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Connection number of set pair analysis based TOPSIS method on intuitionistic fuzzy sets and their application to decision making

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Abstract Intuitionistic fuzzy set plays a significant role to handle the uncertainties in the data during the decision-making process. Keeping the advantage of it in mind, an attempt has been made in the present article for rating the different preferences of the object based on the set pair analysis (SPA). For this, a major component of SPA, known as a connection number, has been constructed based on the preference values and the comprehensive ideal values of the object. An extension of TOPSIS method is further developed, based on the proposed connection number of SPA, to calculate relative-closeness of sets of alternatives which are used to generate the ranking order of the alternatives. A real example is taken to demonstrate the applicability and validity of the proposed methodology.

Keywords Set pair analysis · Connection number · Intuitionistic fuzzy set · TOPSIS · Decision-making problems

1 Introduction

Multi-attribute decision-making (MADM) is one of the most significant and omnipresent real life activity to choose a suitable alternative from those that are needed for realizing a certain goal. Traditionally, it has been assumed that the information regarding accessing the alternatives is taken in the form of real numbers. But in day-to-day life, it is difficult for a decision maker to give his assessments towards the object in crisp values due to ambiguity and incomplete information. Instead, it has become popular that these assessments are presented by a fuzzy set or extensions of the fuzzy set. Fuzzy set (FS) [34], proposed by Zadeh, is a powerful tool to deal with vagueness and has received much attention. After that, researchers have engaged in its extensions such as an intuitionistic fuzzy set (IFS) [2], interval-valued IFS (IVIFS) [1] by adding a degree of non-membership into the analysis. Under these environments, various researchers paid more attention to aggregate the rating values of different alternatives using different aggregation operators. For instance, Xu and Yager [32] presented geometric aggregation operators while Xu [31] presented weighted averaging operators for aggregating different intuitionistic fuzzy numbers (IFNs). Later on, Wang and Liu [28] extended these operators by using Einstein norm operations under IFS environment. Garg [7] presented a generalized intuitionistic fuzzy interactive geometric interaction operator, using Einstein norm operations, for aggregating different intuitionistic fuzzy information. Garg [13], further proposed some series of interactive aggregation operators for IFNs. Hung and Chen [18] presented a fuzzy Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method with the entropy weight to solve the decision-making problems under the intuitionistic fuzzy environment. Garg [8] presented a generalized intuitionistic fuzzy aggregation operator under the intuitionistic multiplicative preference relation instead of intuitionistic

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fuzzy preference relation. Sivaraman et al. [26] presented a score function for ranking the interval-valued intuitionistic fuzzy numbers (IVIFNs). Garg [10] presented generalized improved score function to rank the different IVIFNs. Garg [11, 14] extended the theory of the IFS to the Pythagorean fuzzy set and hence presented their generalized geometric as well as averaging aggregation operators. Apart from that, recently, many authors [4, 9, 10, 15, 16, 21–23] have shown the growing interest in the study of the decision-making problems under different environments by using these above theories.

The above studies have been widely used by the researchers, but credibility is not guaranteed. In order to handle the uncertainties in a more precise way, Zhao [35] introduced the set pair analysis (SPA) theory in which certainty and uncertainty studied as one system. Jiang et al. [20] discussed the basic concept of SPA theory. The main principle of SPA is to analyze the features of set pair and construct a connection number (CN) for them. Wang and Gong [27] proposed a decision-making method based on the set-pair analysis to solve the MADM problems with ascertained criteria weight and criteria value being an interval random variable. Hu and Yang [17] proposed a dynamic stochastic MADM based on cumulative prospect theory and SPA. Xie et al. [30] presented a CN under interval-valued fuzzy set by taking the positive and negative ideal scheme. Kumar and Garg [22] presented a TOPSIS method under the IVIFS environment based on the connection number of the SPA. Apart from these, some researchers [5, 6, 24, 33] also solved the fuzzy decision-making problem under the SPA.

