



Generalized score functions on interval-valued intuitionistic fuzzy sets with preference parameters for different types of decision makers and their application

Fangwei Zhang^{1,2} · Jihong Chen¹ · Yuhua Zhu¹ · Ziyi Zhuang³ · Jiaru Li¹

Published online: 24 May 2018
© Springer Science+Business Media, LLC, part of Springer Nature 2018

Abstract

In this paper, the interval-valued intuitionistic fuzzy sets (IVIFSs) are studied from the viewpoint of the decision makers' preference. Firstly, two series of principles are proposed to guide the ranking of interval-valued intuitionistic fuzzy numbers (IVIFNs), and two kinds of illustrative generalized score functions on IVIFSs are proposed according to the newly proposed principles. Secondly, two kinds of generalized score functions on IVIFSs are proposed based on decision-makers' preference. The two generalized score functions are both of two parameters, which represent the decision makers' attitudinal characters on the classical score values and the classical accuracy values on IVIFNs, respectively. Thirdly, two kinds of generalized score functions on IVIFSs, which are suitable for ranking IVIFNs when there is no information about the importance weights of the classical score values and accuracy values on IVIFNs, are proposed based on integral. Fourthly, three kinds of multi-criteria decision-making (MCDM) methods in interval-valued intuitionistic fuzzy setting are proposed. Finally, an example shows that when a novel generalized score function on IVIFSs is proposed, its suitable application environments should also be pointed out.

Keywords Interval-valued intuitionistic fuzzy sets · Generalized score function · Integral function · Preference parameters · Multi-criteria decision making

1 Introduction

The concept of intuitionistic fuzzy sets (IFSs), a generalization of the concept of fuzzy sets, was first introduced by

[1]. In IFSs, the relationship between an element and a set is described by two numbers, i.e., the membership degree and the non-membership degree of the element to the set, respectively. Thereafter, [2] proposed the notion of interval-valued intuitionistic fuzzy sets (IVIFSs) as an extension of IFSs. In IVIFSs, the membership degree and the non-membership degree of an element to a set are given as interval numbers.

For more than twenty years, the IVIFSs have been studied from multiple perspectives, such as entropy theory, distance theory, score functions, and accuracy functions. For more details on this issue please see [6, 14, 15, 18], etc. Multiple criteria decision making (MCDM) is one of the processes for selecting the optimal alternative from all the feasible alternatives according to some criteria or attributes. When the theories of IFSs or IVIFSs are applied to MCDM, the most important topic worthy of attention is how to compare and rank the IFSs or IVIFSs. Many scholars have paid great attention to this issue. [4] suggested a score function of vague values. Later, [8] defined an accuracy function of vague values. Similar to the aforementioned

✉ Yuhua Zhu
yhzhu@shmtu.edu.cn
Fangwei Zhang
fangweizhang@aliyun.com
Jihong Chen
jhchen@shmtu.edu.cn
Ziyi Zhuang
ziyizhuang@aliyun.com
Jiaru Li
cherishlir@163.com

¹ College of Transport and Communications, Shanghai Maritime University, Shanghai, 201306, China
² School of Transportation, Southeast University, Nanjing 210096, China
³ College of Information Engineering, Shanghai Maritime University, Shanghai, 201306, China

studies, [19] introduced a score function and an accuracy function on IFSs. Furthermore, by using both of them, they recommended a well-known order relation to rank intuitionistic fuzzy numbers (IFNs), and offered a series of measures to indicate the similarity degree between IFNs. Besides, the properties of IFNs are also studied broadly from geometric perspective. For example, [22] proposed a hybrid arithmetic to rank IFNs. It is noteworthy that this order relation is very prospective and outstanding. However, the importance of the score values of IFNs is much more prominent than the accuracy values in this order relation, which leads to relatively strict conditions of application. In the past ten years, some new score functions and accuracy functions have been propounded to rank the IFNs or the interval-valued intuitionistic fuzzy numbers (IVIFNs), where the proposed score functions and accuracy functions are not combined any more, they work alone. For more details on this issue please see [12, 16, 17, 20, 26]. Later, [24] and [23] indicated that there are some difficulties in the proposed score functions or accuracy functions for ranking the IFNs or IVIFNs. Much more recently, [9] provided some optimum techniques to balance between group consensus and individual independence. [7] first developed an intuitionistic fuzzy Bayesian network to obtain the practical attribute weights under uncertain environment in information fusion and dynamic decision making process. The aforementioned studies have enriched the research system on IFSs, which laid a solid foundation for this study.

In general, the essence of generalized score value on an IFS is the “quality” of the membership degree of an element to the given set. From the viewpoint of utility, two factors need to be considered on the comparison between the “qualities” of different membership degrees on IFNs or IVIFNs. One factor is the classical score values, and the other one is the classical accuracy values. The bigger the classical score value of an IVIFN (IFN) is, the greater its utility; the bigger the classical accuracy value of an IVIFN (IFN) is, the greater its utility. Therefore, the coordination of the two factors is worthy of consideration. From the standpoint of probability theory, there are infinite possibilities to combine the classical score value and accuracy value of an IVIFN (IFN). All the existing score functions or accuracy functions are the moment estimation functions of the relationship between the two factors. Therefore, they can only be used in certain environments.

Hong and Choi [8] once indicated that “the relation between the classical score and accuracy functions on IFSs (the original text is ‘vague sets’, which is interlinked with IFSs in essence) is similar to the relation between mean and variance in statistics”. However, the functions, proposed to rank the IVIFSs respectively, are called accuracy function or score function optionally. For uniformity in this study,

the functions used to rank IVIFSs solely are termed as “generalized score function”. Meanwhile, [3] proposed a model which demonstrates the influence driven evolution of experts’ opinions and its convergence properties. Referring to the decision making thoughts proposed by [8] and [3], the comparison between IVIFNs is conducted based on the attitudinal characters of decision-makers in this study. Furthermore, to reduce the risk of errors in the comparing process on IVIFNs (IFNs), this study uses the integral theory to aggregate the infinite possibilities combining the classical score and accuracy values of an IVIFN (IFN).

The rest of this paper is organized as follows. In Section 2, the definitions of IFSs and IVIFSs, as well as the concepts of the classical score function and accuracy function of IFSs and IVIFSs are introduced. In Section 3, two kinds of principles are suggested to guide the ranking of IVIFNs; then, according to the newly proposed principles, two novel generalized score functions on IVIFSs are proposed; subsequently, two kinds of generalized score functions on IVIFSs are proposed based on preference information of decision-makers; thereafter, two kinds of generalized score functions on IVIFSs are proposed based on integral. In Section 4, three kinds of MCDM methods in intuitionistic fuzzy setting are offered. Section 5 provides an example to demonstrate the effectiveness of the proposed methods. Section 6 ends this study with some important conclusions.

2 Preliminaries

In this section, the concepts of IFSs and IVIFSs, the classical score and accuracy functions are reviewed. Primarily, the concepts of IFSs and IVIFSs are introduced as follows.

Definition 1 [1]. Let X be a non-empty set. An IFS A in X is denoted as $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$, where $\mu_A(x) : X \rightarrow [0, 1]$, $\nu_A(x) : X \rightarrow [0, 1]$ with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for any $x \in X$. Here, $\mu_A(x)$ and $\nu_A(x)$ denote the membership degree and the non-membership degree of x to A , respectively. For any $x \in X$, the hesitancy degree of x to A is denoted as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$; the complementary set of A is defined as $A^c = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$.

For convenience, [16] denoted the intuitionistic fuzzy numbers (IFNs) as $A = \langle \mu, \nu \rangle$.

Definition 2 [2]. Let $D[0, 1]$ be the set of all closed sub-intervals of the interval $[0, 1]$ and $X (\neq \Phi)$ be a given set. An IVIFS A in the universe of discourse X is defined as $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X\}$, where $\mu_A(x) : X \rightarrow D[0, 1]$, $\nu_A(x) : X \rightarrow D[0, 1]$ with the condition $0 \leq$

$sup(\mu_A(x)) + sup(v_A(x)) \leq 1$ for any $x \in X$. The intervals $\mu_A(x)$ and $v_A(x)$ represent the membership degree and non-membership degree of the element x to the set A , respectively. Denote the lower and upper end points as $\mu_{AL}(x), \mu_{AU}(x), v_{AL}(x)$, and $v_{AU}(x)$, respectively, i.e.,

$$A = \{x, [\mu_{AL}(x), \mu_{AU}(x)], [v_{AL}(x), v_{AU}(x)] \mid x \in X\}, \tag{1}$$

where $0 \leq \mu_{AU}(x) + v_{AU}(x) \leq 1, 0 \leq \mu_{AL}(x)$ and $0 \leq v_{AL}(x)$.

