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Dynamic uncertain causality graph based on Intuitionistic fuzzy sets and its application to root cause analysis

Li Li[1] · Weichao Yue[1]

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Abstract

Dynamic uncertain causality graph (DUCG), which is based on probability theory, is used for uncertain knowledge representation and reasoning. However, the traditional DUCG has difficulty expressing the causality of the events with crisp numbers. Therefore, an intuitionistic fuzzy set based dynamic uncertain causality graph (IFDUCG) model is proposed in this paper. The model focuses on describing the uncertain event in the form of intuitionistic fuzzy sets, which can handle with the problem of describing vagueness and uncertainty of an event in the traditional model. Then the technique for order preference by similarity to an ideal solution (TOPSIS) method is combined with IFDUCG for knowledge representation and reasoning so as to integrate more abundant experienced knowledge into the model to make the model more reliable. Then some examples are used to validate the proposed method. The experimental results prove that the proposed method is effective and flexible in dealing with the difficulty of the fuzzy event of knowledge representation and reasoning. Furthermore, we make a practical application to root cause analysis of aluminum electrolysis and the results show that the proposed method is available for workers to make decisions.

Keywords Dynamic uncertain causality graph · Knowledge representation and reasoning · Uncertainty · IFDUCG · Intuitionistic fuzzy set

1 Introduction

During the past years, many graphical models for knowledge representation and reasoning have been proposed, including Bayesian network (BN) [1, 2], Petri net(PN) [3–5], artificial neural network (ANN) [6, 7], and so on. These methods have been used in some applications. However, there are some shortcomings. For example, the establishment of these models require precise and sufficient knowledge, while empirical knowledge is usually limited and incomplete in many cases. In addition, BN is NP hard in the process of reasoning for a large and complex system. ANN lacks the ability of interpreting knowledge due to its inference mechanism. For PN, it is easy to cause the problem of state space explosion when there are too many nodes.

As a newly graphical model, dynamic uncertain causality graph model(DUCG) can address the drawback of requirement of complete information. DUCG model is proposed by Zhang [8, 9] for knowledge representation and reasoning. DUCG links events with causality chain by graphic symbols [10] definitely and easily. Meanwhile, a whole DUCG can be decomposed into several small modules as sub-DUCGs which can simplify the complex causality among events. Moreover, it allows incomplete knowledge representation and makes an exact inference at the same time. Thus, it greatly reduces the difficulty of building a knowledge base and reasoning process. Over the past few years, DUCG has been widely applied in fault diagnosis of complex systems such as nuclear power plants, chemical processes, clinical expert systems [11, 12] and so on.

In spite of its advantage over various applications, the basic DUCG model has been criticized for kinds of reasons, mainly including: (1) it only considered the individual events and probabilities, (2) it just addressed the discrete variables and certain evidence, (3) it only addressed the directed acyclic graph, (4)it was limited to model cases of specified causal structure while ignoring statistics knowledge, (5)the knowledge was assigned with a crisp value neglecting the uncertainty. In response, in [13], it addressed that the variables of DUCG can be either acquired from historical data or given by experienced engineers, and the individual events were extended to matrix. Zhang

✉ Li Li
lily122008@163.com

[1] Central South University, Changsha 410083, China

[14] proposed a new approach which took continuous variables and uncertain situations into consideration. In [15] a DUCG with directed cyclic diagram was developed to meet the requirement of feedback among variables. Three kinds of mode of DUCG were put forward in [16] to model complicated cases in which historical statistics knowledge was in different situations with overlap or not and new variables may not be contained in the original information. Zhao [17] proposed a new simplified DUCG model by combining the fuzzy decision tree with DUCG when extracting knowledge.

Despite the fact that much research has been studied to improve the performance of DUCG, little attention has been paid to handle with the uncertainty and fuzzy problem of knowledge in DUCG. Firstly, the probabilities of events are accurate numbers in DUCG which are usually acquired from statistical data or historical data based on the experienced expert. But in many cases various uncertainties are ubiquitous, such as ambiguity, randomness, and time-varying. For example, in the root cause analysis of aluminum reduction production, as the processes of production are accompanied by complex electrochemical reactions, external conditions are changeable, and disturbances are inevitable in the operation, it is difficult to acquire empirical knowledge accurately in the traditional DUCG model. Furthermore, it has low ability and credibility of knowledge representation and inference in the form of accurate numbers in the fuzzy circumstances. Therefore, the purpose of the paper is to propose a new DUCG model to overcome the shortcomings of the traditional model in fuzzy situations.

In this paper, a new type DUCG model is proposed based on intuitionistic fuzzy sets, which is called the intuitionistic fuzzy sets based DUCG (IFDUCG) model. Firstly, the IFDUCG model for characterizing the uncertain event is established based on intuitionistic fuzzy sets (IFs). It contributes to make a good reference of the fuzzy knowledge for the possible causality. Meanwhile, with the uncertain data represented by membership and non-membership degree [18–20], the IFDUCG model can increase the reliability of dealing with fuzzy information to some degrees. Moreover, it is available to find the root cause according to the causal relationship when applied for the root cause analysis of the industrial systems in uncertain situations. Subsequently, an inference mechanism of IFDUCG model based on the TOPSIS [21] method is developed to acquire more empirical knowledge, which can make the model more reliable and flexible in dealing with knowledge ambiguity and uncertainty [22, 23]. Finally, the established model and proposed inference algorithm are demonstrated by simulation and production application. It validates that the new model provides a useful and reliable way for root cause analysis of aluminum reduction production.

Based on the above analyses, the rest of this paper is organized as follows. In Section 2, we briefly review the basic theory of DUCG and operators of intuitionistic fuzzy sets. In Section 3, we propose an IFDUCG model and reasoning algorithm of IFDUCG based on TOPSIS in details. After that, we validate the proposed model by simulation and comparison. In Section 4, the proposed method is applied to solve problems of aluminum electrolytic cell root cause analysis in production. The superiority and validity are demonstrated by numerical examples as well. Conclusions are drawn in Section 5.

2 Preliminaries

In this section, the basic theory of DUCG and operators of intuitionistic fuzzy sets are briefly reviewed.

2.1 DUCG model

DUCG provides an effective way of knowledge representation and inference to the causality relation among observable variables. As shown in Fig. 1, a typical DUCG with directed cyclic cases uses different type of event to express the variables and the uncertain causality links between them [8, 9]. The types are defined as F-, B-, X- in Fig. 1. Usually, the B-type variable or an event drawn as a square represents a parent variable, which is called a basic event and can merely be a cause of other events. The X-type variable or an event drawn as a circle represents a child variable, which represents an intermediate/node event and can be a cause event. The F-type variable is called a connection event variable that represents the uncertain causality relationship from the parent/child variable to the child variable.

In Fig. 1, there is a one-way arc from B_1 to X_2 meaning that B_1 is the cause/parent of X_2. The double-direction arc between X_2 and X_3 denotes that they interact with each other, and this is called a directed cyclic situation. The F-type variable, for example, $F_{2;1}$ means the probability of the event B_1 causing X_2. As shown in Fig. 1, if variable has k states, for example, B_1 has states of B_{11}, $B_{12}, \cdots, B_{1k}$. It means B_{11} has different possibilities to be the cause of X_{21}, $X_{22}, \cdots, X_{2k}$, where the probabilities depend on the value of $F_{21;11}$, $F_{21;12}, \cdots, F_{21;1k}$. It is the same with the states of $B_{12}, \cdots, B_{1k}$ and other variables.

Definition 1 [8, 9] Suppose variable V is either X- or B-type variable, in the case of matrices, the basic model of DUCG is defined as

$$X_n = \sum_i F_{n;i} V_i = \sum_i (r_{n;i}/r_n) A_{n;i} V_i \tag{1}$$

$$F_{n;i} = (r_{n;i}/r_n) A_{n;i} \tag{2}$$

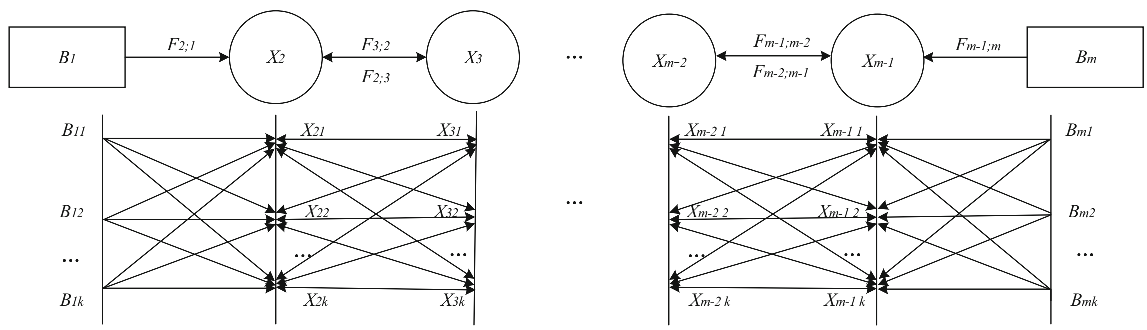

Fig. 1 Example of DUCG model

where the connection event variable $F_{n;i}$ is consist of two parts: the weighting part $(r_{n;i}/r_n)$ and the functional event $A_{n;i}$. $A_{n;i}$ represents the effect event between parent event and child event, while the subscript n represents the variable, the subscript i represents the state of the variable. $r_{n;i}$ represents the degree of association between X_n and V_i, and $r_n = \sum_i r_{n;i}$.

