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A robust correlation coefficient measure of complex intuitionistic fuzzy sets and their applications in decision-making

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Abstract

The objective of this work is to present novel correlation coefficient measures for measuring the relationship between the two complex intuitionistic fuzzy sets (CIFSs). In the existing studies of fuzzy and its extension, the uncertainties present in the data are handled with the help of degrees of membership which are the subset of real numbers, and may lose some useful information and hence consequently affect on the decision results. An alternative to these, complex intuitionistic fuzzy set handles the uncertainties with the degrees whose ranges are extended from real subset to the complex subset with unit disc and hence handle the two-dimensional information in a single set. Thus, motivated by this, we develop correlation and weighted correlation coefficients under the CIFS environment in which pairs of the membership degrees represent the two-dimensional information. Also, some of the desirable properties of it are investigated. Further, based on these measures, a multicriteria decision-making approach is presented under the CIFS environment. Two illustrative examples are taken to demonstrate the efficiency of the proposed approach and validate it by comparing their results with the several existing approaches' results.

Keywords Intuitionistic fuzzy set · Complex intuitionistic fuzzy set · Correlation coefficient · MCDM · Medical diagnosis

1 Introduction

Multicriteria decision making (MCDM) process involves the analysis of a finite set of alternatives and ranking them in terms of how credible they are to decision-maker(s) when all the criteria is considered simultaneously. In this process, the rating values of each alternative include both precise data and experts' subjective information. But, traditionally, it is assumed that the information provided by them are crisp in nature. However, due to the complexity of the system day-by-day, the real-life contains many MCDM problems where the information is either vague, imprecise or uncertain in nature. To deal with it, the theory of fuzzy set (FS) [1] or extended fuzzy sets such as intuitionistic fuzzy set (IFS) [2], interval-valued IFS (IVIFS) [3] are the most successful ones, which characterize the criterion values in terms of membership degrees. From their applications, researchers found that none of these models are able to represent the partial ignorance of the data and its fluctuations at a given phase of time during their execution. Furthermore, in our day-to-day life, uncertainty and vagueness which are present in the data occur concurrently with changes to the phase (periodicity) of the data. Thus, the existing theories are insufficient to consider this information and hence there is an information loss during the process. To overcome it, Ramot et al. [4] initiated a complex fuzzy set (CFS) in which the range of membership function is extended from real number to the complex number with the unit disc. As the complex fuzzy set considers only the membership degree, but doesn't weight on the non-membership portion of the data entities, which likewise assume an equal part in assessing the object in the decision-making process. However, in the real world, it is regularly hard to express the estimation of the membership degree by an exact value in a fuzzy set. In such cases, it might be easier to depict vagueness and uncertainty in the real world using a 2-dimensional information instead of a single one. Consequently, an extension of the existing theories might be extremely valuable to depict the uncertainties because of his/her reluctant judgment in complex decision-making

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problems. For this reason, Alkouri and Salleh [5] extended the concept of CFS to complex intuitionistic fuzzy sets (CIFSs) by adding the degree of complex non-membership functions and studied their basic operations. Henceforth, a CIFS is a more generalized extension of the existing theories such as FSs, IFSs, CFSs. Clearly, the advantage of the CIFSs is that it can contain substantially more data to express the information.

Presently, correlation measure is one of the most important measures which will help not only in comparing one data entity with other but also show the extent of association between them and their direction. Also, CIFSs have powerful ability to model the imprecise and ambiguous information in real-world applications than the existing theories such as FSs, IFSs, and CFSs [6]. Therefore, keeping the advantages of this set and taking the importance of correlation measure, this paper presents the theory of the correlation coefficients among the CIFS. As per our knowledge, in the aforementioned studies, the correlation measures cannot be utilized to handle the CIFS information. Thus, in order to achieve it, we first define the informational energies and the covariance between the two CIFSs that involves both uncertainty and periodicity semantics. Then, based on these, we propose some correlation coefficients for CIFSs and investigate their properties. Further, some weighted correlation coefficients are proposed to address the situations where the element in the set is correlative. Furthermore, we propose a decision-making approach based on the proposed correlation coefficients for CIFSs. The feasibility, as well as superiority of the approach, has been demonstrated through two numerical examples.

To do so, the rest of the manuscript is summarized as follows. Section 2 is a literature review that presents papers on the correlation measures. In Section 3, we briefly present an overview related to the concepts of IFSs, CFSs, and CIFSs. In Section 4, we introduce correlation and weighted correlation coefficients for CIFSs and obtain some properties. In Section 5, we present a multicriteria decision-making approach based on the proposed correlation coefficients under CIFSs environment, where each element of the set is characterized by complex intuitionistic fuzzy numbers. In Section 6, two illustrative examples are presented to discuss the functionality of the proposed approach and compare their results with some of the existing approaches results in Section 7. Finally, Section 8 summarizes this study.

2 Related work

To process an uncertain and imprecise information during the decision-making process, numerable attempts have been made by different researchers in processing the information values using aggregation operators [7–12], information measures [13–16], score and accuracy functions [17, 18] under IFS, IVIFS environments. Among all these concepts, one of the significant ways to solve such type of problems by using the concept of correlation coefficient which provides us with the measurement of the dependence of the two variables. In statistical analysis, one of the important measures is correlation coefficients which give us an idea of the strength and direction of a linear relationship between the pairs of two variables. On the other hand, in fuzzy set theory, these measures determine the degree of the dependency between the two fuzzy sets. In that direction, Gerstenkorn and Manko [19] firstly introduced the concept of coefficient of correlation for measuring the interrelation of IFSs. Later on, Hong and Hwang [20] extended its concept into the probability spaces. Hung and Wu [21] studied the correlation coefficients by using the centroid method. Bustince and Burillo [22] extended the concept of correlation from IFS to IVIFS environment. Zeng and Li [23] presented a decision-making approach based on the correlation coefficients. Garg [24, 25] presented the correlation coefficients for Pythagorean fuzzy sets and intuitionistic multiplicative set respectively. Ye [26] presented the cosine similarity measures for IFSs. Garg [27] presented some improved cosine similarity measures for IFS and applied them to solve the decision-making problems. However, apart from these, some other kinds of the correlation coefficients [28–30] have been proposed by the different researchers and they applied them to solve the multicriteria decision-making problems.

The above measures and their corresponding approaches are widely used by the researchers, but from these studies, it has been analyzed the data under the FSs, IFSs or its generalizations is only able to handle the uncertainty and vagueness that exists in the data. But, simultaneously, none of these existing models are able to represent the partial ignorance of the data and its fluctuations, with changes to the phase (periodicity), at a given phase of time during their execution. To overcome it, Ramot et al. [4] initiated a complex fuzzy set (CFS) in which the range of membership function is extended from real number to the complex number with the unit disc. Ramot et al. [31] generalized traditional fuzzy logic to complex fuzzy logic in which the sets used in reasoning process are complex fuzzy sets, characterized by complex valued membership functions. Greenfield et al. [32] extended the concept of CFS by taking the grade of the membership function as an interval-number rather than single numbers. Furthermore, Alkouri and Salleh [5] extended the concepts of CFS to complex intuitionistic fuzzy sets (CIFSs) by adding the degree of complex non-membership functions and studied their basic operations. Alkouri and Salleh [33] introduced the concepts of complex intuitionistic fuzzy relation, composition, projections and proposed a distance measure between the two CIFSs. Rani and Garg [34] presented some series of distance measures under CIFS environment and their application in the decision-making process. Rani and Garg [35] presented power aggregation operators for different CIFSs. Kumar and Bajaj [36] proposed some distance and entropy measures for complex intuitionistic fuzzy soft sets.

In CIFS theory, membership and non-membership degrees are complex-valued and are represented in polar coordinates. The amplitude term corresponding to the membership (non-membership) degree gives the extent of belongings (not-belongings) of an object in a CIFS and the phase term associated with membership (non-membership) degree gives the additional information, generally related with periodicity. Since, in the existing IFSs theory, it is observed that there is only one parameter to represent the information which results in information loss in some instances. However, in day-to-day life, we come across complex natural phenomena where we need to add the second dimension to the expression of membership and non-membership grades. By introducing this second dimension, the complete information can be projected in one set, and hence loss of information can be avoided. For instance, suppose a certain company decides to set up biometric-based attendance devices (BBADs) in all of its offices spread all over the country. For this, the company consults an expert who gives the information regarding the two-dimensions namely, models of BBADs and their corresponding production dates of BBADs. The task of the company is to select the most optimal model of BBADs with its production date simultaneously. It is obviously seen that such type of problems cannot be modeled accurately by considering both the dimensions simultaneously using the traditional IFS theories. Thus, for such types of problem, there is a need to enhance the existing theories and hence a CIFS environment provides us with an efficient way to handle the two-step judgment scenarios in which an amplitude term may be employed to give a company's decision regarding model of BBADs and the phase terms may be used to represent company's decision regarding the production date of BBADs in the decision making process. Similarly, some other types of examples under CIFSs include the large amounts of data sets that are generated from medical research, as well as government databases for biometric and facial recognition, audio, and images etc. Henceforth, a CIFS is a more generalized extension of the existing theories such as FSs, IFSs, CFSs. Clearly, the advantage of the CIFSs is that it can contain substantially more data to express the information.

Therefore, keeping the advantages of this set and taking the importance of correlation measure, this paper presents the theory of the correlation coefficients among the CIFSs. Also, it is being computed that the several existing correlation measures can be easily obtained from the proposed measures.

3 Preliminaries

In this section, some basic concepts related to the IFSs, CFSs and CIFSs are reviewed over the universal set $U \neq \phi$.

