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Detecting facial emotions using normalized minimal feature vectors and semi-supervised twin support vector machines classifier

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Abstract

In this paper, human facial emotions are detected through normalized minimal feature vectors using semi-supervised Twin Support Vector Machine (TWSVM) learning. In this study, face detection and tracking are carried out using the Constrained Local Model (CLM), which has 66 entire feature vectors. Based on Facial Animation Parameter's (FAPs) definition, entire feature vectors are those things that visibly affect human emotion. This paper proposes the 13 minimal feature vectors that have high variance among the entire feature vectors are sufficient to identify the six basic emotions. Using the Max & Min and Z-normalization technique, two types of normalized minimal feature vectors are formed. The novelty of this study is methodological in that the normalized data of minimal feature vectors fed as input to the semi-supervised multi-class TWSVM classifier to classify the human emotions is a new contribution. The macro facial expression datasets are used by a standard database and several real-time datasets. 10-fold and hold out cross-validation is applied with the cross-database (combining standard and real-time). In the experimental result, using 'One vs One' and 'One vs All' multi-class techniques with 3 kernel functions produce a 36 trained model of each emotion and their validation parameters are calculated. The overall accuracy achieved for 10-fold cross-validation is 93.42 ± 3.25% and hold out cross-validation is 92.05±3.79%. The overall performance (Precision, Recall, F1-score, Error rate and Computation Time) of the proposed model was also calculated. The performance of the proposed model and existing methods were compared and results indicate them to be more reliable than existing models.

Keywords Semi-supervised learning · Minimal feature vectors · Twin support vector machines · Facial animation parameters · Human-computer interaction

1 Introduction

For the past three decades several researchers have displayed more interest in human emotion recognition for Human Computer Interaction (HCI), affecting computing, etc. In [1, 2] and [3] research determined the facial emotion analysis criteria using the still images that had a high recognition rate, but not in video sequences. The work carried out in [4] and [5] established the automatic facial emotion recognition system (FERs) from facial video sequences, which analyse facial emotion through detection and tracking of feature points. From a literature review in [6] and [7] it is suggested that facial emotion is defined by the maximum number of facial feature points with Action Units (AUs) [8]. A maximum feature point for facial emotion creates complex data in computation with less accuracy [6–9]. To overcome this problem, selecting the minimal feature points is essential. Paul Ekman [10] used the Facial Action Coding System (FACS), which defines only muscular movement of a facial feature. FACS also defines the basic six emotions on a human face. However, FACS uses a combination of additional facial features (i.e. the eyebrow, jaw and mouth region, etc.) to define human facial emotions. In FACS, the facial emotion recognition system has more data complexity and high computation time using 40 Action Units (AUs). In this case, Facial Animation Parameters (FAPs) define human facial emotions through the facial feature point movements. Facial Animation Parameters (FAPs) [11] defines facial emotion of action units within 10 groups

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using entire feature points. FAP also defines facial emotions with minimal facial actions. In line with FAPs, this paper concentrates only on minimal feature vectors of human emotion. In [12], the geometric deformable model (CLM) has specific face detection and tracking mechanisms when compared to different face modelling, which is considered state of the art.

From the literature survey [1–3] and [13–21] and [22] the FER system developed by various supervised learning systems achieved good accuracy. However, the robust and automatic facial emotion recognition system is not suitable above those FERs. Therefore, semi supervised learning was chosen for FERs because it is better than supervised learning of FERs. From the survey of [23–25] and [26] the semi-supervised Twin Support Vector Machines (TWSVMs) has a high performance compared to the other classifier models. From the [27, 28] and [29] other semi-supervised learning of facial emotion has less accuracy and moderate performance. In this study, multi-class TWSVMs was used to detect human motions.

The purpose of this study is to detect human emotions using semi-supervised learning with minimal facial feature vectors from videos. Four essential steps are involved in this emotion recognition system. First, Constrained Local Method (CLM) is used for face detection, tracking, and extracting the feature points. Second, the minimal feature vectors are formed based on FAPs of AUs. Third, using normalization, minimal feature vectors are obtained. Finally, the normalized minimal feature vectors are fed as input to the TWSVMs for facial emotion classification. Section 2 describes the methods involving facial emotion systems. Section 3 describes the experimental analysis and validation. Section 4 outlines the experimental results and discussion of the proposed system. Section 5 summarises the conclusion and recommendations for future studies.

2 Facial emotion recognition system

The architecture of the robust facial emotion recognition system is shown in Fig. 1 and contains the following five steps:

2.1 Facial detection and tracking

The non-rigid shape of the face is employed by the Point Distribution Model (PDM). In PDM, 2D vertex meshes is symbolized as dimensionality shaped vectors. PDM is applied through Principal Component Analysis (PCA) for acquiring aligned non-rigid face shapes. Before PCA, the Procrustes analysis is applied for removing the parameters s, R, tx, ty of aligned mesh shapes. The 2D PDM is linearly deformed by the variation of non-rigid shapes and is combined with the transformation, placing the shape in an image frame as shown in (1).

$$\mathbf{x}_i = sR(\tilde{\mathbf{x}}_i) + T_{t_x,t_y} \Leftarrow T_{s,R,t_x,t_y}(\tilde{\mathbf{x}}_i) \tag{1}$$

Where s, R, t_x, t_y denotes as scale, rotation, and translation respectively. Where x_i denotes i^{th} landmark of 2D PDM's location, x_i denotes as mean shape of 2D PDM and pose parameters of PDM represent as p = (s,R,t,q). In CLM fitting, applied Subspace Constrained Mean Shift (SCMS)

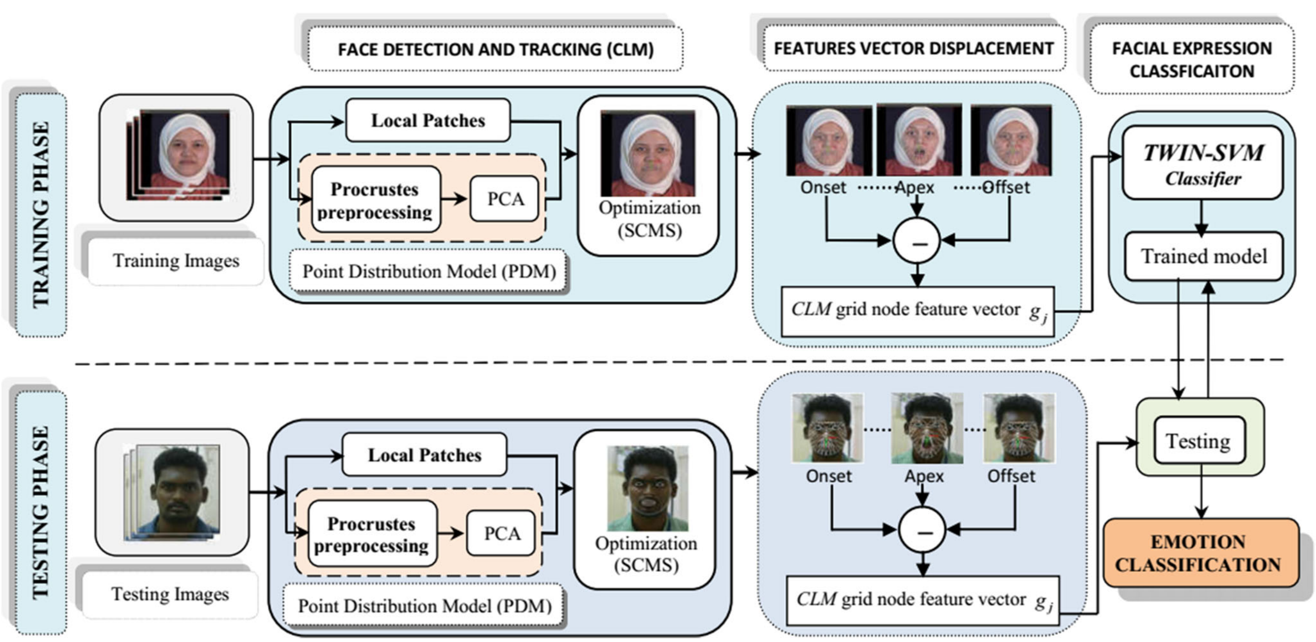


Fig. 1 Facial emotion recognition system architecture

[30] is applied to combine the good local (patch) search and optimised 2D PDM landmark fitting. In an exhaustive local search, using linear logistic regression [31] gives the response maps for the i^{th} landmark position in image frame, which is given in (2).

$$p(l_i = \mathbf{aligned}|I, \mathbf{x}) = \frac{1}{1 + \exp\{\alpha C_i(I, \mathbf{x}) + \beta\}} \tag{2}$$

Where l_i is a discrete random variable that denotes the i^{th} number of iterations, it aligns to correct the PDM landmark value. I denotes the 2D location in local images (x). β is the regression coefficient and denotes as to correct the 2D landmark of local maps. C_i is the linear classifier of a local detector which is defined in (3) with $\mathbf{x}_{i=1}^{m} \in \Omega_{\mathbf{x}_i}$ (image patch) and b_i is shape vector parameters.

$$C_i(I(\mathbf{x}_i)) = w_i^T[I(\mathbf{x}_i); ....; I(\mathbf{x}_m)] + b_i \tag{3}$$

The optimized probabilistic function of local response image for each landmark detection is given in (4). Once the response maps of each landmark of local search have been found that the probabilistic function is maximized.

$$p\left(\{l_i = \mathbf{aligned}\}_{i=1}^{n} |p\right) = \prod_{i=1}^{n} p(l_i = \mathbf{aligned}|) \tag{4}$$

With respect to p. PDM parameters, x_i is parametrized of response images. The active shape model is defined as a summation of weighted least square difference between the maximum response map and peak response of PDM coordinates, which is given in (5).

$$Q(\mathbf{p}) = \sum_{i=1}^{n} w_i \|\mathbf{x}_i - \mu_i\|^2 \tag{5}$$

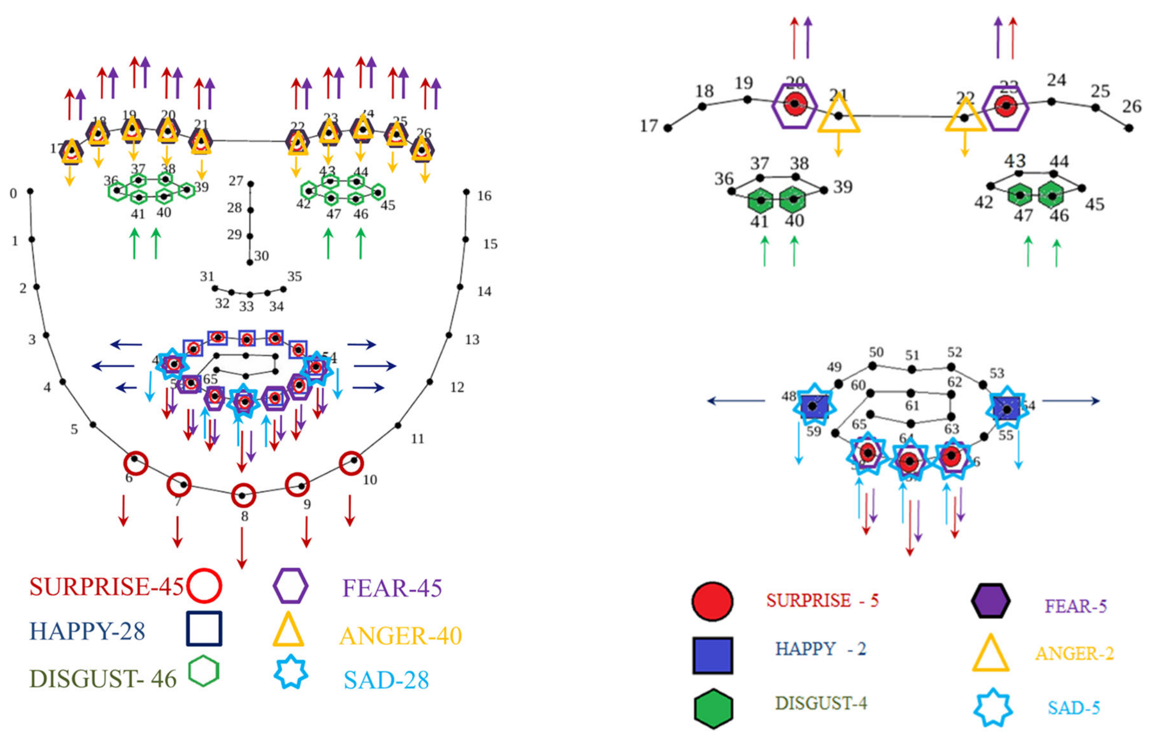
A first order Taylor expansion is applied in (4) for minimising the active shape model, which leads to convergence of the PDM landmark. This is defined in (6).

$$\mathbf{x}_i \approx \mathbf{x}_i^c + \mathbf{J}_i \Delta p \tag{6}$$

And solving the parameter update is defined in (7).

$$\Delta p = \left(\sum_{i=1}^{n} w_i \mathbf{J}_i^T \mathbf{J}_i\right)^{-1} \left(\sum_{i=1}^{n} w_i \mathbf{J}_i^T (\mu_i - \mathbf{x}_i^c)\right) \tag{7}$$

The current parameter is applied in $p \leftarrow p + \Delta p$ to estimate the pose and shape. Here Jacobian is $J = [J_1; \cdots; J_n]$ and current shape parameter update is $x = [x_1^c; \cdots; x_n^c]$. For independent maximisation and non-parametric representation of the Kernel Density Estimate (KDE), [32] is applied for each PDM landmark location in



(a) Enitre Feature vectors (b) Minimal Feature vectors

Fig. 2 Entire and minimal feature vectors of six facial emotions movements

Table 1 Entire feature vectors by FAPs

Emotion	Group(Grp) no affected	No of fpts affected	Total no of fpts affected
Surprise	2, 4, 8	0–16, 17-26, 48–65	45
Happy	2, 4, 8	17–26, 48–65	28
Disgust	2, 3, 4, 9	17–26, 31-35, 36–47, 48-65	46
Fear	2, 4, 8	0–16, 17–26, 48–65	45
Anger	2, 3, 4, 8	17–26, 36-47, 48–65	40
Sad	2, 4, 8	17–26, 48–65	28

the Mean-Shift Algorithm (MSA) [33]. This consists of a fixed point iteration and is defined in (8). Equation (8) is applied iteratively until it reaches convergence Δp

$$\mathbf{x}_i^{\tau+1} \leftarrow \sum_{\mu_i \epsilon \Psi_{\mathbf{x}_i^c}} \frac{\alpha^i_{\mu_i} N\left(\mathbf{x}_i^{(\tau)}; \mu_i, \sigma^2 I\right)}{\sum_{y \epsilon \Psi_{\mathbf{x}_i^c}} \alpha_{y^i} N\left(\mathbf{x}_i^{(\tau)}; y, \sigma^2 I\right)} \mu_i \tag{8}$$

To convert the shape constrained to optimisation, which is defined in (9), MSA uses a two-step strategy: i) compute the mean-shift update for each 2D PDM landmark, ii) constrain the mean-shifted landmark to remain in the PDM parametrization using a least-square fit. The Gauss Newton update for least square PDM constraint is also defined in (9).

$$Q(\mathbf{p}) = \sum_{i=1}^{n} \|\mathbf{x}_i - \mathbf{x}_i^{(\tau+1)}\|^2 \tag{9}$$

The image that has a high probabilistic function of local response obtained from (4) is given as an input to the EM algorithm to find the difference between the peak responses. The Q-function of M-step is defined in (10), and is formed using the linear shape model in (6).

$$\Delta p = \mathbf{J}^{\dagger}\left[\mathbf{x}_1^{(\tau+1)} - \mathbf{x}_1^c; \cdots\cdots; \mathbf{x}_n^{(\tau+1)} - \mathbf{x}_n^c\right] \tag{10}$$

Where $J^{\dagger}$ denotes the pseudo-inverse of J and $x_i^{(\tau+1)}$, it denotes the i^{th} landmark of mean shift update parameters, which is given in (8). The kernel width relaxation and local optima is addressed in the subspace constrained mean shift (SCMS)[30].