In the case of events, it is defined that $B_i = (B_{i1}, B_{i2}, \cdots, B_{ik})$, $X_i = (X_{i1}, X_{i2}, \cdots, X_{ik})$, $B_i, X_i \in R^{k\times 1}$, then the DUCG model can be represented by (3)

$$X_{nk} = \sum_i \sum_j F_{nk;ij} V_{ij} = \sum_i (r_{n;i}/r_n) \sum_j A_{nk;ij} V_{ij} \quad (3)$$

$$F_{nk;ij} = (r_{n;i}/r_n) A_{nk;ij} \quad (4)$$

where X_{nk} means the k-th state of X_n . V_{ij} means the j-th state of V_i. $F_{nk;ij}$ represents the probabilities between event X_{nk} and event V_{ij}. $F_{nk;ij}$ is the member of the connection event variable matrix $F_{n;i}$ and is consist of two parts: the weighting part $(r_{n;i}/r_n)$ and the functional matrix $A_{nk;ij}$. $A_{nk;ij}$ is the member of the functional event $A_{n;i}$ and represents the effect event between event X_{nk} and event V_{ij}, while the subscript n and i represents the variable, the subscript k and j represents the state of the variable. $r_{n;i}$ represents the degree of association between X_{nk} and V_{ij} , and $r_n = \sum_i r_{n;i}$. Thus, the DUCG model can be illustrated as Fig. 2. From this figure, we can know that the X-type variable can be represented by the combination of parent events and weighted events.

Definition 2 [8, 9]. Suppose that the lower case letters indicate the probabilities of the corresponding upper case letter, for example, $a_{nk;ij} = \Pr\{A_{nk;ij}\}$ and $x_{nk} = \Pr\{X_{nk}\}$, and it satisfies $\sum_k a_{nk;ij} = 1$, then

$$\begin{aligned} x_{nk} &= \Pr\{X_{nk}\} = \sum_i (r_{n;i}/r_n) \sum_j a_{nk;ij} \Pr\{V_{ij}\} \\ &= \sum_i (r_{n;i}/r_n) \sum_j a_{nk;ij} v_{ij} \\ &= \sum_i (r_{n;i}/r_n) a_{nk;i} v_i \end{aligned} \quad (5)$$

where $a_{nk;ij}$ and $r_{n;i}$ are commonly given by domain experienced experts, which are explicitly vague related with experienced knowledge. $\Pr\{\cdot\}$ means the probability of the event. According to the (1)–(5), the final expression of X_{nk} will be expanded as the combination with only B-, A-, r- type variable by repeatedly applying the basic (3). Finally, the probability of X_{nk} can be computed by applying (4). Based on the theory, the probability of joint events

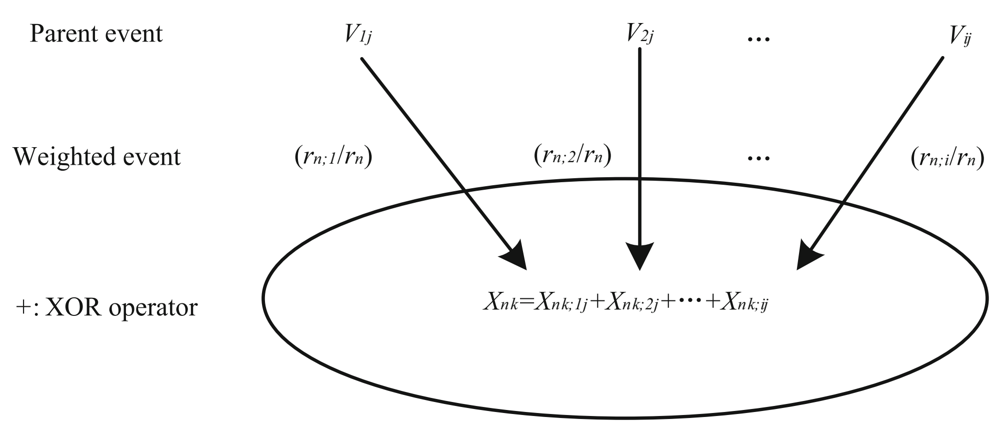

Fig. 2 Illustration of DUCG model

and posterior probability of events can be calculated and simplified by using the assumptions later .

Assumption 1 [14, 15]. Any state of a variable cannot simultaneously result in any state of the same variable. According to the rule of contradiction, an event itself cannot be both cause and consequence simultaneously, so that it breaks the directed cyclic cases in DUCG.

Assumption 2 [14, 15]. During an absorption weighting event, when there is more than one absorption event, they have the same opportunity to absorb the absorption event. Thus, the DUCG model with directed cyclic cases can be simplified by applying the assumptions.

2.2 Intuitionistic fuzzy sets

Definition 3 [23, 24]. Let X be the universe of discourse with a generic element x . Then an IFs M can be defined as

$$M = \{(x, \mu_M(x), \nu_M(x))|x \in X\}$$

where $\mu_M(x)$ and $\nu_M(x)$ are respectively the degrees of membership and non-membership of element x. Meanwhile, $\mu_M(x)$ and $\nu_M(x)$ conform to the rules that $\mu_M(x) \in [0, 1]$, $\nu_M(x) \in [0, 1]$ and $0 \leq \mu_M(x) + \nu_M(x) \leq 1$. In the proposed DUCG model, $\mu_M(x)$ and $\nu_M(x)$ denote the probabilities of occurrence and non-occurrence of the events x respectively. For convenience, the intuitionistic fuzzy number (IFNs) is represented as the form of $a = (\mu_a, \nu_a)$, which conform to the rules $\mu_a \in [0, 1]$, $\nu_a \in [0, 1]$ and $0 \leq \mu_a + \nu_a \leq 1$.

Definition 4 [23, 24]. Suppose that $a_1 = (\mu_1, \nu_1)$ and $a_2 = (\mu_2, \nu_2)$ are IFNs. Then the basic operators are defined as:

(1) $a_1 \oplus a_2 = (\mu_1 + \mu_2 - \mu_1\mu_2, \nu_1\nu_2)$
(2) $a_1 \otimes a_2 = (\mu_1\mu_2, \nu_1 + \nu_2 - \nu_1\nu_2)$
(3) $\lambda a_1 = (1 - (1 - \mu_1)^\lambda, {\nu_1}^\lambda), \lambda > 0$
(4) ${a_1}^\lambda = ({\mu_1}^\lambda, 1 - (1 - \nu_1)^\lambda), \lambda > 0$

where λ is a positive real number. While in DUCG model, expression (1) refers to the probability of which event a_1 and a_2 occur separately, and expression (2) denotes the probability of which event a_1 and a_2 occur simultaneously.

Definition 5 [25, 26]. Suppose that M and N are IFs. Then the division operator is defined as:

$$M/N = \{(x, \mu_{M/N}(x), \nu_{M/N}(x))|x \in X\}$$

where

$$\mu_{M/N}(x) = \begin{cases} \dfrac{\mu_M(x)}{\mu_N(x)}, & \text{if } \mu_M(x) \leq \mu_N(x) \text{ and } \nu_M(x) \geq \nu_N(x) \text{ and } \mu_N(x) \neq 0 \\ & \text{and } \mu_M(x)\nu_N(x) - \mu_N(x)\nu_M(x) \geq \mu_M(x) - \mu_N(x) \\ 1, & \text{others} \end{cases}$$

and

$$\nu_{M/N}(x) = \begin{cases} \frac{\nu_M(x) - \nu_N(x)}{1 - \nu_N(x)}, & \text{if } \mu_M(x) \leq \mu_N(x) \text{ and } \nu_M(x) \geq \nu_N(x) \text{ and } \mu_N(x) \neq 0 \\ & \text{and } \mu_M(x)\nu_N(x) - \mu_N(x)\nu_M(x) \geq \mu_M(x) - \mu_N(x) \\ 0, & \text{others} \end{cases}$$

The division operator is used to compute the conditional probability of the events.

Definition 6 [27, 28]. Let $X = \{x_1, x_2, \ldots x_n\}$ be a finite set, the normalized Hamming distance of IFs M and N is defined as

$$d(M, N) = \frac{1}{2n} \sum_{i=1}^{n} (|\mu_M(x_i) - \mu_N(x_i)| + |\nu_M(x_i) - \nu_N(x_i)| + |\mu_N(x_i) - \mu_M(x_i) + \nu_N(x_i) - \nu_M(x_i)|) \tag{6}$$

Definition 7 [29, 30]. The intuitionistic fuzzy weighted averaging (IFWA) operator is expressed as:

$$\begin{aligned} IFWA(a_1, a_2, \cdots, a_n) &= \lambda_1 a_1 \oplus \lambda_2 a_2 \oplus \cdots \oplus \lambda_n a_n \\ &= \left(1 - \prod_{i=1}^{n} (1 - \mu_i)^{\lambda_i}, \prod_{i=1}^{n} {\nu_i}^{\lambda_i}\right) \end{aligned} \tag{7}$$

where $\lambda = (\lambda_1, \lambda_2, \ldots \lambda_n)^T$ is the weight vector of the IFNs $a_i (i = 1, 2, \ldots, n)$, λ_i subject to $\lambda_i \in [0, 1]$ and $\sum_{i=1}^{n} \lambda_i = 1$.

Definition 8 [29, 30]. The intuitionistic fuzzy weighted geometric (IFWG) operator is expressed as:

$$IFWG(a_1, a_2, \cdots, a_n) = {a_1}^{\lambda_1} \otimes {a_2}^{\lambda_2} \otimes \cdots \otimes {a_n}^{\lambda_n} = \left(\prod_{i=1}^{n} {\mu_i}^{\lambda_i}, 1 - \prod_{i=1}^{n} (1 - \nu_i)^{\lambda_i} \right) \tag{8}$$

where $\lambda_i \in [0, 1]$ and $\sum_{i=1}^{n} \lambda_i = 1$ and $\lambda = (\lambda_1, \lambda_2, \ldots \lambda_n)^T$ is the weight vector of the IFNs $a_i (i = 1, 2, \ldots, n)$. The IFWA and IFWG operators take the weighted information of different events into account, so that they show the different importance degree of different factors in the proposed model.