Definition 1 [2, 9] An Atanassov's Intuitionistic fuzzy set(IFS) S defined on U is an ordered pair given by

$$S = \{(x, \mu_S(x), \nu_S(x)) : x \in U\}, \tag{1}$$

where $\mu_S, \nu_S : U \to [0, 1]$ are real-valued membership and non-membership functions respectively such that $\mu_S(x) + \nu_S(x) \leq 1$ for all $x \in U$. Also, $\pi_S(x) = 1 - \mu_S(x) - \nu_S(x)$ is called the degree of hesitation of x to S. For convenience, this pair of membership and non-membership degrees is called as intuitionistic fuzzy number (IFN) and is denoted by $S = (\mu, \nu)$, where $0 \leq u, v \leq 1$ and $u + v \leq 1$.

Definition 2 [23] For two IFSs $A = \{(x, \mu_A(x), \nu_A(x)) : x \in U\}$ and $B = \{(x, \mu_B(x), \nu_B(x)) : x \in U\}$ defined on $U = \{x_1, x_2, \ldots x_n\}$, [23] defined the correlation between them as

$$C_1(A, B) = \frac{1}{n} \sum_{j=1}^{n} \left(\mu_A(x_j)\mu_B(x_j) + \nu_A(x_j)\nu_B(x_j) + \pi_A(x_j)\pi_B(x_j)\right) \tag{2}$$

and hence defined the correlation coefficient between A and B as:

$$\rho_1(A, B) = \frac{C_1(A, B)}{\sqrt{C_1(A, A) \cdot C_1(B, B)}} \tag{3}$$

and showed that it satisfies the following properties:

1. $\rho_1(A, B) = \rho_1(B, A)$
2. $0 \leq \rho_1(A, B) \leq 1$
3. $A = B \Leftrightarrow \rho_1(A, B) = 1$

Ramot et al. [4] extended the theory of fuzzy set to the complex fuzzy set (CFS) by incorporating the phase angle into the analysis, which has been defined as follows:

Definition 3 [4] A CFS S defined on U is defined as a set of pairs given by

$$S = \{(x, \mu_S(x)) : x \in U\}, \tag{4}$$

where μ_S is a membership function which can assign any element $x \in U$ a complex valued grade of membership. The value of $\mu_S(x)$ lies in a unit circle in the complex plane and is of the form $\mu_S(x) = r_S(x)e^{iw_{r_S}(x)}$ where $i = \sqrt{-1}$, $r_S(x) \in [0, 1]$ and $w_{r_S}(x)$ is real valued.

Later on, [5] extended the concept of CFS to complex intuitionistic fuzzy set (CIFS) by taking the degree of non-membership function into the analysis as follows:

Definition 4 [5] A CIFS S over U is defined as a set given by

$$S = \{(x, \mu_S(x), \gamma_S(x)) : x \in U\}, \tag{5}$$

where $\mu_S : U \to \{a : a \in C, |a| \le 1\}$ and $\gamma_S : U \to \{a : a \in C, |a| \le 1\}$ are complex valued membership and non-membership functions respectively given by:

$$\mu_S(x) = r_S(x)e^{i w_{r_S}(x)}, \qquad \gamma_S(x) = k_S(x)e^{i w_{k_S}(x)}.$$

Here $r_S(x), k_S(x) \in [0, 1]$ such that $r_S(x)+k_S(x) \le 1$. Also $w_{r_S}(x)$ and $w_{k_S}(x)$ are real valued which satisfy the conditions $w_{r_S}(x), w_{k_S}(x) \in [0, 2\pi]$ and $w_{r_S}(x) + w_{k_S}(x) \le 2\pi$ for each $x \in U$. For the sake of convenience, we shall denote the set $\left\{(x, r_S(x)e^{i w_{r_S}(x)}, k_S(x)e^{i w_{k_S}(x)}) : x \in U\right\}$ as $S = \left(r_S(x)e^{i w_{r_S}(x)}, k_S(x)e^{i w_{k_S}(x)}\right)$.

4 Correlation coefficient for CIFSs

In this section, we propose some correlation coefficients for the CIFSs which can be applied in numerous engineering and scientific fields to rank the objects. For it, throughout this paper, we shall use $U = \{x_1, x_2, \ldots x_n\}$ as the universe of discourse.

Let $A = \left(r_A(x)e^{i w_{r_A}(x)}, k_A(x)e^{i w_{k_A}(x)}\right)$ and $B = \left(r_B(x)e^{i w_{r_B}(x)}, k_B(x)e^{i w_{k_B}(x)}\right)$ be two CIFSs defined on U. Then, the informational energies of two CIFSs A and B are defined as

$$T(A) = \sum_{j=1}^{n} \left(r_A^2(x_j) + k_A^2(x_j) + \frac{1}{4\pi^2} \left(w_{r_A}^2(x_j) + w_{k_A}^2(x_j) \right) \right), \tag{6}$$

$$T(B) = \sum_{j=1}^{n} \left(r_B^2(x_j) + k_B^2(x_j) + \frac{1}{4\pi^2} \left(w_{r_B}^2(x_j) + w_{k_B}^2(x_j) \right) \right). \tag{7}$$

The correlation of the CIFSs A and B is defined as

$$C(A, B) = \sum_{j=1}^{n} \left[\begin{array}{l} r_A(x_j)r_B(x_j) + \frac{1}{4\pi^2} w_{r_A}(x_j) w_{r_B}(x_j) \\ + k_A(x_j)k_B(x_j) + \frac{1}{4\pi^2} w_{k_A}(x_j) w_{k_B}(x_j) \end{array} \right]. \tag{8}$$

From Eq. (8), it is clearly seen that correlation of CIFSs satisfies the following properties:

(P1) $C(A, B) = C(B, A)$
(P2) $C(A, A) = T(A)$

Then, based on these, we defined the correlation coefficient between CIFSs A and B, as follows:

Definition 5 If $A = (r_A(x)e^{i w_{r_A}(x)}, k_A(x)e^{i w_{k_A}(x)})$ and $B = (r_B(x)e^{i w_{r_B}(x)}, k_B(x)e^{i w_{k_B}(x)})$ be two CIFSs defined on U, then the correlation coefficient between them is denoted by $K_1(A, B)$ and is defined as

$$K_1(A, B) = \frac{C(A, B)}{\sqrt{T(A) \times T(B)}} = \frac{\sum_{j=1}^{n} \left(\begin{array}{l} r_A(x_j)r_B(x_j) + \frac{1}{4\pi^2} \left(w_{r_A}(x_j) w_{r_B}(x_j) \right) \\ + k_A(x_j)k_B(x_j) + \frac{1}{4\pi^2} \left(w_{k_A}(x_j) w_{k_B}(x_j) \right) \end{array} \right)}{\left\{ \begin{array}{l} \sqrt{\sum_{j=1}^{n} \left(r_A^2(x_j) + k_A^2(x_j) + \frac{1}{4\pi^2} \left(w_{r_A}^2(x_j) + w_{k_A}^2(x_j) \right) \right)} \\ \times \sqrt{\sum_{j=1}^{n} \left(r_B^2(x_j) + k_B^2(x_j) + \frac{1}{4\pi^2} \left(w_{r_B}^2(x_j) + w_{k_B}^2(x_j) \right) \right)} \end{array} \right\}} \tag{9}$$

Theorem 1 *The correlation coefficient K_1 between two CIFSs A and B defined on U satisfies the following properties:*

(P1) $0 \le K_1(A, B) \le 1$.
(P2) $K_1(A, B) = K_1(B, A)$.
(P3) $K_1(A, B) = 1$, *if* $A = B$.
(P4) *If* $A \subseteq B \subseteq C$ *then,* $K_1(A, C) \le K_1(A, B)$ *and* $K_1(A, C) \le K_1(B, C)$ *for CIFS C defined on U.*

Proof Let $A = \left(r_A(x)e^{i w_{r_A}(x)}, k_A(x)e^{i w_{k_A}(x)}\right)$ and $B = \left(r_B(x)e^{i w_{r_B}(x)}, k_B(x)e^{i w_{k_B}(x)}\right)$ be two CIFSs defined on U. Then, we have

1. The inequality $K_1(A, B) \ge 0$ is obvious due to $C(A, B) \ge 0$ is obtained from the Eq. (8). Now we shall prove $K_1(A, B) \le 1$. For it, based on the Definition 5, we get

$$K_1(A,B) = \frac{C(A,B)}{\sqrt{C(A,A)\times C(B,B)}}$$
$$= \frac{\sum_{j=1}^{n}\begin{pmatrix} r_A(x_j)r_B(x_j) + \frac{1}{4\pi^2}w_{r_A}(x_j)w_{r_B}(x_j) \\ +k_A(x_j)k_B(x_j) + \frac{1}{4\pi^2}w_{k_A}(x_j)w_{k_B}(x_j)\end{pmatrix}}{\begin{Bmatrix} \sqrt{\sum_{j=1}^{n}\left(r_A^2(x_j)+k_A^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}^2(x_j)+w_{k_A}^2(x_j)\right)\right)} \\ \times\sqrt{\sum_{j=1}^{n}\left(r_B^2(x_j)+k_B^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_B}^2(x_j)+w_{k_B}^2(x_j)\right)\right)}\end{Bmatrix}}$$
$$= \frac{\begin{pmatrix} \sum_{j=1}^{n} r_A(x_j)r_B(x_j) + \sum_{j=1}^{n}\left(\frac{w_{r_A}(x_j)}{2\pi}\right)\left(\frac{w_{r_B}(x_j)}{2\pi}\right) \\ +\sum_{j=1}^{n} k_A(x_j)k_B(x_j) + \sum_{j=1}^{n}\left(\frac{w_{k_A}(x_j)}{2\pi}\right)\left(\frac{w_{k_B}(x_j)}{2\pi}\right)\end{pmatrix}}{\begin{Bmatrix} \sqrt{\sum_{j=1}^{n}\left(r_A^2(x_j)+k_A^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}^2(x_j)+w_{k_A}^2(x_j)\right)\right)} \\ \times\sqrt{\sum_{j=1}^{n}\left(r_B^2(x_j)+k_B^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_B}^2(x_j)+w_{k_B}^2(x_j)\right)\right)}\end{Bmatrix}}$$