For any $x \in X$, the interval intuitionistic index of x to A is denoted as

$$\pi_A(x) = [\pi_{AL}(x), \pi_{AU}(x)] = [1 - \mu_{AU}(x) - v_{AU}(x), 1 - \mu_{AL}(x) - v_{AL}(x)]. \tag{2}$$

The complementary set A^c of an IVIFS A is defined as

$$A^c = \{x, [v_{AL}(x), v_{AU}(x)], [\mu_{AL}(x), \mu_{AU}(x)] \mid x \in X\}. \tag{3}$$

Xu and Yager [19] called the pair $(\mu_A(x), v_A(x))$ as an IVIFN, and simplified it as $\tilde{\alpha} = ([a, b], [c, d])$, where $[a, b] \subset [0, 1], [c, d] \subset [0, 1], b + d \leq 1$. Thereafter, [19] proposed the classical score and accuracy functions on IFNs as follows.

Definition 3 [19] Let $\alpha = (\mu, v)$ be an IFN, then the classical score function on α is defined as $S(\alpha) = \mu - v$.

Definition 4 [19] Let $\alpha = (\mu, v)$ be an IFN, then the classical accuracy function on α is defined as $H(\alpha) = \mu + v$.

By using the classical score and accuracy functions, [16] proposed a well-known approach to rank IFNs as follows.

Definition 5 [16] Let $\alpha_i = (\mu_i, v_i)(i = 1, 2)$ be two IFNs, and let $S(\alpha_i)$ and $H(\alpha_i)$ be the score values and the accuracy values of $\alpha_i(i = 1, 2)$, respectively. Then the following conditions hold:

- (i) if $S(\alpha_1) > S(\alpha_2)$, then $\alpha_1 \succ \alpha_2$;
- (ii) if $S(\alpha_1) = S(\alpha_2)$, then,
 - (1) if $H(\alpha_1) > H(\alpha_2)$, then $\alpha_1 \succ \alpha_2$;
 - (2) if $H(\alpha_1) = H(\alpha_2)$, then $\alpha_1 = \alpha_2$;
 - (3) if $H(\alpha_1) < H(\alpha_2)$, then $\alpha_1 \prec \alpha_2$.

The most significant contributions of Definition 5 is that it gives a procedure to compare different IFNs. Furthermore, the following conclusions are obtained under interval-valued intuitionistic fuzzy environments.

Definition 6 [17] Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN, then the score function of $\tilde{\alpha}$ is defined as $S_{xu}(\tilde{\alpha}) = \frac{1}{2}(a +$

$b - c - d)$, and the accuracy function of $\tilde{\alpha}$ is defined as $H_{xu}(\tilde{\alpha}) = \frac{1}{2}(a + b + c + d)$.

3 Main results

3.1 Problem analysis

In this subsection, the IVIFNs are studied through data analysis. Take an IVIFN $\tilde{\alpha} = ([a, b], [c, d])$ for example, at present, there are many kinds of generalized score functions or accuracy functions on $\tilde{\alpha}$. For example, [12, 16, 17, 20] and [26] proposed some kinds of them. Since the differences between different IVIFNs mainly depend on two parameters. One parameter is $S_{xu}(\tilde{\alpha})$, which represents the closeness between the IVIFN $\tilde{\alpha}$ and the fuzzy concept ‘‘excellence’’; the other parameter is $H_{xu}(\tilde{\alpha})$, which represents the credibility of the IVIFN $\tilde{\alpha}$. It is difficult to find a sole function to rank all the IVIFNs without self-contradictory in some cases. For example, when comparing two IVIFNs $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$ and $\alpha_2 = ([a_2, b_2], [c_2, d_2])$, we should compare not only the difference between $S_{xu}(\tilde{\alpha}_1)$ and $S_{xu}(\tilde{\alpha}_2)$, but also the difference between $H_{xu}(\tilde{\alpha}_1)$ and $H_{xu}(\tilde{\alpha}_2)$. In the comparing process, how important is $S_{xu}(\cdot)$, and how important is $H_{xu}(\cdot)$? Is $S_{xu}(\cdot)$ more or less important than $H_{xu}(\cdot)$? Practically, the importance degrees of $S_{xu}(\cdot)$ and $H_{xu}(\cdot)$ are relative rather than absolute. Different decision-makers have different viewpoints on them, and any certain viewpoint on them is only suitable for comparing the IVIFNs in certain decision making circumstances. Therefore, when a novel generalized score or accuracy function on IVIFSs is proposed, its application scope should also be pointed out.

Frequently, fuzziness problems lack the illustrating information that leads us to suitable generalized score functions or accuracy functions. Therefore, it is of great interest to propose the fundamental principles to evaluate the effectiveness of these generalized score or accuracy functions. In the following subsection, some principles are introduced on the basis of practice. Furthermore, under the guidance of these fundamental principles, two kinds of illustrative generalized score functions on IVIFSs are proposed.

3.2 Applicable conditions of generalized score functions on IVIFSs

In this subsection, the applicable conditions of generalized score functions on IVIFSs are proposed through two theorems. Thereafter, two specific generalized score functions on IVIFS which satisfy the aforementioned two theorems are introduced.

Theorem 1 Let $\tilde{\alpha}_i = ([a_i, b_i], [c_i, d_i]) (i = 1, 2)$ be two IVIFNs, and let $\tilde{S}(\cdot)$ be the generalized score function which is proposed to rank IVIFNs by itself. Then $\tilde{S}(\cdot)$ should satisfy the following properties:

- (i) if $(a_1 - d_1) - (b_2 - c_2) > 0, (a_1 + c_1) - (b_2 + d_2) > 0$, then, $\tilde{S}(\tilde{\alpha}_1) > \tilde{S}(\tilde{\alpha}_2)$;
- (ii) if $(b_1 - c_1) - (a_2 - d_2) < 0, (b_1 + d_1) - (a_2 + c_2) < 0$, then, $\tilde{S}(\tilde{\alpha}_1) < \tilde{S}(\tilde{\alpha}_2)$;
- (iii) if $\tilde{S}(\tilde{\alpha}_1) > \tilde{S}(\tilde{\alpha}_2)$, then, $\tilde{S}(\tilde{\alpha}_1^c) < \tilde{S}(\tilde{\alpha}_2^c)$;
- (iv) $\tilde{S}(\tilde{\alpha}_i) \in [0, 1]$, for any $i = 1, 2$;
- (v) $\tilde{S}([0, 0], [1, 1]) = 0, \tilde{S}([1, 1], [0, 0]) = 1$.

Analogously, for $i = 1, 2$, when $a_i = b_i$, and $c_i = d_i$, we have the following theorem.

Theorem 2 Let $\alpha_i = (\mu_i, \nu_i) (i = 1, 2)$ be two IFNs, and let $S(\cdot)$ be the generalized score function which is proposed to rank IFNs by itself. Then $S(\cdot)$ should satisfy the following properties:

- (i) if $(\mu_1 - \nu_1) - (\mu_2 - \nu_2) > 0, (\mu_1 + \nu_1) - (\mu_2 + \nu_2) > 0$, then, $S(\alpha_1) > S(\alpha_2)$;
- (ii) if $(\mu_1 - \nu_1) - (\mu_2 - \nu_2) < 0, (\mu_1 + \nu_1) - (\mu_2 + \nu_2) < 0$, then, $S(\alpha_1) < S(\alpha_2)$;
- (iii) if $S(\alpha_1) > S(\alpha_2)$, then, $S(\alpha_1^c) < S(\alpha_2^c)$;
- (iv) $S(\alpha_i) \in [0, 1]$, for any $i = 1, 2$;
- (v) $S((0, 1)) = 0, S((1, 0)) = 1$.

It is noteworthy that Theorem 1 and 2 are consistent with Definition 5. However, Definition 5 is a kind of method to compare IVIFSs. Different from Definition 5, Theorem 1 and 2 are mainly used to evaluate the generalized score functions which rely on one parameter. Meanwhile, Theorem 1 and 2 can not be used to compare IVIFSs directly in many cases, since they only work under very specific conditions. For example, when comparing two IVIFNs $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ by any generalized score function, the condition (i) in Thm. 1 means that if the classical score value of $\tilde{\alpha}_1$ is larger than that of $\tilde{\alpha}_2$, and the classical accuracy value of $\tilde{\alpha}_1$ is larger than that of $\tilde{\alpha}_2$ hold at the same time, $\tilde{S}(\tilde{\alpha}_1)$ should be prior to $\tilde{S}(\tilde{\alpha}_2)$.