2.2 Feature vectors displacement

The geometric deformable model (CLM) is carried out to perform face detection, tracking and extraction from video. The feature vectors displacement ($\mathbf{d}_{i,j}$) is defined as the facial feature point movement between the consecutive frame by frame sequence. ($\mathbf{d}_{i,j}$) is the difference between the grid node displacements of the first to i^{th} node coordinates. The feature vectors displacement is given in (11).

$$\mathbf{d}_{i,j} = \begin{bmatrix} \Delta\mathbf{x}_{i,j} \\ \Delta\mathbf{y}_{i,j} \end{bmatrix} = \sum_{i,j=1}^{F,N} \begin{bmatrix} \mathbf{a}_{11} - \mathbf{a}_{12} & \mathbf{a}_{12} - \mathbf{a}_{13} & \cdots & \mathbf{a}_{1,j+1} - \mathbf{a}_{1,j+2} \\ \mathbf{a}_{21} - \mathbf{a}_{22} & \mathbf{a}_{22} - \mathbf{a}_{23} & \cdots & \mathbf{a}_{2,j+1} - \mathbf{a}_{2,j+2} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{a}_{i+1,1} - \mathbf{a}_{i+1,2} & \cdots & \cdots & \mathbf{a}_{m,n} - \mathbf{a}_{m,n+1} \end{bmatrix} \tag{11}$$

$i = 1, \ldots, F,\ j = 1, \ldots, N$, where $\Delta\mathbf{x}_{i,j}$, $\Delta\mathbf{y}_{i,j}$ are x, y axis coordinates of grid node displacement of the i^{th} node in the j^{th} frame, respectively. F is the number of grid node (F = 66 no. of nodes of CLM) and N is the number of extracted facial frames from video. The grid deformation feature vector displacement $\mathbf{g}_j$ consists of feature vector displacement of the every geometric grid node $\mathbf{d}_{i,j}$, $\mathbf{g}_j$ which is given in (12).

$$\mathbf{g}_j = \left[\mathbf{d}_{1,j}, \mathbf{d}_{2,j}, \cdots\cdots, \mathbf{d}_{E,j}\right]^T \tag{12}$$

2.3 Minimal feature vectors displacement

The pictorial representations of feature points that are affected by different emotions are shown in Fig. 2. The entire feature vectors and minimal feature vectors of the six basic emotions are shown in Fig. 2a and b, respectively.

Table 2 Minimal feature vectors

Emotion	Grp no affd	No of fpts affd	Tot.no of fpts affd	minimal feature variance range	other fpts diff variance range
Surprise	4,8	(20, 23) & (56–58)	5	ebw (5.5–61.9) & olp (3.6–53.9)	(20–55.2) & (28.4–43.9)
Happy	8	48 & 54	2	clp(1.9–9.7)	(1.5–5.6)
Disgust	3	40, 41, 46 & 47	4	eld (1.9–18.6)	(15.5–16.4)
Fear	4,8	(20, 23) & (56–58)	5	ebw (1.2–18.5) & olp (0.2–11.9)	(11.7–17.8) & (6.2–10.2)
Anger	4	21 & 22	2	ebw (1.4–32)	(15.4–29.3)
Sad	2,8	(48, 54 & (56–58))	5	clp & olp(0.07–3.5)	(0.04–2.26)

In [11], [34] describe the groups, number of FAPs, and textual description of emotion, which are affected by the six basic emotions. Based on the definition of FAPs, which is illustrated in Tables 1 and 2, it provides the information regarding the number of feature points and group numbers that are affected by the six basic emotions. Table 1 provides information about the entire set of feature points involved in six emotions. Insight of any emotion is given as onset, where staring points of emotion, apex, where emotion reaches peak position, and offset, where emotion returns to the neutral state. The entire set of feature vectors of any emotion is the displacement between the onset and offset phase. From the literature [6, 9] and [35], the entire vectors displacement has a high data redundancy, less accuracy, and low data computation. To increase the accuracy and reduce computation time, the minimal feature vectors displacement is chosen. Any emotion that reaches the apex phase from onset phase is defined as the peak response. The minimal feature vectors displacements are chosen based on the high variance values among feature points during the peak

Table 3 The hold-out validation result of Surprise eyebrow in both multi-classifier and normalization

	Norm	One vs One	Kernel	Acc(%)	Pre	Rec	F1-sco	Err.rte	Comp.Time (sec)
One vs One	Max & Min	Sur vs Hap	Lin	91.87	0.71	0.67	0.69	0.32	7.73
		Sur vs Ang	Lin	89.33	0.57	0.82	0.68	0.17	1.95
		Sur vs Dis	Lin	91.87	0.68	0.73	0.71	0.26	2.09
		Sur vs Fea	Lin	91.74	0.82	0.49	0.62	0.50	1.84
		Sur vs Sad	Lin	91.74	0.71	0.65	0.68	0.34	1.87
		Sur vs Hap	Poly	91.61	0.70	0.66	0.68	0.33	2.95
		Sur vs Ang	Poly	90.34	0.61	0.75	0.68	0.24	1.97
		Sur vs Dis	Poly	91.36	0.68	0.67	0.68	0.32	2.11
		Sur vs Fea	Poly	91.49	0.68	0.70	0.69	0.29	1.81
		Sur vs Sad	Poly	91.74	0.73	0.60	0.69	0.39	2.37
		Sur vs Hap	RBF	91.87	0.72	0.65	0.67	0.34	3.12
		Sur vs Ang	RBF	89.96	0.59	0.82	0.69	0.17	2.21
		Sur vs Dis	RBF	90.47	0.62	0.74	0.69	0.25	2.31
		Sur vs Fea	RBF	91.61	0.70	0.65	0.68	0.34	2.37
		Sur vs Sad	RBF	91.61	0.71	0.64	0.68	0.35	2.26
	Z-norm	Sur vs Hap	Lin	91.23	0.67	0.69	0.68	0.30	2.17
		Sur vs Ang	Lin	85.26	0.47	0.89	0.62	0.10	2.24
		Sur vs Dis	Lin	86.53	0.50	0.86	0.63	0.14	2.05
		Sur vs Fea	Lin	91.49	0.72	0.60	0.66	0.39	1.83
		Sur vs Sad	Lin	90.34	0.62	0.74	0.68	0.25	1.71
		Sur vs Hap	Poly	88.44	0.54	0.83	0.66	0.16	2.80
		Sur vs Ang	Poly	77.64	0.37	0.95	0.54	0.04	2.09
		Sur vs Dis	Poly	89.33	0.57	0.82	0.68	0.17	2.19
		Sur vs Fea	Poly	80.05	0.39	0.84	0.53	0.15	2.22
		Sur vs Sad	Poly	87.04	0.51	0.85	0.64	0.15	2.05
		Sur vs Hap	RBF	88.18	0.54	0.82	0.65	0.17	2.81
		Sur vs Ang	RBF	60.36	0.25	0.99	0.4	0.01	2.23
		Sur vs Dis	RBF	84.63	0.46	0.88	0.61	0.11	2.23
		Sur vs Fea	RBF	91.49	0.74	0.57	0.65	0.43	2.77
		Sur vs Sad	RBF	87.8	0.53	0.80	0.64	0.19	2.50
One vs All	Max & Min	Sur vs All	Lin	91.86	0.77	0.56	0.65	0.43	2.29
		Sur vs All	Poly	91.48	0.73	0.58	0.65	0.41	2.22
		Sur vs All	RBF	91.74	0.80	0.51	0.63	0.48	2.72
	Z-norm	Sur vs All	Lin	91.61	0.71	0.64	0.68	0.35	2.08
		Sur vs All	Poly	89.58	0.58	0.78	0.67	0.21	2.61
		Sur vs All	RBF*	95.29	0.83	0.82	0.83	0.26	1.73

response of an emotion. In Table 2, the high variance range of minimal feature is calculated and tabulated. The major feature movement for six facial emotions are eyebrow (ebw), outer lip (olp), eyelids (eld) and corner lip (clp). Table 2, shows the differences of variance range of the other than minimal feature points that are also tabulated.

Based on the definition of FAPs and the facts stated above, surprise emotion involves 45 entire feature points. In those 45 feature points, 5 feature points have a high variance compared to the remaining 40 feature points. Similarly, for all the emotions, the minimal feature vectors points are determined and are tabulated in Table 2. Table 2 provides details of the minimal feature points of six emotions that are affected. The minimal feature points from eyebrow, mouth, and eyelids are observed. To obtain effective performance and improve accuracy, less data redundancy

Table 4 The hold-out validation result of Surprise mouth in both multi-classifier and normalization

	Norm	One vs One	Kernel	Acc(%)	Pre	Rec	F1-sco	Err.rte	Comp.Time (sec)
One vs One	Max & Min	Sur vs Hap	Lin	90.1	0.91	0.3	0.45	0.7	2.46
		Sur vs Ang	Lin	95.2	0.95	0.69	0.8	0.31	2.42
		Sur vs Dis	Lin	90.7	0.85	0.39	0.53	0.61	2.62
		Sur vs Fea	Lin	91	0.78	0.47	0.59	0.53	2.31
		Sur vs Sad	Lin	91	0.76	0.49	0.6	0.51	2.14
		Sur vs Hap	Poly	90.5	0.73	0.48	0.58	0.52	2.61
		Sur vs Ang	Poly	90.3	0.71	0.5	0.59	0.5	2.56
		Sur vs Dis	Poly	90.9	0.78	0.46	0.58	0.54	2.48
		Sur vs Fea	Poly	91	0.67	0.67	0.67	0.33	2.35
		Sur vs Sad	Poly	90.5	0.72	0.5	0.59	0.5	2.52
		Sur vs Hap	RBF	90.3	0.9	0.33	0.48	0.67	3.022
		Sur vs Ang	RBF	90.9	0.76	0.48	0.59	0.52	2.89
		Sur vs Dis	RBF	90.9	0.76	0.48	0.59	0.52	3.04
		Sur vs Fea	RBF	91.1	0.68	0.67	0.67	0.33	3.98
		Sur vs Sad	RBF	90.7	0.74	0.5	0.6	0.5	2.73
	Z-norm	Sur vs Hap	Lin	90.8	0.74	0.48	0.58	0.52	2.08
		Sur vs Ang	Lin	89.2	0.96	0.22	0.35	0.78	1.93
		Sur vs Dis	Lin	91.4	0.81	0.45	0.58	0.55	2.34
		Sur vs Fea	Lin	91.1	0.85	0.42	0.57	0.58	2.48
		Sur vs Sad	Lin	89.9	0.7	0.46	0.56	0.54	2.40
		Sur vs Hap	Poly	89.4	0.65	0.5	0.56	0.5	2.20
		Sur vs Ang	Poly	90.5	0.67	0.6	0.63	0.4	2.21
		Sur vs Dis	Poly	88.8	0.58	0.65	0.61	0.35	3.01
		Sur vs Fea	Poly	70.5	0.28	0.75	0.41	0.25	2.36
		Sur vs Sad	Poly	85.1	0.45	0.84	0.59	0.16	2.34
		Sur vs Hap	RBF	91	0.79	0.46	0.58	0.54	2.49
		Sur vs Ang	RBF	90.5	0.7	0.54	0.61	0.46	2.71
		Sur vs Dis	RBF	89.2	0.62	0.53	0.57	0.47	2.56
		Sur vs Fea	RBF	90.5	0.64	0.68	0.66	0.32	2.46
		Sur vs Sad	RBF	91	0.7	0.6	0.65	0.4	2.74
One vs All	Max & Min	Sur vs All	Lin	95.10	0.90	0.71	0.8	0.28	2.39
		Sur vs All	Poly	95.2	0.91	0.72	0.8	0.28	2.86
		Sur vs All	RBF	95.48	0.90	0.75	0.82	0.25	2.83
	Z-norm	Sur vs All	Lin	90.20	0.76	0.40	0.53	0.59	2.03
		Sur vs All	Poly	90.3	0.7	0.51	0.59	0.49	2.32
		Sur vs All	RBF*	96.4	0.93	0.79	0.86	0.20	1.75

and computation time are selected by the minimal feature points. From (11) and (12), the minimal feature vectors displacements are obtained.