3 The proposed IFDUCG model

3.1 Model establishment

In traditional DUCG, the probability of basic events and connection events in the DUCG model is often considered to be an exact number. However, in many cases, the events are complicated, variable and uncertain due to lack of information or the limited experienced knowledge, which will result in the fact that the traditional DUCG model fails in handling with the uncertain problems with incomplete knowledge. In this paper, the DUCG model combined with intuitionistic fuzzy set is proposed to provide an effective method to describe the uncertain events, which is helpful for dealing with the problems in traditional DUCG model.

Let B_1, B_4 be basic event variables, X_2, X_3 be node event variables, and $F_{2,1}$, $F_{2,3}$, $F_{3,2}$, $F_{3,4}$ connection event variables. And there is a one-way arc from B_1 to X_2 and B_4 to X_3, and double-direction arc between X_2 and X_3. Then $F_{2;1}$ is the probability from the variable B_1 to variable X_2, $F_{2;3}$ is the probability from the variable X_3 to variable X_2, $F_{3;2}$ is the probability from the variable X_2 to variable X_3, $F_{3;4}$ is the probability from the variable B_4 to variable X_3. Suppose that every event variables have three states, for example, B_1 has states of B_{11}, B_{12}, B_{13}. It means B_{11} has different possibilities to be the cause of X_{21}, X_{22}, X_{23}, where the probabilities depend on the variables of $F_{21;11}$, $F_{21;12}$, $F_{21;13}$. It is the same with the states of other variables. The causal relationship between different variables can be shown in Fig. 3.

The process of the establishment of IFDUCG model consists of the following steps:

(1) Construct the causal relationship among different events of IFDUCG based on knowledge of experienced experts firstly. In order to express the model clearly, the example shown in Fig. 1 is simplified as Fig. 3. Then we can acquire the relations among B_1, X_2, X_3, B_4 and form the causality diagram according to the given information and basic rules of DUCG;

(2) Obtain the expression of node events according to the causality, as shown in Fig. 3, the node events of X_2, X_3 are expressed as (9)–(10).

$$X_2 = F_{2;1} B_1 + F_{2;3} X_3 \tag{9}$$

$$X_3 = F_{3;4} B_4 + F_{3;2} X_2 \tag{10}$$

The basic principle of the conventional DUCG is that variables can be obtained as crisp numbers from experts. But the information is usually uncertain and fuzzy due to the time pressure, scarcity of information, and the technicians limited knowledge about the professional problem. Thus, the F- type variables take the form of intuitionistic fuzzy sets. The membership and non-membership degrees of intuitionistic fuzzy number can be acquired by triangular function, trapezoid function, normal function and so on, which are based on the knowledge of domain technicians. For example, $f_{2;1}$ can be expressed as (11).

$$f_{2;1} = \Pr(F_{2;1}) = \begin{pmatrix} f_{21;11} & f_{21;12} & f_{21;13} \\ f_{22;11} & f_{22;12} & f_{22;13} \\ f_{23;11} & f_{23;12} & f_{23;13} \end{pmatrix} = \begin{pmatrix} <\mu_{21;11}, \nu_{21;11}> & <\mu_{21;12}, \nu_{21;12}> & <\mu_{21;13}, \nu_{21;13}> \\ <\mu_{22;11}, \nu_{22;11}> & <\mu_{22;12}, \nu_{22;12}> & <\mu_{22;13}, \nu_{22;13}> \\ <\mu_{23;11}, \nu_{23;11}> & <\mu_{23;12}, \nu_{23;12}> & <\mu_{23;13}, \nu_{23;13}> \end{pmatrix} \tag{11}$$

(3) Expand the expression according to the rules mentioned in Section 2 and then simplify it by removing the unrelated variables from the expression. As a result, the final expression of any event is the combination of the basic event and the connection event, for example, the event of X_2 is expanded as (12).

$$X_2 = F_{2;1} B_1 + F_{2;3} (F_{3;4} B_4 + F_{3;2} X_2) = F_{2;1} B_1 + F_{2;3} F_{3;4} B_4 + F_{2;3} F_{3;2} X_2 \tag{12}$$

With refer to the assumptions in Section 2, X_2 can be simplified as (13).

$$X_2 = F_{2;1} B_1 + F_{2;3} F_{3;4}^{\{2\}} B_4 \tag{13}$$

thus X_{2k} can be expressed as (14).

$$X_{2k} = F_{2k;1} B_1 + F_{2k;3} F_{3k;4}^{\{2\}} B_4, \quad k = 1, 2, 3 \tag{14}$$

then we can get (15).

$$x_{2k} = \Pr(X_{2k}) = (r_{2;1}/r_2) a_{2k;1} b_1 + (r_{2;3}/r_2) a_{2k;3} \times (r_{3;4}/r_3^{\{2\}}) a_{3k;4} b_4, \quad k = 1, 2, 3 \tag{15}$$

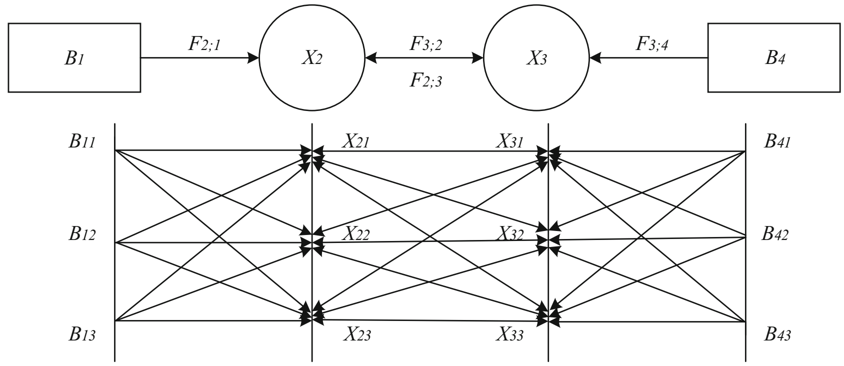

Fig. 3 Example of typical IFDUCG model

where

$$
\begin{aligned}
r_2 &= r_{2;1} + r_{2;3} \\
r_3 &= r_{3;2} + r_{3;4} \\
r_3^{\{2\}} &= r_{3;4} \\
a_{2k;1} &= (<\mu'_{2k;11}, \nu'_{2k;11}> \quad <\mu'_{2k;12}, \nu'_{2k;12}> \quad <\mu'_{2k;13}, \nu'_{2k;13}>) \\
a_{2k;3} &= (<\mu'_{2k;31}, \nu'_{2k;31}> \quad <\mu'_{2k;32}, \nu'_{2k;32}> \quad <\mu'_{2k;33}, \nu'_{2k;33}>) \\
a_{3k;4} &= (<\mu'_{3k;41}, \nu'_{3k;41}> \quad <\mu'_{3k;42}, \nu'_{3k;42}> \quad <\mu'_{3k;43}, \nu'_{3k;43}>)
\end{aligned}
$$

$$
b_1 = \begin{pmatrix} <\mu''_{11}, \nu''_{11}> \\ <\mu''_{12}, \nu''_{12}> \\ <\mu''_{13}, \nu''_{13}> \end{pmatrix}
$$

and

$$
b_4 = \begin{pmatrix} <\mu''_{41}, \nu''_{41}> \\ <\mu''_{42}, \nu''_{42}> \\ <\mu''_{43}, \nu''_{43}> \end{pmatrix}
$$

(4) Compute the probability of the interested events and then sort the results based on the intuitionistic fuzzy theory. In the case of uncertain probabilities quantified by intuitionistic fuzzy sets, probabilistic inference are made by posteriori probabilities which is defined as (16).

$$
\Pr\{H_{kj}|E\} = \Pr\{H_{kj}E\} / \Pr\{E\} \tag{16}
$$

where H_{kj} represents the case related to B-, X- type events, and E represents the given events related to B-, X- type events. $\Pr\{\cdot\}$ means the probability of the event. Suppose that it is interested in the events of $B_{1,2}|X_{2,2}X_{3,2}$, and $B_{4,2}|X_{2,2}X_{3,2}$ in Fig. 3, namely, we want to know which is more likely to cause the event of $X_{2,2}X_{3,2}$ between $B_{1,2}$ and $B_{4,2}$. Thus we can compute the posteriori probabilities of $B_{1,2}|X_{2,2}X_{3,2}$, and $B_{4,2}|X_{2,2}X_{3,2}$ in the form of intuitionistic fuzzy numbers and then compare the results based on the computation rules of the intuitionistic fuzzy theory [31–33].

$$
\Pr\{B_{1,2}|X_{2,2}X_{3,2}\} = \Pr\{B_{1,2}X_{2,2}X_{3,2}\} / \Pr\{X_{2,2}X_{3,2}\}
$$

3.2 Reasoning process of IFDUCG model based on TOPSIS

IFDUCG considers the fuzzy knowledge, involving more abundant information than traditional DUCG. In order to combine more domain information with IFDUCG model, a TOPSIS [34] method is incorporated into IFDUCG for knowledge reasoning and decision making in the paper, which locate optimal and non-optimal results for the rules based on the experience of experts to provide users more efficient decision options. The framework based on a combined application of the method of IFDUCG and TOPSIS is shown in Fig. 4. The procedure for IFDUCG model based on TOPSIS is given as follows: (1) Construct the intuitionistic fuzzy matrix according to the opinions of experienced experts. Suppose $\tilde{r}_{ij} = (\mu_{ij}, \nu_{ij})$ be an intuitionistic fuzzy number, then the matrix can be acquired as (17).