It is evident from the above-mentioned literature survey that authors have conducted the SPA under the fuzzy environment only. But in day-to-day life, it is difficult for decision-makers to give the preference towards the object in terms of the only favorable membership function. In contrast to this, the decision-maker has usually preferred to give their rating value toward the alternatives in terms of favorable degrees as well as the rejection simultaneously. As far as we know, the study of the SPA, CN and the MADM problem, based on the intuitionistic fuzzy, has not been reported yet in the existing academic literature. Therefore, it is a growing research topic to apply these in MADM to rank and obtain the best alternative under IFS environment. Meanwhile, we also provide a TOPSIS method based on the connection degree of the SPA, whose aim is to achieve the optimal solution.

To do so, the rest of the manuscript is summarized as follows. Section 2 gives some overviews on IFS and SPA theories. In Section 3, a TOPSIS method for MADM has been presented under the SPA in which the assessments related to the attributes are taken in the form of intuitionistic fuzzy numbers. An example to illustrate the approach has been described in Section 4 and the results have been compared with the existing methods. Finally, Section 5 concludes the paper.

2 Preliminaries

In this section, some basic concepts about the IFSs and SPA are defined.

2.1 Intuitionistic fuzzy set (IFS)

Definition 1 Let X be a non-empty reference set, an IFS [2] A in X is defined as $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$,where $\mu_A(x)$ and $\nu_A(x)$ are all subsets of [0, 1], and represent the membership and the non-membership degrees of x to A. For any $x \in X$, $\mu_A(x) + \nu_A(x) \leq 1$, and in turn, the intuitionistic index of x to A is defined as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, the complementary set A^c of A is defined as $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle \mid x \in X\}$. Usually, the pair $\langle \mu_A(x), \nu_A(x)\rangle$ is called an intuitionistic fuzzy number (shortened by IFN), and it is often simplified as $\alpha = \langle u, v\rangle$ where $u \in [0, 1]$, $v \in [0, 1]$, $u + v \leq 1$. The score value corresponding to IFN α is defined as $sc(\alpha) = u - v$.

Definition 2 To rank the different IFNs, a score S and an accuracy H functions[32] can be represented as $S(\alpha) = u_\alpha - v_\alpha$ and $H(\alpha) = u_\alpha + v_\alpha$ for an IFN $\alpha = \langle u_\alpha, v_\alpha\rangle$. Thus, based on these functions, an order relation between two IFNs $\alpha = \langle u_\alpha, v_\alpha\rangle$ and $\beta = \langle u_\beta, v_\beta\rangle$ is stated as, if $S(\alpha) \geq S(\beta)$ then $\alpha \geq \beta$ and if $S(\alpha) = S(\beta)$ then compute their accuracy functions. If $H(\alpha) \geq H(\beta)$ then $\alpha \geq \beta$ and if $H(\alpha) = H(\beta)$ then α and β represent the same information, denoted by $\alpha = \beta$.

2.2 Set pair analysis (SPA)

Definition 3 Set pair analysis (SPA) is the modern uncertainty theory developed by Zhao [35], which overlaps the other uncertainty theories such as the probability, vague, rough and fuzzy. It provides a different way to express the uncertainty in which certainty and uncertainty treat as integrated certain- uncertain system of an object. A set pair $H(A, B)$ consists of two interdependent sets A and B under the problem W. The most important feature of the SPA is to analyze the system on "identical", "discrepancy" and "contrary" features of given problem W and set up a connection number for them. In it, if out of the total number of features (N), the identity and contrary features are denoted by S and P respectively such that $F = N - S - P$ is neither identity nor contrary of the sets A and B then the connection number (μ) is represented as

$$\mu = a + bi + cj$$

where $a = S/N, b = F/N$ and $c = P/N$ represents the "identity", "discrepancy" and "contrary" degrees such that $0 < a, b, c \leq 1$ and $a + b + c = 1$; $i \in [-1, 1]$ and $j = -1$ are the coefficients of "discrepancy and contrary" degrees, respectively.