2.4 Normalization of feature vectors displacement

For data scaling, normalization is applied on the minimal feature vectors displacement to increase the feature scaling. In this paper, Max & Min and Z-normalization are applied and compared to obtain the feature scaling as defined in (13). Using normalization, the normalized data of minimal feature vectors displacement is formed.

$$\begin{aligned} &\textbf{Max \& Min}_{(-1,1)} \quad f(x) = 2\left(\frac{x - Max(x)}{Max(x) - Min(x)}\right) - 1 \\ &\textbf{Z-norm}_{(-1,1)} \quad f(x) = \frac{x-\mu}{\sigma} \end{aligned} \tag{13}$$

Table 5 The hold-out validation result of Happy mouth in both multi-classifier and normalization

	Norm	One vs One	Kernel	Acc(%)	Pre	Rec	F1-sco	Err.rte	Comp.Time (sec)
One vs One	Max & Min	Hap vs Ang	Lin	91.89	0.94	0.75	0.83	0.25	2.49
		Hap vs Dis	Lin	93.31	0.9	0.85	0.87	0.15	2.08
		Hap vs Fea	Lin	81.47	0.6	0.92	0.73	0.08	1.88
		Hap vs Sad	Lin	91.76	0.94	0.74	0.83	0.26	1.93
		Hap vs Sur	Lin	92.15	0.93	0.77	0.84	0.23	1.97
		Hap vs Ang	Poly	93.69	0.93	0.83	0.88	0.17	2.88
		Hap vs Dis	Poly	93.44	0.89	0.86	0.88	0.14	2.11
		Hap vs Fea	Poly	90.99	0.79	0.91	0.85	0.09	2.08
		Hap vs Sad	Poly	93.56	0.93	0.83	0.87	0.17	2.11
		Hap vs Sur	Poly	92.92	0.9	0.83	0.86	0.17	2.12
		Hap vs Ang	RBF	93.44	0.9	0.85	0.88	0.15	2.50
		Hap vs Dis	RBF	92.79	0.86	0.88	0.87	0.12	2.10
		Hap vs Fea	RBF	91.51	0.81	0.91	0.85	0.09	2.20
		Hap vs Sad	RBF	92.92	0.86	0.89	0.87	0.11	2.17
		Hap vs Sur	RBF	90.73	0.79	0.9	0.84	0.1	2.02
	Z-norm	Hap vs Ang	Lin	91.89	0.93	0.75	0.83	0.12	2.73
		Hap vs Dis	Lin	93.31	0.9	0.85	0.87	0.15	1.97
		Hap vs Fea	Lin	81.08	0.6	0.92	0.73	0.08	2.01
		Hap vs Sad	Lin	91.89	0.94	0.75	0.83	0.25	1.75
		Hap vs Sur	Lin	92.02	0.93	0.76	0.84	0.24	1.92
		Hap vs Ang	Poly	92.79	0.85	0.89	0.87	0.11	2.51
		Hap vs Dis	Poly	91.25	0.8	0.9	0.85	0.1	2.39
		Hap vs Fea	Poly	88.93	0.75	0.9	0.82	0.1	1.98
		Hap vs Sad	Poly	93.32	0.87	0.88	0.88	0.12	1.98
		Hap vs Sur	Poly	92.79	0.85	0.89	0.87	0.11	3.11
		Hap vs Ang	RBF	87.8	0.72	0.9	0.8	0.1	2.25
		Hap vs Dis	RBF	92.54	0.84	0.89	0.87	0.11	1.99
		Hap vs Fea	RBF	89.58	0.76	0.91	0.83	0.09	2.01
		Hap vs Sad	RBF	92.41	0.84	0.89	0.86	0.11	2.76
		Hap vs Sur	RBF	85.84	0.68	0.92	0.78	0.08	2.04
One vs All	Max & Min	Hap vs All	Lin	91.7	0.93	0.74	0.83	0.25	3.93
		Hap vs All	Poly	93.8	0.93	0.83	0.88	0.17	2.78
		Hap vs All	RBF	92.4	0.93	0.78	0.85	0.22	2.47
	Z-norm	Hap vs All	Lin	91.89	0.94	0.76	0.84	0.24	2.70
		Hap vs All	Poly	92.3	0.83	0.9	0.86	0.14	2.71
		Hap vs All	RBF	95.64	0.93	0.9	0.92	0.09	2.47

2.5 Facial emotion classifier-bilinear classifier

In this system, the TWSVM classifier is used to classify the basic six facial emotions. The normalized data of minimal feature vectors displacement is given as input to the two classes TWSVM [23, 25] and [26] to classify facial emotion. Using TWSVM, the two non-parallel hyperplanes for each class is constructed to solve the quadratic programming problem. In this case, let $g_j = \{(x_i, y_i)\}$; $i = 1, \ldots, k$; $x \in \Re^n$; $y_i \in \{-1, +1\}$ is the training dataset of normalized minimal feature vectors displacement. The separating hyperplane of linear data is defined in (14). Separating the minimal feature vectors displacement $\mathbf{g}_j = \Re^L$ in the order of positive and negative class is formed by the Karush-Kuhn Tucker (K.K.T) conditions, which is provided in (14).

$$\begin{aligned} f(x)_+ &= x^T \cdot w_+ + b_+ = 0 \\ f(x)_- &= x^T \cdot w_- + b_- = 0 \end{aligned} \tag{14}$$

Table 6 The hold-out validation result of Fear eyebrow in both multi-classifier and normalization

	Norm	One vs One	Kernel	Acc(%)	Pre	Rec	F1-sco	Err.rte	Comp.Time (sec)
One vs One	Max & Min	Fea vs Hap	Lin	76.5	0.16	0.22	0.19	0.78	2.10
		Fea vs Ang	Lin	58.7	0.15	0.52	0.23	0.48	1.94
		Fea vs Dis	Lin	72	0.13	0.22	0.16	0.78	1.79
		Fea vs Sad	Lin	76.6	0.16	0.22	0.19	0.78	2.10
		Fea vs Sur	Lin	18.2	0.12	0.92	0.21	0.08	1.91
		Fea vs Hap	Poly	76.7	0.16	0.22	0.19	0.78	2.36
		Fea vs Ang	Poly	69	0.17	0.4	0.24	0.6	2.02
		Fea vs Dis	Poly	72.6	0.17	0.32	0.22	0.68	1.97
		Fea vs Sad	Poly	78.3	0.16	0.19	0.17	0.81	1.90
		Fea vs Sur	Poly	21.5	0.11	0.81	0.2	0.19	1.87
		Fea vs Hap	RBF	76.7	0.16	0.22	0.19	0.78	2.34
		Fea vs Ang	RBF	60.7	0.16	0.53	0.24	0.47	2.15
		Fea vs Dis	RBF	68	0.17	0.41	0.24	0.59	2.05
		Fea vs Sad	RBF	77	0.16	0.22	0.19	0.78	2.03
		Fea vs Sur	RBF	20.3	0.11	0.82	0.2	0.18	1.98
	Z-norm	Fea vs Hap	Lin	72.8	0.17	0.33	0.22	0.67	2.707
		Fea vs Ang	Lin	39	0.13	0.73	0.22	0.27	2.03
		Fea vs Dis	Lin	64.9	0.15	0.42	0.22	0.58	1.92
		Fea vs Sad	Lin	70.9	0.16	0.34	0.22	0.66	1.99
		Fea vs Sur	Lin	18.9	0.11	0.85	0.22	0.15	1.66
		Fea vs Hap	Poly	72.7	0.17	0.33	0.22	0.67	2.06
		Fea vs Ang	Poly	63.5	0.15	0.45	0.23	0.55	1.94
		Fea vs Dis	Poly	74.5	0.17	0.28	0.21	0.72	3.08
		Fea vs Sad	Poly	57.6	0.14	0.51	0.22	0.49	1.96
		Fea vs Sur	Poly	31.9	0.1	0.61	0.18	0.39	1.99
		Fea vs Hap	RBF	75.2	0.17	0.27	0.21	0.73	6.32
		Fea vs Ang	RBF	55.4	0.15	0.6	0.25	0.4	2.43
		Fea vs Dis	RBF	62.6	0.16	0.48	0.24	0.52	2.40
		Fea vs Sad	RBF	73.6	0.17	0.32	0.22	0.68	2.80
		Fea vs Sur	RBF	33.4	0.14	0.84	0.23	0.16	2.29
One vs All	Max & Min	Fea vs All	Lin	81.45	0.37	0.75	0.49	0.25	3.74
		Fea vs All	Poly	72.17	0.23	0.6	0.34	0.41	2.29
		Fea vs All	RBF	83.74	0.41	0.78	0.54	0.22	3.61
	Z-norm	Fea vs All	Lin	62.64	0.16	0.49	0.24	0.51	2.69
		Fea vs All	Poly	82.08	0.38	0.76	0.51	0.24	2.63
		Fea vs All	RBF*	89.58	0.54	0.87	0.67	0.13	2.14

Where w^T is the weight vector and b is a bias. The objective function of linear TWSVM is corresponding to one class and constraints to the other class are defined by (15).

$$\begin{array}{ll} \mathbf{min}(w_+, b_+, \xi) & \frac{1}{2}\|Xw_+ + e_+b_+\|^2 + c_1e_-^T\xi \\ \mathbf{s.t} & -(Yw_+ + e_-b_+) + \xi \geq e_-, \quad \xi \geq 0 \\ \mathbf{min}(w_-, b_-, \xi) & \frac{1}{2}\|Yw_- + e_-b_-\|^2 + c_2e_+^T\eta \\ \mathbf{s.t} & -(Xw_- + e_+b_-) + \xi \geq e_+, \quad \eta \geq 0 \end{array} \tag{15}$$

Where c_+, c_- are a penalty parameter and are slack variables. e_+, e_- are vectors of suitable dimensions. Let $H = [X_+^T\ e_+^T]$ and $G = [X_-^T\ e_-^T]$. In (16), the Lagrangian and Wolf dual problem are formulated and they obtained from the equation of TWSVM.

$$\begin{array}{ll} \mathbf{max}_\alpha & e_-^T\alpha - \frac{1}{2}\alpha^T G(H^T H)^{-1}G^T\alpha \\ \mathbf{s.t} & 0 \leq \alpha \leq c_1e_- \\ \mathbf{max}_\beta & e_+^T\beta - \frac{1}{2}\beta^T H(G^T G)^{-1}H^T\beta \\ \mathbf{s.t} & 0 \leq \beta \leq c_2e_+ \end{array} \tag{16}$$

Where Lagrangian multipliers are α and β. In (17) introduces a term δI ($\delta > 0$) to avoid becoming singular and the ill-conditioning of $H^T G$. Where I denotes a regularisation

Table 7 The hold-out validation result of Fear mouth in both multi-classifier and normalization

	Norm	One vs One	Kernel	Acc(%)	Pre	Rec	F1-sco	Err.rte	Comp.Time (sec)
One vs One	Max & Min	Fea vs Hap	Lin	79	0.21	0.27	0.23	0.73	1.92
		Fea vs Ang	Lin	89.2	0.54	0.8	0.64	0.2	1.86
		Fea vs Dis	Lin	76	0.2	0.32	0.24	0.68	2.19
		Fea vs Sad	Lin	84.7	0.22	0.11	0.14	0.89	2.60
		Fea vs Sur	Lin	19.5	0.13	0.97	0.23	0.03	2.91
		Fea vs Hap	Poly	38	0.12	0.67	0.21	0.33	2.78
		Fea vs Ang	Poly	65.6	0.17	0.47	0.25	0.53	2.25
		Fea vs Dis	Poly	84	0.2	0.11	0.14	0.89	2.69
		Fea vs Sad	Poly	49.1	0.14	0.61	0.22	0.39	2.12
		Fea vs Sur	Poly	22	0.12	0.85	0.21	0.15	2.13
		Fea vs Hap	RBF	44.6	0.13	0.63	0.22	0.37	2.61
		Fea vs Ang	RBF	72.3	0.18	0.35	0.23	0.65	1.94
		Fea vs Dis	RBF	84.9	0.23	0.11	0.15	0.89	2.19
		Fea vs Sad	RBF	70.9	0.18	0.38	0.24	0.62	2.07
		Fea vs Sur	RBF	21.9	0.12	0.85	0.21	0.15	2.32
	Z-norm	Fea vs Hap	Lin	48.2	0.12	0.53	0.22	0.47	3.71
		Fea vs Ang	Lin	84.3	0.11	0.04	0.06	0.96	2.47
		Fea vs Dis	Lin	71.8	0.18	0.37	0.24	0.63	2.35
		Fea vs Sad	Lin	85.3	0.05	0.01	0.02	0.99	1.94
		Fea vs Sur	Lin	18.7	0.13	0.99	0.23	0.01	1.96
		Fea vs Hap	Poly	52.6	0.13	0.52	0.21	0.48	2.47
		Fea vs Ang	Poly	82.6	0.16	0.11	0.13	0.89	2.54
		Fea vs Dis	Poly	70.9	0.15	0.31	0.22	0.69	2.08
		Fea vs Sad	Poly	83.8	0.23	0.15	0.18	0.85	2.06
		Fea vs Sur	Poly	39.3	0.12	0.62	0.22	0.38	3.19
		Fea vs Hap	RBF	61.2	0.13	0.38	0.19	0.62	2.72
		Fea vs Ang	RBF	69.5	0.14	0.29	0.19	0.71	2.03
		Fea vs Dis	RBF	70.6	0.06	0.1	0.07	0.9	2.22
		Fea vs Sad	RBF	70.2	0.15	0.3	0.20	0.7	2.51
		Fea vs Sur	RBF	22.3	0.12	0.83	0.21	0.17	2.04
One vs All	Max & Min	Fea vs All	Lin	92	0.64	0.8	0.71	0.2	2.51
		Fea vs All	Poly	86.2	0.46	0.77	0.57	0.23	2.97
		Fea vs All	RBF	86.1	0.46	0.76	0.57	0.24	2.39
	Z-norm	Fea vs All	Lin	32.5	0.11	0.66	0.19	0.34	2.76
		Fea vs All	Poly	67.9	0.16	0.39	0.23	0.61	3.05
		Fea vs All	RBF*	93.8	0.67	0.98	0.79	0.02	2.39

identity matrix. The function of non-parallel hyperplane of TWSVM from the α and β value is defined in (17).

$$\begin{aligned} \mathbf{u}_1 &= -(H^T H + \delta I)^{-1} G^T \alpha \\ \mathbf{u}_2 &= -(G^T G + \delta I)^{-1} H^T \beta \end{aligned} \tag{17}$$

Where $u_{i=1,2} = [w_{\pm}^T b_{\pm}]^T$. From (17), the weight vector and bias value of optimal hyperplane of linear TWSVM is obtained. For validation, a new data sample is assigned to class 'i' by the decision function, as given in (18). A decision surface classified in the new data depends upon whether its distance is closer to hyperplane (18).

$$\textbf{Class (i)} = \arg\min_{i=1,2} \frac{|x^T w_{(i)} + b_{(i)}|}{\|w_{(i)}\|} \tag{18}$$

For Non-Linear cases of TWSVM, the training data are examined with the Kernel Function such as Linear, Polynomial and Radial Base Function (RBF) [23–25] and [26]. The Multi class TWSVM [25, 26] and [36] has two approaches: 'One vs One' and 'One vs All'. The 'One

Table 8 The hold-out validation result of Anger eyebrow in both multi-classifier and normalization