Now, by using Cauchy-Schwarz inequality which states that, in Euclidean space R^n with standard inner product, we have:

$$\left(\sum_{j=1}^{n} u_j v_j\right)^2 \leq \left(\sum_{j=1}^{n} u_j^2\right)\left(\sum_{j=1}^{n} v_j^2\right)$$

where $u=(u_1,u_2,\ldots u_n)$ and $v=(v_1,v_2,\ldots v_n)\in R^n$ and equality holds if and only if u and v are linearly dependent vectors. Therefore,

$$K_1(A,B) \leq \frac{\begin{pmatrix} \sqrt{\sum_{j=1}^{n} r_A^2(x_j)}\sqrt{\sum_{j=1}^{n} r_B^2(x_j)} + \sqrt{\sum_{j=1}^{n}\left(\frac{w_{r_A}(x_j)}{2\pi}\right)^2}\sqrt{\sum_{j=1}^{n}\left(\frac{w_{r_B}(x_j)}{2\pi}\right)^2} \\ \sqrt{\sum_{j=1}^{n} k_A^2(x_j)}\sqrt{\sum_{j=1}^{n} k_B^2(x_j)} + \sqrt{\sum_{j=1}^{n}\left(\frac{w_{k_A}(x_j)}{2\pi}\right)^2}\sqrt{\sum_{j=1}^{n}\left(\frac{w_{k_B}(x_j)}{2\pi}\right)^2}\end{pmatrix}}{\begin{Bmatrix} \sqrt{\sum_{j=1}^{n}\left(r_A^2(x_j)+k_A^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}^2(x_j)+w_{k_A}^2(x_j)\right)\right)} \\ \times\sqrt{\sum_{j=1}^{n}\left(r_B^2(x_j)+k_B^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_B}^2(x_j)+w_{k_B}^2(x_j)\right)\right)}\end{Bmatrix}}$$

By taking the notations, $\sum_{j=1}^{n} r_A^2(x_j)=a$, $\sum_{j=1}^{n} r_B^2(x_j)=b$, $\sum_{j=1}^{n} k_A^2(x_j)=c$, $\sum_{j=1}^{n} k_B^2(x_j)=d$, $\sum_{j=1}^{n}\left(\frac{w_{r_A}(x_j)}{2\pi}\right)^2=p$, $\sum_{j=1}^{n}\left(\frac{w_{r_B}(x_j)}{2\pi}\right)^2=q$, $\sum_{j=1}^{n}\left(\frac{w_{k_A}(x_j)}{2\pi}\right)^2=r$ and $\sum_{j=1}^{n}\left(\frac{w_{k_B}(x_j)}{2\pi}\right)^2=s$, the above inequality reduces to

$$K_1(A,B) \leq \frac{\sqrt{ab}+\sqrt{cd}+\sqrt{pq}+\sqrt{rs}}{\sqrt{(a+c+p+r)(b+d+q+s)}}$$

Therefore,

$$
\begin{aligned}
(K_1(A,B))^2-1 &\le \frac{\left(\sqrt{ab}+\sqrt{cd}+\sqrt{pq}+\sqrt{rs}\right)^2}{(a+c+p+r)(b+d+q+s)}-1\\
&= \frac{\left(\sqrt{ab}+\sqrt{cd}+\sqrt{pq}+\sqrt{rs}\right)^2-(a+c+p+r)(b+d+q+s)}{(a+c+p+r)(b+d+q+s)}\\
&= \frac{\left(\begin{array}{l} ab+cd+pq+rs+2\sqrt{abcd}+2\sqrt{pqrs}+2\sqrt{abpq}+2\sqrt{abrs}\\ +2\sqrt{cdpq}+2\sqrt{cdrs}-ab-ad-aq-as-cb-cd-cq\\ -cs-pb-pd-pq-ps-rb-rd-rq-rs \end{array}\right)}{(a+c+p+r)(b+d+q+s)}\\
&= -\left(\frac{\begin{array}{l}(\sqrt{ad}-\sqrt{bc})^2+(\sqrt{ps}-\sqrt{qr})^2+(\sqrt{aq}-\sqrt{bp})^2\\ +(\sqrt{as}-\sqrt{br})^2+(\sqrt{cq}-\sqrt{dp})^2+(\sqrt{cs}-\sqrt{dr})^2\end{array}}{(a+c+p+r)(b+d+q+s)}\right)\\
&\le 0
\end{aligned}
$$

Hence, $K_1^2(A,B)\le 1$ which implies $K_1(A,B)\le 1$. So, $0\le K_1(A,B)\le 1$.

2. For any two CIFSs A and B, we have

$$
\begin{aligned}
K_1(A,B) &= \frac{\sum_{j=1}^{n}\left(\begin{array}{l} r_A(x_j)r_B(x_j)+\frac{1}{4\pi^2}w_{r_A}(x_j)w_{r_B}(x_j)\\ +k_A(x_j)k_B(x_j)+\frac{1}{4\pi^2}w_{k_A}(x_j)w_{k_B}(x_j)\end{array}\right)}{\left\{\begin{array}{l}\sqrt{\sum_{j=1}^{n}\left(r_A^2(x_j)+k_A^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}^2(x_j)+w_{k_A}^2(x_j)\right)\right)}\\ \times\sqrt{\sum_{j=1}^{n}\left(r_B^2(x_j)+k_B^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_B}^2(x_j)+w_{k_B}^2(x_j)\right)\right)}\end{array}\right\}}\\
&= \frac{\sum_{j=1}^{n}\left(\begin{array}{l} r_B(x_j)r_A(x_j)+\frac{1}{4\pi^2}w_{r_B}(x_j)w_{r_A}(x_j)\\ +k_B(x_j)k_A(x_j)+\frac{1}{4\pi^2}w_{k_B}(x_j)w_{k_A}(x_j)\end{array}\right)}{\left\{\begin{array}{l}\sqrt{\sum_{j=1}^{n}\left(r_B^2(x_j)+k_B^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_B}^2(x_j)+w_{k_B}^2(x_j)\right)\right)}\\ \times\sqrt{\sum_{j=1}^{n}\left(r_A^2(x_j)+k_A^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}^2(x_j)+w_{k_A}^2(x_j)\right)\right)}\end{array}\right\}}\\
&= K_1(B,A)
\end{aligned}
$$

3. If $A=B$ this implies that $r_A(x_j)=r_B(x_j)$, $k_A(x_j)=k_B(x_j)$, $w_{r_A}(x_j)=w_{r_B}(x_j)$ and $w_{k_A}(x_j)=w_{k_B}(x_j)$ for all j and thus from Eq. (9), it follows that $K_1(A,B)=1$.
4. Geometrically, if $A\subseteq B\subseteq C$, then the angle between A and C should be larger than the angle between B and C for any element x_j and $\cos\theta$ is decreasing function within interval $[0,\frac{\pi}{2}]$. Therefore, $K_1(A,C)\le K_1(A,B)$ and $K_1(A,C)\le K_1(B,C)$.

Hence, the theorem holds. □

Example 1 Let $U=\{x_1,x_2,x_3\}$ be the universal set and $A=\{(x_1,0.6e^{i2\pi(0.7)},0.2e^{i2\pi(0.2)}),(x_2,0.7e^{i2\pi(0.5)},0.3e^{i2\pi(0.4)}),(x_3,0.5e^{i2\pi(0.4)},0.4e^{i2\pi(0.1)})\}$, $B=\{(x_1,0.5e^{i2\pi(0.6)},0.1e^{i2\pi(0.2)}),(x_2,0.7e^{i2\pi(0.4)},0.1e^{i2\pi(0.4)}),(x_3,0.6e^{i2\pi(0.5)},0.3e^{i2\pi(0.4)})\}$ are two CIFSs defined on the universal set U. By applying Eq. (6), we obtain the informational energy of A as:

$$
\begin{aligned}
T(A) &= \sum_{j=1}^{n}\left(r_A^2(x_j)+k_A^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}^2(x_j)+w_{k_A}^2(x_j)\right)\right)\\
&= (0.6)^2+(0.2)^2+\frac{1}{4\pi^2}\left[(2\pi\times 0.7)^2+(2\pi\times 0.2)^2\right]
\end{aligned}
$$

$$
\begin{aligned}
&+(0.7)^2+(0.3)^2+\frac{1}{4\pi^2}\left[(2\pi\times 0.5)^2+(2\pi\times 0.4)^2\right]\\
&+(0.5)^2+(0.4)^2+\frac{1}{4\pi^2}\left[(2\pi\times 0.4)^2+(2\pi\times 0.1)^2\right]\\
=&\ 0.36+0.04+0.49+0.04+0.49+0.09+0.25\\
&+0.16+0.25+0.16+0.16+0.01\\
=&\ 2.5
\end{aligned}
$$

Similarly, the informational energy of CIFS B is:

$$
\begin{aligned}
T(B) =& \sum_{j=1}^{n}\left(r_B^2(x_j)+k_B^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_B}^2(x_j)+w_{k_B}^2(x_j)\right)\right)\\
=&\ (0.5)^2+(0.1)^2+\frac{1}{4\pi^2}\left[(2\pi\times 0.6)^2+(2\pi\times 0.2)^2\right]\\
&+(0.7)^2+(0.1)^2+\frac{1}{4\pi^2}\left[(2\pi\times 0.4)^2+(2\pi\times 0.4)^2\right]\\
&+(0.6)^2+(0.3)^2+\frac{1}{4\pi^2}\left[(2\pi\times 0.5)^2+(2\pi\times 0.4)^2\right]\\
=&\ 0.25+0.01+0.36+0.04+0.49+0.01+0.16\\
&+0.16+0.36+0.09+0.25+0.16\\
=&\ 2.34
\end{aligned}
$$