To illustrate Theorem 1 and 2 more clearly, two kinds of illustrative novel generalized score functions are introduced as follows.

Definition 7 Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN. Then, two generalized score functions $F_1(\cdot)$ and $F_2(\cdot)$ on IVIFNs are proposed as

$$F_1(\tilde{\alpha}) = \frac{1}{2} \left[s(\tilde{\alpha}) \cdot \sqrt{abs\left(\frac{(a-c)+(b-d)}{2}\right) \cdot \frac{(a+c)+(b+d)}{2}} + 1 \right], \tag{4}$$

$$F_2(\tilde{\alpha}) = \frac{1}{2} \left\{ \frac{s(\tilde{\alpha})}{2} \cdot \left[abs\left(\frac{(a-c)+(b-d)}{2}\right) + \frac{(a+c)+(b+d)}{2} \right] + 1 \right\}, \tag{5}$$

where $s(\tilde{\alpha}) = sgn\left[\frac{(a-c)+(b-d)}{2}\right]$; $sgn(\cdot)$ represents the sign function; $abs(\cdot)$ represents the absolute value function. Obviously, $0 \leq F_1(\cdot), F_2(\cdot) \leq 1$.

Obviously, $F_1(\cdot)$ and $F_2(\cdot)$ satisfy the five properties of Theorem 1 or Theorem 2. Besides, though $F_1(\cdot)$ and $F_2(\cdot)$ consider the classical score and accuracy values of IVIFSs without distinction, the score values of $F_1(\cdot)$ and $F_2(\cdot)$ on the same given IVIFN are usually different. To illustrate the effectiveness of the score functions $F_1(\cdot)$ and $F_2(\cdot)$, an example is introduced.

Case 1 Assume that there are two IVIFNs, where

$$\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1]) = ([0.65, 0.75], [0.15, 0.20]),$$

$$\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2]) = ([0.45, 0.50], [0.10, 0.15]),$$

please compare them.

On one hand, by calculation, it is obtained that

$$(a_1 - d_1) - (b_2 - c_2) = 0.45 - 0.40 = 0.05 > 0,$$

$$(a_1 + c_1) - (b_2 + d_2) = 0.80 - 0.60 = 0.20 > 0.$$

Therefore, according to the property (i) in Theorem 1, their ranking is as $\tilde{\alpha}_1 > \tilde{\alpha}_2$.

On the other hand, by using (4), it is obtained that $F_1(\tilde{\alpha}_1) = 0.839, F_1(\tilde{\alpha}_2) = 0.729$. Similarly, by using (5), it is obtained that $F_2(\tilde{\alpha}_1) = 0.850, F_2(\tilde{\alpha}_2) = 0.738$. Therefore, it holds that $F_1(\alpha_1) > F_1(\alpha_2), F_2(\alpha_1) > F_2(\alpha_2)$. As a result, it is concluded that $\tilde{\alpha}_1 > \tilde{\alpha}_2$, which satisfies the property (i) of Theorem 1. Meanwhile, by using (4), it is obtained that $F_1(\tilde{\alpha}_1^c) = 0.161, F_1(\tilde{\alpha}_2^c) = 0.271$. Similarly, by using (5), it is obtained that $F_2(\tilde{\alpha}_1^c) = 0.150, F_2(\tilde{\alpha}_2^c) = 0.263$. Therefore, it holds that $F_1(\alpha_2^c) > F_1(\alpha_1^c)$. As a result, it is concluded that $\tilde{\alpha}_2^c > \tilde{\alpha}_1^c$, which satisfies the property (iii) of Theorem 1.

3.3 Two kinds of generalized score functions on IVIFSs based on preference information of decision-makers

Similar to $F_1(\cdot)$ and $F_2(\cdot)$, there are many generalized score functions which were proposed to compare IVIFSs. Since the differences between different IVIFNs mainly depend on two parameters, the classical score value, and the classical accuracy value, it is considered that any given generalized score function on IVIFSs reflects a certain relationship between the classical score function and the classical accuracy function. Therefore, in order to serve different types of decision makers targetedly, two kinds of generalized score functions on IVIFSs are suggested based on preference information of decision-makers. The first kind of generalized score function on IVIFSs is based on geometric weighted operators; while the second one is based

on arithmetic weighted operator. Besides, these two kinds of generalized score functions are of two parameters. More details are as follows.

Definition 8 Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN. A novel generalized weighted geometric average score function $F_3(\cdot)$ on IVIFNs is defined as

$$F_3(\tilde{\alpha}) = \frac{1}{2} \left\{ s(\tilde{\alpha}) \cdot \left[abs \left(\frac{(a-c) + (b-d)}{2} \right) \right]^t \cdot \left[\frac{(a+c) + (b+d)}{2} \right]^{1-t} + 1 \right\}, \quad (6)$$

$$\text{where } s(\tilde{\alpha}) = \text{sgn} \left[\frac{(a-c) + (b-d)}{2} \right], t \in [0, 1].$$

Meanwhile, a novel generalized weighted arithmetic average score function with parameters is proposed as follows.

Definition 9 Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN, a novel generalized weighted arithmetic average score function $F_4(\cdot)$ on IVIFNs is defined as

$$F_4(\tilde{\alpha}) = \frac{1}{2} \left\{ s(\tilde{\alpha}) \cdot \left[t * abs \left(\frac{(a-c) + (b-d)}{2} \right) + (1-t) * \frac{(a+c) + (b+d)}{2} \right] + 1 \right\}, \quad (7)$$

$$\text{where } s(\tilde{\alpha}) = \text{sgn} \left[\frac{(a-c) + (b-d)}{2} \right], t \in [0, 1].$$

To illustrate the effectiveness of the generalized score functions $F_3(\cdot)$ and $F_4(\cdot)$, an example is introduced.

Case 2 Assume that there are two IVIFNs, where

$$\tilde{\alpha}_3 = ([a_3, b_3], [c_3, d_3]) = ([0.15, 0.20], [0.05, 0.15]),$$

$$\tilde{\alpha}_4 = ([a_4, b_4], [c_4, d_4]) = ([0.40, 0.45], [0.35, 0.40]),$$

please compare them.

By (6), they get that $F_3(\tilde{\alpha}_3) = 0.5 * (0.075^t * 0.275^{1-t} + 1)$, $F_3(\tilde{\alpha}_4) = 0.5 * (0.050^t * 0.800^{1-t} + 1)$. It is proved that when $0 \leq t < 0.724$, they hold that $F_2(\alpha_3) < F_2(\alpha_4)$, while when $0.724 < t \leq 1$, they hold that $F_2(\alpha_3) > F_2(\alpha_4)$.

By (7), they get that $F_4(\tilde{\alpha}_3) = 0.5 * (1.275 - 0.2t)$, $F_4(\tilde{\alpha}_4) = 0.5 * (1.8 - 0.75t)$. It is proved that when $0 \leq t < 0.954$, they hold that $F_4(\alpha_3) < F_4(\alpha_4)$, while when $0.954 < t \leq 1$, they hold that $F_4(\alpha_3) > F_4(\alpha_4)$.

This case shows that the generalized score functions $F_3(\cdot)$ and $F_4(\cdot)$ have their own characteristics. Besides, there are some common properties between these two functions: (i) t represents the decision makers' attitudinal character on the classical score values of IVIFNs, where the bigger the parameter t , the larger the influence of

the classical score values of IVIFNs on them; (ii) $1 - t$ represents the decision makers' attitudinal character on the classical accuracy values of IVIFNs, where the bigger the parameter $1 - t$, the larger the influence of the classical accuracy values of IVIFNs on them.

3.4 Two kinds of generalized score functions on IVIFSs based on integral

In the following, two kinds of generalized score functions on IVIFSs are proposed based on integral. When there is no information about the relationship between the classical score function and the classical accuracy function, these generalized score functions are available.