Definition 4 Let $\mu_1 = a_1 + b_1 i + c_1 j$ and $\mu_2 = a_2 + b_2 i + c_2 j$ be any two CNs, then

(i) $\mu_1 = \mu_2 \Leftrightarrow a_1 = a_2, b_1 = b_2, c_1 = c_2$
(ii) $\mu_1 \leq \mu_2 \Leftrightarrow a_1 \leq a_2, b_1 \geq b_2$

Many researchers have been researching various approaches with more advantages under certain limitations to MADM. Analysis of information corresponding to the decision maker is always occurring with random, fuzzy or variant uncertainty. There are various theories to handle the uncertainties, but they do not define uncertainty exactly. SPA handles the certainty and uncertainty with quantitative analysis based on "identity"-"discrepancy"-"contrary" degree of CN. It has analytic characteristic and simple mathematical representation with bright physical significance.

3 Connection number based TOPSIS approach

Hwang and Yoon [19] introduced the TOPSIS method to find out the best alternative based on the shortest distance from an ideal solution. In this section, TOPSIS method has been presented under IFS environment by using SPA and its connection number.

Suppose there are 'm' alternatives $A_1, A_2, \ldots, A_m$ and 'n' attributes $G_1, G_2, \ldots, G_n$ whose normalized weight vector is $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$. Decision makers have evaluated these alternatives under each attribute and given their preferences in terms of IFNs $\alpha_{kt} = \langle \widetilde{u}_{kt}, \widetilde{v}_{kt} \rangle$, and hence formulated an intuitionistic fuzzy decision matrix $D = (\langle \widetilde{u}_{kt}, \widetilde{v}_{kt} \rangle)_{m \times n}$ where $\widetilde{u}_{kt} \in (0, 1]$ and $\widetilde{v}_{kt} \in (0, 1]$ such that $\widetilde{u}_{kt} + \widetilde{v}_{kt} \leq 1$. In order to balance the physical dimensions of these ratings, this matrix D is converted into the normalized matrix $R = (r_{kt})_{m \times n}$ where $r_{kt} = \langle u_{kt}, v_{kt} \rangle$ has been obtained as follows.

$$r_{kt} = \begin{cases} \alpha_{kt} \ ; & \text{for benefit type criteria} \\ \alpha_{kt}^c \ ; & \text{for cost type criteria} \end{cases} \tag{1}$$

where $\alpha_{kt}^c = \langle \widetilde{v}_{kt}, \widetilde{u}_{kt} \rangle$ is the complement of IFN $\alpha_{kt} = \langle \widetilde{u}_{kt}, \widetilde{v}_{kt} \rangle$. Based on the matrix R, the positive ideal scheme (PIS) and negative ideal scheme (NIS) of the alternatives are computed and denoted by $A^+ = \langle u_t^+, v_t^+ \rangle = \langle \max_k u_{kt}, \min_k v_{kt} \rangle$ and $A^- = \langle u_t^-, v_t^- \rangle = \langle \min_k u_{kt}, \max_k v_{kt} \rangle$ respectively, for all $t = 1, 2, \ldots, n$. Thus, corresponding to it, connection number of the set pairs $H(r_{kt}, A^+)$ and $H(r_{kt}, A^-)$, denoted by μ_{kt}^+ and μ_{kt}^- respectively, is defined as

$$\mu_{kt}^+ = a_{kt}^+ + c_{kt}^+ j \tag{2}$$

and

$$\mu_{kt}^- = a_{kt}^- + c_{kt}^- j \tag{3}$$

where $a_{kt}^+ = \left(\frac{u_{kt}}{u_t^+} \times \frac{v_t^+}{v_{kt}} \right)$ and $a_{kt}^- = \left(\frac{u_t^-}{u_{kt}} \times \frac{v_{kt}}{v_t^-} \right)$ are identity degrees with proximity to PIS and NIS respectively, while $c_{kt}^+ = \left(\frac{u_t^+ - u_{kt}}{u_t^+} \times \frac{v_{kt} - v_t^+}{v_{kt}} \right)$ and $c_{kt}^- = \left(\frac{u_{kt} - u_t^-}{u_{kt}} \times \frac{v_t^- - v_{kt}}{v_t^-} \right)$ are contrary degrees which is remote from PIS and NIS respectively.