	Norm	One vs One	Kernel	Acc(%)	Pre	Rec	F1-sco	Err.rte	Comp.Time (sec)
One vs One	Max & Min	Ang vs Hap	Lin	79	0.29	0.28	0.29	0.72	2.88
		Ang vs Dis	Lin	84.1	0.38	0.08	0.13	0.92	3.12
		Ang vs Fea	Lin	73.8	0.32	0.65	0.43	0.35	1.68
		Ang Vs Sad	Lin	77.8	0.29	0.34	0.31	0.66	2.27
		Ang vs Sur	Lin	41.8	0.2	0.95	0.33	0.05	1.77
		Ang vs Hap	Poly	74.2	0.3	0.54	0.39	0.46	2.24
		Ang vs Dis	Poly	84.1	0.2	0.02	0.03	0.98	2.01
		Ang vs Fea	Poly	58.2	0.25	0.91	0.4	0.09	2.11
		Ang Vs Sad	Poly	76.7	0.3	0.4	0.34	0.6	2.45
		Ang vs Sur	Poly	39.6	0.2	0.96	0.32	0.04	2.04
		Ang vs Hap	RBF	79	0.3	0.29	0.29	0.71	3.56
		Ang vs Dis	RBF	84	0.18	0.02	0.03	0.98	2.83
		Ang vs Fea	RBF	70	0.31	0.8	0.45	0.2	2.87
		Ang Vs Sad	RBF	77.6	0.3	0.36	0.33	0.64	2.10
		Ang vs Sur	RBF	47.8	0.22	0.95	0.35	0.05	2.27
	Z-norm	Ang vs Hap	Lin	79.9	0.29	0.24	0.26	0.76	2.09
		Ang vs Dis	Lin	83.5	0.34	0.1	0.16	0.9	1.84
		Ang vs Fea	Lin	77.5	0.29	0.34	0.31	0.66	1.70
		Ang Vs Sad	Lin	76.4	0.3	0.44	0.36	0.56	2.07
		Ang vs Sur	Lin	67.9	0.3	0.84	0.44	0.16	1.86
		Ang vs Hap	Poly	74.2	0.32	0.61	0.42	0.39	2.87
		Ang vs Dis	Poly	80.3	0.24	0.14	0.18	0.86	3.70
		Ang vs Fea	Poly	58.4	0.26	0.92	0.40	0.08	2.27
		Ang Vs Sad	Poly	69.5	0.31	0.83	0.45	0.17	2.49
		Ang vs Sur	Poly	50.7	0.22	0.92	0.36	0.08	2.39
		Ang vs Hap	RBF	78.9	0.29	0.28	0.28	0.72	3.21
		Ang vs Dis	RBF	83.7	0.36	0.1	0.16	0.9	3.03
		Ang vs Fea	RBF	72.9	0.31	0.65	0.42	0.35	2.18
		Ang Vs Sad	RBF	75.2	0.3	0.47	0.36	0.53	3.23
		Ang vs Sur	RBF	61.6	0.27	0.9	0.41	0.1	2.09
One vs All	Max & Min	Ang vs All	Lin	79.7	0.39	0.6	0.47	0.4	2.93
		Ang vs All	Poly	76.7	0.33	0.54	0.41	0.46	2.50
		Ang vs All	RBF	90	0.66	0.68	0.67	0.32	2.76
	Z-norm	Ang vs All	Lin	74.7	0.3	0.5	0.38	0.5	2.60
		Ang vs All	Poly	75.3	0.3	0.46	0.36	0.54	3.58
		Ang vs All	RBF*	91.9	0.66	0.86	0.75	0.14	2.35

vs One' approach is a 'divide and conquer' approach, consisting of building one TWSVM class for each pair of subclasses. The 'One vs All' is a 'single approach' consisting of built TWSVM which is one class versus all other classes. This paper carried out both approaches of multi class TWSVM and it proved more effective in the 'One vs All' approach.

3 Experimental analysis and validation

3.1 Experimental settings

The FAPs [11], [34] 's definition has a combination of AU, for emotion identification, with entire and minimal feature vectors. This is shown in Fig. 2. From the FAPS definition

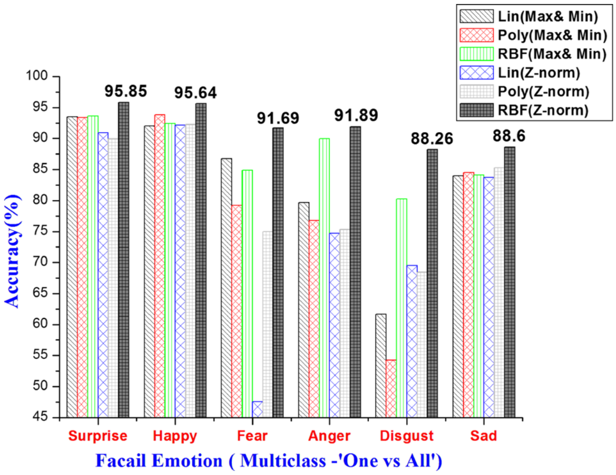
Table 9 The hold-out validation result of Disgust eyelids in both multi-classifier and normalization

	Norm	One vs One	Kernel	Acc(%)	Pre	Rec	F1-sco	Err.rte	Comp.Time (sec)
One vs One	Max &Min	Dis vs Hap	Lin	60.8	0.24	0.78	0.37	0.22	1.90
		Dis vs Ang	Lin	53.5	0.23	0.9	0.36	0.1	1.78
		Dis vs Fea	Lin	53.2	0.23	0.93	0.37	0.07	2.04
		Dis vs Sad	Lin	58.6	0.24	0.82	0.37	0.18	1.85
		Dis vs Sur	Lin	35.9	0.18	0.99	0.31	0.01	1.94
		Dis vs Hap	Poly	55.7	0.23	0.87	0.36	0.13	2.17
		Dis vs Ang	Poly	59.7	0.24	0.82	0.37	0.18	1.84
		Dis vs Fea	Poly	44.5	0.2	0.97	0.34	0.03	2.05
		Dis vs Sad	Poly	61.4	0.25	0.81	0.38	0.19	1.99
		Dis vs Sur	Poly	41	0.2	0.99	0.33	0.01	2.00
		Dis vs Hap	RBF	73.5	0.32	0.71	0.44	0.29	2.50
		Dis vs Ang	RBF	65.7	0.27	0.78	0.4	0.22	2.09
		Dis vs Fea	RBF	56.8	0.23	0.87	0.37	0.13	2.22
		Dis vs Sad	RBF	69.7	0.29	0.75	0.42	0.25	2.37
		Dis vs Sur	RBF	54.6	0.23	0.89	0.36	0.11	2.45
	Z-norm	Dis vs Hap	Lin	57.8	0.24	0.85	0.37	0.15	2.28
		Dis vs Ang	Lin	67.7	0.28	0.75	0.41	0.25	2.05
		Dis vs Fea	Lin	43.9	0.21	0.98	0.34	0.02	1.92
		Dis vs Sad	Lin	70	0.3	0.74	0.42	0.26	2.08
		Dis vs Sur	Lin	32.9	0.18	0.99	0.3	0.01	2.16
		Dis vs Hap	Poly	69.6	0.29	0.75	0.42	0.25	2.11
		Dis vs Ang	Poly	61.7	0.25	0.78	0.38	0.22	2.02
		Dis vs Fea	Poly	66.1	0.27	0.75	0.4	0.25	1.99
		Dis vs Sad	Poly	67.7	0.28	0.76	0.41	0.24	1.97
		Dis vs Sur	Poly	60.6	0.25	0.8	0.38	0.2	2.08
		Dis vs Hap	RBF	81.5	0.41	0.58	0.48	0.42	2.76
		Dis vs Ang	RBF	70.7	0.3	0.74	0.43	0.26	2.46
		Dis vs Fea	RBF	67.9	0.28	0.71	0.4	0.29	3.09
		Dis vs Sad	RBF	77.8	0.37	0.68	0.48	0.32	2.63
		Dis vs Sur	RBF	57.9	0.24	0.82	0.37	0.18	2.29
One vs All	Max & Min	Dis vs All	Lin	61.7	0.25	0.81	0.38	0.19	2.79
		Dis vs All	Poly	54.3	0.23	0.91	0.37	0.09	2.31
		Dis vs All	RBF	80.3	0.4	0.68	0.5	0.32	3.64
	Z-norm	Dis vs All	Lin	69.5	0.29	0.75	0.42	0.25	3.33
		Dis vs All	Poly	68.4	0.29	0.76	0.42	0.24	2.15
		Dis vs All	RBF*	88.27	0.56	0.92	0.7	0.07	2.07

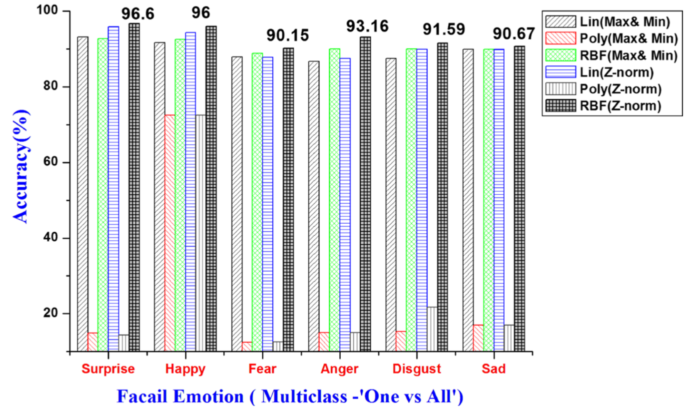
[34], the entire feature vectors displacement is in the form of the neutral to peak response then returning to the neutral state (i.e. expressive time episode = onset, apex and offset phase). In this system, the geometric deformable grid node (CLM) has L = 66*2 = 132 dimensions. The feature vectors displacement is employed for the set of six emotions (i.e. Surprise (SUR), Happy (HAP), Disgust (DIS), Fear (FEA), Angry (ANG) and Sad (SAD)) using TWSVM. In this proposed system, CLM and TWSVM are developed in C++ with open framework and Matlab2014b, which are implemented with an Intel i5 processor. In both training and testing phases, standard databases such as the MMI facial expression database [37], Oulu Database [38], CK [39], Extended CK+ database [40] and Mahonb Laughter database [41] are used. A few real-time databases were also used [24] in which the video rate was 25 frames/sec. In training and testing phases, let $\mathbf{g}_j$ be the feature vectors displacements of the extracted face image sequence from the standard database as $i = 1, \ldots, N$, $N = 6$, emotions in face, which is shown in Fig. 2. The normalized data of

Table 10 The hold-out validation result of Sad mouth in both multi-classifier and normalization

	Norm	One vs One	Kernel	Acc(%)	Pre	Rec	F1-sco	Err.rte	Comp.Time (sec)
One vs One	Max & Min	Sad vs Hap	Lin	79.6	0.4	0.34	0.37	0.66	3.74
		Sad vs Ang	Lin	76.3	0.3	0.26	0.36	0.74	2.52
		Sad vs Fea	Lin	67.6	0.22	0.34	0.27	0.66	1.95
		Sad vs Dis	Lin	82.3	0.49	0.28	0.28	0.72	1.93
		Sad vs Sur	Lin	54	0.27	0.92	0.41	0.08	2.20
		Sad vs Hap	Poly	83.5	0.53	0.49	0.51	0.51	2.54
		Sad vs Ang	Poly	68	0.32	0.72	0.39	0.28	2.29
		Sad vs Fea	Poly	78.2	0.42	0.65	0.51	0.35	2.11
		Sad vs Dis	Poly	49	0.25	0.93	0.44	0.07	2.16
		Sad vs Sur	Poly	41.7	0.23	0.98	0.37	0.02	2.48
		Sad vs Hap	RBF	85.4	0.63	0.4	0.51	0.6	3.41
		Sad vs Ang	RBF	72.3	0.34	0.62	0.4	0.38	2.67
		Sad vs Fea	RBF	84.1	0.56	0.43	0.49	0.57	2.44
		Sad vs Dis	RBF	53.1	0.26	0.9	0.44	0.1	2.19
		Sad vs Sur	RBF	46.2	0.24	0.96	0.39	0.04	2.53
	Z-norm	Sad vs Hap	Lin	78.6	0.36	0.27	0.31	0.73	3.22
		Sad vs Ang	Lin	74	0.31	0.39	0.39	0.61	2.76
		Sad vs Fea	Lin	57	0.22	0.59	0.33	0.41	1.89
		Sad vs Dis	Lin	65	0.28	0.63	0.34	0.37	1.74
		Sad vs Sur	Lin	78.5	0.3	0.17	0.21	0.83	1.90
		Sad vs Hap	Poly	84.9	0.6	0.42	0.49	0.58	2.32
		Sad vs Ang	Poly	78.9	0.4	0.39	0.51	0.61	1.88
		Sad vs Fea	Poly	83.5	0.54	0.36	0.43	0.64	1.89
		Sad vs Dis	Poly	80.8	0.46	0.58	0.42	0.42	2.03
		Sad vs Sur	Poly	67.1	0.33	0.86	0.48	0.14	3.63
		Sad vs Hap	RBF	79	0.43	0.65	0.52	0.35	3.06
		Sad vs Ang	RBF	66.2	0.3	0.7	0.47	0.3	2.33
		Sad vs Fea	RBF	69.5	0.33	0.69	0.44	0.31	2.11
		Sad vs Dis	RBF	68.6	0.34	0.8	0.42	0.2	2.63
		Sad vs Sur	RBF	52.2	0.26	0.91	0.4	0.09	2.19
One vs All	Max & Min	Sad vs All	Lin	84	0.63	0.22	0.32	0.78	3.75
		Sad vs All	Poly	84.5	0.65	0.25	0.36	0.75	3.64
		Sad vs All	RBF	84.1	0.64	0.22	0.33	0.78	3.04
	Z-norm	Sad vs All	Lin	83.7	0.78	0.1	0.18	0.9	2.16
		Sad vs All	Poly	85.3	0.75	0.24	0.36	0.76	2.84
		Sad vs All	RBF*	88.6	0.83	0.44	0.57	0.56	1.61



(a) Hold-out cross validation of Accuracy



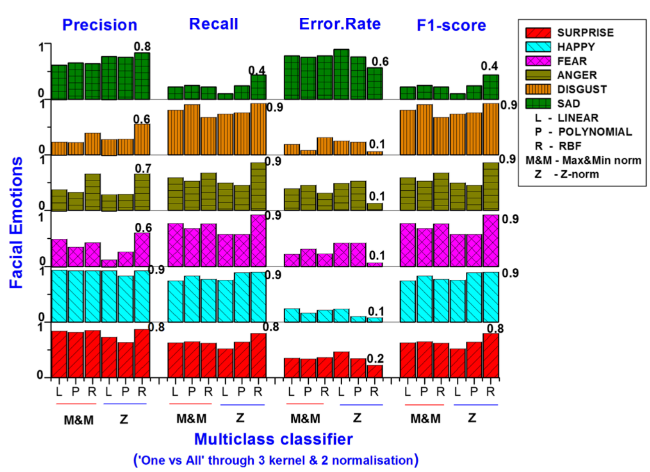
(b) 10-fold cross validation of Accuracy

Fig. 3 Bar chart representation of Accuracy of the proposed system ('One vs All') using both validation

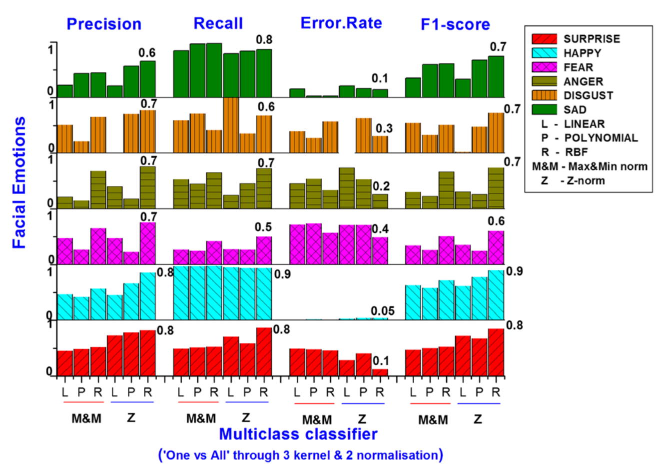
minimal feature vectors is fed as input to the multi-class TWSVM to classify facial emotions.