Definition 10 Let $\tilde{A} = ([a, b], [c, d])$ be an IVIFN. Then, two kinds of generalized score functions on IVIFNs are proposed as

$$F_5(\tilde{A}) = \frac{1}{2} \left(1 + \int_0^1 s(\tilde{\alpha}) \cdot \left[abs \left(\frac{(a-c) + (b-d)}{2} \right) \right]^t \cdot \left[\frac{(a+c) + (b+d)}{2} \right]^{1-t} dt \right), \quad (8)$$

$$F_6(\tilde{A}) = \frac{1}{2} \left(1 + \int_0^1 s(\tilde{\alpha}) \cdot \left[t \cdot abs \left(\frac{(a-c) + (b-d)}{2} \right) + (1-t) \cdot \frac{(a+c) + (b+d)}{2} \right] dt \right), \quad (9)$$

$$\text{where } s(\tilde{\alpha}) = \text{sgn} \left[\frac{(a-c) + (b-d)}{2} \right].$$

3.5 Supplement explanations

(1) The relationships between $F_1(\cdot)$ and $F_3(\cdot)$, and between $F_2(\cdot)$ and $F_4(\cdot)$ are summarized as follows.

Proposition 1 Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN. When $t = 0.5$, the generalized score function $F_3(\cdot)$ on $\tilde{\alpha}$ is reduced to the generalized score function $F_1(\cdot)$, i.e., $F_3(\tilde{\alpha})|_{t=0.5} = F_1(\tilde{\alpha})$; the generalized score function $F_4(\cdot)$ on $\tilde{\alpha}$ is reduced to the generalized score function $F_2(\cdot)$, i.e., $F_4(\tilde{\alpha})|_{t=0.5} = F_2(\tilde{\alpha})$.

(2) The relationships between $F_3(\cdot)$ and $S_{xu}(\cdot)$, and between $F_4(\cdot)$ and $S_{xu}(\cdot)$ are summarized as follows.

Proposition 2 Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ be two IVIFNs. When $t = 1$ and $S_{xu}(\alpha_1) \neq S_{xu}(\alpha_2)$, it is obtained that

$$[S_{xu}(\tilde{\alpha}_1) - S_{xu}(\tilde{\alpha}_2)] \cdot [F_3(\tilde{\alpha}_1) - F_3(\tilde{\alpha}_2)] > 0, \quad (10)$$

$$[S_{xu}(\tilde{\alpha}_1) - S_{xu}(\tilde{\alpha}_2)] \cdot [F_4(\tilde{\alpha}_1) - F_4(\tilde{\alpha}_2)] > 0; \quad (11)$$

when $t = 1$ and $S_{xu}(\tilde{\alpha}_1) = S_{xu}(\tilde{\alpha}_2)$, it is obtained that $F_3(\tilde{\alpha}_1) = F_3(\tilde{\alpha}_2)$, and $F_4(\tilde{\alpha}_1) = F_4(\tilde{\alpha}_2)$; when $t = 1$, $a_1 = b_1 = 1$, $c_1 = d_1 = 0$, it is obtained that $F_3(\tilde{\alpha}_1) = F_4(\tilde{\alpha}_1) = 1$; when $t = 1$, $a_1 = b_1 = 0$, $c_1 = d_1 = 1$, it is obtained that $F_3(\tilde{\alpha}_1) = F_4(\tilde{\alpha}_1) = 0$.

(3) The relationships between $F_3(\cdot)$ and $H_{xu}(\cdot)$, and between $F_4(\cdot)$ and $H_{xu}(\cdot)$ are summarized as follows.

Proposition 3 Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ be two IVIFNs. When $t = 0$ and $H_{xu}(\tilde{\alpha}_1) \neq H_{xu}(\tilde{\alpha}_2)$, it is obtained that

$$[H_{xu}(\tilde{\alpha}_1) - H_{xu}(\tilde{\alpha}_2)] \cdot [F_3(\tilde{\alpha}_1) - F_3(\tilde{\alpha}_2)] > 0, \tag{12}$$

$$[H_{xu}(\tilde{\alpha}_1) - H_{xu}(\tilde{\alpha}_2)] \cdot [F_4(\tilde{\alpha}_1) - F_4(\tilde{\alpha}_2)] > 0; \tag{13}$$

when $t = 0$, $s(\tilde{\alpha}_1) = s(\tilde{\alpha}_2)$, and $H_{xu}(\tilde{\alpha}_1) = H_{xu}(\tilde{\alpha}_2)$, it is obtained that $F_3(\tilde{\alpha}_1) = F_3(\tilde{\alpha}_2)$, $F_4(\tilde{\alpha}_1) = F_4(\tilde{\alpha}_2)$; when $t = 0$, $s(\tilde{\alpha}_1) = s(\tilde{\alpha}_2) = 1$, $a_1 + c_1 = 1$, $b_1 + d_1 = 1$, it is obtained that $F_3(\tilde{\alpha}_1) = F_4(\tilde{\alpha}_1) = 1$; when $t = 0$, $s(\tilde{\alpha}_1) = s(\tilde{\alpha}_2) = -1$, $a_1 + c_1 = 1$, $b_1 + d_1 = 1$, it is obtained that $F_3(\tilde{\alpha}_1) = F_4(\tilde{\alpha}_1) = 0$.

(4) The relationships between $F_3(\cdot)$ and $F_4(\cdot)$ are summarized as follows.

Proposition 4 Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ be two IVIFNs. For any given $t \in [0, 1]$, it doesn't always hold that

$$[F_3(\alpha_1) - F_3(\alpha_2)] \cdot [F_4(\alpha_1) - F_4(\alpha_2)] > 0. \tag{14}$$

(5) There are some important special cases on $F_5(\cdot)$ and $F_6(\cdot)$ as follows.

Proposition 5 Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN. When $a = b = 1$, $c = d = 0$, it is obtained that $F_5(\tilde{\alpha}) = F_6(\tilde{\alpha}) = 1$; when $a = b = 0$, $c = d = 1$, it is obtained that $F_5(\tilde{\alpha}) = F_6(\tilde{\alpha}) = 0$.

Studying the aforementioned propositions, it shows that $F_1(\cdot)$, $F_2(\cdot)$, $F_3(\cdot)$, $F_4(\cdot)$, $F_5(\cdot)$ and $F_6(\cdot)$ are all suitable to be used as generalized score functions.

(6) The relationships between the newly proposed generalized score functions are shown in Fig. 1.

4 Three kinds of MCDM methods on IVIFSs

At present, the intuitionistic fuzzy sets are widely used to solve MCDM problems (more details please see [11, 21, 25], etc). In this paper, the studied MCDM problem in interval-valued intuitionistic fuzzy setting is introduced as follows.

Let $A = \{A_1, A_2, \dots, A_m\}$ be a discrete set of alternatives, $C = \{C_1, C_2, \dots, C_n\}$ be a set of decision making criteria. Let $W = \{w_1, w_2, \dots, w_n\}$ be the weight vector of the criteria, where $w_j \in [0, 1]$, $\sum_{j=1}^n w_j = 1$, $j = 1, 2, \dots, n$. Let $\tilde{Y} = (\tilde{\alpha}_{ij})_{m \times n} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])_{m \times n}$ be an interval-valued intuitionistic fuzzy decision matrix, where $[a_{ij}, b_{ij}]$ indicates the degree that the alternative A_i satisfies the criterion C_j , whereas $[c_{ij}, d_{ij}]$ indicates the degree that the alternative A_i does not satisfy the criterion C_j . Besides, $[a_{ij}, b_{ij}] \subset [0, 1]$, $[c_{ij}, d_{ij}] \subset [0, 1]$, $b_{ij} + d_{ij} \leq 1$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$. And then, for any $i = 1, 2, \dots, m$, the characteristic of the alternative A_i is expressed by an IVIFS: $\tilde{A}_i = \{(C_j, [a_{ij}, b_{ij}], [c_{ij}, d_{ij}]) \mid C_j \in C\}$. Under the above conditions, how to rank the alternatives, and how to select the optimal alternative? To

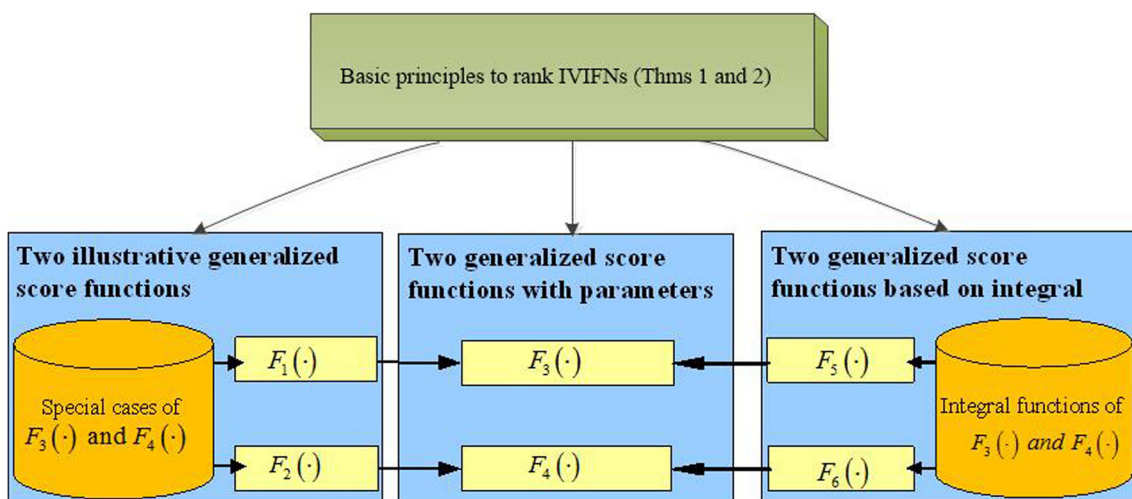


Fig. 1 The relationships between the newly proposed generalized score functions

solve this problem, three kinds of MCDM methods are proposed.