By using Eq. (8), the correlation between CIFSs A and B is computed as:

$$
\begin{aligned}
C(A,B) =& \sum_{j=1}^{n}\begin{pmatrix} r_A(x_j)r_B(x_j)+\frac{1}{4\pi^2}w_{r_A}(x_j)w_{r_B}(x_j)\\ +k_A(x_j)k_B(x_j)+\frac{1}{4\pi^2}w_{k_A}(x_j)w_{k_B}(x_j)\end{pmatrix}\\
=&\ 0.6\times 0.5+0.2\times 0.1+\frac{1}{4\pi^2}\left(2\pi(0.7)\times 2\pi(0.6)+2\pi(0.2)\times 2\pi(0.2)\right)\\
&+0.7\times 0.7+0.3\times 0.1+\frac{1}{4\pi^2}\left(2\pi(0.5)\times 2\pi(0.4)+2\pi(0.4)\times 2\pi(0.4)\right)\\
&+0.5\times 0.6+0.4\times 0.3+\frac{1}{4\pi^2}\left(2\pi(0.4)\times 2\pi(0.5)+2\pi(0.1)\times 2\pi(0.4)\right)\\
=&\ 0.30+0.02+0.42+0.04+0.49+0.03+0.20+0.16+0.30\\
&+0.12+0.20+0.04\\
=&\ 2.32
\end{aligned}
$$

Thus, the correlation coefficient between A and B is given by Eq. (9) as

$$
\begin{aligned}
K_1(A,B) =& \frac{C(A,B)}{\sqrt{T(A)\times T(B)}}\\
=& \frac{2.32}{\sqrt{2.5\times 2.34}}=0.9592
\end{aligned}
$$

Definition 6 Let $A=\left(r_A(x)e^{iw_{r_A}(x)}, k_A(x)e^{iw_{k_A}(x)}\right)$ and $B=\left(r_B(x)e^{iw_{r_B}(x)}, k_B(x)e^{iw_{k_B}(x)}\right)$ be two CIFSs defined on U. Then, the correlation coefficient, denoted by K_2, is defined as

$$
\begin{aligned}
K_2(A,B) =& \frac{C(A,B)}{\max\{T(A),T(B)\}}\\
=& \frac{\sum_{j=1}^{n}\begin{pmatrix} r_A(x_j)r_B(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}(x_j)w_{r_B}(x_j)\right)\\ +k_A(x_j)k_B(x_j)+\frac{1}{4\pi^2}\left(w_{k_A}(x_j)w_{k_B}(x_j)\right)\end{pmatrix}}{\max\left\{\begin{array}{l}\sum_{j=1}^{n}\left(r_A^2(x_j)+k_A^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}^2(x_j)+w_{k_A}^2(x_j)\right)\right),\\ \sum_{j=1}^{n}\left(r_B^2(x_j)+k_B^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_B}^2(x_j)+w_{k_B}^2(x_j)\right)\right)\end{array}\right\}}
\end{aligned}
\tag{10}
$$

Theorem 2 *The correlation coefficient of two CIFSs A and B, as defined in* Eq. (10)*, satisfies the following properties:*

(P1) $0\le K_2(A,B)\le 1$.

(P2) $K_2(A,B)=K_2(B,A)$.

(P3) $K_2(A,B)=1$ *if* $A=B$.

(P4) *If* $A\subseteq B\subseteq C$ *then,* $K_2(A,C)\le K_2(A,B)$ *and* $K_2(A,C)\le K_2(B,C)$ *for any CIFS C defined on* U.

Proof Since $A, B \in$ CIFSs, then $0 \leq r_A(x_j), k_A(x_j) \leq 1, 0 \leq w_{r_A}, w_{k_A} \leq 2\pi$ and $r_A(x_j) + k_A(x_j) \leq 1$ and $w_{r_A} + w_{k_A} \leq 2\pi$ for all $x_j \in U$ and thus from Eq. (10), we can obtain the inequality $K_2(A, B) \geq 0$. The inequality $K_2(A, B) \leq 1$ can be proven directly by using the well-known Cauchy-Schwarz inequality:

$$\sum_{j=1}^{n} a_j b_j \leq \sqrt{\left(\sum_{j=1}^{n} a_j^2\right) \cdot \left(\sum_{j=1}^{n} b_j^2\right)} \tag{11}$$

with equality if and only if the two vectors $a = (a_1, a_2, \ldots, a_n)$ and $b = (b_1, b_2, \ldots, b_n)$ are linearly dependent.

In fact, by Eq. (11), we have

$$\sum_{j=1}^{n} a_j b_j \leq \sqrt{\left(\sum_{j=1}^{n} a_j^2\right) \cdot \left(\sum_{j=1}^{n} b_j^2\right)} \leq \sqrt{\left(\max\left\{\sum_{j=1}^{n} a_j^2, \sum_{j=1}^{n} b_j^2\right\}\right)^2} = \max\left\{\sum_{j=1}^{n} a_j^2, \sum_{j=1}^{n} b_j^2\right\}$$

and from Eq. (10), it follows that $0 \leq K_2(A, B) \leq 1$ which completes the proof of (P1). In addition, by Eq. (10), we have

$$\begin{aligned} K_2(A, B) &= \frac{\sum_{j=1}^{n} \begin{pmatrix} r_A(x_j) r_B(x_j) + \frac{1}{4\pi^2}\left(w_{r_A}(x_j) w_{r_B}(x_j)\right) \\ + k_A(x_j) k_B(x_j) + \frac{1}{4\pi^2}\left(w_{k_A}(x_j) w_{k_B}(x_j)\right) \end{pmatrix}}{\max\left\{ \begin{array}{l} \sum_{j=1}^{n}\left(r_A^2(x_j) + k_A^2(x_j) + \frac{1}{4\pi^2}\left(w_{r_A}^2(x_j) + w_{k_A}^2(x_j)\right)\right), \\ \sum_{j=1}^{n}\left(r_B^2(x_j) + k_B^2(x_j) + \frac{1}{4\pi^2}\left(w_{r_B}^2(x_j) + w_{k_B}^2(x_j)\right)\right) \end{array} \right\}} \\ &= \frac{\sum_{j=1}^{n} \begin{pmatrix} r_B(x_j) r_A(x_j) + \frac{1}{4\pi^2}\left(w_{r_B}(x_j) w_{r_A}(x_j)\right) \\ + k_B(x_j) k_A(x_j) + \frac{1}{4\pi^2}\left(w_{k_B}(x_j) w_{k_A}(x_j)\right) \end{pmatrix}}{\max\left\{ \begin{array}{l} \sum_{j=1}^{n}\left(r_B^2(x_j) + k_B^2(x_j) + \frac{1}{4\pi^2}\left(w_{r_B}^2(x_j) + w_{k_B}^2(x_j)\right)\right), \\ \sum_{j=1}^{n}\left(r_A^2(x_j) + k_A^2(x_j) + \frac{1}{4\pi^2}\left(w_{r_A}^2(x_j) + w_{k_A}^2(x_j)\right)\right) \end{array} \right\}} \\ &= K_2(B, A) \end{aligned}$$

Thus, (P2) also holds. Similarly, we can complete the proofs of (P3) and (P4).

Hence, the theorem holds. □

From Definition 5 and Definition 6, we observe that the correlation coefficient defined by Eq. (9) uses the geometric mean of the informational energies of the CIFSs A and B, and the correlation coefficient defined by Eq. (10) applies the maximum between them. For the optimistic decision makers, they tend to use the correlation coefficient defined by Eq. (9). Contrary to the optimistic decision makers, the pessimistic decision makers tend to apply the correlation coefficient defined by Eq. (10).

In the above defined formulas for calculating coefficient of correlation, equal importance is given to all the elements of the universal set. But in real life situations, this may not be always possible. Some elements in the universal set are more important than the others. So we must take into account the proper weightage given to the various elements of the universal set. In the following, we propose a weighted correlation coefficient between CIFSs. Let $\xi = (\xi_1, \xi_2, \ldots \xi_n)^T$ be the weight vector corresponding to the elements x_j $(j = 1, 2, \ldots, n)$ with $\xi_j > 0$ and $\sum_{j=1}^{n} \xi_j = 1$. Then, we extend the above defined correlation coefficients K_1 and K_2 to weighted correlation coefficients, K_3 and K_4 respectively, as follows:

$$\begin{aligned} K_3(A, B) &= \frac{C_w(A, B)}{\sqrt{T_w(A) \times T_w(B)}} \\ &= \frac{\sum_{j=1}^{n}\left(\xi_j \begin{bmatrix} r_A(x_j) r_B(x_j) + \frac{1}{4\pi^2}\left(w_{r_A}(x_j) w_{r_B}(x_j)\right) \\ + k_A(x_j) k_B(x_j) + \frac{1}{4\pi^2}\left(w_{k_A}(x_j) w_{k_B}(x_j)\right) \end{bmatrix}\right)}{\left\{ \begin{array}{l} \sqrt{\sum_{j=1}^{n}\left(\xi_j\left[r_A^2(x_j) + k_A^2(x_j) + \frac{1}{4\pi^2}\left(w_{r_A}^2(x_j) + w_{k_A}^2(x_j)\right)\right]\right)} \\ \times \sqrt{\sum_{j=1}^{n}\left(\xi_j\left[r_B^2(x_j) + k_B^2(x_j) + \frac{1}{4\pi^2}\left(w_{r_B}^2(x_j) + w_{k_B}^2(x_j)\right)\right]\right)} \end{array} \right\}} \end{aligned} \tag{12}$$

and

$$K_4(A,B) = \frac{C_w(A,B)}{\max\{T_w(A),T_w(B)\}} = \frac{\sum_{j=1}^{n}\left(\xi_j\left[\begin{array}{l} r_A(x_j)r_B(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}(x_j)w_{r_B}(x_j)\right) \\ +k_A(x_j)k_B(x_j)+\frac{1}{4\pi^2}\left(w_{k_A}(x_j)w_{k_B}(x_j)\right)\end{array}\right]\right)}{\max\left\{\begin{array}{l}\sum_{j=1}^{n}\left(\xi_j\left[r_A^2(x_j)+k_A^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}^2(x_j)+w_{k_A}^2(x_j)\right)\right]\right), \\ \sum_{j=1}^{n}\left(\xi_j\left[r_B^2(x_j)+k_B^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_B}^2(x_j)+w_{k_B}^2(x_j)\right)\right]\right)\end{array}\right\}} \tag{13}$$