3.2 Training and testing

The 10-fold and hold out cross-validation technique is applied in both training and testing phases. A new database is formed through the fusion of existing standard databases such as MMI, Oulu, CK, CK+, Mahnob and a few real-time datasets. The normalized minimal feature vectors of cross-databases are given to hold out validation. In cross- validation, 80% of normalized datasets are given to the training phase and the remaining 20% of datasets is used as a testing phase for validating the facial emotion classifier. In validation, the multi-class TWSVM of 3 kernel cases (Linear, Polynomial and RBF) with both normalization are evaluated. In hold out cross-validation, 3 kernel case of penalty and kernel parameters values such as c1, c2 and $\gamma = 0.5$ is applied. In 10-fold cross-validation, the grid search mechanism is formulated for attaining the optimal parameters such as Cost (c1 and c2) and Gamma (γ) value in the training phase. The range of Cost (c1and c2) is $10^{-5}, 10^{-4.5}, ..., 10^{4.5}, 10^{5}$ and Gamma (γ) is $2^{-5}, 2^{-4.5}, ..., 2^{4.5}, 2^{5}$ are applied in the training phase. From the training phase, the optimal value of Cost and Gamma value is formed by the optimal trained model and then testing the unknown data of six emotions. In cross-validation, the linear case is considered where Cost value



(a) Hold-out cross validation of Precision, Recall, Error rate,F1-score



(b) 10-fold cross validation of Precision, Recall, Error rate,F1-score

Fig. 4 Bar chart representation of Precision, Recall, Error.rate,F1-score of the proposed system ('One vs All') using both validation and non-linear case (Poly, RBF), whereas Cost and Gamma value are considered. The Confusion Matrix of 'One vs One' and 'One vs All' is employed in both phases.

From the results of the Confusion Matrix, validation parameters such as True Positive (TP), True Negative (TN), False Positive (FP), False Negative (FN), Accuracy (Acc), Precision (Pre), Recall (Rec), F1-score (F1-sco), Error rate (Err.rte), and Computational Time (Comp. Time) are calculated. For each emotion, 18 models of multi-class ('One vs One' and 'One vs All' with 3 kernel) with one normalization method are calculated. A total of 36 models are calculated by Max & Min and Z-normalization through the multi class TWSVM classifier. The computation time of training and testing phase of all basic emotions are also calculated.

4 Experimental results and discussion

4.1 Experimental results

4.1.1 Hold out cross validation

Max-Min and Z-norm is applied on minimal feature vector displacement to produce the global normalized data which is fed as input to multi-class classifiers and validated with the hold out method for each kernel namely Linear, Polynomial (Poly) and RBF. In cases of 'One vs One' and 'One vs All', the validation parameter (accuracy, precision, recall, F1-score, error rate) of the six basic emotions are computed and shown in Tables 3, 4, 5, 6, 7, 8, 9, 10. From the Table 3–10, it is inferred that 'One vs All' multi classes have higher performance than the 'One vs One' multi classes. From the Table 3–10, the bar chart representation of validation parameter of proposed model ('One vs All') are plotted and shown in Figs. 3, 4, 5. In Figs. 3–5, relations between the validation parameters, Multi class and facial emotions. From Fig. 3a, it is inferred that the six emotions: Surprise (95.85%), Happy (95.6%), Disgust (88.26%), Fear (91.69%), Anger (91.89%) and Sad (88.6%) have high accuracy, which is achieved in **RBF kernel** of **'One vs All'** using **Z-norm**. Similarly, the validation parameter such as Precision, Recall, Error rate, F1-score, Computation Time for training and testing phase are also shown in Table 3–10, is calculated. From Tables 3–10 and Figs. 3a, 4a and 5, it is inferred that **RBF kernel** of **'One vs All'** using **Z-norm** has high Precision, high Recall, high F1-score, less Error rate and less computational time compared to other kernel (multi class) of normalization. The overall performance of the proposed model which was higher in **RBF ('One vs All')** and **Z-norm** are calculated and tabulated in Table 19. From Table 19, the overall validation parameter such as Accuracy is **92.05± 3.79%**, Precision is **0.75± 0.18**, Recall is **0.68±0.25**, Error rate is **0.32±0.25**, F1-score is **2.74± 0.98** and computation time **2.05±0.43** sec of basic six emotions are achieved.

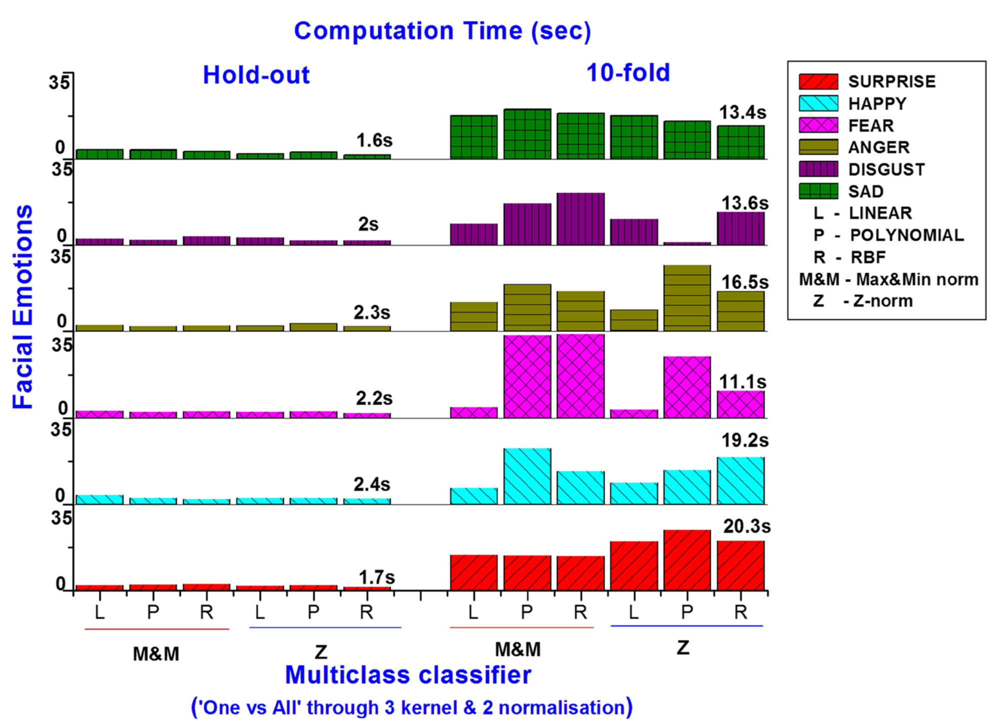


Fig. 5 Bar chart representation of Computation time of the proposed system (One vs All) using both validation

Table 11 The 10-fold validation result of Surprise eyebrow in both multi-classifier and normalization

	Norm	One vs One	Kernel	10-fold				Acc(%)	Pre	Rec	F1-sco	Err.rte	Time (s)
				(Acc(%),	Cost(c1, c2)	Gamma(γ)	Time(s)						
One vs One	Max & Min	Sur vs Hap	Lin	94	2.5,1.5	–	34.78	87.43	0.65	0.97	0.78	0.03	1.36
		Sur vs Ang	Lin	97.2	3.5,1	–	27.81	85.12	0.75	0.91	0.83	0.09	1.42
		Sur vs Dis	Lin	84.8	1.5,0.5	–	26.34	74.51	0.89	0.72	0.8	0.28	0.95
		Sur vs Fea	Lin	100	3,1.5	–	28.02	81.98	0.7	0.93	0.8	0.07	0.79
		Sur vs Sad	Lin	92.3	0.5,2	–	30.05	81.45	0.65	0.93	0.76	0.08	0.93
		Sur vs Hap	Poly	94	2.5,2	3.5	36.35	89.82	0.72	0.98	0.83	0.02	2.28
		Sur vs Ang	Poly	97.2	0.5,1	1.5	26.27	85.95	0.77	0.92	0.84	0.08	0.99
		Sur vs Dis	Poly	81.8	0.5,1	0.5	25.88	75.49	0.77	0.79	0.78	0.21	1.06
		Sur vs Fea	Poly	91.7	0.5,1	1	26.65	88.29	0.88	0.89	0.88	0.11	1.04
		Sur vs Sad	Poly	89.7	0.5,1	3	28.35	81.45	0.67	0.9	0.77	0.1	1.08
		Sur vs Hap	RBF	66	2,0.5	3.5	38.62	87.43	0.65	0.97	0.78	0.03	1.29
		Sur vs Ang	RBF	52.8	1.5,1.5	1.5	31.05	88.43	0.86	0.89	0.88	0.11	1.43
		Sur vs Dis	RBF	54.8	1.5,0.5	3.5	29.09	73.53	0.68	0.81	0.74	0.19	1.22
		Sur vs Fea	RBF	52.8	1.5,1.5	1.5	31.25	88.29	0.86	0.91	0.88	0.09	1.25
		Sur vs Sad	RBF	56.4	2,0.5	2	32.03	81.45	0.67	0.9	0.77	0.1	1.41
	Z-norm	Sur vs Hap	Lin	96	1.5,0.5	–	33.83	83.23	0.53	0.97	0.68	0.03	1.45
		Sur vs Ang	Lin	91.7	0.5,0.5	–	30.77	93.39	0.93	0.93	0.93	0.07	1.05
		Sur vs Dis	Lin	97.2	2.5,0.5	–	26.37	85.59	0.74	0.98	0.84	0.02	1.01
		Sur vs Fea	Lin	87.9	1,1	–	24.29	77.45	0.74	0.84	0.79	0.16	1.07
		Sur Vs Sad	Lin	94.9	2.5,0.5	–	27.13	79.03	0.58	0.94	0.72	0.06	0.94
		Sur vs Hap	Poly	96	1.5,1	3.5	33.8	93.41	0.86	0.94	0.9	0.06	2.13
		Sur vs Ang	Poly	100	2.5,1	3.5	26.73	89.26	0.88	0.89	0.88	0.11	1.26
		Sur vs Dis	Poly	100	0.5,1	3	27.63	90.99	0.86	0.96	0.91	0.04	1.25
		Sur vs Fea	Poly	93.5	0.5,1.5	0.5	25.32	78.43	0.88	0.77	0.82	0.23	1.04
		Sur vs Sad	Poly	94.9	3,1	3.5	26.9	82.26	0.95	0.74	0.83	0.26	1.39
		Sur vs Hap	RBF	66	2,0.5	3.5	41.07	91.62	0.79	0.96	0.87	0.04	2.45
		Sur vs Ang	RBF	52.8	1.5,1.5	1.5	33.14	89.26	0.79	0.98	0.87	0.02	1.75
		Sur vs Dis	RBF	52.8	1.5,1.5	1.5	35.67	91.89	0.88	0.96	0.92	0.04	1.86
		Sur vs Fea	RBF	54.8	1.5,0.5	2.5	29.56	69.61	0.58	0.83	0.68	0.18	1.93
		Sur vs Sad	RBF	56.4	2,0.5	2	31.77	79.03	0.58	0.94	0.72	0.06	1.53
One vs All	Max & Min	Sur vs All	Lin	93.4	0.5,1	–	105.6	85.14	0.46	0.5	0.5	0.5	14.8
		Sur vs All	Poly	14.2	0.5,0.5	3.5	95.18	85.64	0.49	0.52	0.5	0.48	14.4
		Sur vs All	RBF	93.4	1,3.5	1	84.16	86.15	0.53	0.53	0.5	0.47	14.3
	Z-norm	Sur vs All	Lin	95.9	0.5,1	–	119.8	91.94	0.74	0.71	0.72	0.29	20.2
		Sur vs All	Poly	14.2	0.5,0.5	3.5	194.8	89.17	0.79	0.59	0.68	0.41	24.9
		Sur vs All	RBF*	97.5	3,3.5	3	125.4	95.71	0.83	0.9	0.9	0.13	20.3

4.1.2 10-fold cross validation

The 10-fold cross-validation result of system shown in Tables 11, 12, 13, 14, 15, 16, 17, 18. Similarly, used Max & Min and Z-normalization with 3 kernel to attain the 36 optimal trained model of the FER system. Table 11–18 shows the 10 fold cross-validation of accuracy, cost, gamma, and validation parameter result of tested data. In Table 11, it is inferred that the RBF kernel of the 'One vs All' model achieved high performance using 10-fold validation of z-normalization data. From the result of the 10-fold RBF kernel model is the cost and gamma value for creating the optimal trained model for the surprise eyebrow feature. Then the tested data is given to the optimal trained

Table 12 The 10-fold validation result of Surprise mouth in both multi-classifier and normalization