- (1) By using the generalized score function $F_1(\cdot)$ or $F_2(\cdot)$, a series of novel MCDM methods are proposed as follows.

Step 1 Calculate the generalized score values of all the elements of the matrix $\tilde{Y} = (\tilde{\alpha}_{ij})_{m \times n}$ by (4) (or (5)), and denote the results as $\tilde{Y}_1 = (F_1(\tilde{\alpha}_{ij}))_{m \times n}$ (or $\tilde{Y}_2 = (F_2(\tilde{\alpha}_{ij}))_{m \times n}$).

Step 2 For any $j = 1, 2, \dots, n$, by using the thoughts of Shannon entropy method [25], the weight value for the attribute C_j is calculated, and the weight vector for C is denoted as $W = (w_1, w_2, \dots, w_n)$, where

$$w_j = \frac{\sum_{i=1}^m \sqrt{\frac{1}{2}[(1-\pi_{\tilde{\alpha}_{ij}L})^2 + (1-\pi_{\tilde{\alpha}_{ij}U})^2]}}{\sum_{j=1}^n \sum_{i=1}^m \sqrt{\frac{1}{2}[(1-\pi_{\tilde{\alpha}_{ij}L})^2 + (1-\pi_{\tilde{\alpha}_{ij}U})^2]}}, j \in \{1, 2, \dots, n\}. \quad (15)$$

Step 3 Calculate the comprehensive attribute value of each alternative. For any $i = 1, 2, \dots, m$, denote the comprehensive attribute value of A_i as Z_i , where $Z_i = \sum_{j=1}^n w_j \cdot F_1(\tilde{\alpha}_{ij})$ (or $Z_i = \sum_{j=1}^n w_j \cdot F_2(\tilde{\alpha}_{ij})$).

Step 4 Rank all the alternatives and select the optimal alternative A^* by comparing the comprehensive attribute values Z_1, Z_2, \dots, Z_m .

- (2) By using the generalized score functions $F_3(\cdot)$ or $F_4(\cdot)$, a series of novel MCDM methods are proposed as follows.

Step 1 Determine the parameter t by decision makers according to the given decision making environments.

Step 2 Calculate the generalized score values of all the elements of the matrix $\tilde{Y} = (\tilde{\alpha}_{ij})_{m \times n}$ by (6) (or (7)), and denote the results as $\tilde{Y}_3 = (F_3(\tilde{\alpha}_{ij}))_{m \times n}$ (or $\tilde{Y}_4 = (F_4(\tilde{\alpha}_{ij}))_{m \times n}$).

Step 3 For any $j = 1, 2, \dots, n$, by (15), the weight values for all the attributes $C_j (j = 1, 2, \dots, n)$ are calculated, and the weight vector for C is denoted as $W = (w_1, w_2, \dots, w_n)$.

Step 4 Calculate the comprehensive attribute value of each alternative. For any $i = 1, 2, \dots, m$, denote the comprehensive attribute value of A_i as Z'_i , where $Z'_i = \sum_{j=1}^n w_j \cdot F_3(\tilde{\alpha}_{ij})$ (or $Z'_i = \sum_{j=1}^n w_j \cdot F_4(\tilde{\alpha}_{ij})$).

Step 5 By comparing the values Z'_1, Z'_2, \dots, Z'_m , the optimal alternative $A^{*'}$ is obtained.

- (3) By using the generalized score functions $F_5(\cdot)$ or $F_6(\cdot)$, a series of novel MCDM methods are proposed as follows.

Step 1 Calculate the generalized score values of all the elements of the matrix $\tilde{Y} = (\tilde{\alpha}_{ij})_{m \times n}$ by (8) (or (9)), and denote the results as $\tilde{Y}_5 = (F_5(\tilde{\alpha}_{ij}))_{m \times n}$ (or $\tilde{Y}_6 = (F_6(\tilde{\alpha}_{ij}))_{m \times n}$).

Step 2 For any $j = 1, 2, \dots, n$, by (15), the weight values for all the attribute $C_j (j = 1, 2, \dots, n)$ are calculated, and the weight vector for C is denoted as $W = (w_1, w_2, \dots, w_n)$.

Step 3 Calculate the comprehensive attribute value of each alternative. For any $i = 1, 2, \dots, m$, denote the comprehensive attribute value of A_i as Z''_i , where $Z''_i = \sum_{j=1}^n w_j \cdot F_5(\tilde{\alpha}_{ij})$ (or $Z''_i = \sum_{j=1}^n w_j \cdot F_6(\tilde{\alpha}_{ij})$).

Step 4 By comparing the values $Z''_1, Z''_2, \dots, Z''_m$, the optimal alternative $A^{*''}$ is obtained.

In the next section, an example is introduced to illustrate the effectiveness of the proposed generalized score functions and the proposed MCDM methods.

5 Supplier selection example

Since supply chains depend on the time compression, flexible response and unit cost reduction, suppliers play an important role in the whole supply chain. Suppliers are also critical in providing essential components and resources for goods in today's globalization supply chain networks. The number of components comprising a finished product may be small or immense as in aircraft manufacturing and other complex products. Even in the case of simpler products, such as bread, all the ingredients of it may travel across the globe as inputs into production processes. Suppliers are also decision-makers and they compete with one another to provide components to downstream manufacturing firms. Thus, the optimized performance will be elusive without building a good relationship with a reliable supplier, establishing a system to evaluate performance and offer improvements on cost reduction, which will have a significant impact on the whole supply chain.

Up to now, many researchers have paid attentions to supplier selection problem. Among the studies, some aim to enhance the competitiveness or to decrease the costs in supply chain from suppliers. For example, [13] studied the model for decreasing the costs in supply chain. This model reveals the importance of suppliers in the supply chain

management, particularly the role of alternative suppliers, and tries to compensate the weakness of the supply chain. Considering supplier selection is an important issue in supply chain management, [5] proposed a series of AHP methods based on a novel effective and feasible representation of uncertain informations, which extend the classical AHP method. Meanwhile, [10] proposed an integrated approach of fuzzy multi attribute utility theory and multi-objective programming for rating and selecting the best green suppliers. However, the aforementioned studies did not consider the supplier selection problem in a framework which is composed of positive and negative factors. With that in consideration, this study applies IFSs to solve supplier selection problem.

Assuming that a manufacturing company is searching for the best global supplier for one of its most critical parts used in assembling process, the attributes which are considered in the selection process are: f_1 , overall cost of the product; f_2 , quality of the product; f_3 , service performance of supplier; f_4 , supplier’s profile; and f_5 , risk factor. Here, f_1 and f_5 are cost type attributes, whereas $f_2, f_3,$ and f_4 are benefit type attributes. The alternative set is $A = \{A_1, A_2, A_3, A_4\}$. A decision maker group is formed to evaluate the characteristics of each potential global supplier with respect to each attribute on the fuzzy concept “excellence”. By aggregating the decision making results of each decision maker, a matrix $\tilde{Y} = (\tilde{y}_{ij})_{4 \times 5}$ is obtained as

$$Y = \begin{matrix} & f_1 & f_2 & f_3 \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} & \left(\begin{matrix} \langle [0.482, 0.627], [0.177, 0.301] \rangle & \langle [0.300, 0.451], [0.354, 0.481] \rangle & \langle [0.626, 0.726], [0.126, 0.226] \rangle \\ \langle [0.349, 0.475], [0.355, 0.503] \rangle & \langle [0.131, 0.314], [0.292, 0.506] \rangle & \langle [0.324, 0.424], [0.403, 0.503] \rangle \\ \langle [0.358, 0.506], [0.300, 0.427] \rangle & \langle [0.626, 0.775], [0.100, 0.225] \rangle & \langle [0.554, 0.776], [0.100, 0.224] \rangle \\ \langle [0.300, 0.427], [0.358, 0.506] \rangle & \langle [0.100, 0.225], [0.626, 0.775] \rangle & \langle [0.100, 0.224], [0.554, 0.776] \rangle \end{matrix} \right) \\ & f_4 & f_5 \\ & \left(\begin{matrix} \langle [0.500, 0.625], [0.100, 0.275] \rangle & \langle [0.158, 0.356], [0.381, 0.552] \rangle \\ \langle [0.174, 0.299], [0.557, 0.679] \rangle & \langle [0.626, 0.751], [0.126, 0.226] \rangle \\ \langle [0.427, 0.605], [0.227, 0.327] \rangle & \langle [0.525, 0.650], [0.201, 0.326] \rangle \\ \langle [0.227, 0.327], [0.427, 0.605] \rangle & \langle [0.201, 0.326], [0.525, 0.650] \rangle \end{matrix} \right) \end{matrix}$$