Now, the connection number for making an overall decision to select the best alternative from the given alternatives proximity to PIS and remote from NIS is defined as

$$\mu_{kt} = a_{kt} + c_{kt} j \tag{4}$$

where $a_{kt} = a_{kt}^+ \times c_{kt}^-$ represents the "identity degree" proximity to PIS or remote from NIS while $c_{kt} = c_{kt}^+ \times a_{kt}^-$ represents the "contrary degree" which is remote from PIS and proximity to NIS.

The connection number of set pair composed of each alternative under the set of attribute weights $\omega_t, t = 1, 2, \ldots, n$ is defined as

$$\mu_{A_k} = a_k + c_k j \tag{5}$$

where $a_k = \sum_{t=1}^{n} \omega_t a_{kt}$ represents the overall "identity degree" between the alternative A_k and PIS, while $c_k = \sum_{t=1}^{n} \omega_t c_{kt}$ represents the overall "contrary degree" between the alternative A_k and NIS. Hence, the relative closeness degree of an alternative A_k is defined as:

$$T(\mu_{A_k}) = \frac{a_k}{a_k + c_k} \tag{6}$$

In a nutshell, after combination of all the above demonstrations, our proposed decision-making method on the basis of the connection number of SPA has been summarized as follows.

Step 1: Normalize the decision matrix, if needed, by using (1) for each alternative.
Step 2: Determine the PIS and NIS of the alternative.
Step 3: Utilize (4) to determine the CN of each alternative proximity to PIS and remote from NIS.
Step 4: Determine the relative weighted CN by using (5).
Step 5: Rank the alternative based on coefficient degree $T(\mu_{A_k})$ as defined in (6) and hence choose the best one(s).

4 Illustrative example

The above mentioned approach has been illustrated with a practical example of the DM which can be read as:

The Kedarnath valley, along with other parts of the state of Uttarakhand in northern India, was hit with an unprecedented flash floods in 2013. Large number of roads, which connect the Kedarnath valley to the other parts of Uttarakhand, had been destroyed in this flood. In this context, Uttrakhand government had to take a considerable number of road building projects either to maintain the roads already built or to undertake new roads. These projects were carried out by a limited number of well-established contractors, and the selection process was on the basis of bid price alone. In recent years, the use of multi-attribute decision making methods have been demanded for increased project complexity, technical capability, higher performance, safety and financial requirements. For this, Uttarakhand government issued the notice in the newspapers, and considered the six attribute required for contractor selection, namely, tender price (G_1), completion time (G_2), technical capability (G_3), financial status (G_4), contractor background (G_5), reference from previous project (G_6) and assigned the weights of relative importance of each attributes as $\omega = (0.3, 0.2, 0.15, 0.1, 0.15, 0.1)^T$ on the basis of decision maker's preferences. The four contractors taken as in the form of the alternatives, namely, Jaihind Road Builders Pvt. Ltd. (A_1), J.K. Construction (A_2), Buildquick Infrastructure Pvt. Ltd. (A_3), Relcon Intraprojects Ltd. (A_4) bid for these projects. Then, the objective of the Government is to choose the best contractor among them for the task. In order to fulfill it, they evaluated these and gave their preferences in term of intuitionistic fuzzy numbers which are summarized in Table 1.

4.1 By proposed approach

The main procedure steps for obtaining the best alternative(s) by utilizing the developed approach are summarized as follows:

Step 1: Since G_1 and G_2 are the cost types attributes, so by using (1) we get the normalized decision-matrix summarized in Table 2.