	Norm	One vs One	Kernel	10-fold				Acc(%)	Pre	Rec	F1-sco	Err.rte	Time (s)
				Acc(%),	Cost(c1, c2)	Gamma(γ)	Time(s)						
One vs One	Max & Min	Sur vs Hap	Lin	84.62	0.5,2.5	–	52.24	78.53	0.47	0.87	0.61	0.13	2.49
		Sur vs Ang	Lin	92.11	2.5,3	–	41.87	64.41	0.9	0.59	0.71	0.41	1.49
		Sur vs Dis	Lin	91.43	1,1.5	–	30.88	81.2	0.64	0.97	0.77	0.03	1.01
		Sur vs Fea	Lin	90.91	2.5,2.5	–	28.4	74.51	0.9	0.72	0.8	0.28	1.27
		Sur vs Sad	Lin	95	3,2	–	35.44	87.69	0.81	0.9	0.85	0.1	2.27
		Sur vs Hap	Poly	88	0.5,1.5	1	43.49	80.98	0.53	0.89	0.67	0.11	2.88
		Sur vs Ang	Poly	94.44	2,1	2	58.13	71.19	0.41	1	0.59	0	2.01
		Sur vs Dis	Poly	91.43	3.5,1	3.5	36.1	76.92	0.9	0.71	0.79	0.29	4.11
		Sur vs Fea	Poly	91.43	3,2	3	28.03	73.53	0.97	0.69	0.81	0.31	1.59
		Sur vs Sad	Poly	95	2,2	2	34.48	73.08	0.41	0.96	0.58	0.04	1.7
		Sur vs Hap	RBF	63.46	2,0.5	3.5	49.36	78.53	0.47	0.87	0.61	0.13	3.01
		Sur vs Ang	RBF	52.78	0.5,0.5	3.5	34.95	74.58	0.48	1	0.65	0	1.43
		Sur vs Dis	RBF	54.29	2.5,0.5	3.5	45.8	82.91	0.84	0.82	0.83	0.18	2.92
		Sur vs Fea	RBF	57.58	2.5,0.5	3.5	31.6	74.51	0.9	0.72	0.8	0.28	1.35
		Sur vs Sad	RBF	52.5	1.5,0.5	3	44.26	80	0.57	0.97	0.72	0.03	2.09
	Z-norm	Sur vs Hap	Lin	89.8	1,1	–	34.52	77.64	0.41	0.88	0.56	0.12	1.47
		Sur vs Ang	Lin	97.22	2,0.5	–	33.34	75	0.48	1	0.65	0	1.53
		Sur vs Dis	Lin	94.29	3.5,3	–	34.76	71.3	0.41	1	0.58	0	2.27
		Sur vs Fea	Lin	90.91	0.5,1	–	33.04	77	0.63	0.95	0.75	0.05	1.34
		Sur Vs Sad	Lin	94.74	0.5,1	–	29.54	87.5	0.73	0.98	0.84	0.02	0.75
		Sur vs Hap	Poly	93.88	2,1.5	2.5	37.23	77.02	0.36	0.95	0.52	0.05	2.83
		Sur vs Ang	Poly	97.22	0.5,1	0.5	58.86	85.34	0.8	0.88	0.84	0.12	2.05
		Sur vs Dis	Poly	94.29	3.5,1	3.5	41.36	78.26	0.57	0.97	0.72	0.03	1.6
		Sur vs Fea	Poly	93.94	1,1.5	2.5	25.1	79	0.8	0.82	0.81	0.18	0.88
		Sur vs Sad	Poly	97.37	0.5,1	3.5	29.19	85.16	0.77	0.88	0.82	0.12	1.71
		Sur vs Hap	RBF	65.31	2,0.5	3	39.77	77.64	0.41	0.88	0.56	0.12	7.03
		Sur vs Ang	RBF	52.78	1.5,1.5	1.5	33.14	86.21	0.71	1	0.83	0	1.98
		Sur vs Dis	RBF	51.52	0.5,0.5	3.5	47.4	83.48	0.7	0.95	0.8	0.05	1.58
		Sur vs Fea	RBF	54.84	1.5,0.5	3	29.31	79	0.7	0.91	0.79	0.09	1.71
		Sur vs Sad	RBF	55.26	1.5,1.5	1.5	33.6	77.34	0.48	1	0.65	0	5
One vs All	Max & Min	Sur vs All	Lin	92.74	0.5,0.5	–	100.2	90.02	0.33	0.89	0.49	0.05	17.3
		Sur vs All	Poly	15.57	0.5,0.5	3.5	133.1	89.78	0.33	0.9	0.48	0.1	29.4
		Sur vs All	RBF	91.94	3,2.5	3	110.5	87.53	0.16	0.9	0.26	0.1	20.6
		Sur vs All	Lin	95.8	0.5,3.5	–	90.9	90.15	0.36	0.87	0.51	0.13	12.1
	Z-norm	Sur vs All	Poly	14.53	0.5,0.5	3.5	108.6	90.15	0.38	0.84	0.52	0.16	21.5
		Sur vs All	RBF*	95.8	2,1.5	3.5	77.76	95.44	0.8	0.95	0.84	0.11	16.8

model achieving high validation parameter when compared to other multi class models of surprise eyebrow. Similarly, in Tables 11–18, it is inferred that the RBF kernel of 'One vs All' model achieved high performance using 10-fold validation of z-normalization data. Somehow emotion has attained high accuracy in different optimal models but the performance is less than the RBF Trained model. From Fig. 3b, it is inferred the six emotions such as **Surprise (96.6%), Happy (96%), Disgust(91.6%), Fear(90.15%), Anger(93.16%) and Sad(90.67%)** has high accuracy is achieved in **RBF kernel** of **'One vs All'** using **Z-norm**. From Table 11–18 and Figs. 3b, 4b and 5 this inference the

Table 13 The 10-fold validation result of Happy mouth in both multi-classifier and normalization

	Norm	One vs One		10-fold				Acc(%)	Pre	Rec	F1-sco	Err.rte	Time (s)
				(Acc(%),	Cost(c1, c2)	Gamma(γ)	Time(s)						
One vs One	Max & Min	Hap vs Ang	Lin	92.31	2,0.5	–	67.97	84.94	0.76	1	0.87	0	3.66
		Hap vs Dis	Lin	90.2	3,0.5	–	53.53	86.06	0.96	0.84	0.9	0.16	3.66
		Hap vs Fea	Lin	95.92	1.5,0.5	–	37.97	89.4	0.96	0.89	0.93	0.11	2
		Hap vs Sad	Lin	94.23	2,0.5	–	48.86	79.66	0.67	0.99	0.8	0.01	1.23
		Hap vs Sur	Lin	96	0.5,0.5	–	49.58	83.95	0.78	0.97	0.86	0.03	3.63
		Hap vs Ang	Poly	94.23	2,2.5	2	51.31	89.16	0.83	1	0.91	0	3.89
		Hap vs Dis	Poly	93.88	3,2	3	49.7	81.21	0.98	0.78	0.87	0.22	2.66
		Hap vs Fea	Poly	93.88	2,1.5	2	63.97	89.4	0.97	0.89	0.93	0.11	4.69
		Hap vs Sad	Poly	96.3	2.5,1.5	3	51.19	83.05	0.73	0.99	0.84	0.01	2.25
		Hap vs Sur	Poly	96	0.5,1	2.5	60.73	86.42	0.98	0.84	0.9	0.16	2.83
		Hap vs Ang	RBF	66	2,0.5	3.5	50.74	88.55	0.82	1	0.9	0	3.14
		Hap vs Dis	RBF	67.35	2,0.5	3.5	65	86.06	0.94	0.85	0.9	0.15	7.11
		Hap vs Fea	RBF	70.21	2.5,0.5	3.5	37.08	85.43	0.98	0.84	0.9	0.16	2.28
		Hap vs Sad	RBF	63.46	2,0.5	3.5	50.08	86.44	0.78	0.99	0.87	0.01	2.74
		Hap vs Sur	RBF	68.75	2,0.5	3.5	42.96	86.42	0.83	0.96	0.89	0.04	2.86
	Z-norm	Hap vs Ang	Lin	93.15	2,2.5	–	44.76	83.13	0.74	1	0.85	0	2.04
		Hap vs Dis	Lin	93.88	3.5,2.5	–	54.14	84.24	0.97	0.82	0.89	0.18	2.23
		Hap vs Fea	Lin	93.88	2,0.5	–	34.53	90.07	0.96	0.9	0.93	0.1	1.6
		Hap vs Sad	Lin	94.44	2,0.5	–	70	78.53	0.64	1	0.78	0	1.96
		Hap vs Sur	Lin	93.88	1,0.5	–	40.05	80.13	0.75	0.96	0.84	0.04	1.95
		Hap vs Ang	Poly	90.38	0.5,1	0.5	38.55	93.37	0.94	0.95	0.95	0.05	1.9
		Hap vs Dis	Poly	85.71	3.5,3.5	3.5	67.49	86.67	0.92	0.88	0.9	0.13	4.69
		Hap vs Fea	Poly	93.88	3,1	3	39.19	90.73	0.96	0.91	0.94	0.09	2.25
		Hap vs Sad	Poly	94.44	3,1	2	40.98	93.22	0.92	0.96	0.94	0.04	2.55
		Hap vs Sur	Poly	93.88	2,1	3	44.2	91.39	0.92	0.95	0.94	0.05	3.81
		Hap vs Ang	RBF	66	2,0.5	3.5	43.55	89.76	0.84	1	0.91	0	1.91
		Hap vs Dis	RBF	85.71	3.5,3.5	3.5	67.49	86.67	0.92	0.88	0.9	0.13	4.69
		Hap vs Fea	RBF	70.21	2.5,0.5	3.5	41.93	88.74	0.98	0.87	0.92	0.13	2.5
		Hap vs Sad	RBF	63.46	2.5,0.5	3.5	69.77	89.27	0.84	0.98	0.9	0.02	2.32
		Hap vs Sur	RBF	70.21	2,0.5	3.5	49.63	88.74	0.98	0.87	0.92	0.13	2.42
One vs All	Max & Min	Hap vs All	Lin	91.67	0.5,2	–	116.8	85.64	0.47	0.98	0.64	0.02	6.78
		Hap vs All	Poly	72.5	3,0.5	3.5	123	84.38	0.42	0.98	0.59	0.02	22.8
		Hap vs All	RBF	92.5	0.5,1	1	113.6	88.41	0.58	0.98	0.73	0.02	13.7
	Z-norm	Hap vs All	Lin	94.33	1.5,1	–	108.7	85.14	0.46	0.96	0.62	0.04	8.99
		Hap vs All	Poly	72.5	3,0.5	3.5	164.1	90.18	0.67	0.95	0.78	0.05	14.1
		Hap vs All	RBF*	96	3.5,1	3.5	140.9	95.21	0.9	0.95	0.91	0.05	19.3

RBF kernel model has high optimal performance for all six basic emotions when compared to the other model.

The overall performance of proposed model that is higher in RBF ('One vs All') and Z-norm are calculated and tabulated in Table 19. From Table 19, the overall validation parameter such as Accuracy is **93.42±3.25%** (10-fold) and **92.56±3.02%** (test), Precision is **0.76± 0.11**, Recall is **0.73±0.22**, Error. Rate is **0.27±0.22**, F1-score is **20.76±0.22** and computation time **15.08±4.08** sec of basic six emotions are achieved. From the experiments, it is found that the minimal feature vector which is specified in Table 2 is good enough to classify the basic six emotions such as

Table 14 The 10-fold validation result of Fear eyebrow in both multi-classifier and normalization

	Norm	One vs One	Kernel	10-fold				Acc(%)	Pre	Rec	F1-sco	Err.rte	Time (s)
				(Acc(%),	Cost(c1, c2)	Gamma(γ)	Time(s)						
One vs One	Max & Min	Fea vs Hap	Lin	85.11	0.5,0.5	–	51.11	78.06	0.29	0.87	0.43	0.13	1.41
		Fea vs Ang	Lin	81.82	1.5,0.5	–	30.12	73.39	0.49	0.79	0.6	0.21	0.93
		Fea vs Dis	Lin	84.85	1,0.5	–	44.14	74.75	0.58	0.81	0.68	0.19	1.36
		Fea vs Sad	Lin	75	1,1.5	–	42.62	66.96	0.22	0.83	0.35	0.17	1.57
		Fea vs Sur	Lin	90.02	0.5,1	–	40.55	74.51	0.73	0.7	0.72	0.3	0.91
		Fea vs Hap	Poly	87.23	0.5,1	0.5	56.94	77.42	0.24	0.92	0.39	0.08	2.14
		Fea vs Ang	Poly	81.82	0.5,1	1	30.24	77.06	0.58	0.81	0.68	0.19	0.91
		Fea vs Dis	Poly	84.85	2.5,1.5	2.5	43.83	73.74	0.56	0.81	0.66	0.19	2.06
		Fea vs Sad	Poly	75	0.5,1	0.5	47.16	66.96	0.22	0.83	0.35	0.17	1.28
		Fea vs Sur	Poly	87.1	1.5,1	1.5	39.66	66.67	0.84	0.58	0.69	0.42	1.52
		Fea vs Hap	RBF	68.09	2.5,0.5	3.5	58.8	73.55	0.09	1	0.16	0	6.67
		Fea vs Ang	RBF	54.55	1.5,0.5	3.5	31.9	73.39	0.49	0.79	0.6	0.21	1.01
		Fea vs Dis	RBF	54.55	1.5,0.5	3.5	46.76	71.72	0.49	0.81	0.61	0.19	1.85
		Fea vs Sad	RBF	58.33	1.5,0.5	3.5	62.61	70.54	0.33	0.83	0.48	0.17	3.58
		Fea vs Sur	RBF	51.61	1.5,0.5	3.5	46.89	74.51	0.73	0.7	0.72	0.3	1.69
	Z-norm	Fea vs Hap	Lin	89.36	0.5,1	–	89.04	72.9	0.67	0.53	0.59	0.47	2.82
		Fea vs Ang	Lin	87.88	0.5,3.5	–	75.19	71.56	0.73	0.63	0.68	0.37	1.81
		Fea vs Dis	Lin	87.88	2.5,0.5	–	69.54	70.71	0.71	0.67	0.69	0.33	2.52
		Fea vs Sad	Lin	83.12	1.5,0.5	–	101.2	68.11	0.56	0.29	0.38	0.71	3.77
		Fea vs Sur	Lin	83.87	0.5,0.5	–	63.23	69.61	0.38	0.85	0.52	0.15	1.64
		Fea vs Hap	Poly	90.19	2,3.5	2	87.55	74.84	0.53	0.57	0.55	0.43	8.4
		Fea vs Ang	Poly	90.04	0.5,1	1.5	83.51	73.39	0.73	0.66	0.69	0.34	2.73
		Fea vs Dis	Poly	54.55	1.5,0.5	3.5	185.4	68.69	0.64	0.66	0.65	0.34	4.09
		Fea vs Sad	Poly	87.01	3,1	3	125.2	19.69	0.87	0.16	0.28	0.84	4.58
		Fea vs Sur	Poly	90.32	1,3.5	1.5	66.21	68.63	0.44	0.74	0.56	0.26	2.54
		Fea vs Hap	RBF	68.09	2.5,0.5	3.5	98.9	76.13	0.38	0.65	0.48	0.35	11.7
		Fea vs Ang	RBF	54.55	1.5,0.5	3.5	80.72	71.56	0.64	0.66	0.65	0.34	3.69
		Fea vs Dis	RBF	54.55	1.5,0.5	3.5	78.68	75.76	0.64	0.78	0.71	0.22	2.94
		Fea vs Sad	RBF	19.48	0.5,0.5	3.5	194.1	62.6	0.67	0.27	0.39	0.73	3.74
		Fea vs Sur	RBF	51.61	1.5,0.5	3.5	79.04	59.8	0.49	0.55	0.52	0.45	2.7
One vs All	Max & Min	Fea vs All	Lin	87.5	1,0.5	–	156.7	80.1	0.51	0.29	0.37	0.71	4.92
		Fea vs All	Poly	12.5	0.5,0.5	3.5	238.3	83.63	0.22	0.25	0.24	0.75	66.5
		Fea vs All	RBF	88.33	1,1.5	1	176	85.39	0.64	0.41	0.5	0.59	57.7
	Z-norm	Fea vs All	Lin	87.5	1.5,0.5	–	151.7	81.61	0.56	0.32	0.41	0.68	5
		Fea vs All	Poly	12.5	0.5,0.5	3.5	379.3	85.64	0.24	0.32	0.28	0.68	19.7
		Fea vs All	RBF*	90.17	1.5,1	3.5	213.3	88.66	0.76	0.48	0.59	0.52	5.36

Surprise (eyebrow+outer lip), Happy (corner lip), Disgust (eyelids), Fear (eyebrow + outer lip), Anger (eyebrow) and Sad (corner lip+outer lip).