By using the generalized score function $F_3(\cdot), F_4(\cdot), F_5(\cdot),$ or $F_6(\cdot),$ the given problem is solved, specific details are as follows.

Step 1 Since there is no information about the attitudinal characters of the decision-makers on the classical

score functions and the accuracy functions on IVIFSs, the parameter t is dealt with as a variable, and $t \in [0, 1]$.

Step 2 By using (6), (7), (8), and (10), the matrix $\tilde{Y} = (\tilde{y}_{ij})_{4 \times 5}$ is calculated, respectively. The calculation results are as follows.

$$Y_{3,t} = \begin{matrix} & f_1 & f_2 & f_3 \\ \left(\begin{matrix} F_{3,t}(\langle [0.482, 0.627], [0.177, 0.301] \rangle) & F_{3,t}(\langle [0.300, 0.451], [0.354, 0.481] \rangle) & F_{3,t}(\langle [0.626, 0.726], [0.126, 0.226] \rangle) \\ F_{3,t}(\langle [0.349, 0.475], [0.355, 0.503] \rangle) & F_{3,t}(\langle [0.131, 0.314], [0.292, 0.506] \rangle) & F_{3,t}(\langle [0.324, 0.424], [0.403, 0.503] \rangle) \\ F_{3,t}(\langle [0.358, 0.506], [0.300, 0.427] \rangle) & F_{3,t}(\langle [0.626, 0.775], [0.100, 0.225] \rangle) & F_{3,t}(\langle [0.554, 0.776], [0.100, 0.224] \rangle) \\ F_{3,t}(\langle [0.300, 0.427], [0.358, 0.506] \rangle) & F_{3,t}(\langle [0.100, 0.225], [0.626, 0.775] \rangle) & F_{3,t}(\langle [0.100, 0.224], [0.554, 0.776] \rangle) \end{matrix} \right) \\ & f_4 & f_5 \\ \left(\begin{matrix} F_{3,t}(\langle [0.500, 0.625], [0.100, 0.275] \rangle) & F_{3,t}(\langle [0.158, 0.356], [0.381, 0.552] \rangle) \\ F_{3,t}(\langle [0.174, 0.299], [0.557, 0.679] \rangle) & F_{3,t}(\langle [0.626, 0.751], [0.126, 0.226] \rangle) \\ F_{3,t}(\langle [0.427, 0.605], [0.227, 0.327] \rangle) & F_{3,t}(\langle [0.525, 0.650], [0.201, 0.326] \rangle) \\ F_{3,t}(\langle [0.227, 0.327], [0.427, 0.605] \rangle) & F_{3,t}(\langle [0.201, 0.326], [0.525, 0.650] \rangle) \end{matrix} \right) \end{matrix}$$

$$\begin{aligned}
 Y_{4,t} = & \\
 & \begin{matrix} & f_1 & & f_2 & & f_3 \\ \left(\begin{array}{l} F_{4,t}(\langle [0.482, 0.627], [0.177, 0.301] \rangle) \quad F_{4,t}(\langle [0.300, 0.451], [0.354, 0.481] \rangle) \quad F_{4,t}(\langle [0.626, 0.726], [0.126, 0.226] \rangle) \\ F_{4,t}(\langle [0.349, 0.475], [0.355, 0.503] \rangle) \quad F_{4,t}(\langle [0.131, 0.314], [0.292, 0.506] \rangle) \quad F_{4,t}(\langle [0.324, 0.424], [0.403, 0.503] \rangle) \\ F_{4,t}(\langle [0.358, 0.506], [0.300, 0.427] \rangle) \quad F_{4,t}(\langle [0.626, 0.775], [0.100, 0.225] \rangle) \quad F_{4,t}(\langle [0.554, 0.776], [0.100, 0.224] \rangle) \\ F_{4,t}(\langle [0.300, 0.427], [0.358, 0.506] \rangle) \quad F_{4,t}(\langle [0.100, 0.225], [0.626, 0.775] \rangle) \quad F_{4,t}(\langle [0.100, 0.224], [0.554, 0.776] \rangle) \end{array} \right) \\
 & \begin{matrix} & f_4 & & f_5 \\ \left(\begin{array}{l} F_{4,t}(\langle [0.500, 0.625], [0.100, 0.275] \rangle) \quad F_{4,t}(\langle [0.158, 0.356], [0.381, 0.552] \rangle) \\ F_{4,t}(\langle [0.174, 0.299], [0.557, 0.679] \rangle) \quad F_{4,t}(\langle [0.626, 0.751], [0.126, 0.226] \rangle) \\ F_{4,t}(\langle [0.427, 0.605], [0.227, 0.327] \rangle) \quad F_{4,t}(\langle [0.525, 0.650], [0.201, 0.326] \rangle) \\ F_{4,t}(\langle [0.227, 0.327], [0.427, 0.605] \rangle) \quad F_{4,t}(\langle [0.201, 0.326], [0.525, 0.650] \rangle) \end{array} \right) \\
 Y_5 = & \begin{pmatrix} 0.7594 & 0.3722 & 0.8302 & 0.7705 & 0.7077 \\ 0.3944 & 0.3230 & 0.3408 & 0.2065 & 0.8369 \\ 0.6487 & 0.8439 & 0.8258 & 0.7310 & 0.7729 \\ 0.3514 & 0.1562 & 0.1742 & 0.2691 & 0.2272 \end{pmatrix}, \\
 Y_6 = & \begin{pmatrix} 0.7753 & 0.2913 & 0.8380 & 0.7813 & 0.7335 \\ 0.2855 & 0.3003 & 0.2735 & 0.1908 & 0.8445 \\ 0.7163 & 0.8503 & 0.8325 & 0.7580 & 0.7938 \\ 0.2838 & 0.1498 & 0.2123 & 0.2420 & 0.2063 \end{pmatrix}.
 \end{aligned}$$

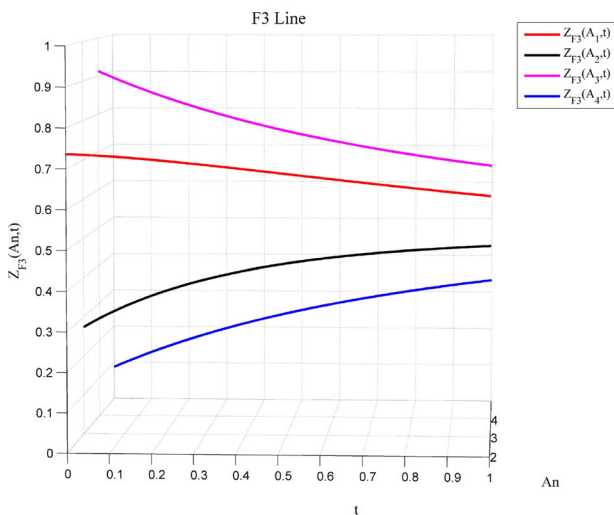
Step 3 By using (15), the attribute weight vector corresponding to each attribute is obtained as

$$W = (0.199, 0.195, 0.206, 0.197, 0.203).$$

Step 4 The comprehensive attribute values of the four alternatives are calculated, and the results are obtained as