Step 2: Based on this normalized data, PIS and NIS of the alternative are evaluated and are given as

$$A^+ = \{\langle 0.7, 0.3\rangle, \langle 0.4, 0.4\rangle, \langle 0.8, 0.2\rangle, \langle 0.5, 0.2\rangle, \langle 0.8, 0.1\rangle \langle 0.7, 0.2\rangle\}$$
$$A^- = \{\langle 0.2, 0.6\rangle, \langle 0.1, 0.9\rangle, \langle 0.5, 0.4\rangle, \langle 0.5, 0.2\rangle, \langle 0.4, 0.4\rangle, \langle 0.2, 0.8\rangle\}$$

Step 3: CNs of each alternative, under the set of each criterion, are computed by using (4) and their corresponding values are summarized in Table 3.

Step 4: Corresponding to weight vector ω, the relative weighted CN of each alternative is obtained by using (5) as $\mu_{A_1} = 0.2591 + 0.0234j$; $\mu_{A_2} = 0.0860 + 0.1009j$, $\mu_{A_3} = 0.1205 + 0.1048j$ and $\mu_{A_4} = 0.0837 + 0.2249j$.

Step 5: By utilizing (6), we get the overall performance value of each alternative A_k as $T(\mu_{A_1}) = 0.9171$, $T(\mu_{A_2}) = 0.4600$, $T(\mu_{A_3}) = 0.5350$ and $T(\mu_{A_4}) = 0.2712$ and hence the best alternative is A_1.

4.2 Validity test of the proposed approach

Since, practically it is not possible to determine which one is the best suitable alternative for a given decision-making problem, therefore Wang and Triantaphyllou [29] established the following test criteria to evaluate the validity of MADM methods.

Test criterion 1: An effective MADM method should not change the indication of the best alternative on replacing a non-optimal alternative by another worse alternative without changing the relative importance of each decision criteria.

Test criterion 2: An effective MADM method should follow transitive property.

Test criterion 3: When a MADM problem is decomposed into smaller problems and same MADM method is applied on smaller problems to rank the alternatives, combined ranking of the alternatives should be identical

Table 1 Decision matrix of the alternatives in the form of Intuitionistic fuzzy number

	G_1	G_2	G_3	G_4	G_5	G_6
A_1	$\langle 0.3, 0.7\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.5, 0.2\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.6, 0.4\rangle$
A_2	$\langle 0.5, 0.3\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.7, 0.2\rangle$
A_3	$\langle 0.5, 0.4\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.6, 0.2\rangle$
A_4	$\langle 0.6, 0.2\rangle$	$\langle 0.4, 0.3\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.4, 0.4\rangle$	$\langle 0.2, 0.8\rangle$

Table 2 Normalized decision matrix

	G_1	G_2	G_3	G_4	G_5	G_6
A_1	$\langle 0.7, 0.3\rangle$	$\langle 0.4, 0.5\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.5, 0.2\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.6, 0.4\rangle$
A_2	$\langle 0.3, 0.5\rangle$	$\langle 0.2, 0.8\rangle$	$\langle 0.5, 0.4\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.7, 0.2\rangle$
A_3	$\langle 0.4, 0.5\rangle$	$\langle 0.1, 0.9\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.6, 0.2\rangle$
A_4	$\langle 0.2, 0.6\rangle$	$\langle 0.3, 0.4\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.4, 0.4\rangle$	$\langle 0.2, 0.8\rangle$

to the original ranking of un-decomposed problem.

The validity of the proposed aggregation operators, based MADM method, is tested using these test criteria.