$$
R_{m\times n} = \begin{pmatrix} <\mu_{11}, \nu_{11}> & <\mu_{12}, \nu_{12}> & \cdots & <\mu_{1n}, \nu_{1n}> \\ <\mu_{21}, \nu_{21}> & <\mu_{22}, \nu_{22}> & \cdots & <\mu_{2n}, \nu_{2n}> \\ \vdots & \vdots & \ddots & \vdots \\ <\mu_{m1}, \nu_{m1}> & <\mu_{m2}, \nu_{m2}> & \cdots & <\mu_{mn}, \nu_{mn}> \end{pmatrix} \tag{17}
$$

According to IFDUCG model, we can get the posteriori probabilities by means of intuitionistic fuzzy rules, which constitutes the elements of intuitionistic fuzzy matrix. The elements are calculated by combining IFWA and IFWG operators.

$$
\tilde{r}_{ij} = (\mu_{ij}, \nu_{ij}) = \mathop{\oplus}_{p=1}^{s} \left(\hat{\lambda}_p (\tilde{a}_{p1}^{\lambda_{p1}} \otimes \tilde{a}_{p2}^{\lambda_{p2}} \otimes \cdots \otimes \tilde{a}_{pq}^{\lambda_{pq}})\right), \quad q = 1, 2, \ldots, t
$$

(2) Obtain intuitionistic fuzzy ideal solution. Let $\hat{r} = (1, 0)$ be the ideal solution, namely, the incidence probability of events in IFDUCG is 1, then we can obtain the reference matrix as (18).

$$
\bar{R}_{m\times n} = \begin{pmatrix} \hat{r} & \hat{r} & \cdots & \hat{r} \\ \hat{r} & \hat{r} & \cdots & \hat{r} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{r} & \hat{r} & \cdots & \hat{r} \end{pmatrix} \tag{18}
$$

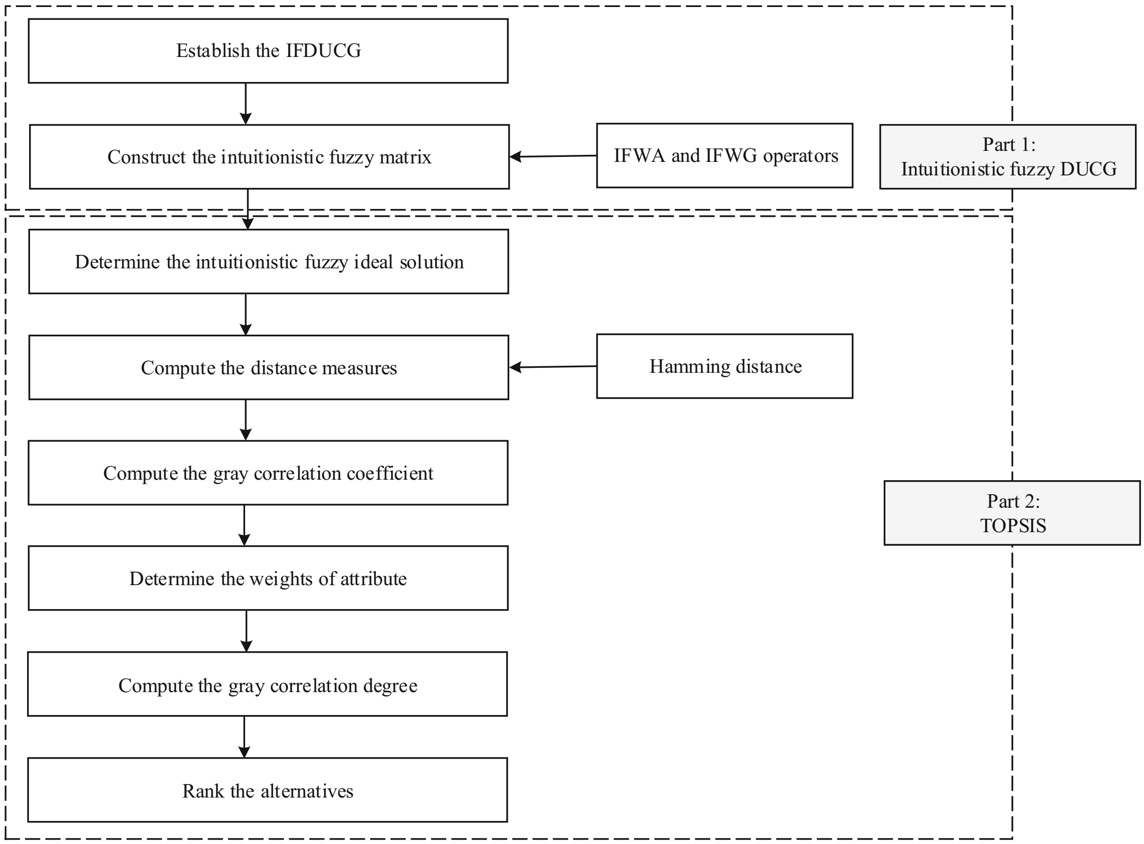

Fig. 4 The framework based on a combined application of the method of IFDUCG and TOPSIS

(3) Compute the distance of intuitionistic fuzzy numbers between $\tilde{r}_{ij}$ and $\hat{r}$ according to the expression of Hamming distance(HD) shown as (6).

(4) Compute the gray correlation coefficient (GCC) between the intuitionistic fuzzy number $\tilde{r}_{ij}$ and the intuitionistic fuzzy ideal solution $\hat{r}$, where the gray correlation coefficient [34] ξ_{ij} is defined as (19).

$$\xi_{ij} = \frac{\Delta_{\min} + \rho\Delta_{\max}}{\Delta_{ij} + \rho\Delta_{\max}}, \quad i = 1, 2, \ldots, m; \ j = 1, 2, \ldots, n \tag{19}$$

where $\Delta_{\min} = \min\{\Delta_{ij}, i = 1, 2, \cdots, m; j = 1, 2, \cdots, n\}$, $\Delta_{\max} = \max\{\Delta_{ij}, i = 1, 2, \cdots, m; j = 1, 2, \cdots, n\}$, $\Delta_{ij} = d\{\tilde{r}_{ij}, \hat{r}\}$, $\rho \in [0, 1]$. In general, the value of ρ is 0.5.Usually, the larger the value ξ_{ij}, the closer the value $\tilde{r}_{ij}$ to the ideal solution, which denotes the greater the probability of occurrence in IFDUCG.

(5) Determine the weights of attribute. The attribute may not be equally important. Let ω_j be the grade of importance and it can be given by the domain expert¡¯s knowledge. The more important the event in IFDUCG, the larger the coefficient of attribute.

(6) Compute the gray correlation degree [34] ψ_i which is defined as (20).

$$\psi_i = \sum_{j=1}^{n} \omega_j \xi_{ij}, i = 1, 2, \cdots, m. \tag{20}$$

(7) Rank the alternatives. After the value of gray correlation degree (GCD) of every alternative is calculated, rank the alternatives based on descending orders of ψ_i. The greater the value of ψ_i , the higher probability of the events.

3.3 Validity of IFDUCG model

To demonstrate the effectiveness and superiority of the IFDUCG model, we did some experiments where we compared the performance of the proposed method with the well-known approaches of Bayesian network(BN) [2], Petri nets (PN) [4, 5] and DUCG [14].

For comparison, the typical instances are selected from the illustrative example in the reference [5]. Then we applied BN, DUCG and IFDUCG algorithm for a fault diagnosis reasoning system respectively. Based on the mentioned information in reference [5], we mapped the fault

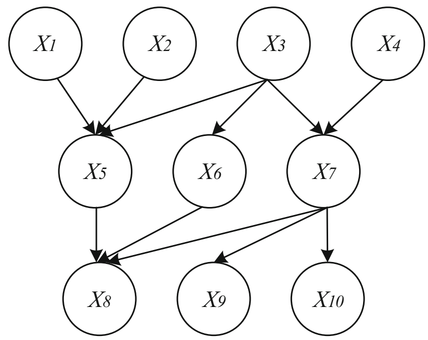

Fig. 5 Bayesian network of the example

diagnosis system into BN, DUCG and IFDUCG which are shown in Figs. 5 and 6. Although the schematic diagram of DUCG and IFDUCG are the same, the reasoning process are different from each other.

By the experience on Bayesian network learning presented in reference [2], we know that the reasoning process of BN relates with the prior and conditional probabilities of events. And for a given event A, the full probability of event can be obtained by [2]

$$\mathrm{P}\{A\} = \sum_{i=1}^{n} \mathrm{P}\{B_i\} / \mathrm{P}\{A|B_i\}$$

According to the knowledge and experiences given by experts in [5], the prior and conditional probabilities tables of BN can be shown as Tables 1, 2 and 3. Then we can obtain the full probability of event as Table 4. From Table 4, it is easy to obtain that the event of X_8 is the most possible fault in the system.

Based on DUCG model presented in reference [14], we describe the knowledge by modules, which are composed of child variables and its parent variables, as shown in Fig. 6.

