')$.

Thus, $E_{RSVNN}(r) = 1 - 2D(r, r') \leq E_{RSVNN}(l) = 1 - 2D(l, r')$. If $D(r, r') \geq D(l, r')$, $E_{RSVNN}(r) \leq E_{RSVNN}(l)$, that is, Theorem 3 (3) is established.

(P4) Because $r^c = \langle (F_{1r}, F_{2r}, \cdots, F_{kr}), (1 - I_{1r}, 1 - I_{2r}, \cdots, 1 - I_{kr}), (T_{1r}, T_{2r}, \cdots, T_{kr})\rangle$,

$$E_{RSVNN}(r) = 1-2D\left(r, r'\right) = 1-2\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|T_{jr}(x_i)-0.5\right|^{\lambda}+\left|I_{jr}(x_i)-0.5\right|^{\lambda}+\left|F_{jr}(x_i)-0.5\right|^{\lambda}\right)\right\}^{1/\lambda},$$

$$E_{RSVNN}(r^c) = 1-2D\left(r^c, r'\right) = 1-2\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{3}\left(\left|F_{jr}(x_i)-0.5\right|^{\lambda}+\left|1-I_{jr}(x_i)-0.5\right|^{\lambda}+\left|T_{jr}(x_i)-0.5\right|^{\lambda}\right)\right\}^{1/\lambda}.$$

Thus, we can get $E_{RSVNN}(r) = E_{RSVNN}(r^c)$.

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