4.2.1 Validity test of the proposed approach using test criterion 1

In order to test the validity of the proposed approach under test criterion 1, the following decision matrix is obtained by interchanging the degree of the membership and non-membership grades of alternative A_2 (non-optimal alternative) and A_4 (worse alternative) in the original decision matrix, then the original decision matrix is transferred to

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \left(\begin{array}{ccc} G_1 & G_2 & G_3 \\ \langle 0.3, 0.7\rangle & \langle 0.5, 0.4\rangle & \langle 0.8, 0.2\rangle \\ \langle 0.3, 0.5\rangle & \langle 0.2, 0.8\rangle & \langle 0.4, 0.5\rangle \\ \langle 0.5, 0.4\rangle & \langle 0.9, 0.1\rangle & \langle 0.8, 0.2\rangle \\ \langle 0.2, 0.6\rangle & \langle 0.3, 0.4\rangle & \langle 0.2, 0.8\rangle \\ G_4 & G_5 & G_6 \\ \langle 0.5, 0.2\rangle & \langle 0.8, 0.1\rangle & \langle 0.6, 0.4\rangle \\ \langle 0.1, 0.9\rangle & \langle 0.3, 0.6\rangle & \langle 0.2, 0.7\rangle \\ \langle 0.9, 0.1\rangle & \langle 0.8, 0.2\rangle & \langle 0.6, 0.2\rangle \\ \langle 0.2, 0.7\rangle & \langle 0.4, 0.4\rangle & \langle 0.8, 0.2\rangle \end{array} \right)$$

Since the relative importance of the criteria remained unchanged in modified problem then the proposed TOPSIS method has been implemented to find out the best alternative and hence the weighted CN of each alternative is obtained as $\mu_{A_1} = 0.2381 + 0.0117j$, $\mu_{A_2} = 0.1532 + 0.2385j$, $\mu_{A_3} = 0.2304 + 0.2133$ and $\mu_{A_4} = 0.1402 + 0.1412j$. Thus, the overall performance value of each alternative is computed by using (6), as $T(A_1) = 0.9532$, $T(A_2) = 0.3911$, $T(A_3) = 0.5193$ and $T(A_4) = 0.4983$ respectively. According to the descending order of these values, the alternatives are ranked as $A_1 \succ A_3 \succ A_4 \succ A_2$. Since the indication of the best alternative is again A_1 which is same as that of the original decision-making problem, therefore it is confirmed that the proposed method does not change the indication of the best alternative when a non-optimal alternative is replaced by another worst alternative. Hence the proposed TOPSIS method is valid under *test criterion 1* established by Wang and Triantaphyllou [29].

4.2.2 Validity test of the proposed approach using test criterion 2 and test criterion 3

In order to test validity of proposed method using test criterion 2 and test criterion 3, original decision-making problem is decomposed into a set of smaller MADM problems $\{A_1, A_2, A_4\}$, $\{A_1, A_3, A_4\}$ and $\{A_2, A_3, A_4\}$. By following the steps of proposed method, ranking orders of these subproblems are obtained as $A_1 \succ A_2 \succ A_4$, $A_1 \succ A_3 \succ A_4$ and $A_3 \succ A_2 \succ A_4$ respectively. Now, if ranking orders of the alternatives of these sub-problems are combined together, then we get the final ranking order is $A_1 \succ A_3 \succ A_2 \succ A_4$ which is identical to the ranking of un-decomposed MADM problem and exhibits transitive property. Hence the proposed method is valid under the *test criterion 2 and test criterion 3* established by Wang and Triantaphyllou [29].

4.3 Comparative study

In order to compare the performance of the proposed approach with some existing approaches under the IFS environment, we conducted a comparative analysis based on different approaches as given by the authors in [3, 10–12,

Table 3 Connection number of each alternative

	G_1	G_2	G_3	G_4	G_5	G_6
A_1	$0.3571 + 0.0000j$	$0.2667 + 0.0000j$	$0.1875 + 0.0000j$	$0.0000 + 0.2222j$	$0.3750 + 0.0000j$	$0.1429 + 0.0119j$
A_2	$0.0143 + 0.1270j$	$0.0139 + 0.1111j$	$0.0000 + 0.1875j$	$0.2222 + 0.0000j$	$0.0208 + 0.0833j$	$0.5357 + 0.0000j$
A_3	$0.0286 + 0.0714j$	$0.0000 + 0.4167j$	$0.1875 + 0.0000j$	$0.2222 + 0.0000j$	$0.1250 + 0.0000j$	$0.4286 + 0.0000j$
A_4	$0.0000 + 0.3571j$	$0.2778 + 0.0000j$	$0.1875 + 0.0000j$	$0.0000 + 0.0794j$	$0.0000 + 0.3750j$	$0.0000 + 0.5357j$

21, 25, 26, 31, 32]. The results corresponding to these are summarized in Table 4.