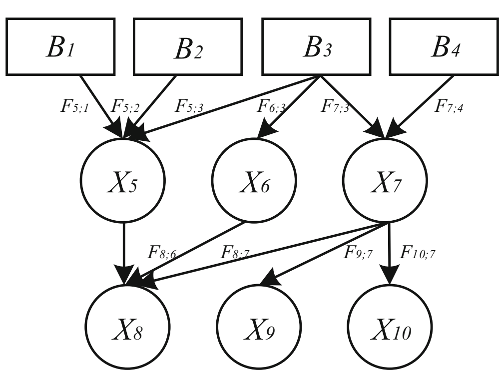

Fig. 6 DUCG of the example

Table 1 Prior probability of event

	X_1	X_2	X_3	X_4	X_5	X_6	X_7
Prior probability	0.8	0.9	0.7	0.8	0.2	0.3	0.2

According to the given rules of DUCG model, we can obtain that

$$\begin{aligned} X_8 &= F_{8;5}X_5 + F_{8;6}X_6 + F_{8;7}X_7 \\ &= F_{8;5}(F_{5;1}B_1 + F_{5;2}B_2 + F_{5;3}B_3) + F_{8;6}F_{6;3}B_3 \\ &\quad + F_{8;7}(F_{7;3}B_3 + F_{7;4}B_4) \\ &= F_{8;5}F_{5;1}B_1 + F_{8;5}F_{5;2}B_2 + (F_{8;5}F_{5;3} + F_{8;6}F_{6;3} \\ &\quad + F_{8;7}F_{7;3})B_3 + F_{8;7}F_{7;4}B_4 \end{aligned} \tag{21}$$

$$\begin{aligned} X_9 &= F_{9;7}(F_{7;3}B_3 + F_{7;4}B_4) \\ &= F_{9;7}F_{7;3}B_3 + F_{9;7}F_{7;4}B_4 \end{aligned} \tag{22}$$

$$\begin{aligned} X_{10} &= F_{10;7}(F_{7;3}B_3 + F_{7;4}B_4) \\ &= F_{10;7}F_{7;3}B_3 + F_{10;7}F_{7;4}B_4 \end{aligned} \tag{23}$$

Given that the knowledge parameters of DUCG model are set as Tables 5 and 6. The results are shown in Table 7. From Table 7, we can draw that the fault of X_8 may occur with the biggest probability of 0.4052.

Taking the same example as mentioned above, we establish the IFDUCG model as follows.

(1) According to the event rules referred in the paper, and based on Fig. 6, we can obtain event X_8, X_9 and X_{10} as (24)–(26).

$$\begin{aligned} X_8 &= F_{8;5}X_5 + F_{8;6}X_6 + F_{8;7}X_7 \\ &= F_{8;5}(F_{5;1}B_1 + F_{5;2}B_2 + F_{5;3}B_3) + F_{8;6}F_{6;3}B_3 \\ &\quad + F_{8;7}(F_{7;3}B_3 + F_{7;4}B_4) \\ &= F_{8;5}F_{5;1}B_1 + F_{8;5}F_{5;2}B_2 + (F_{8;5}F_{5;3} \\ &\quad + F_{8;6}F_{6;3} + F_{8;7}F_{7;3})B_3 + F_{8;7}F_{7;4}B_4 \end{aligned} \tag{24}$$

$$\begin{aligned} X_9 &= F_{9;7}(F_{7;3}B_3 + F_{7;4}B_4) \\ &= F_{9;7}F_{7;3}B_3 + F_{9;7}F_{7;4}B_4 \end{aligned} \tag{25}$$

$$\begin{aligned} X_{10} &= F_{10;7}(F_{7;3}B_3 + F_{7;4}B_4) \\ &= F_{10;7}F_{7;3}B_3 + F_{10;7}F_{7;4}B_4 \end{aligned} \tag{26}$$

(2) Events are described as IFNs based on the definitions of linguistic terms in [36]. Then the events are represented as Tables 8 and 9. According to (24)–(26),

Table 2 Conditional probability of event

	X_1	X_2	X_3	X_4
X_5	0.5	0.3	0.2	0.0
X_6	0.0	0.0	0.5	0.0
X_7	0.0	0.0	0.3	0.2

Table 3 Conditional probability of event

	X_5	X_6	X_7
X_8	0.4	0.2	0.2
X_9	0.0	0.0	0.45
X_{10}	0.0	0.0	0.15

Table 4 Full probability of event

	X_8	X_9	X_{10}
Full probability	0.2796	0.1431	0.0954

Table 5 Probability of event

	B_1	B_2	B_3	B_4
Probability	0.8	0.9	0.7	0.8

Table 6 The value of parameter

Parameter	Value	Parameter	Value
$a_{5;1}$	0.5	$r_{5;1}$	0.3
$a_{5;2}$	0.3	$r_{5;2}$	0.4
$a_{5;3}$	0.2	$r_{5;3}$	0.6
$a_{6;3}$	1.0	$r_{6;3}$	0.2
$a_{7;3}$	0.5	$r_{7;3}$	0.2
$a_{7;4}$	0.5	$r_{7;4}$	0.4
$a_{8;5}$	0.9	$r_{8;5}$	0.1
$a_{8;6}$	0.8	$r_{8;6}$	0.2
$a_{8;7}$	0.9	$r_{8;7}$	0.2
$a_{9;7}$	0.95	$r_{9;7}$	0.2
$a_{10;7}$	0.85	$r_{10;7}$	0.6

Table 7 Probability of event

	X_8	X_9	X_{10}
Probability	0.4052	0.3642	0.3258

Table 8 IFNs description of event

	IFNs
B_1	$< 0.75, 0.15 >$
B_2	$< 0.85, 0.10 >$
B_3	$< 0.65, 0.25 >$
B_4	$< 0.75, 0.15 >$

Table 9 The intuitionistic fuzzy representation of Parameter

Parameter	Intuitionistic fuzzy representation
$a_{5;1}$	$< 0.50, 0.40 >$
$a_{5;2}$	$< 0.25, 0.65 >$
$a_{5;3}$	$< 0.15, 0.80 >$
$a_{6;3}$	$< 0.95, 0.05 >$
$a_{7;3}$	$< 0.50, 0.40 >$
$a_{7;4}$	$< 0.50, 0.40 >$
$a_{8;5}$	$< 0.85, 0.10 >$
$a_{8;6}$	$< 0.75, 0.15 >$
$a_{8;7}$	$< 0.85, 0.10 >$
$a_{9;7}$	$< 0.95, 0.05 >$
$a_{10;7}$	$< 0.75, 0.15 >$

Table 10 Results of IFDUCG

	IFNs	GCC	GCD
X_8	$< 0.3509, 0.5130 >$	1.0000	0.1000
X_9	$< 0.3408, 0.5338 >$	0.9901	0.0990
X_{10}	$< 0.2690, 0.5830 >$	0.9252	0.0925

Table 11 Results of different methods

Methods	Comparative results
BN	$P(X_8) > P(X_9) > P(X_{10})$
FPN	$P(X_8) > P(X_9) > P(X_{10})$
LRPN	$P(X_8) > P(X_9) > P(X_{10})$
DUCG	$P(X_8) > P(X_9) > P(X_{10})$
IFDUCG	$P(X_8) > P(X_9) > P(X_{10})$

we can have the intuitionistic fuzzy representation of event X_8, X_9 and X_{10} as shown in Table 10 by applying intuitionistic fuzzy operators.

(3) Given that the ideal solution is $\hat{r} =< 1, 0 >$, then we compute the Hamming distance between the given intuitionistic fuzzy numbers and the ideal solution as (6).

$$d(X_8, \hat{r}) = \frac{1}{2}(|1 - 0.3509| + |0 - 0.5130| + |1 - 0.3509 + 0 - 0.5130|) = 0.6491$$

$$d(X_9, \hat{r}) = 0.6592$$

$$d(X_{10}, \hat{r}) = 0.7310$$

(4) Compute the gray correlation coefficient (GCC) between the intuitionistic fuzzy number and the intuitionistic fuzzy ideal solution as Eq. (9).

$$\Delta_{\min} = 0.6491$$

$$\Delta_{\max} = 0.7310$$

$$\xi_{X_8} = \frac{0.6491 + 0.5 * 0.7310}{0.6491 + 0.5 * 0.7310} = 1.000$$

$$\xi_{X_9} = 0.9901$$

$$\xi_{X_{10}} = 0.9252$$

(5) Supposed that the attributes are equally important in the system. Given that $\omega_j = 0.1$, then we can calculate the gray correlation degree (GCD) by (20).

(6) Rank the GCD based on descending orders. It is obvious that $\psi_{X_8} > \psi_{X_9} > \psi_{X_{10}}$. Then we know that the probability of event X_8 is bigger than event X_9 and X_{10}. The result conforms to the one presented in [5].

Table 11 presents the results of different methods. From Table 11, we know that the same ranking results is produced by the approaches of BN, FPN, LRPN,DUCG and IFDUCG. The comparison results indicate that the established IFDUCG model is effective. Morever, the proposed IFDUCG model has advantages over the other methods which can be listed as follows. First, by introducing IFs, the proposed IFDUCG model can successfully deal with the kinds of intuitionistic information in knowledge representation. The IFs considering both membership and non-membership of an event are rather appropriate for the description of the uncertain information. In this way, the proposed model provides a better way for experts to express their preference. Second, through the different integrated operators of IFs and the combination with TOPSIS, the IFDUCG model can consider more abundant information in the process of information inference and reduce the information loss to a certain extent. Thus, it contributes in providing much more reliable references for making decisions. As a result, conclusion can be drawn that the IFDUCG proposed in this paper is effective.