$$\begin{aligned}
 Z_{F_3}(A_1, t) &= \frac{1}{2} (0.199 \times (1 + 0.32^t \times 0.79^{1-t}) + 0.195 \times (1 - 0.04^t \times 0.79^{1-t}) + 0.206 \times (1 + 0.50^t \times 0.85^{1-t}) \\
 &\quad + 0.197 \times (1 + 0.38^t \times 0.75^{1-t}) + 0.203 \times (1 + 0.21^t \times 0.72^{1-t})), \\
 Z_{F_3}(A_2, t) &= \frac{1}{2} (0.199 \times (1 - 0.02^t \times 0.84^{1-t}) + 0.195 \times (1 - 0.18^t \times 0.62^{1-t}) + 0.206 \times (1 - 0.08^t \times 0.83^{1-t}) \\
 &\quad + 0.197 \times (1 - 0.38^t \times 0.86^{1-t}) + 0.203 \times (1 + 0.51^t \times 0.87^{1-t})), \\
 Z_{F_3}(A_3, t) &= \frac{1}{2} (0.199 \times (1 + 0.07^t \times 0.80^{1-t}) + 0.195 \times (1 + 0.54^t \times 0.86^{1-t}) + 0.206 \times (1 + 0.50^t \times 0.83^{1-t}) \\
 &\quad + 0.197 \times (1 + 0.24^t \times 0.79^{1-t}) + 0.203 \times (1 + 0.32^t \times 0.85^{1-t})), \\
 Z_{F_3}(A_4, t) &= \frac{1}{2} (0.199 \times (1 - 0.07^t \times 0.80^{1-t}) + 0.195 \times (1 - 0.54^t \times 0.86^{1-t}) + 0.206 \times (1 - 0.50^t \times 0.83^{1-t}) \\
 &\quad + 0.197 \times (1 - 0.24^t \times 0.79^{1-t}) + 0.203 \times (1 - 0.32^t \times 0.85^{1-t})), \\
 Z_{F_4}(A_1, t) &= 0.7361 - 0.0999 \times t, \\
 Z_{F_4}(A_2, t) &= 0.2751 + 0.2135 \times t, \\
 Z_{F_4}(A_3, t) &= 0.9130 - 0.2457 \times t, \\
 Z_{F_4}(A_4, t) &= 0.0870 + 0.1005 \times t,
 \end{aligned}$$

(a) The calculation results obtained by F_3 .



(b) The calculation results obtained by F_4 .

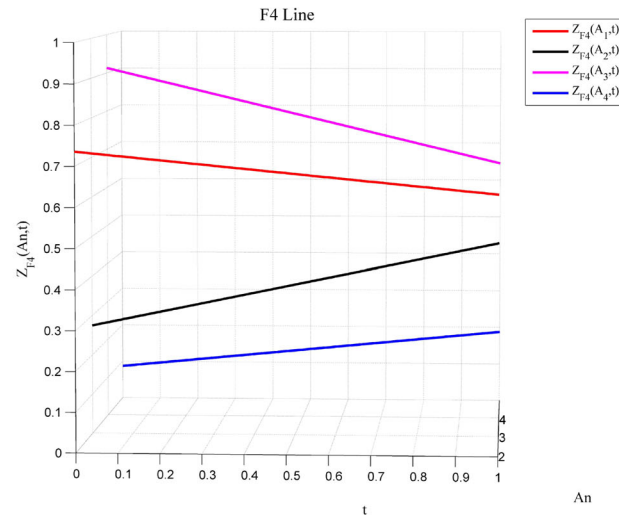


Fig. 2 The comprehensive attribute values calculated by $F_3(\cdot)$ and $F_4(\cdot)$

where $t \in [0, 1]$. For convenience, the sketch maps of them are shown in Fig. 2.

As shown in Fig. 2, the results are consistent on the whole that the optimal alternative is A_3 , and the ranking is $Z(A_3) > Z(A_1) > Z(A_2) > Z(A_4)$. Meanwhile, by using the generalized score function $F_5(\cdot)$ and $F_6(\cdot)$, the given problem can be solved as $Z_{F_5}(A_1) = 0.6901$, $Z_{F_5}(A_2) = 0.4223$, $Z_{F_5}(A_3) = 0.7646$, $Z_{F_5}(A_4) = 0.2354$, and $Z_{F_6}(A_1) = 0.6865$, $Z_{F_6}(A_2) = 0.3808$, $Z_{F_6}(A_3) = 0.7903$, $Z_{F_6}(A_4) = 0.2189$, respectively. Then, it gets $Z(A_3) > Z(A_1) > Z(A_2) > Z(A_4)$, and the optimal alternative is also A_3 .

It is interesting that the aforementioned comparing results are consistent on the whole that A_3 is the optimal alternative. This phenomenon has a certain component and an accidental component. Specifically, the ranking result obtained by $F_5(\cdot)$ is determined by that obtained by $F_3(\cdot)$ while the ranking result obtained by $F_6(\cdot)$ is determined by that obtained by $F_4(\cdot)$. Meanwhile, it is a coincidence that the ranking results obtained by $F_3(\cdot)$ and $F_4(\cdot)$ are the same in this example. Though there are some common properties on $F_3(\cdot)$ and $F_4(\cdot)$, they have their own characteristics. In essence, $F_3(\cdot)$ is a kind of exponential function, while $F_4(\cdot)$ is a kind of linear function. Therefore, $F_3(\cdot)$ is more sensitive to extreme large or small classical score or accuracy values than $F_4(\cdot)$, which could lead to inconsistent comparing results obtained by $F_3(\cdot)$ and $F_4(\cdot)$ in some cases.

Besides, this study has studied the problem using the method proposed by [4], and the calculation results are obtained as $Z(A_1) = 0.139$, $Z(A_2) = 0.533$, $Z(A_3) = 0.298$, and $Z(A_4) = 0.355$. Obviously, it gets $Z(A_2) >$

$Z(A_4) > Z(A_3) > Z(A_1)$, and the optimal alternative as A_2 .

By comparing the novel score functions and score functions proposed by [4], it is found that the internal mechanism of them is different. In particular, by using operations of fetching maximum or minimum, and operations of union and intersection, the techniques proposed in [4] put emphasis on extreme large or small decision making values. On the whole, [4] is more suitable to deal with acute problems. In contrast, the novel generalized score functions proposed in this study emphasize the subjective attitude of the decision-makers. Usually, different decision viewpoints bring different decision results. Therefore, decision makers should choose suitable decision making methods according to their different decision making problem.

6 Conclusion

Since any given generalized score function is only suitable for comparing the IVIFNs in certain circumstances, the IVIFS is studied from the viewpoint of the types of decision-makers in this study. According to the illustrative examples, some conclusions are shown as follows.

- (i) Two series of principles are proposed to evaluate generalized score functions on IVIFNs. Furthermore, according to the newly proposed principles, two novel generalized score functions on IVIFSs are suggested. However, Theorem 1 and 2 are only periodical results. As the internal mechanism of IVIFSs is exposed further, more theorems would be proposed in the future.

- (ii) Two kinds of generalized score functions on IVIFSs are proposed based on preference information of decision-makers. Both of the two kinds of generalized score functions are of two parameters, which represent the decision makers' attitudinal characters on the classical score function and the classical accuracy function on IVIFNs, respectively. Thereafter, two kinds of generalized score functions, which are suitable for ranking the IVIFNs when there is no information about the relationship between the classical score values and the classical accuracy values on IVIFNs, are suggested based on integral. For the next step, the parameters in $F_3(\cdot)$ $F_4(\cdot)$ would be studied in details. Moreover, the comparing methods on generalized score functions would be studied as well.
- (iii) Three kinds of MCDM methods in intuitionistic fuzzy setting are proposed, and the effectiveness of these methods is illustrated by an example. The example shows that any given generalized score function is only suitable for comparing the IVIFNs in certain circumstances, and when there is little comparing information on IVIFNs, the integral-based generalized score functions can be used to reduce the risk of errors. For the next step, more applications of the novel generalized score functions would be carried on in the port management and shipping optimization fields.

Acknowledgments The Fangwei Zhang's work is partially supported by the National Natural Science Foundation of China (51508319, 61374195), the research program of the National Special Authorized Social Science Fund of China (07@ZH005), the Nature and Science Fund from Zhejiang Province Ministry of Education (Y201327642). The Jihong Chen's work is partially supported by the National Natural Science Foundation of China (51409157). The authors are thankful to Prof. Lihua Luo, Mrs. Sifan Li, Mrs. Hang Tian, Mr. Yajun Deng, College of Transport and Communications, Shanghai Maritime University, Mrs. Yujuan Xu, School of Economics and Management, Shanghai Maritime University, Mrs. Shujun Xiao, Mr. Boyi Yu, Mrs. Chun Chen, Logistics Engineering College, Shanghai Maritime University. They have provided useful guidance for this manuscript. Specifically, Fangwei Zhang conceived the generalized score functions on interval-valued intuitionistic fuzzy sets; Jihong Chen and Jiaru Li pointed out that the novel score functions are suitable to make supplier decision making; Yuhua Zhu and Jiaru Li analyzed and calculated the example data; Fangwei Zhang, Jiaru Li and Ziyi Zhuang wrote the manuscript; Fangwei Zhang, Sifan Li, Boyi Yu, Yujuan Xu, and Qiang Li revised the manuscript. All the authors declare no conflict of interest.