From these comparative studies, it has been concluded that the results, computed by the existing approaches, coincide with the proposed one which validate the proposed approach. Therefore, the proposed technique can be suitably utilized to solve the decision-making problem better than the other existing measures.

4.4 Superiority of the proposed approach

In this section, we present some counter examples which show that the existing TOPSIS methods under the IFS environment fail to rank the given alternatives while the proposed approach can overcome their shortcoming.

Example 1 Consider a decision-making problem in which there are two alternatives denoted by A_1 and A_2 which are evaluated by an expert under the set of three different attributes denoted by G_1, G_2 and G_3. The objective of the problem is to find out the best alternative under the given set. In order to do so, an expert evaluated these alternatives and gave their preferences in terms of intuitionistic fuzzy numbers which are summarized as follows:

$$D = \begin{array}{c c} & \begin{array}{ccc} G_1 & G_2 & G_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} \langle 0.5, 0.2\rangle & \langle 0.6, 0.3\rangle & \langle 0.3, 0.2\rangle \\ \langle 0.4, 0.3\rangle & \langle 0.6, 0.2\rangle & \langle 0.4, 0.3\rangle \end{bmatrix} \end{array} \tag{7}$$

Based on this decision-matrix, by utilizing the existing TOPSIS approach [4] to find out the best alternative, the following steps are to be executed as:

(Step 1:) The information related to the alternatives is represented in the form of the decision matrix D as given in (7).

(Step 2:) The positive and negative ideal solutions of these two alternatives are found as $A^+ = \{\langle 0.5, 0.2\rangle, \langle 0.6, 0.2\rangle, \langle 0.4, 0.2\rangle\}$ and $A^- = \{\langle 0.4, 0.3\rangle, \langle 0.6, 0.3\rangle, \langle 0.3, 0.3\rangle\}$ respectively.

(Step 3:) Based on these values, the distance measure values between the alternatives $A_i (i = 1, 2)$ from its ideals values are computed as $d(A_1, A^+) = 0.0667$, $d(A_2, A^+) = 0.0667$, $d(A_1, A^-) = 0.0667$ and $d(A_2, A^-) = 0.0667$.

(Step 4:) The relative closeness coefficient $C(\cdot)$ of each alternative is $C(A_1) = \frac{d(A_1,A^-)}{d(A_1,A^-)+d(A_1,A^+)} = 0.5$ and $C(A_2) = \frac{d(A_2,A^-)}{d(A_2,A^-)+d(A_2,A^+)} = 0.5$. Since, $C(A_1) = C(A_2)$ and hence we conclude that the existing TOPSIS approach is unable to rank the given alternatives.

On the other hand, if we utilize the proposed approach for above considered data, then we get the relative closeness degrees of each alternative i.e., $T(\mu_{A_1}) = 1$ and $T(\mu_{A_2}) = 0$. Since $T(\mu_{A_1}) > T(\mu_{A_2})$ and hence conclude that the alternative A_1 is better than A_2. Therefore, the proposed approach is suitably working in those cases where the existing TOPSIS method fails.