4 Industrial application

4.1 IFDUCG used for root cause analysis of aluminum reduction cell condition

The proposed IFDUCG model is applied to root cause analysis of aluminum reduction cell condition. In general, in the industrial process of aluminum reduction, an abnormal aluminum reduction cell condition may appear randomly, and it is expected to judge what happens, which can be provided to workers to decide what can be done to adjust the current condition to a normal one. Firstly, some typical parameters are selected to build the IFDUCG model, which includes operating parameters of the setting voltage (SV), the numbers of AlF_3 addition (NoA) and aluminum tapping (AlT), and the state parameters of bath temperature (BaT), molecular ratio (MoR), aluminum level (AL) and electrolyte level (EL). Based on the causality of these parameters given by knowledgeable experts, the IFDUCG model can be drawn as Fig. 7, among which the root node is of fuzzification, for example, the state of SV is often divided into three grades, namely, high (greater than 4.25V), normal (4.05-4.25V), low (less than 4.05V). Similarly, the other parameters can also be divided into three grades. Usually, the division of the unobservable state is determined by domain technician.

Usually, membership function is a valid method to describe fuzzy number. Triangular fuzzy number is introduced in the paper to express the membership and non-membership. The triangular fuzzy function [35] is defined as (27)–(28).

$$\mu(x) = \begin{cases} (x - \underline{a})\omega_a/(a - \underline{a}), & \underline{a} \le x < a \\ \omega_a, & x = a \\ (x - \overline{a})\omega_a/(a - \overline{a}), & a < x < \overline{a} \\ 0, & x < \underline{a} \text{ or } x > \overline{a} \end{cases} \quad (27)$$

$$\nu(x) = \begin{cases} [a - x + u_a(x - \underline{a})]/(a - \underline{a}), & \underline{a} \le x < a \\ u_a, & x = a \\ [a - x + u_a(x - \overline{a})]/(a - \overline{a}), & a < x < \overline{a} \\ 1, & x < \underline{a} \text{ or } x > \overline{a} \end{cases} \quad (28)$$

According to (27)–(28), suppose that w_a=0.9, u_a=0.1 and the setting voltage is 4.09V, then the intuitionistic fuzzy representation of the setting voltage is $< 0.18, 0.82 >$, $< 0.72, 0.28 >$, $< 0.00, 1.00 >$ when the state is high, normal and low respectively, as shown in Table 12. Similarly, the intuitionistic fuzzy representation of the parameters can be acquired according to the aluminum reduction cell condition. The explanation of the parameter is shown in Table 13.

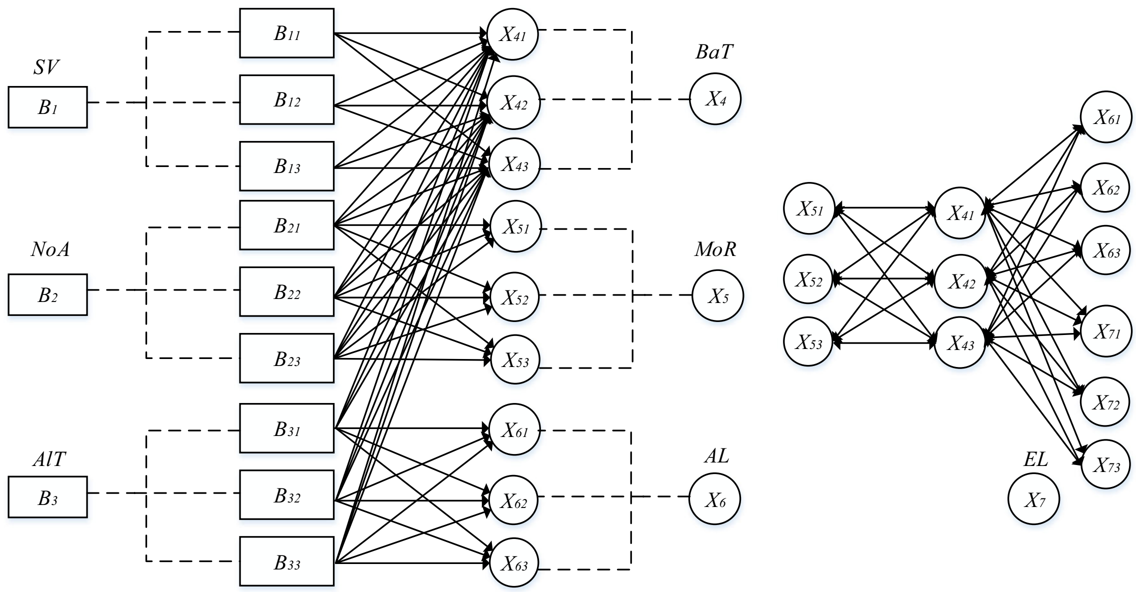

Fig. 7 The IFDUCG model based on aluminum reduction production

4.2 Illustration of the proposed approach

(1) Acquire the expression of node events based on Fig. 7 and then simplify them as (29)–(32) according to the assumption mention in the Section 2.

$$\begin{aligned} X_{4k} &= F_{4k;1}B_1+F_{4k;2}B_2+F_{4k;3}B_3 + F_{4k;5k}X_{5k} + F_{4k;6k}X_{6k} + F_{4k;7k}X_{7k} \\ &= F_{4k;1}B_1 + F_{4k;2}B_2 + F_{4k;3}B_3 + F_{4k;5k}(F_{5k;2}B_2 + F_{5k;4k}X_{4k}) \\ &\quad +F_{4k;6k}(F_{6k;3}B_3 + F_{6k;4k}X_{4k}) + F_{4k;7k}F_{7k;4k}X_{4k} \\ &= F_{4k;1}B_1 + (F_{4k;2} + F_{4k;5k}F^{\{4\}}_{5k;2})B_2 + (F_{4k;3} + F_{4k;6k}F^{\{4\}}_{6k;3})B_3 \end{aligned} \tag{29}$$

$$\begin{aligned} X_{5k} &= F_{5k;4k}F^{\{5\}}_{4k;1}B_1 + \left(F_{5k;4k}F^{\{5\}}_{4k;2} + F_{5k;2}\right)B_2 \\ &\quad + \left(F_{5k;4k}F^{\{5\}}_{4k;3} + F_{5k;4k}F^{\{5\}}_{4k;6k}F^{\{4\}}_{6k;3}\right)B_3 \end{aligned} \tag{30}$$

$$\begin{aligned} X_{6k} &= F_{6k;4k}F^{\{6\}}_{4k;1}B_1 + \left(F_{6k;4k}F^{\{6\}}_{4k;2} + F_{6k;4k}F^{\{6\}}_{4k;5k}F^{\{4\}}_{5k;2}\right) \\ &\quad \times B_2 + (F_{6k;3} + F_{6k;4k}F^{\{6\}}_{4k;3})B_3 \end{aligned} \tag{31}$$

$$\begin{aligned} X_{7k} &= F_{7k;4k}F^{\{7\}}_{4k;1}B_1 + \left(F_{7k;4k}F^{\{7\}}_{4k;2} + F_{7k;4k}F^{\{7\}}_{4k;5k}F^{\{4\}}_{5k;2}\right)B_2 \\ &\quad + \left(F_{7k;4k}F^{\{7\}}_{4k;3} + F_{7k;4k}F^{\{7\}}_{4k;6k}F^{\{4\}}_{6k;3}\right)B_3 \end{aligned} \tag{32}$$

(2) Get the intuitionistic fuzzy representation of variables.

Table 12 Intuitionistic fuzzy representation of the parameters

Linguistic terms	SV	NoA	AlT
High	$< 0.18, 0.82 >$	$< 0.10, 0.70 >$	$< 0.20, 0.20 >$
Normal	$< 0.72, 0.28 >$	$< 0.75, 0.20 >$	$< 0.85, 0.15 >$
Low	$< 0.00, 1.00 >$	$< 0.00, 1.00 >$	$< 0.00, 1.00 >$

According to the basic and aggregated operators of intuitionistic fuzzy numbers, we can calculate the intuitionistic fuzzy number of parameters under different states, for example, when bath temperature is normal (X_{42}), molecular ratio normal (X_{52}), aluminum level high (X_{61}) and electrolyte level low (X_{73}), then we need to acquire the intuitionistic fuzzy representation of different states of the setting voltage, the numbers of AlF_3 addition and aluminum tapping, namely the fuzzy representation of posterior probability of events $B_{1k}|X_{42}X_{52}X_{61}X_{73}$, $B_{2k}|X_{42}X_{52}X_{61}X_{73}$ and $B_{3k}|X_{42}X_{52}X_{61}X_{73}$ where k=1,2,3. The results are shown as Table 14.

(3) Suppose that the intuitionistic fuzzy ideal solution is $< 1, 0 >$, then we can compute the Hamming distance between intuitionistic fuzzy numbers and the ideal solution. The results are shown as Table 15.

(4) Compute the gray correlation coefficient (GCC) between the intuitionistic fuzzy number and the intuitionistic fuzzy ideal solution. The results are shown as Table 16.

(5) Determine the weights w_i, then compute the gray correlation degree (GCD). Weights are decided according to the technicians assessments in the experiment. The results are shown as Table 17.

From Table 17, we can know that the gray correlation degree of the event of $B_{31}|X_{42}X_{52}X_{61}X_{73}$ acquires the greatest value among all, which manifests that B_{31} is the most likely happening events when in the evidence of $X_{42}X_{52}X_{61}X_{73}$, namely, the most likely state is that aluminum tapping (AlT) is in high state when the state parameters

Table 13 The explanation of the variables

Variables	Explanation in detail
Setting Voltage (SV)	The proper voltage can keep the energy of the cell balanced.
Numbers of AlF_3 addition (NoA)	The suitable NoA can keep the energy and the material of the cell balanced. It can have influence on BT and MoR.
Aluminum Tapping (AlT)	The suitable AlT can keep the energy and the material of the cell balanced. It can have influence on BT and MoR.
Bath Temperature (BT)	It can have an effect on the operation of cell.
Molecular Ratio (MoR)	It influences the alumina solubility.
Aluminum Level (AL)	The advisable AL can keep the energy of the cell balanced and the SV stable.
Electrolyte Level (EL)	It can keep the stability of heat and the cell itself.