It can be easily verified that, if $\xi=(1/n,1/n,\ldots,1/n)^T$ then Eqs. (12) and (13) reduce to the correlation coefficients given in Eqs. (9) and (10) respectively. Further, can be deduced that the correlation coefficients K_3 and K_4 between CIFSs A and B also satisfies the property of $0 \le K_3(A,B) \le 1$ and $0 \le K_4(A,B) \le 1$.

Theorem 3 *Let A and B be two CIFSs defined on U. If $\xi=(\xi_1,\xi_2,\ldots,\xi_n)^T$ be the weight vector corresponding to $x_j,(j=1,2,\ldots,n)$ with $\xi_j>0$ and $\sum_{j=1}^{n}\xi_j=1$ then the weighted correlation coefficient $K_3(A,B)$ between the two CIFSs A and B defined in* Eq. (12)*, satisfies the following properties:*

(P1) $0 \le K_3(A,B) \le 1$
(P2) $K_3(A,B)=K_3(B,A)$
(P3) $K_3(A,B)=1$ *if* $A=B$
(P4) *If* $A\subseteq B\subseteq C$ *then,* $K_3(A,C)\le K_3(A,B)$ *and* $K_3(A,C)\le K_3(B,C)$ *for any CIFS C defined on U.*

Proof The properties (P2)-(P4) are straightforward, so we omit to proof here. Now, we shall proof only (P1) property. For it, let $A=(r_A(x)e^{iw_{r_A}(x)},k_A(x)e^{iw_{k_A}(x)})$ and $B=(r_B(x)e^{iw_{r_B}(x)},k_B(x)e^{iw_{k_B}(x)})$ be two CIFSs defined on U. From the Eq. (12), it is clearly seen that, $K_3(A,B)\ge 0$. So, we will prove only $K_3(A,B)\le 1$.

$$\begin{aligned}K_3(A,B) &= \frac{C_w(A,B)}{\sqrt{T_w(A)\times T_w(B)}} \\ &= \frac{\sum_{j=1}^{n}\left(\xi_j\left[\begin{array}{l} r_A(x_j)r_B(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}(x_j)w_{r_B}(x_j)\right) \\ +k_A(x_j)k_B(x_j)+\frac{1}{4\pi^2}\left(w_{k_A}(x_j)w_{k_B}(x_j)\right)\end{array}\right]\right)}{\left\{\begin{array}{l}\sqrt{\sum_{j=1}^{n}\left(\xi_j\left[r_A^2(x_j)+k_A^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}^2(x_j)+w_{k_A}^2(x_j)\right)\right]\right)} \\ \times\sqrt{\sum_{j=1}^{n}\left(\xi_j\left[r_B^2(x_j)+k_B^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_B}^2(x_j)+w_{k_B}^2(x_j)\right)\right]\right)}\end{array}\right\}} \\ &= \frac{\left(\begin{array}{l}\sum_{j=1}^{n}\left(\sqrt{\xi_j}r_A(x_j)\right)\left(\sqrt{\xi_j}r_B(x_j)\right)+\sum_{j=1}^{n}\frac{1}{4\pi^2}\left(\sqrt{\xi_j}w_{r_A}(x_j)\right)\left(\sqrt{\xi_j}w_{r_B}(x_j)\right) \\ +\sum_{j=1}^{n}\left(\sqrt{\xi_j}k_A(x_j)\right)\left(\sqrt{\xi_j}k_B(x_j)\right)+\sum_{j=1}^{n}\frac{1}{4\pi^2}\left(\sqrt{\xi_j}w_{k_A}(x_j)\right)\left(\sqrt{\xi_j}w_{k_B}(x_j)\right)\end{array}\right)}{\left\{\begin{array}{l}\sqrt{\sum_{j=1}^{n}\left(\xi_j\left[r_A^2(x_j)+k_A^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}^2(x_j)+w_{k_A}^2(x_j)\right)\right]\right)} \\ \times\sqrt{\sum_{j=1}^{n}\left(\xi_j\left[r_B^2(x_j)+k_B^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_B}^2(x_j)+w_{k_B}^2(x_j)\right)\right]\right)}\end{array}\right\}}\end{aligned}$$

By using Cauchy-Schwarz inequality, we obtain

$$K_3(A,B) \le \frac{\left(\begin{array}{l}\sqrt{\sum_{j=1}^{n}\xi_j r_A^2(x_j)}\sqrt{\sum_{j=1}^{n}\xi_j r_B^2(x_j)} + \sqrt{\sum_{j=1}^{n}\xi_j\left(\frac{w_{r_A}(x_j)}{2\pi}\right)^2}\sqrt{\sum_{j=1}^{n}\xi_j\left(\frac{w_{r_B}(x_j)}{2\pi}\right)^2}\\ +\sqrt{\sum_{j=1}^{n}\xi_j k_A^2(x_j)}\sqrt{\sum_{j=1}^{n}\xi_j k_B^2(x_j)} + \sqrt{\sum_{j=1}^{n}\xi_j\left(\frac{w_{k_A}(x_j)}{2\pi}\right)^2}\sqrt{\sum_{j=1}^{n}\xi_j\left(\frac{w_{k_B}(x_j)}{2\pi}\right)^2}\end{array}\right)}{\left\{\begin{array}{l}\sqrt{\sum_{j=1}^{n}\left(\xi_j\left[r_A^2(x_j)+k_A^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_A}^2(x_j)+w_{k_A}^2(x_j)\right)\right]\right)}\\ \times\sqrt{\sum_{j=1}^{n}\left(\xi_j\left[r_B^2(x_j)+k_B^2(x_j)+\frac{1}{4\pi^2}\left(w_{r_B}^2(x_j)+w_{k_B}^2(x_j)\right)\right]\right)}\end{array}\right\}}$$

By using the following notations

$$\sum_{j=1}^{n}\xi_j r_A^2(x_j)=a,\ \sum_{j=1}^{n}\xi_j r_B^2(x_j)=b,\ \sum_{j=1}^{n}\xi_j\left(\frac{w_{r_A}(x_j)}{2\pi}\right)^2=p,\ \sum_{j=1}^{n}\xi_j k_A^2(x_j)=c,$$
$$\sum_{j=1}^{n}\xi_j k_B^2(x_j)=d,\ \sum_{j=1}^{n}\xi_j\left(\frac{w_{r_B}(x_j)}{2\pi}\right)^2=q,\ \sum_{j=1}^{n}\xi_j\left(\frac{w_{k_A}(x_j)}{2\pi}\right)^2=r,\ \sum_{j=1}^{n}\xi_j\left(\frac{w_{k_B}(x_j)}{2\pi}\right)^2=s,$$

we can reduce the above inequality to

$$K_3(A,B) \le \frac{\sqrt{ab}+\sqrt{cd}+\sqrt{pq}+\sqrt{rs}}{\sqrt{(a+c+p+r)(b+d+q+s)}}$$

Therefore,

$$\begin{aligned}(K_3(A,B))^2-1 &\le \frac{\left(\sqrt{ab}+\sqrt{cd}+\sqrt{pq}+\sqrt{rs}\right)^2}{(a+c+p+r)(b+d+q+s)}-1\\ &= \frac{\left(\sqrt{ab}+\sqrt{cd}+\sqrt{pq}+\sqrt{rs}\right)^2-(a+c+p+r)(b+d+q+s)}{(a+c+p+r)(b+d+q+s)}\\ &= \frac{\left(\begin{array}{l}ab+cd+pq+rs+2\sqrt{abcd}+2\sqrt{pqrs}+2\sqrt{abpq}\\ +2\sqrt{abrs}+2\sqrt{cdpq}+2\sqrt{cdrs}-ab-ad-aq-as-cb\\ -cd-cq-cs-pb-pd-pq-ps-rb-rd-rq-rs\end{array}\right)}{(a+c+p+r)(b+d+q+s)}\\ &= -\left(\frac{\begin{array}{l}(\sqrt{ad}-\sqrt{bc})^2+(\sqrt{ps}-\sqrt{qr})^2+(\sqrt{aq}-\sqrt{bp})^2\\ +(\sqrt{as}-\sqrt{br})^2+(\sqrt{cq}-\sqrt{dp})^2+(\sqrt{cs}-\sqrt{dr})^2\end{array}}{(a+c+p+r)(b+d+q+s)}\right)\\ &\le 0\end{aligned}$$

which implies that $K_3(A,B) \le 1$. Hence, $0 \le K_3(A,B) \le 1$. □

Theorem 4 *The correlation coefficient of two CIFSs A and B, as defined in* Eq. (13) *i.e., K_4, satisfies the same properties as those in Theorem 2.*

Proof The proof is similar to Theorem 2, so we omit here. □

5 MCDM approach based on the proposed correlation coefficients

In this section, we utilize the proposed correlation coefficients of CIFSs to present the multicriteria decision making method.