4.2 Discussion

From Table 20 the comparison of various classifier models for recognition of facial emotion shows supervised and semi-supervised models, databases, and accuracy of the model. From Table 20, Uddin [30] have supervised classifiers as LDP+HMM and LDP-PCA+HMM used for facial emotion recognition, which achieved accuracy are 82.91% and 87.50% respectively. Yu [14] has used SVM+PCA supervised classifier for facial emotion recognition achieved 75.50% accuracy. Saeed [15] has applied a supervised SVM classifier for facial emotion recognition that achieved 83% accuracy. Wan [16] applied a supervised SVM classifier for facial emotion recognition that achieved 80% accuracy.

Table 15 The 10-fold validation result of Fear mouth in both multi-classifier and normalization

	Norm	One vs One	Kernel	10-fold				Acc(%)	Pre	Rec	F1-sco	Err.rte	Time (s)
				(Acc(%),	Cost(c1, c2)	Gamma(γ)	Time(s)						
One vs One	Max & Min	Fea vs Hap	Lin	74.47	0.5,0.5	–	33.22	71.14	0.07	0.6	0.12	0.4	1.14
		Fea vs Ang	Lin	66.67	0.5,1	–	26.3	61.54	0.34	0.58	0.43	0.42	0.65
		Fea vs Dis	Lin	68.75	0.5,0.5	–	25.68	59.22	0.18	0.57	0.28	0.43	0.62
		Fea vs Sad	Lin	71.43	0.5,0.5	–	27.75	65.52	0.23	0.63	0.33	0.38	0.71
		Fea vs Sur	Lin	90.91	1.5,1	–	30.9	80.39	0.89	0.72	0.8	0.28	1.1
		Fea vs Hap	Poly	78.72	3.5,2	3.5	40.5	70.47	0	0	0	0	2.48
		Fea vs Ang	Poly	72.73	2,2.5	2	28.93	62.5	0.55	0.56	0.55	0.44	1.76
		Fea vs Dis	Poly	68.75	0.5,1	3	27.65	59.22	0.41	0.53	0.46	0.47	0.78
		Fea vs Sad	Poly	74.29	1,1	1	29.5	65.52	0.32	0.58	0.41	0.42	5.13
		Fea vs Sur	Poly	90.91	3,1.5	3	30.49	74.51	0.52	0.82	0.64	0.18	1.88
		Fea vs Hap	RBF	68.09	2.5,0.5	3.5	43.72	70.47	0	0	0	0	6.95
		Fea vs Ang	RBF	54.55	1.5,0.5	3.5	33.17	42.31	1	0.42	0.59	0.58	4.51
		Fea vs Dis	RBF	53.13	1.5,0.5	3.5	31.75	57.28	0	0	0	0	5.1
		Fea vs Sad	RBF	57.14	1.5,0.5	3.5	33.61	62.07	0	0	0	0	7.38
		Fea vs Sur	RBF	54.55	1.5,0.5	3.5	34.53	83.33	0.95	0.74	0.83	0.26	1.27
	Z-norm	Fea vs Hap	Lin	78.26	0.5,1.5	–	31.08	73.83	0.14	0.86	0.24	0.14	1.05
		Fea vs Ang	Lin	75.76	0.5,1.5	–	31.71	62.5	0.48	0.57	0.52	0.43	1
		Fea vs Dis	Lin	68.75	0.5,1	–	25.05	58.25	0.18	0.53	0.27	0.47	0.58
		Fea vs Sad	Lin	74.29	2,1	–	26.99	62.07	0	0	0	0	0.88
		Fea vs Sur	Lin	90.55	1.5,0.5	–	33.7	75	1	0.64	0.78	0.36	1.25
		Fea vs Hap	Poly	84.78	2.5,2.5	2.5	37.98	70.47	0	0	0	0	2.49
		Fea vs Ang	Poly	75.76	2,1.5	2	28.77	44.23	0.98	0.43	0.6	0.57	2.18
		Fea vs Dis	Poly	68.75	0.5,1	1.5	26.96	60.19	0.3	0.57	0.39	0.43	0.79
		Fea vs Sad	Poly	80	0.5,2	0.5	29.78	68.97	0.43	0.63	0.51	0.37	0.96
		Fea vs Sur	Poly	90.55	2.5,1.5	2.5	27.88	83	0.91	0.75	0.82	0.25	1.22
		Fea vs Hap	RBF	67.39	2.5,0.5	3.5	41.44	70.47	0	0	0	0	15.1
		Fea vs Ang	RBF	54.55	1.5,0.5	3.5	32.74	64.42	0.48	0.6	0.53	0.4	3.2
		Fea vs Dis	RBF	53.13	1.5,0.5	3.5	32.67	57.28	0	0	0	0	2.55
		Fea vs Sad	RBF	57.14	1.5,0.5	3.5	35.03	62.93	0.02	1	0.04	0	4.06
		Fea vs Sur	RBF	51.61	1.5,0.5	3.5	32.4	83	0.95	0.74	0.83	0.26	1.65
One vs All	Max & Min	Fea vs All	Lin	88.33	0.5,2	–	57.73	80.1	0.45	0.27	0.34	0.73	4.49
		Fea vs All	Poly	12.5	0.5,0.5	3.5	105.8	82.37	0.34	0.27	0.3	0.73	0.97
		Fea vs All	RBF	89.33	1,1	3.5	69.47	86.68	0.6	0.44	0.53	0.56	10.6
	Z-norm	Fea vs All	Lin	88.14	2.5,1	–	63.61	80.1	0.41	0.25	0.31	0.75	2.29
		Fea vs All	Poly	12.71	0.5,0.5	3.5	114.7	83.12	0.22	0.24	0.23	0.76	30.9
		Fea vs All	RBF*	90.14	0.5,1	2.5	83.18	90.43	0.76	0.53	0.63	0.47	16.8

Mohammadian [17] used a supervised SVM classifier for facial emotion recognition which achieved 83.90% accuracy. Ren [18] applied the Fuzzy+SVM for facial emotion recognition that achieved an 81.4% accuracy. Jiang [19] used a SRC classifier for facial emotion recognition which achieved an 80% accuracy. Papachristou [20] applied an SSL classifier for facial emotion recognition with 71.14% (CK, CK+) and 71.84% (BU) accuracy. Nikitidis [21] used a MMPP classifier for facial emotion recognition with 80.9% accuracy. Owusu [1] has used SVM of facial emotion system was attained 97.57% (JAFFE), 92.33% (Yale) accuracy. Patil [2] has developed Patch-LDSMTNN for Facial emotion system of accuracy are 98.33% (JAFFE), 99.27% (CMU-AMP), 98.14% (ORL), 98.44% (Yale), 98.49% (CK) was achieved. Siddiqi [3] has developed robust facial emotion system with help HMM 95% (You tube images)

Table 16 The 10-fold validation result of Anger eyebrow in both multi-classifier and normalization

	Norm	One vs One	Kernel	10-fold				Acc(%)	Pre	Rec	F1-sco	Err.rte	Time (s)
				(Acc(%),	Cost(c1, c2)	Gamma(γ)	Time(s)						
One vs One	Max & Min	Ang vs Hap	Lin	88	1,0.5	–	43.57	82.18	0.66	0.82	0.73	0.18	1.39
		Ang vs Fea	Lin	63.89	1,0.5	–	35.72	54.24	1	0.54	0.7	0.46	1.67
		Ang vs Dis	Lin	93.94	2,0.5	–	45.08	73.39	0.91	0.72	0.8	0.28	1.59
		Ang vs Sad	Lin	92.31	2.5,0.5	–	32.65	83.21	0.7	0.94	0.8	0.06	1.2
		Ang vs Sur	Lin	94.12	3.5,0.5	–	39.31	85.95	0.95	0.81	0.88	0.19	1.31
		Ang vs Hap	Poly	88	0.5,3	0.5	56.07	81.03	0.83	0.71	0.76	0.29	2.08
		Ang vs Fea	Poly	69.44	1.5,1.5	1.5	35.1	54.24	1	0.54	0.7	0.46	3.39
		Ang vs Dis	Poly	93.94	2,2	3	31.82	68.81	0.61	0.81	0.7	0.19	1.78
		Ang vs Sad	Poly	89.74	0.5,3.5	0.5	34.04	81.68	0.66	0.95	0.78	0.05	1.15
		Ang vs Sur	Poly	94.12	0.5,1	3.5	41.33	83.47	0.98	0.77	0.86	0.23	1.39
		Ang vs Hap	RBF	64	2,0.5	3.5	59.51	83.33	0.61	0.91	0.73	0.09	3.82
		Ang vs Fea	RBF	50	0.5,0.5	3.5	40.19	54.24	0.36	0.64	0.46	0.36	1.27
		Ang vs Dis	RBF	54.55	1.5,0.5	3.5	33.87	74.31	0.94	0.71	0.81	0.29	1.29
		Ang vs Sad	RBF	53.85	2.5,2	3.5	36.1	83.21	0.7	0.94	0.8	0.06	1.81
		Ang vs Sur	RBF	52.94	1.5,3.5	2	48.89	86.78	0.97	0.82	0.89	0.18	1.94
	Z-norm	Ang vs Hap	Lin	90	1,0.5	–	39.89	79.89	0.48	0.94	0.64	0.06	1.17
		Ang vs Fea	Lin	52.78	1.5,0.5	–	29.53	54.24	1	0.54	0.7	0.46	0.77
		Ang vs Dis	Lin	90.91	0.5,0.5	–	29.68	70.64	0.64	0.82	0.72	0.18	0.7
		Ang vs Sad	Lin	92.31	2.5,0.5	–	33.67	53.44	0.05	1	0.09	0	1.3
		Ang vs Sur	Lin	97.06	2.5,1	–	30.83	69.42	1	0.63	0.78	0.37	1.34
		Ang vs Hap	Poly	94	1,1	2.5	44.57	69.54	0.17	1	0.29	0	2.07
		Ang vs Fea	Poly	61.11	0.5,1.5	0.5	29.96	55.93	1	0.55	0.71	0.45	0.87
		Ang vs Dis	Poly	81.82	2.5,1	2.5	29.84	69.72	0.94	0.67	0.78	0.33	1.31
		Ang vs Sad	Poly	97.44	1,1.5	1	34.41	84.73	0.69	1	0.81	0	1.22
		Ang vs Sur	Poly	94.12	0.5,1	1	28.35	72.73	1	0.66	0.8	0.34	1.04
		Ang vs Hap	RBF	64	2,0.5	3.5	43.63	64.94	0.05	1	0.09	0	2.7
		Ang vs Fea	RBF	50	0.5,0.5	3.5	34.48	40.68	0.52	0.46	0.49	0.54	0.98
		Ang vs Dis	RBF	54.55	1.5,0.5	3.5	31.97	57.8	0.3	0.95	0.45	0.05	1.38
		Ang vs Sad	RBF	53.85	1.5,0.5	3.5	37.04	52.67	0.03	1	0.06	0	1.37
		Ang vs Sur	RBF	52.94	2,2	3	32.86	63.64	1	0.59	0.74	0.41	1.29
One vs All	Max & Min	Ang vs All	Lin	86.67	0.5,1.5	–	68.42	84.38	0.22	0.54	0.31	0.46	12.2
		Ang vs All	Poly	15	0.5,0.5	3.5	113.9	83.38	0.16	0.45	0.23	0.55	19.4
		Ang vs All	RBF	90	0.5,1	0.5	81.49	86.65	0.53	0.6	0.67	0.4	16.7
	Z-norm	Ang vs All	Lin	87.5	0.5,0.5	–	67.74	80.1	0.41	0.25	0.31	0.75	9.01
		Ang vs All	Poly	15	0.5,0.5	3.5	118.2	83.38	0.19	0.46	0.27	0.54	27.2
		Ang vs All	RBF*	93.17	0.5,2	0.5	80.07	91.69	0.77	0.73	0.75	0.36	16.5

accuracy was achieved. Cohen [27] have a three semi supervised classifier for facial emotion that achieved accuracy are Navies Bayes (69.10 ± 1.44%), Tree-Augmented Naive Bayes classifier (69.30 ± 1.44%), and Stochastic search (74.80 ± 1.36%). Rifai [28] has semi-supervised classifier is CC-NET+CDA+SVM, which achieved 85% accuracy of facial emotion recognition. Jihang [29] achieved 82.68% and 87.71% accuracy using Transfer Learning Adaptive Boosting semi-supervised learning for facial emotions. From Table 20, [1], [2],[3], it is inferrred that it has a higher accuaracy than our proposed model, but the drawback is differences in types of datasets. Our proposed FERs model

Table 17 The 10-fold validation result of Disgust eyelids in both multi-classifier and normalization