Table 14 The intuitionistic fuzzy representation of events

Events	IFN
$B_{11}\|X_{42}X_{52}X_{61}X_{73}$	$< 0.7883, 0.2100 >$
$B_{12}\|X_{42}X_{52}X_{61}X_{73}$	$< 0.7091, 0.2390 >$
$B_{13}\|X_{42}X_{52}X_{61}X_{73}$	$< 0.0000, 1.0000 >$
$B_{21}\|X_{42}X_{52}X_{61}X_{73}$	$< 0.7065, 0.1010 >$
$B_{22}\|X_{42}X_{52}X_{61}X_{73}$	$< 0.7091, 0.1810 >$
$B_{23}\|X_{42}X_{52}X_{61}X_{73}$	$< 0.0000, 1.0000 >$
$B_{31}\|X_{42}X_{52}X_{61}X_{73}$	$< 0.8895, 0.0100 >$
$B_{32}\|X_{42}X_{52}X_{61}X_{73}$	$< 0.7994, 0.0895 >$
$B_{33}\|X_{42}X_{52}X_{61}X_{73}$	$< 0.0000, 1.0000 >$

Table 15 The Hamming distance between different events

Events	HD
$B_{11}\|X_{42}X_{52}X_{61}X_{73}$	0.2108
$B_{12}\|X_{42}X_{52}X_{61}X_{73}$	0.2650
$B_{13}\|X_{42}X_{52}X_{61}X_{73}$	1.0000
$B_{21}\|X_{42}X_{52}X_{61}X_{73}$	0.1702
$B_{22}\|X_{42}X_{52}X_{61}X_{73}$	0.2360
$B_{23}\|X_{42}X_{52}X_{61}X_{73}$	1.0000
$B_{31}\|X_{42}X_{52}X_{61}X_{73}$	0.0603
$B_{32}\|X_{42}X_{52}X_{61}X_{73}$	0.1450
$B_{33}\|X_{42}X_{52}X_{61}X_{73}$	1.0000

Table 16 The GCC among different IFNs

Events	GCC
$B_{11}\|X_{42}X_{52}X_{61}X_{73}$	0.7881
$B_{12}\|X_{42}X_{52}X_{61}X_{73}$	0.7324
$B_{13}\|X_{42}X_{52}X_{61}X_{73}$	0.3735
$B_{21}\|X_{42}X_{52}X_{61}X_{73}$	0.8359
$B_{22}\|X_{42}X_{52}X_{61}X_{73}$	0.7613
$B_{23}\|X_{42}X_{52}X_{61}X_{73}$	0.3735
$B_{31}\|X_{42}X_{52}X_{61}X_{73}$	1.0000
$B_{32}\|X_{42}X_{52}X_{61}X_{73}$	0.8685
$B_{33}\|X_{42}X_{52}X_{61}X_{73}$	0.3735

Table 17 The GCD of different events

Events	GCD
$B_{11}\|X_{42}X_{52}X_{61}X_{73}$	0.0778
$B_{12}\|X_{42}X_{52}X_{61}X_{73}$	0.1099
$B_{13}\|X_{42}X_{52}X_{61}X_{73}$	0.0486
$B_{21}\|X_{42}X_{52}X_{61}X_{73}$	0.1170
$B_{22}\|X_{42}X_{52}X_{61}X_{73}$	0.0914
$B_{23}\|X_{42}X_{52}X_{61}X_{73}$	0.0482
$B_{31}\|X_{42}X_{52}X_{61}X_{73}$	0.1987
$B_{32}\|X_{42}X_{52}X_{61}X_{73}$	0.0955
$B_{33}\|X_{42}X_{52}X_{61}X_{73}$	0.0336

Table 18 The situations of abnormal cell conditions

Group \ Variable	BaT	MoR	AL	EL
1	High	High	Normal	Normal
2	High	High	Low	Normal
3	High	High	Low	High
4	Normal	High	Low	Normal
5	Normal	High	Normal	Normal
6	High	High	Normal	High
7	Normal	Normal	Low	High
8	Normal	High	Low	High
9	Normal	High	Normal	High
10	Normal	Normal	Normal	High
11	High	Normal	High	Low
12	Normal	High	High	Low
13	Normal	Normal	High	Low
14	Normal	High	High	Normal
15	Normal	Normal	High	Normal
16	Low	Normal	Normal	Normal
17	Low	Low	Normal	Normal
18	High	Normal	Low	Normal
19	High	Normal	Low	High
20	High	Normal	Normal	Normal

of BaT is normal, MoR is normal, $AL4$ is high, and EL is low. Meanwhile, it can be acquired the same results from the experts according to the actual production. Thus, it shows that the result based on the developed method is consistent with the one provided by experts.

In order to demonstrate the effectiveness of the proposed method, the reasoning results of the method is compared with the actual situation of aluminum reduction production. 20 groups of abnormal cell conditions are adopted for analysis and demonstration as shown in Table 18. The results based on the proposed method and the actual production given by experts are shown as Table 19.

Based on the reasoning process, it can be inferred that the abnormal state along with the maximum gray correlation degree denotes the most likely cause of the cell condition in Table 19. Then it can be concluded that the root cause is the state possessed the maximum GCD value. And a conclusion can be drawn from the consistency in Table 19 that 19 groups obtain the same result with the results given by domain technicians among 20 groups.

4.3 Comparative analysis and discussion

To further demonstrate the validity of the proposed IFDUCG in root cause analysis of aluminum reduction cell condition, we do a comparative analysis with the method of fuzzy-Bayesian network for root cause analysis proposed in [37].

The results acquired through IFDUCG are displayed in Table 19. From Table 19, we can calculate that the accuracy of IFDUCG model achieves 95% which is as high as the accuracy of method in [37]. In comparison with the method in [37], the FBN defines the uncertainty of the variables on the model with a conditional probability table (CPT) while the construction of CPT requires a large amount of statistical data. Meanwhile, the expression and reasoning of the knowledge applicable to FBN in the case of single assignment is not applicable in the case of multiple assignments, and FBN requires complete knowledge. In contrast, IFDUCG allows for incomplete knowledge representation which can greatly reduce the workload and difficulty of building knowledge bases and reasoning. What's more, IFDUCG is suitable for both single assignment and multiple assignment situations. Most importantly, the IFDUCG can provide the events with possibility and impossibility as shown in Table 14, which is more credible than that of FBN with only possibility. It provides a better way for experts to express their preference by IFDUCG model. Therefore, it can draw a conclusion that the proposed method is more effective and useful in the application of the root cause determination of abnormal aluminum reduction cell conditions.

According to the analysis mentioned above, the advantages of the proposed IFDUCG can be summarized as follows:

(1) Experts' knowledge is hard to precisely express by crisp numbers under uncertain environment, however, intuitionistic fuzzy sets are effective methods to handle with these circumstances. Therefore, the study on DUCG model based on IFs is considerably important.

(2) In the proposed method, fuzziness can be reflected in the membership and non-membership degree of intuitionistic fuzzy sets. In this way, the vagueness of the original information can be kept and completely applied for final reasoning. Therefore, the proposed method is more competent in uncertain information representation and reasoning than the other methods considered, wherein the fuzziness in variable is ignored or distorted.

(3) The IFDUCG is based on intuitionistic fuzzy sets, which has been demonstrated to be effective in the DUCG. The proposed IFDUCG combined with TOPSIS captures more cognitive uncertainty of the engineers without knowledge distortion and loss. The influence of knowledge distortion and loss on the final result is important, and it can be surveyed by experiments. Therefore, the proposed method increases the reliability and stability of dealing uncertain information to a certain extent.

Table 19 The comparison of results

Group	SV			NoA			AlT			Result reasoned by model	Result given by experts	Consistency
	High	Normal	Low	High	Normal	Low	High	Normal	Low			
1	0.0503	0.0731	0.1800	0.0704	0.0465	0.0975	0.0603	0.1048	0.0630	SV Low	SV Low	Y
2	0.0557	0.1461	0.2200	0.0708	0.1096	0.1151	0.0607	0.0892	0.0763	SV Low	SV Normal	N
3	0.1463	0.0589	0.0433	0.0646	0.0265	0.0367	0.0517	0.0384	0.0900	SV High	SV High	Y
4	0.1598	0.1411	0.0636	0.1198	0.1059	0.0631	0.1482	0.0821	0.0440	SV High	SV High	Y
5	0.2100	0.0736	0.0659	0.0987	0.0492	0.0557	0.1622	0.1076	0.0456	SV High	SV High	Y
6	0.0876	0.1117	0.0592	0.2140	0.1192	0.0587	0.1109	0.0697	0.0410	NoA High	NoA High	Y
7	0.0800	0.0653	0.0580	0.1500	0.0495	0.0494	0.0626	0.0955	0.0405	NoA High	NoA High	Y
8	0.0857	0.1277	0.0575	0.2200	0.1138	0.0571	0.1030	0.0800	0.0398	NoA High	NoA High	Y
9	0.0887	0.0653	0.0584	0.1760	0.0406	0.0494	0.1094	0.0997	0.0405	NoA High	NoA High	Y
10	0.0581	0.0964	0.0575	0.1560	0.1138	0.0571	0.0745	0.0808	0.0398	NoA High	NoA High	Y
11	0.0940	0.0748	0.0594	0.0891	0.0374	0.0503	0.1150	0.0835	0.0412	AlT High	AlT High	Y
12	0.0984	0.0977	0.0623	0.0925	0.1118	0.0619	0.1540	0.0857	0.0432	AlT High	AlT High	Y
13	0.1000	0.0655	06270	0.1298	0.0469	0.0530	0.1800	0.1004	0.0434	AlT High	AlT High	Y
14	0.0932	0.1500	0.0535	0.1194	0.1018	0.0531	0.1760	0.0912	0.0370	AlT High	AlT High	Y
15	0.0924	0.0820	0.0576	0.1193	0.0483	0.0487	0.1660	0.0983	0.0399	AlT High	AlT High	Y
16	0.0971	0.1057	0.0594	0.1379	0.0940	0.0590	0.1500	0.1014	0.0412	AlT High	AlT High	Y
17	0.0895	0.0716	0.0589	0.1360	0.0460	0.0499	0.1870	0.0899	0.0408	AlT High	AlT High	Y
18	0.0788	0.1099	0.0486	0.1170	0.0914	0.0482	0.1987	0.0955	0.0336	AlT High	AlT High	Y
19	0.0871	0.0641	0.0574	0.0860	0.0473	0.0485	0.2000	0.0979	0.0397	AlT High	AlT High	Y
20	0.0932	0.1500	0.0535	0.1350	0.1057	0.0531	0.1864	0.0751	0.0370	AlT High	AlT High	Y