For a multi criteria decision-making with complex intuitionistic fuzzy information, assume that there are m different alternatives denoted by $A_1, A_2, \ldots, A_m$ which have to be evaluated under the set of the n criteria denoted by $C_1, C_2, \ldots, C_n$. Assume that an expert is invited to evaluate these alternatives under the set of each criteria. Further, assume that the importance of each criteria is considered in the form of the weight vector as $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^T$ such that $\xi_q > 0$ and $\sum_{q=1}^{n} \xi_q = 1$. Consider an expert provide the values under the CIFS environment and these values can be considered as a complex intuitionistic fuzzy element. The rating values corresponding to each alternative are represented in the form of CIFS $A_p(p = 1, 2, \ldots, m)$ as follows

$$A_p = \left\{\left(C_q, r_{pq}(C_q)e^{iw_{r_{pq}}(C_q)}, k_{pq}(C_q)e^{iw_{k_{pq}}(C_q)}\right) | \, q = 1, 2, \ldots, n\right\};$$
$$p = 1, 2, \ldots, m.$$

where $r_{pq}(C_q) \in [0, 1]$ represent the satisfaction degree of the alternative A_p towards the criteria C_q and $k_{pq}(C_q)$ represent the possible degree of the rejection for the alternative A_p under the criteria C_q. For convenience, we denote this CIFS by $\alpha_{pq} = \left(r_{pq}e^{iw_{r_{pq}}}, k_{pq}e^{iw_{k_{pq}}}\right)$ where $r_{pq}, k_{pq} \in [0, 1]$, $w_{r_{pq}}, w_{k_{pq}} \in [0, 2\pi]$ and $r_{pq} + k_{pq} \leq 1$, $w_{r_{pq}} + w_{k_{pq}} \leq 2\pi$ for $p = 1, 2, \ldots, m$; $q = 1, 2, \ldots, n$ and call it as complex intuitionistic fuzzy numbers (CIFNs). Then, we utilize the following steps based on the proposed correlation coefficients for solving the MCDM problems under the CIFSs environment.

Step 1: Construct the ideal reference set to find the best alternative in the decision set whose rating values are taken under the CIFSs environment. We denote such reference set by B.

Step 2: Construct the decision matrix based on the collective information of the alternatives $A_p(p = 1, 2, \ldots, m)$ under the set of criteria $C_q(q = 1, 2, \ldots, n)$ as provided by an expert in terms of CIFNs $\alpha_{pq} = \left(r_{pq}e^{iw_{r_{pq}}}, k_{pq}e^{iw_{k_{pq}}}\right)$. We denote such matrix as $D = (\alpha_{pq})_{m \times n}$ which can be represented as

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{pmatrix} \alpha_{11} & \alpha_{12} & \ldots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \ldots & \alpha_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \ldots & \alpha_{mn} \end{pmatrix} \end{array}$$

Step 3: Calculate the correlation coefficient between the alternatives $A_p(p = 1, 2, \ldots, m)$ and the reference set B by using either K_1 or K_2 or by using weighted correlation coefficients K_3 or K_4, to compute the degree of the relationship among the alternatives.

Step 4: Rank the alternatives based on the index values of correlation coefficients as obtained from $\arg\max\{K\}$. As larger the value of correlation coefficients, the better is the alternative $A_p(p = 1, 2, \ldots, m)$.

6 Illustrative examples

In order to demonstrate the above mentioned approach based on correlation coefficients, we present two illustrative examples which are described as follows.

6.1 Example 1: Decision-making problem

An earthquake of 7.8 magnitude, also called as Gorkha earthquake, racked Nepal on 25 April 2015 at a depth of approximately 15km and lasted nearly fifty seconds and its epicenter was about 21 miles east southest of Lamjung and 48 miles northwest of Kathmandu and its focus was 9.3 miles underground and it destroyed thousands of houses across many districts of the country with entire villages flattened especially near the epicenter. An aftershock occurred on 12 May 2015 in Nepal which heightened the fears and tensions among the affected people. The two earthquakes together resulted in many damages, economic losses in almost 35 districts out of which 5 regions were severely affected namely: A_1 : Lalitpur, A_2 : Kathmandu, A_3 : Gorkha, A_4 : Bhaktapur and A_5 : Makwanpur. An earthquake relief camp decided to help victims of the earthquake in these five different regions $A_p(p = 1, 2, \ldots, 5)$ of the affected area with a different intensity of an earthquake. The panel decided that they will plan their budget by considering the four basic needs of victims, considered as criterion, namely C_1(Food), C_2(Shelter), C_3(Clothes) and C_4(Medical requirements) and decided to allocate the budget firstly to the most affected region so that by initial efforts only, a large strata of people get relief. The weight vector corresponding to these basic needs is taken as $\xi = (0.30, 0.25, 0.15, 0.30)^T$. Clearly, according to the intensity of the earthquake, the basic needs of victims will be affected and changed. The target of this problem is to find the most affected region out of $A_1, A_2, \ldots, A_5$ so as to allocate the proper budget and all the necessary facilities to them. To achieve this, we utilize the developed approach to rank the regions and the best one(s) can be found by implementing the steps of the proposed approach as follows:

Step 1: Assume that an expert gives their preference in terms of maximum possible needs of all the five regions over the each basic need $C_q(q = 1, 2, 3, 4)$

as a complex intuitionistic fuzzy set. The rating values of this set, denoted by B and called as a "reference set" in order to evaluate the given five regions are summarized as below:

$$B = \left\{ \begin{array}{l} \left(C_1, 0.7e^{i2\pi(0.5)}, 0.1e^{i2\pi(0.3)}\right), \left(C_2, 0.4e^{i2\pi(0.6)}, 0.5e^{i2\pi(0.2)}\right), \\ \left(C_3, 0.5e^{i2\pi(0.5)}, 0.3e^{i2\pi(0.1)}\right), \left(C_4, 0.8e^{i2\pi(0.7)}, 0.2e^{i2\pi(0.1)}\right) \end{array} \right\}$$

Step 2: An expert evaluates each region $A_p(p = 1, 2, \ldots, 5)$ individually and estimate the requirements under the set of criteria $C_q(q = 1, 2, 3, 4)$. Their rating values towards each region is expressed in terms of CIFNs whose values are summarized as follows.

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccc} \quad\quad C_1 \quad\quad & \quad\quad C_2 \quad\quad & \quad\quad C_3 \quad\quad & \quad\quad C_4 \quad\quad \end{array} \\ \begin{pmatrix} \left(0.6e^{i2\pi(0.7)}, 0.1e^{i2\pi(0.2)}\right) & \left(0.9e^{i2\pi(0.8)}, 0.1e^{i2\pi(0.1)}\right) & \left(0.5e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.4)}\right) & \left(0.6e^{i2\pi(0.4)}, 0.2e^{i2\pi(0.1)}\right) \\ \left(0.4e^{i2\pi(0.2)}, 0.3e^{i2\pi(0.1)}\right) & \left(0.5e^{i2\pi(0.3)}, 0.1e^{i2\pi(0.1)}\right) & \left(0.6e^{i2\pi(0.4)}, 0.2e^{i2\pi(0.3)}\right) & \left(0.8e^{i2\pi(0.6)}, 0.1e^{i2\pi(0.2)}\right) \\ \left(0.7e^{i2\pi(0.7)}, 0.1e^{i2\pi(0.2)}\right) & \left(0.4e^{i2\pi(0.6)}, 0.3e^{i2\pi(0.1)}\right) & \left(0.7e^{i2\pi(0.7)}, 0.1e^{i2\pi(0.1)}\right) & \left(0.6e^{i2\pi(0.5)}, 0.3e^{i2\pi(0.4)}\right) \\ \left(0.7e^{i2\pi(0.6)}, 0.3e^{i2\pi(0.3)}\right) & \left(0.4e^{i2\pi(0.9)}, 0.2e^{i2\pi(0.1)}\right) & \left(0.7e^{i2\pi(0.7)}, 0.2e^{i2\pi(0.3)}\right) & \left(0.5e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.6)}\right) \\ \left(0.2e^{i2\pi(0.8)}, 0.5e^{i2\pi(0.1)}\right) & \left(0.7e^{i2\pi(0.3)}, 0.3e^{i2\pi(0.3)}\right) & \left(0.6e^{i2\pi(0.5)}, 0.1e^{i2\pi(0.3)}\right) & \left(0.6e^{i2\pi(0.5)}, 0.3e^{i2\pi(0.4)}\right) \end{pmatrix} \end{array}$$

Step 3a: By applying the correlation coefficient K_1 as given in Eq. (9) between the alternatives $A_p(p=1, 2, 3, 4, 5)$ and the reference set B, we can obtain their measurement values as $K_1(A_1, B) = 0.8936$, $K_1(A_2, B) = 0.9068$, $K_1(A_3, B) = 0.9424$, $K_1(A_4, B) = 0.8830$ and $K_1(A_5, B) = 0.8450$. On the other hand, by utilizing the correlation coefficient K_2 given in Eq. (10), their corresponding measurement values are $K_2(A_1, B) = 0.8722$, $K_2(A_2, B) = 0.7522$, $K_2(A_3, B) = 0.9316$, $K_2(A_4, B) = 0.8228$ and $K_2(A_5, B) = 0.8251$.