References

- Atanassov K (1986) Intuitionistic fuzzy sets. *Fuzzy Sets Syst* 20(1):87–96
- Atanassov K, Gargov G (1989) Interval valued intuitionistic fuzzy sets. *Fuzzy Sets Syst* 31(3):343–349
- Capuano N, Chiclana F, Fujita H, Herrera-Viedma E, Loia V (2017) Fuzzy group decision making with incomplete information guided by social influence. *IEEE T Fuzzy Syst.* <https://doi.org/10.1109/TFUZZ.2017.2744605>
- Chen SM, Tan JM (1994) Handling multicriteria fuzzy decision making problems based on vague set theory. *Fuzzy Sets Syst* 67(2):163–172
- Deng X, Hu Y, Deng Y, Mahadevan S (2013) Supplier selection using AHP methodology extended by D numbers. *Expert Syst Appl* 41(1):156–167
- Farhadinia B (2014) An efficient similarity measure for intuitionistic fuzzy sets. *Soft Comput* 18:85–94. <https://doi.org/10.1007/s00500-013-1035-5>
- Hao ZN, Xu ZS, Zhao H, Fujita H (2017) A dynamic weight determination approach based on the intuitionistic fuzzy bayesian network and its application to emergency decision making. *IEEE T Fuzzy Syst.* <https://doi.org/10.1109/TFUZZ.2017.2755001>
- Hong DH, Choi CH (2000) Multicriteria fuzzy decision-making problems based on vague set theory. *Fuzzy Sets Syst* 114(1):103–113
- Wu J, Dai LF, Chiclana F, Fujita H, Herrera-Viedma E (2018) A minimum adjustment cost feedback mechanism based consensus model for group decision making under social network with distributed linguistic trust. *Inform Fusion* 41:232–242
- Kannan D, Khodaverdi R, Olfat L, Jafarian A, Diabat A (2013) Integrated fuzzy multi criteria decision making method and multi-objective programming approach for supplier selection and order allocation in a green supply chain. *J Clean Prod* 47:355–367
- Meng FY, Chen XY, Zhang Q (2015) An approach to interval-valued intuitionistic uncertain linguistic multi-attribute group decision making. *Int J Mach Learn Cyb* 6:859–871
- Nayagam VLG, Muralikrish S, Sivaraman G (2011) Multicriteria decision-making method based on interval-valued intuitionistic fuzzy sets. *Expert Syst Appl* 38(3):1464–1467
- Olaf D (2014) The model for decreasing the costs in supply chain with importance of suppliers. *Global J Human-social Sci: Econ* 1:31–41
- Song Y, Wang X (2015) A new similarity measure between intuitionistic fuzzy sets and the positive definiteness of the similarity matrix. *Pattern Anal Appl.* <https://doi.org/10.1007/s10044-015-0490-2>
- Wei GW, Wang HJ, Lin R (2011) Application of correlation coefficient to interval-valued intuitionistic fuzzy multiple attribute decision-making with incomplete weight information fuzzy multiattribute group decision making method based on entropy and similarity measure. *Knowl Inf Syst* 26(2):337–349
- Xu ZS (2007) Intuitionistic fuzzy aggregation operators. *IEEE Trans Fuzzy Syst* 15(6):1179–1187
- Xu ZS (2007) Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making. *Control Decis* 22(2):215–219
- Xu ZS, Cai XQ (2010) Recent advances in intuitionistic fuzzy information aggregation. *Fuzzy Optim Decis Making* 9(4):359–381
- Xu ZS, Yager RR (2006) Some geometric aggregation operators based on intuitionistic fuzzy sets. *Int J Gen Syst* 35(4):417–433
- Ye J (2009) Multi-criteria fuzzy decision-making method based on a novel accuracy function under interval-valued intuitionistic fuzzy environment. *Expert Syst Appl* 36(3):6899–6902
- Ye J (2013) Multiple attribute group decision-making methods with completely unknown weights in intuitionistic fuzzy setting and interval-valued intuitionistic fuzzy setting. *Group Decis Negot* 22(2):173–188
- Ye J (2017) Intuitionistic fuzzy hybrid arithmetic and geometric aggregation operators for the decision-making of mechanical design schemes. *Appl Intell* 47:743–751

23. Zhang FW, Ge YE, Garg H, Luo LH (2017) Commentary on A new generalized improved score function of interval-valued intuitionistic fuzzy sets and applications in expert systems". *Appl Soft Comput* 52:48–52
24. Zhang FW, Xu SH (2017) Remarks to Fuzzy multicriteria decision making method based on the improved accuracy function for interval-valued intuitionistic fuzzy sets". *Soft Comput* 21(9):2263–2268
25. Zhang FW, Xu SH (2016) Multiple attribute group decision making method based on utility theory under interval-valued intuitionistic fuzzy environment. *Group Decis Negot* 25(6):1261–1275
26. Zhao SP, Liang CY, Zhang JL (2017) Some intuitionistic trapezoidal fuzzy aggregation operators based on Einstein operations and their application in multiple attribute group decision making. *Int J Mach Learn & Cyber* 8(2):547–569

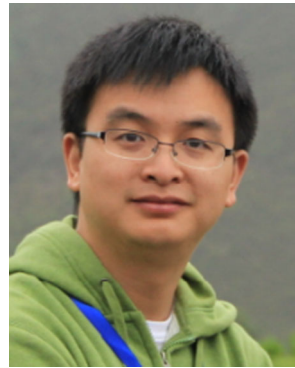


Fangwei Zhang is an Associate Professor of College of Transportation and Communications, Shanghai Maritime University, and visiting scholar at National University of Singapore, Singapore. He has written one monograph and published extensively on issues relating to artificial intelligence, intelligent decision making and management and played a lead role, or acted as an Expert Advisor, in a number of research projects.

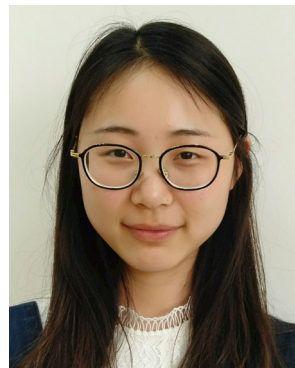


Jihong Chen is a Professor and Department Chair of Waterway Transportation, College of Transportation and Communications, Shanghai Maritime University, Maritime expert member at the Science and Technology Committee of Shanghai Municipal Transport Commission, Shanghai, China, Director of the Chinese Association of Productivity Science, and visiting scholar at Rutgers University, USA. He holds the degree of Bachelor, Master and Ph.D of engineering. He has written one monograph, three works of joint authorship

and co-edited one textbooks. He has published more than 120 papers on the topic relating to shipping and transportation and played a lead role, or acted as an Expert Advisor, in a number of projects. He has directed and involved in over 70 research projects with an approval of state, province or ministry, other businesses and enterprises, among which, over 20 won provincial and other academy awards.



Yuhua Zhu is a lecturer of College of Transport and Communications, Shanghai Maritime University. His current research interests include the shipping finance and port management. He has published more than 10 papers in various journals.



Ziyi Zhuang received the B.S. degree in college of Communication Engineering, Dalian Jiaotong University, China, in 2016, and currently is studying for M.S. degree in college of Information Engineering, in Shanghai Maritime University, China. She has published one paper in Symmetry as co-author, one utility model patent authorized, and won the first prize in Liaoning Province in the Freescale Intelligent Car competition. Her current research interests

include Traffic Engineering, Underwater Acoustical Networks, Underwater Communications and Networking, etc.



Jiaru Li is a student in College of Transport and Communications, Shanghai Maritime University, China. She was sent to Central South University as an exam-exempted postgraduate. She won the first prize in a national logistics management competition in 2017, and her major work was to optimize the storage and transportation of a hazardous chemicals E-commerce. Jiaru Li has published around 5 academic papers during the college. Her research interests include: Logistics Management, Traffic Engineering, Supply Chain, etc.