5 Conclusions

Based on the fact that most real-world situations involve uncertain and vague information provided by experienced experts, a new model named IFDUCG model was proposed in this paper to solve the fuzzy problem in the DUCG model. The fuzzy parameters of the IFDUCG model were identified from the domain engineers in the form of intuitionistic fuzzy sets. Later on, IFDUCG model based on TOPSIS method had been presented for knowledge representation and inference. Finally, numerical experiments and industrial applications have been conducted to verify the validity of the developed method. The experimental results showed that the proposed method had fine granularity and representation ability with the superiority of intuitionistic fuzzy sets considering both membership degree and non-membership degree. What's more, it had considerable guiding significance for actual production. However, in the future, it was expected that the new approach would provide a novel solution for more practical problems and had more available applications as well.

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References

1. Seixas FL, Zadrozny B, Laks J et al (2014) A Bayesian network decision model for supporting the diagnosis of dementia, Alzheimers disease and mild cognitive impairment. Comput Biol Med 51:140–158
2. Zhao Y, Xiao F, Wang S (2013) An intelligent chiller fault detection and diagnosis methodology using Bayesian belief network. Energy Build 57:278–288
3. Yang B, Li H (2018) A novel dynamic timed fuzzy Petri nets modeling method with applications to industrial processes. Expert Syst Appl 97:276–289
4. Liu HC, You JX, You XY et al (2016) Linguistic reasoning petri nets for knowledge representation and reasoning. IEEE Trans Syst Man Cybern Syst 46:499–511
5. Liu HC, Lin QL, Mao LX (2013) Dynamic Adaptive Fuzzy Petri Nets for Knowledge Representation and Reasoning, vol 43
6. Ben Ali J, Fnaiech N, Saidi L et al (2015) Application of empirical mode decomposition and artificial neural network for automatic bearing fault diagnosis based on vibration signals. Appl Acoust 89:16–27
7. Ahmed R, Sayed ME, Gadsden SA et al (2015) Automotive internal-combustion-engine fault detection and classification using artificial neural network techniques. IEEE Trans Veh Technol 64:21–33
8. Zhang Q (2015) Dynamic uncertain causality graph for knowledge representation and reasoning: continuous variable, uncertain evidence, and failure forecast. IEEE Trans Syst Man Cybern Syst 45:990–1003
9. Zhang Q (2010) A New Methodology to Deal with Dynamical Uncertain Causalities (I): The Static Discrete DAG Case. Chin J Comput 625—651:33

10. Wang HC, Zhang Q (2005) Fault diagnosis based on the fuzzy causality diagram, Microelectronics and Computer
11. Zhang Q, Geng S (2015) Dynamic uncertain causality graph applied to dynamic fault diagnoses of large and complex systems. IEEE Trans Reliab 64:910–927
12. Geng S, Zhang Q (2015) Clinical diagnosis expert system based on dynamic uncertain causality graph, Information Technology and Artificial Intelligence Conference IEEE, 233–237
13. Zhang Q, Dong C, Cui Y et al (2014) Dynamic uncertain causality graph for knowledge representation and probabilistic reasoning: statistics base, matrix, and application. IEEE Trans Neural Networks and Learn Syst 25:645–663
14. Zhang Q (2015) Dynamic uncertain causality graph for knowledge representation and probabilistic reasoning: directed cyclic graph and joint probability distribution. IEEE Trans Neural Networks and Learn Syst 26:1503–1517
15. Zhang Q (2012) Dynamic Uncertain Causality Graph for Knowledge Representation and Reasoning: Discrete DAG Cases. J Comput Sci Technol 27:1–23
16. Zhang Q, Yao Q (2017) Dynamic uncertain causality graph for knowledge representation and reasoning: Utilization of statistical data and domain knowledge in complex cases, IEEE Trans, Neural Networks and Learning Systems. https://doi.org/10.1109/TNNLS.2017.2673243
17. Zhao (2017) Optimization of a dynamic uncertain causality graph for fault diagnosis in nuclear power plant. Nuclear Tech
18. Song Y, Wang X, Zhu J et al (2018) Sensor dynamic reliability evaluation based on evidence theory and intuitionistic fuzzy sets. Appl Intell 48(11):3950–3962
19. Song Y, Wang X, Wu W et al (2017) Uncertainty measure for Atanassov¡¯s intuitionistic fuzzy sets. Appl Intell 46(4):757–774
20. Ye J (2017) Intuitionistic fuzzy hybrid arithmetic and geometric aggregation operators for the decision-making of mechanical design schemes. Appl Intell 47(3):743–751
21. Kumar K, Garg H (2017) Connection number of set pair analysis based TOPSIS method on intuitionistic fuzzy sets and their application to decision making. Applied Intelligence, 1–8
22. Song Y, Wang X, Lei L et al (2015) A novel similarity measure on intuitionistic fuzzy sets with its applications. Appl Intell 252–261(2):42
23. Xu Z (2010) Choquet integrals of weighted intuitionistic fuzzy information. Inform Sci 180:726–736
24. Garg H (2017) Novel intuitionistic fuzzy decision making method based on an improved operation laws and its application. Eng Appl Artif Intel 60:164–174
25. Beloslav R, Krassimir T (2010) Operation division by n over intuitionistic fuzzy sets
26. Chen T (2007) Remarks on the subtraction and division operations over intuitionistic fuzzy sets and interval-valued fuzzy sets. Int J Fuzzy Syst 9:169–172
27. He Y, Chen H, Zhou L et al (2014) Intuitionistic fuzzy geometric interaction averaging operators and their application to multi-criteria decision making. Inf Sci 259:142–159
28. Su W, Yang Y, Zhang C et al (2013) Intuitionistic fuzzy decision-making with similarity measures and OWA operator. Int J Uncertainty Fuzziness Knowledge Based Syst 21:245–262
29. Wang W, Liu X (2012) Intuitionistic fuzzy information aggregation using Einstein operations. IEEE Trans Fuzzy Syst 20:923–938
30. Wei G (2010) Some induced geometric aggregation operators with intuitionistic fuzzy information and their application to group decision making. Appl Soft Comput 10:423–431
31. Hao Z, Xu Z, Zhao H et al (2018) A Dynamic Weight Determination Approach Based on the Intuitionistic Fuzzy Bayesian Network and Its Application to Emergency Decision Making. IEEE Trans Fuzzy Syst 26:1893–1907
32. Liu P, Chen SM, Liu J (2017) Multiple attribute group decision making based on intuitionistic fuzzy interaction partitioned Bonferroni mean operators. Inform Sci 411:98–121
33. Hao Z, Xu Z, Zhao H et al (2017) Novel intuitionistic fuzzy decision making models in the framework of decision field theory. Inf Fusion 33:57–70
34. Boran FE, Genc S, Kurt M et al (2009) A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. Expert Syst Appl 36:11363–11368
35. Li D (2010) A ratio ranking method of triangular intuitionistic fuzzy numbers and its application to MADM problems. Computers and Mathematics with Applications 60:1557–1570
36. Zhang SF, Liu SY (2011) A GRA-based intuitionistic fuzzy multi-criteria group decision making method for personnel selection. Expert Systems with Applications 38(9):11401–11405
37. Yue W, Chen X, Gui W et al (2017) A knowledge reasoning Fuzzy-Bayesian network for root cause analysis of abnormal aluminum electrolysis cell condition. Front Chem Sci Eng 11:414–428

Li Li received the B.S. in communication engineering from Huaihua college, Huaihua, China, in 2012, and M.S. in communications and information systems from Liaoning university of technology, Jinzhou, China, in 2015. She is currently pursuing the Ph.D. degree in the college of automation from Central South University, Changsha, China. Her current research interests include knowledge representation and reasoning, artificial intelligence, root cause analysis.

Weichao Yue received the B.S. in electrical engineering and automatization from Zhengzhou University of Light Industry, Zhengzhou, China, in 2011, and M.S. in control theory and control engineering from Henan Polytechnic University, Jiaozuo, China, in 2014. He is pursuing the Ph.D. from Central South University, Changsha, China. His research interests include process control, artificial intelligence, knowledge representation and reasoning.