	Norm	One vs One	Kernel	10-fold				Acc(%)	Pre	Rec	F1-sco	Err.rte	Time (s)
				(Acc(%),	Cost(c1, c2)	Gamma(γ)	Time(s)						
One vs One	Max & Min	Dis vs Hap	Lin	79.59	0.5,0.5	–	59.74	74.1	0.97	0.59	0.73	0.41	2.24
		Dis vs Ang	Lin	85.71	0.5,0.5	–	34.04	75.41	0.98	0.67	0.8	0.33	1.5
		Dis vs Fea	Lin	88.24	1,0.5	–	29.68	81.13	0.98	0.76	0.86	0.24	1.25
		Dis vs Sad	Lin	83.78	0.5,0.5	–	46.16	71.76	0.97	0.62	0.76	0.38	3.57
		Dis vs Sur	Lin	90.91	0.5,0.5	–	30.41	83.62	1	0.76	0.86	0.24	0.82
		Dis vs Hap	Poly	85.71	0.5,1.5	0.5	103.6	59.04	1	0.47	0.64	0.53	2.54
		Dis vs Ang	Poly	91.89	1.5,2.5	2	32.73	78.69	0.95	0.71	0.81	0.29	1.55
		Dis vs Fea	Poly	79.41	2.5,1	2.5	31.17	81.13	0.98	0.76	0.86	0.24	1.39
		Dis vs Sad	Poly	87.18	1.5,1	1.5	64.19	74.05	0.8	0.69	0.74	0.31	4.08
		Dis vs Sur	Poly	84.85	3,1	3.5	35.09	91.38	0.97	0.88	0.92	0.12	3.41
		Dis vs Hap	RBF	64.71	2,0.5	3.5	81.76	71.69	0.28	0.81	0.42	0.19	3.23
		Dis vs Ang	RBF	51.43	0.5,0.5	3.5	34.51	80.33	0.95	0.73	0.83	0.27	1.15
		Dis vs Fea	RBF	56.25	1.5,0.5	3.5	32.58	79.25	0.98	0.74	0.84	0.26	1.33
		Dis vs Sad	RBF	53.85	1.5,0.5	3.5	50.71	73.28	0.9	0.65	0.76	0.35	2.14
		Dis vs Sur	RBF	54.55	2,0.5	2	44.25	88.79	0.98	0.83	0.9	0.17	2.36
	Z-norm	Dis vs Hap	Lin	84.31	0.5,2.5	–	38.59	76.36	0.46	0.79	0.58	0.21	1.05
		Dis vs Ang	Lin	86.49	2,3	–	44.17	77.69	0.81	0.75	0.78	0.25	1.63
		Dis vs Fea	Lin	82.35	0.5,1.5	–	39.19	71.43	0.98	0.67	0.79	0.33	1.13
		Dis vs Sad	Lin	89.74	2.5,1	–	42.05	70.77	0.66	0.68	0.67	0.32	1.87
		Dis vs Sur	Lin	91.43	3.5,0.5	–	43.87	59.72	0.51	1	0.67	0	1.34
		Dis vs Hap	Poly	85.71	2.5,1.5	2.5	103.7	70.91	0.2	0.92	0.33	0.08	4.64
		Dis vs Ang	Poly	86.49	2.5,1	3.5	29.64	75.21	0.51	0.97	0.67	0.03	1.28
		Dis vs Fea	Poly	87.5	3,2	3	33.84	73.33	0.71	0.79	0.75	0.21	1.77
		Dis vs Sad	Poly	84.62	1.5,2.5	1.5	62.94	68.46	0.81	0.62	0.7	0.38	3.68
		Dis vs Sur	Poly	91.14	2.5,1	3.5	47.17	86.11	0.85	0.98	0.91	0.02	2.6
		Dis vs Hap	RBF	64.71	2.5,0.5	3.5	84.84	73.33	0.66	0.62	0.64	0.38	10.3
		Dis vs Ang	RBF	51.43	0.5,0.5	3	44.85	76.86	0.75	0.77	0.76	0.23	1.61
		Dis vs Fea	RBF	56.25	3.5,0.5	3.5	55.27	72.38	1	0.67	0.8	0.33	6.03
		Dis vs Sad	RBF	53.85	1.5,0.5	3.5	67.94	68.46	0.83	0.61	0.71	0.39	4.93
		Dis vs Sur	RBF	54.55	2,0.5	2	52.85	93.06	0.93	0.98	0.96	0.02	3.87
One vs All	Max & Min	Dis vs All	Lin	87.5	0.5,0.5	–	78.47	87.53	0.52	0.6	0.55	0.4	9.02
		Dis vs All	Poly	15.25	0.5,0.5	3.5	206.1	87.03	0.22	0.72	0.33	0.28	17.3
		Dis vs All	RBF	90	3,1	3	154.2	81.75	0.66	0.42	0.52	0.58	21.5
	Z-norm	Dis vs All	Lin	89.92	1,0.5	–	56.19	85.5	0.02	1	0.03	0	10.8
		Dis vs All	Poly	21.69	0.5,0.5	3.5	83.67	77.5	0.71	0.37	0.48	0.63	1.36
		Dis vs All	RBF*	91.6	0.5,1	0.5	66.52	91.5	0.78	0.69	0.73	0.31	13.7

are used video datasets of facial emotions not in images. Most of the existing model are used images datasets and few model are used videos are highlited in Table 20. From Table 20, our proposed model has high accuracy and high performance than the state of arts of FER system. Our proposed model has attained the good performance for facial emotions using semi-supervised lerning from the video sequence compared to other existing model. In this work, compared to other models for a semi-supervised classifier for facial emotion, TWSVM achieved a higher accuracy and better performance than Cohen [27], Rifai [28] and Jihang [29].

In this work, the research produces an overall computation time of **2.05± 0.43 sec(hold out) and 15.08 ± 4.08 sec(10-fold)** in training and testing phases of basic six emotion classification. However Dapgony's [42] work shows the

Table 18 The 10-fold validation result of Sad mouth in both multi-classifier and normalization

	Norm	One vs One	Kernel	10-fold				Acc(%)	Pre	Rec	F1-sco	Err.rte	Time (s)
				(Acc(%),	Cost(c1, c2)	Gamma(γ)	Time(s)						
One vs One	Max & Min	Sad vs Hap	Lin	82.69	0.5,0.5	–	39.14	70.06	0.29	0.91	0.44	0.09	1.07
		Sad vs Ang	Lin	81.58	1,1	–	38.38	69.7	0.71	0.73	0.72	0.27	1.31
		Sad vs Fea	Lin	82.86	3.5,1.5	–	30.17	62.07	1	0.62	0.77	0.38	2.03
		Sad vs Dis	Lin	78.38	0.5,0.5	–	30.12	56.49	0.25	0.86	0.39	0.14	0.72
		Sad vs Sur	Lin	94.44	1,3	–	31.99	82.03	0.99	0.76	0.86	0.24	1.05
		Sad vs Hap	Poly	84.62	2,2.5	2	43.39	70.06	0.31	0.88	0.45	0.12	2.63
		Sad vs Ang	Poly	86.84	1,2	1	32.12	62.88	0.99	0.6	0.74	0.4	1.23
		Sad vs Fea	Poly	88.57	0.5,1.5	3	30.93	76.72	0.97	0.74	0.84	0.26	0.97
		Sad vs Dis	Poly	81.08	0.5,2	0.5	31.18	69.47	0.6	0.8	0.68	0.2	1.12
		Sad vs Sur	Poly	94.44	2.5,1	2.5	31.52	79.69	0.99	0.74	0.85	0.26	1.4
		Sad vs Hap	RBF	61.54	2,0.5	3.5	47.04	66.67	0.18	1	0.31	0	3.6
		Sad vs Ang	RBF	52.63	1.5,0.5	3.5	34.84	60.61	0.99	0.58	0.73	0.42	1.97
		Sad vs Fea	RBF	57.14	1.5,0.5	3.5	33.86	84.48	0.92	0.85	0.88	0.15	1.45
		Sad vs Dis	RBF	54.05	2,0.5	3.5	34.24	54.96	1	0.55	0.71	0.45	4.13
		Sad vs Sur	RBF	55.56	2.5,3	3.5	34.95	80.47	0.99	0.75	0.85	0.25	1.61
	Z-norm	Sad vs Hap	Lin	86.54	0.5,1	–	37.43	73.45	0.56	0.73	0.63	0.27	0.97
		Sad vs Ang	Lin	84.21	1.5,1.5	–	30.36	59.85	0.97	0.58	0.73	0.42	1.27
		Sad vs Fea	Lin	88.57	2.5,1	–	29.3	65.52	0.47	0.94	0.63	0.06	1.65
		Sad vs Dis	Lin	83.78	2.5,3	–	29.05	54.96	1	0.55	0.71	0.45	2.08
		Sad vs Sur	Lin	100	0.5,0.5	–	30.72	73.44	0.97	0.69	0.8	0.31	0.79
		Sad vs Hap	Poly	88.46	1.5,1	1.5	42.42	79.1	0.9	0.68	0.78	0.32	2.43
		Sad vs Ang	Poly	86.84	2,1	2	31.51	59.85	0.97	0.58	0.73	0.42	2.28
		Sad vs Fea	Poly	97.14	1.5,1.5	1.5	29.92	82.76	0.83	0.88	0.86	0.12	1.3
		Sad vs Dis	Poly	83.78	2.5,2	3	30.75	56.49	0.97	0.56	0.71	0.44	2.73
		Sad vs Sur	Poly	97.22	0.5,1	3.5	29.95	83.59	0.83	0.87	0.85	0.13	1.12
		Sad vs Hap	RBF	61.54	2,0.5	3.5	49.1	77.4	0.53	0.86	0.66	0.14	3.2
		Sad vs Ang	RBF	52.63	1.5,0.5	3.5	37.44	72.73	0.58	0.88	0.7	0.13	1.83
		Sad vs Fea	RBF	57.14	1.5,0.5	3.5	36.5	78.45	0.9	0.78	0.84	0.22	1.5
		Sad vs Dis	RBF	54.05	2,0.5	2	35.04	79.39	0.88	0.78	0.82	0.22	2.31
		Sad vs Sur	RBF	55.56	2,3.5	2	36.04	84.38	0.96	0.8	0.87	0.2	1.8
One vs All	Max & Min	Sad vs All	Lin	89.83	0.5,0.5	–	71.5	85.13	0.22	0.84	0.35	0.16	17.6
		Sad vs All	Poly	16.95	0.5,0.5	3.5	100.5	89.4	0.43	0.97	0.59	0.03	20.2
		Sad vs All	RBF	89.83	0.5,1.5	0.5	89.34	89.6	0.44	0.97	0.61	0.03	18.6
	Z-norm	Sad vs All	Lin	89.83	0.5,3	–	79.7	84.6	0.21	0.79	0.33	0.33	17.6
		Sad vs All	Poly	16.95	0.5,0.5	3.5	104.2	90.2	0.57	0.84	0.67	0.16	15.4
		Sad vs All	RBF*	90.68	0.5,1	1.5	75.57	91.7	0.65	0.85	0.74	0.15	13.4

(*)-highlighted as the higher performance achieved in FERs

computation time of 11.75±7.5 sec and Guo's [43] work shows a computation time of 3.0±0.25 sec for training and testing phase of emotional classifier. Patil's [2] system shows the computation time of 4 sec. This shows the computational time is reduced in our model when compared to the other models are shown in the Table 21 . From the experiments result, the 10-fold cross validation found above 90% accuracy of all basic six facial emotions and the hold out cross validation found that Disgust and Sad has less than 90% accuracy when compared to other emotions (Surprise, Happy, Fear, and Anger). This may be improved by local normalized data with cross-validation applied in the minimal feature vector of TWSVM classifier. The optimal model can be achieved by validating the above experiment with

Table 19 The overall performance of proposed model (RBF-'One vs All'-Z-norm)

Validation parameters	Overall performance-holdout	Overall performance-10 fold
Accuracy(%)	92.05 ± 3.79%	93.42 ± 3.25%(10-fold), 92.56 ± 3.02%(test data)
Precision	0.75 ± 0.18	0.76 ± 0.11(test data)
Recall	0.68 ± 0.25	0.73 ± 0.22(test data)
Error.rate	0.32 ± 0.25	0.27 ± 0.22(test data)
F1-score	0.68 ± 0.25	0.76 ± 0.22(test data)
Computation time(sec)	2.05 ± 0.43 s	15.08 ± 4.08 s (10-fold+test data)

Table 20 Comparison of proposed model and state of the arts of facial emotion

Learning	Model	Classifier	Database	Type	Overall. Accuracy(%)
Supervised	Owusu [1]	SVM	JAFFE,Yale	Images	97.57%,92.33%
	Patil [2]	NN	JAFFE, CMU-AMP, ORL, Yale, CK	Images	98.33%, 99.27%, 98.14%, 98.44%, 98.49%
	Siddiqi [3]	HMM	Youtube	Images	95%
	Uddin [13]	HMM(RGB)	FER	Videos	82.91%
		HMM(RGB)			87.50%
	Yu [14]	SVM+PCA	JAFFE,CK,GWI	Images	75.50%
	Saeed [15]	SVM	CK+,BU-4DFE	Images and videos	83%
	Wan [16]	SVM	CK	Images	80%
	Mohammadian [17]	SVM+HMM	CK+	Images	83.90%
	Ren [18]	Fuzzy+SVM	BU3DFE	Videos	81.4%
	Jiang [19]	SRC	CK+,JAFFE	Images	80%
	Papachristou [20]	SSL	CK,CK+,BU	Images and videos	71.14%,71.84%
	Nikitidis [21]	MMPP	CK,CK+	Images	80.9%
Semi-supervised	Cohen [27]	NB	CK,CK+	Images	69.10%±1.44
		TANB			69.30%±1.44
		SSS			74.80%±1.36
	Rifai [28]	CDA+SVM	Toronto Face	Images	85/%±0.47
	Jiang [29]	TLAB	RaFD, BHU	Videos	83.68%,87.71%
	Proposed model	TWSVM	Cross Database (CK, CK+, MMI Oulu,MAHNOB, Realtime)	Videos	92.05±3.79%, 93.42 ± 3.25%(10-fold), 92.56 ± 3.02%(test data)∗

Table 21 Comparison of the execution time of proposed and existing models

Model	Overall comp.time (sec)
Dapgony [42]	11.75 ± 7.5
Guo [43]	3.0 ± 0.25
Patil [2]	4
*Proposed model	2.05± 0.43(hold out),15.08 ± 4.08 s(10-fold+test) ∗

(k-fold, leave-one-out and bootstrapping with grid search) minimal feature vectors of TWSVM classifier.

5 Conclusion

This paper proposes the classification of facial emotion with minimal facial features of geometric deformable nodes with a semi-supervised classifier. In this system, it is demonstrated that those minimal feature vectors with a semi supervised classifier has high accuracy and less computation time. In this paper, the hold-out validation and 10-fold cross validation of a fused global normalized minimal feature vector is applied in Multi TWSVM to determine the validation parameters of the proposed system. From the comparison of validation parameters, using **RBF kernel ('One vs All') with Z-normalization** have achieved high accuracy of all facial emotions compared to other kernels with normalization. From the proposed model, good performance and accuracy are achieved with the comparison of proposed and existing models, which achieves a better performance than the existing model. This work can be extended for micro and subtle expression as well. This work can go much deeper by applying different cross-validation, local normalized features and feature selection optimization. This work can go in real time application of Human Computer Interactions.

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Conflict of Interest The authors declare that they have no conflict of interest.

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