Step 3b: If we assign the weight vector $\xi = (0.30, 0.25, 0.15, 0.30)^T$ to the criteria, then by utilizing a weighted correlation coefficient K_3 as given in Eq. (12) to compute the measurement values between the alternatives $A_p(p = 1, 2, \ldots, 5)$ and set B, we get $K_3(A_1, B) = 0.8965$, $K_3(A_2, B) = 0.9087$, $K_3(A_3, B) = 0.9439$, $K_3(A_4, B) = 0.8747$ and $K_3(A_5, B) = 0.8351$. Similarly, by using correlation coefficient K_4, we get their corresponding results are $K_4(A_1, B) = 0.8920$, $K_4(A_2, B) = 0.7379$, $K_4(A_3, B) = 0.9299$, $K_4(A_4, B) = 0.8429$ and $K_4(A_5, B) = 0.8058$.

Step 4: From these computed measurement values, we conclude that the ranking order of the regions $A_p(p = 1, 2, \ldots, 5)$ is $A_3 \succ A_2 \succ A_1 \succ A_4 \succ A_5$, where "$\succ$" stands for "preferred to" when K_1 correlation coefficient index has been used while $A_3 \succ A_1 \succ A_5 \succ A_4 \succ A_2$ when K_2 index has been used. Similarly, the ranking order of the region by considering the weight factor into the account is $A_3 \succ A_2 \succ A_1 \succ A_4 \succ A_5$ and $A_3 \succ A_1 \succ A_4 \succ A_5 \succ A_2$ respectively, when either the correlation coefficient K_3 or K_4 is utilized. It is observed from this analysis that the ranking order of the region is different for the different indices. However, the best alternative i.e., the most affected area remains same (A_3) while the worst changes according to the optimistic to pessimistic behavior. Hence, based on the behavior of the decision makers' toward the ranking order related to optimistic and pessimistic behavior, they can choose the desired one accordingly. For instance, related to an optimistic decision maker's behavior, they tend to prefer A_4 over A_5 due to ranking order $A_4 \succ A_5$ while a pessimistic attitude will choose towards the region, they tend to choose A_5 over A_4 regions to allocate the funds due to $A_5 \succ A_4$. Therefore, based on the attitude behavioral characteristic of the decision maker, they can use the best and worst region to allocate the budget.

6.2 Example 2: Medical diagnosis

Consider a decision-making problem with respect to the medical diagnosis which consists of a set of four diseases $Q = \{Q_1$(Viral fever), Q_2(Malaria), Q_3(Typhoid), Q_4(Stomach Problem)$\}$ and a set of symptoms $S = \{s_1$(Temperature), s_2(HeadAche), s_3(Stomach Pain), s_4(Cough)$\}$. Assume that the patient P, with respect to all the symptoms, has been evaluated by an expert in order to find which diseases are patient affected by the most. So, the target of this decision-making problem is to diagnose the disease of the patient P among Q_1, Q_2, Q_3, Q_4. For it, we utilized the steps of the proposed approach to obtain the suitable ranking of the diagnoses and are summarized as follows:

Step 1: The patient P is treated as a reference set and an expert gave their preferences with respect to all the symptoms in terms of CIFSs and is represented by the following set:

$$P = \left\{ \begin{array}{l} \left(s_1, 0.8e^{i2\pi(0.6)}, 0.1e^{i2\pi(0.2)}\right), \left(s_2, 0.9e^{i2\pi(0.7)}, 0.1e^{i2\pi(0.2)}\right), \\ \left(s_3, 0.7e^{i2\pi(0.8)}, 0.2e^{i2\pi(0.1)}\right), \left(s_4, 0.6e^{i2\pi(0.5)}, 0.2e^{i2\pi(0.4)}\right) \end{array} \right\}$$

Step 2: The rating values of each diagnosis $Q_p (p = 1, 2, 3, 4)$ are expressed by a doctor (called as an expert) under the set of symptoms $s_q (q = 1, 2, 3, 4)$ and are summarized as a complex intuitionistic fuzzy decision matrix D as

$$D = \begin{array}{c} \\ Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{array} \begin{array}{c} \begin{array}{cccc} \quad s_1 \quad & \quad s_2 \quad & \quad s_3 \quad & \quad s_4 \quad \end{array} \\ \begin{pmatrix} \left(0.8e^{i2\pi(0.7)}, 0.1e^{i2\pi(0.2)}\right) & \left(0.9e^{i2\pi(0.6)}, 0.1e^{i2\pi(0.2)}\right) & \left(0.7e^{i2\pi(0.8)}, 0.2e^{i2\pi(0.1)}\right) & \left(0.8e^{i2\pi(0.7)}, 0.2e^{i2\pi(0.1)}\right) \\ \left(0.6e^{i2\pi(0.4)}, 0.1e^{i2\pi(0.5)}\right) & \left(0.4e^{i2\pi(0.9)}, 0.5e^{i2\pi(0.1)}\right) & \left(0.5e^{i2\pi(0.5)}, 0.3e^{i2\pi(0.3)}\right) & \left(0.4e^{i2\pi(0.9)}, 0.5e^{i2\pi(0.1)}\right) \\ \left(0.3e^{i2\pi(0.8)}, 0.3e^{i2\pi(0.1)}\right) & \left(0.8e^{i2\pi(0.3)}, 0.1e^{i2\pi(0.6)}\right) & \left(0.7e^{i2\pi(0.6)}, 0.2e^{i2\pi(0.2)}\right) & \left(0.2e^{i2\pi(0.7)}, 0.8e^{i2\pi(0.2)}\right) \\ \left(0.5e^{i2\pi(0.3)}, 0.4e^{i2\pi(0.6)}\right) & \left(0.3e^{i2\pi(0.1)}, 0.6e^{i2\pi(0.3)}\right) & \left(0.8e^{i2\pi(0.3)}, 0.1e^{i2\pi(0.5)}\right) & \left(0.1e^{i2\pi(0.3)}, 0.6e^{i2\pi(0.5)}\right) \end{pmatrix} \end{array}$$

Step 3a: By applying the correlation coefficient K_1 between the set $Q_p (p = 1, 2, 3, 4)$ and the patient P, we get their measurement values are $K_1(Q_1, P) = 0.9800$, $K_1(Q_2, P) = 0.8582$, $K_1(Q_3, P) = 0.8446$ and $K_1(Q_4, P) = 0.7037$. On the other hand, if we utilize the correlation coefficient K_2, then their corresponding measurement values are $K_2(Q_1, P) = 0.9412$, $K_2(Q_2, P) = 0.8109$, $K_3(Q_3, P) = 0.8132$ and $K_4(Q_4, P) = 0.5923$.

Step 3b: If we assign the weightage to the set of symptoms $s_q (q = 1, 2, 3, 4)$ as $\xi = (0.30, 0.20, 0.10, 0.40)^T$ then by applying the weighted correlation coefficient K_3 and K_4, we get their respectively measurement values are $K_3(Q_1, P) = 0.9696$, $K_3(Q_2, P) = 0.8486$, $K_3(Q_3, P) = 0.8008$, $K_3(Q_4, P) = 0.6980$ and $K_4(Q_1, P) = 0.9015$, $K_4(Q_2, P) = 0.8433$, $K_4(Q_3, P) = 0.7935$, $K_4(Q_4, P) = 0.5969$.

Step 4: Based on the optimal values of the diseases, we conclude that its ranking order is $Q_1 \succ Q_2 \succ Q_3 \succ Q_4$ when the K_1 correlation coefficient index has been used, while $Q_1 \succ Q_3 \succ Q_2 \succ Q_4$ when K_2 index has been used. From this analysis, it is concluded that the patient P suffer from the Q_1 diseases. Further, from these ranking orders, we observe that when decision maker utilize the K_1 correlation coefficient by keeping his mind towards the optimistic view, then the second most diseases affected to the patient is Q_2. On the other hand, if the decision maker's attitude towards the diseases is pessimistic in nature, they will tend towards Q_3 to be the second most diseases affected to the patient P. Similarly, we get the ranking order of the diseases affected to the patient corresponding to the utilization of K_3 and K_4 as $Q_1 \succ Q_2 \succ Q_3 \succ Q_4$.

7 Comparative analysis

In this section, we compare the performance of the proposed measures with some of the existing approaches under the CIFS as well as IFS environment. The detailed analysis of the above considered examples is explained as below.

7.1 Comparative studies of Example 1 under CIFS environment

In order to compare the proposed approach results with some of the existing approaches [33, 34] under the CIFS environment, an analysis has been conducted for the considered data. The results corresponding to these approaches are summarized as follows:

(i) If we utilize the distance measure, denoted by d_1, as proposed by [33] to the considered data, then corresponding to each region the measurement values from the reference set B are $d_1(A_1, B) = 0.1817$, $d_1(A_2, B) = 0.1917$, $d_1(A_3, B) = 0.1400$, $d_1(A_4, B) = 0.2167$ and $d_1(A_5, B) = 0.2600$. Since $d_1(A_3, B) < d_1(A_1, B) < d_1(A_2, B) < d_1(A_4, B) < d_1(A_5, B)$ and hence the ranking order of the given regions is $A_3 \succ A_1 \succ A_2 \succ A_4 \succ A_5$. Thus, it has been computed that that A_3 is the most affected region.

(ii) If we utilize the weighted Euclidean distance measure, denoted by d_2, as defined by [34] to the considered problem, then the measurement values of each region are computed as $d_2(A_1, B) = 0.1871, d_2(A_2, B) = 0.1803, d_2(A_3, B) = 0.1374, d_2(A_4, B) = 0.2086$ and $d_2(A_5, B) = 0.2225$. Since $d_2(A_3, B) < d_2(A_2, B) < d_2(A_1, B) < d_2(A_4, B) < d_2(A_5, B)$ and hence ranking order of the region is $A_3 \succ A_2 \succ A_1 \succ A_4 \succ A_5$. Therefore, we conclude that A_3 is again the most affected region.

From these comparative studies, it is concluded that the best region obtained from the proposed measure coincides with the existing measures hence it validates the feasibility of the approach under the CIFS environment.

7.2 Comparative studies of Example 1 under IFS environment

In order to compare the proposed approach results with the results obtained under IFSs environment, we conducted an analysis by executing some of the existing approaches [23, 26, 28, 29] to the considered data. Since IFS is a special case of the CIFS, so we firstly convert the CIFS environment data into the IFS environment data by setting phase term corresponding to each criteria to 0 in every CIFN. Then, based on these existing approaches, we conduct the following analysis to compute the most affected region(s) under the IFS environment.

(i) If we apply the correlation coefficient, denoted by ρ_1, as defined by [23] to the considered problem, then we get the measurement value of each region is $\rho_1(A_1, B) = 0.8740, \rho_1(A_2, B) = 0.8874, \rho_1(A_3, B) = 0.9442, \rho_1(A_4, B) = 0.8822$ and $\rho_1(A_5, B) = 0.8262$. Since $\rho_1(A_3, B) > \rho_1(A_2, B) > \rho_1(A_4, B) > \rho_1(A_1, B) > \rho_1(A_5, B)$ and hence we conclude that A_3 is the most affected region.

(ii) If we apply the correlation coefficient, denoted by ρ_2, as defined by [26] on the reduced data, then we get $\rho_2(A_1, B) = 0.8808, \rho_2(A_2, B) = 0.9106, \rho_2(A_3, B) = 0.9551, \rho_2(A_4, B) = 0.9246$ and $\rho_2(A_5, B) = 0.8250$. Since, $\rho_2(A_3, B) > \rho_2(A_4, B) > \rho_2(A_2, B) > \rho_2(A_1, B) > \rho_2(A_5, B)$ and hence ranking order is $A_3 \succ A_4 \succ A_2 \succ A_1 \succ A_5$. Thus, we conclude that A_3 is the most affected region which coincides with the proposed one.

(iii) If we utilize correlation coefficient (ρ_3) as presented by [28], then the corresponding indices to each region are computed as $\rho_3(A_1, B) = -0.4603, \rho_3(A_2, B) = 0.0000, \rho_3(A_3, B) = 0.5198, \rho_3(A_4, B) = 0.1143$ and $\rho_3(A_5, B) = -0.6336$. Since, $\rho_3(A_3, B) > \rho_3(A_4, B) > \rho_3(A_2, B) > \rho_3(A_1, B) > \rho_3(A_5, B)$ and hence again the most affected region is A_3 and it coincides with the proposed measure results.

(iv) By utilizing the similarity measure [29], denoted by ρ_4, on the considered data, we get their measurement values are $\rho_4(A_1, B) = 0.8221, \rho_4(A_2, B) = 0.8527, \rho_4(A_3, B) = 0.8827, \rho_4(A_4, B) = 0.8492$ and $\rho_4(A_5, B) = 0.7562$. From it, we get the ranking order of the regions is $A_3 \succ A_2 \succ A_4 \succ A_1 \succ A_5$ and hence we conclude that the most affected region is again A_3.

Thus, from this analysis, it has been clearly seen that the results computed by the existing approaches coincides with the proposed one and hence it supports the proposed results.

7.3 Comparative studies of Example 2 under CIFS environment

To compare the performance of the proposed approach with some of the existing approaches under the CIFSs environment, an analysis has been done and their results are summarized as follows.

(i) By applying the approach of [33] using distance measures, denoted by d_1 to the considered data, we get the measurement values of each disease as $d_1(Q_1, P) = 0.0967, d_1(Q_2, P) = 0.2717, d_1(Q_3, P) = 0.2867$ and $d_1(Q_4, P) = 0.3550$. From these values, we observed that $d_1(Q_1, P) < d_1(Q_2, P) < d_1(Q_3, P) < d_1(Q_4, P)$ and hence conclude that the patient P suffers from disease Q_1.

(ii) By utilizing the distance measure (d_2) as proposed by [34] to the considered problem, then the measurement values for each disease are computed as $d_2(Q_1, P) = 0.1194, d_2(Q_2, P) = 0.2291, d_2(Q_3, P) = 0.2669$ and $d_2(Q_4, P) = 0.3004$. Since measurement value of Q_1 is minimum among all these and hence we conclude that patient P suffers from disease Q_1 which again coincides with the proposed measure results.

7.4 Comparative studies of Example 2 under IFS environment

In order to validate the efficiency of the proposed approach under the IFS environment, we conducted an analysis based on some of the existing correlation coefficients [23, 26, 28, 29]. The results corresponding to its are summarized as follows.

(i) If we utilize correlation coefficient (ρ_1) as proposed by [23], then their measurement values for each diagnosis are summarized as $\rho_1(Q_1, P) = 0.9856, \rho_1(Q_2, P) = 0.8461, \rho_1(Q_3, P) = 0.7959$ and $\rho_1(Q_4, P) = 0.7258$. From it, we conclude that $\rho_1(Q_1, P) > \rho_1(Q_2, P) > \rho_1(Q_3, P) > \rho_1(Q_4, P)$ and hence the patient P suffers from disease Q_1.

(ii) By applying the correlation coefficient(ρ_2) as defined by [26] to the considered data then we get $\rho_2(Q_1, P) = 0.9912$, $\rho_2(Q_2, P) = 0.8585$, $\rho_2(Q_3, P) = 0.7265$ and $\rho_2(Q_4, P) = 0.6645$. Thus $\rho_2(Q_1, P) > \rho_2(Q_2, P) > \rho_2(Q_3, P) > \rho_2(Q_4, P)$. From it, we conclude that patient P suffers from disease Q_1.
(iii) If we apply the correlation coefficient(ρ_3) as proposed by [28] to the data, then the measurement values are obtained as $\rho_3(Q_1, P) = 0.8485$, $\rho_3(Q_2, P) = 0.1907$, $\rho_3(Q_3, P) = 0.6608$ and $\rho_3(Q_4, P) = -0.0690$. Thus $\rho_3(Q_1, P) > \rho_3(Q_3, P) > \rho_3(Q_2, P) > \rho_3(Q_4, P)$ which implies that the ranking order of $Q_p (p = 1, 2, 3, 4)$ is $Q_1 \succ Q_3 \succ Q_2 \succ Q_4$. From it, we conclude that patient P suffers from disease Q_1.
(iv) On applying the similarity measure (ρ_4) as defined by [29] on the considered information, then we get $\rho_4(Q_1, P) = 0.9642$, $\rho_4(Q_2, P) = 0.7394$, $\rho_4(Q_3, P) = 0.7725$ and $\rho_4(Q_4, P) = 0.6538$. As, $\rho_4(Q_1, P) > \rho_4(Q_3, P) > \rho_4(Q_2, P) > \rho_4(Q_4, P)$ and hence we conclude that patient P suffers from disease Q_1.

Thus, from the above analysis, it has been seen that results computed by the proposed approach coincide with the existing approaches which validates the feasibility of the proposed approach.

7.5 Advantages of the proposed approach

From the existing studies and the proposed measures, we address the following merits of the proposed method to solve the decision-making problem under the CIFS environment.

(i) A complex intuitionistic fuzzy set is a generalization of the existing studies such as complex fuzzy sets [4], intuitionistic fuzzy sets [2], fuzzy set [1] by considering much more information related to an object during the process and to handle the two-dimensional information in a single set. For instance, CIFS contains information (both the membership and non-membership degrees are complex valued) with amplitude and phase terms than the CFS (contains only complex valued membership degree), IFS (with a real-valued membership and non-membership degrees and only considered amplitude term), FS (with only crisp membership degrees with amplitude term only). Thus, the proposed correlation coefficients under CIFSs environment are more generalized than the existing correlation coefficients [19–30].
(ii) It is revealed from the present study that the correlation coefficients under IFSs, FSs [19–30] are the special cases of the proposed measures. Thus, the proposed correlation coefficients can be equivalently utilized to solve the MCDM problem under these existing environment by setting phase term to be zero while the existing measures [19, 21–23, 26, 27] are unable to solve the problems under the environment considered in the present paper.
(iii) The major advantages of the proposed decision-making approach are to consider the much more information to access the alternative to reduce the information loss. Further, the correlation coefficients based on the optimistic and pessimistic with or without weighting factor will help the decision maker to select the best alternative(s) more accurately. In other words, we can say that the proposed correlation coefficients will give the various choices to the decision makers based on their optimistic and pessimistic behavior towards the decision-making process.

8 Conclusion

In this article, an attempt has been made to present different kinds of the coefficients of correlation for decision-making process under the complex intuitionistic fuzzy set environment. Earlier, various existing coefficients of correlation have been defined under the IFSs environment where the range of their corresponding membership and non-membership degrees is the subset of the real numbers. But this condition has been relaxed in the present manuscript by considering the CIFSs where the ranges of the membership degrees are extended from the real numbers to the complex numbers with the unit disc. Therefore, considered environment models the information in a better way for time-periodic problems and has the ability to handle two-dimensional information in the single set. Keeping these points in view, under this environment, we present various coefficients of correlation and studied their properties in detail. Further, based on them, a decision-making approach is presented to find the best alternative in the CIFSs environment. Two numerical examples are taken for illustrating the developed approach and their results are compared with some of the existing correlation measures to show the validity of it. From the studies, we conclude that the proposed approach can be efficiently used in decision-making problems where two-dimensional information is clubbed in a single set. Also, it is observed that the existing correlation measures under the IFSs environment can be taken as a special case of the proposed measure. In the future, we will extend the study of CIFS to present some aggregation operators and the results of this paper to complex interval-valued IFS, type-2 fuzzy set, Pythagorean fuzzy set, multiplicative fuzzy set [37–40].

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