Example 2 Consider another decision-making problem with two alternatives A_1 and A_2 which are evaluated under the set of the different attributes G_1, G_2 and G_3 whose weight vector is $\omega = (0.24, 0.40, 0.36)^T$. An expert evaluated these alternatives and gave their preferences in terms of intuitionistic fuzzy numbers, which are represented in the form of decision-matrix $D = (d_{kt})_{2\times 3}$, as follows:

$$D = \begin{array}{c c} & \begin{array}{ccc} G_1 & G_2 & G_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} \langle 0.510, 0.361\rangle & \langle 0.660, 0.250\rangle & \langle 0.530, 0.290\rangle \\ \langle 0.532, 0.400\rangle & \langle 0.360, 0.520\rangle & \langle 0.760, 0.120\rangle \end{bmatrix} \end{array}$$

If we utilize the intuitionistic fuzzy weighted averaging (IFWA) operators [31] to aggregate these alternatives, then the aggregating values of IFNs obtained are $\langle 0.5829, 0.2880\rangle$ and $\langle 0.5829, 0.2880\rangle$ respectively, of the alternatives A_1 and A_2. Thus, we get the same values for both the alternatives and hence decision-makers will be unable to choose the best one for their decision.

Table 4 Comparative analysis

	Overall values of the alternatives				Ranking order
Xu [31]	0.4074	0.1425	0.2925	0.0435	$A_1 \succ A_3 \succ A_2 \succ A_4$
Xu and Yager [32]	0.3122	−0.0973	−0.1147	−0.1527	$A_1 \succ A_2 \succ A_3 \succ A_4$
Joshi and Kumar [21]	0.7886	0.4423	0.5637	0.3793	$A_1 \succ A_3 \succ A_2 \succ A_4$
Garg [10]	0.6647	0.4419	0.4393	0.3948	$A_1 \succ A_2 \succ A_4 \succ A_3$
Sahin [25]	0.6647	0.4419	0.4393	0.3948	$A_1 \succ A_2 \succ A_4 \succ A_3$
Bai [3]	0.6561	0.4415	0.4931	0.4042	$A_1 \succ A_3 \succ A_2 \succ A_4$
Sivaraman et al. [26]	0.5247	0.3020	0.3085	0.2538	$A_1 \succ A_3 \succ A_2 \succ A_4$
Garg [12]	−0.2260	−0.4172	−0.2389	−0.5215	$A_1 \succ A_3 \succ A_2 \succ A_4$
Garg [11]	0.2907	−0.0928	−0.1105	−0.1309	$A_1 \succ A_2 \succ A_3 \succ A_4$

On the other hand, if we apply the proposed decision-making method on this data, then the relative closeness degrees corresponding to alternative A_1 and A_2 are $T(\mu_{A_1}) = 0.5965$ and $T(\mu_{A_2}) = 0.4035$ respectively. Hence, it is concluded that the alternative A_1 is better than that of alternative A_2.

4.5 Advantages of proposed method

According to the above comparative analysis to address the decision-making problems, the proposed approach has the following advantages:

(a) The IFS is characterized by the degrees of the membership and non-membership of an element such that their sum is less than 1. However, there may be situations in which IFS theory is unable to provide the whole information about the situation. On the other hand, SPA theory provides with an alternate way to deal with the certainty and uncertainty with quantitative analysis of "identity", "discrepancy" and "contrary" degree of the connection number such that the sum of their degree is equal to one. Therefore, SPA theory is more suitable for real scientific and engineering applications.

(b) The proposed approach represents the intuitionistic fuzzy information using connection degrees, which can be simultaneously described the degrees of membership, non-membership and hesitation degree with a simple mathematical depiction. Based on it, we can compute the connection degree without any transformation and hence it can effectively avoid the loss of information.

(c) The results obtained by the proposed methods might be more accurate as it takes the hesitation degree into account. The proposed method is more generalized and suitable to solve the real-life problem more accurately than the existing ones.

5 Conclusion

In the present paper, the connection number based TOPSIS method to solve the decision-making problem has been proposed where the preferences of the attribute values are represented in the form of IFNs. In this approach, by using a collective information data and the ideal scheme alternative, a connection number corresponding to each alternative under attribute is constructed which is proximate to PIS and remote from NIS. Based on these CNs, a weighted connection number of each alternative scheme is determined and hence the relative closeness degree of each alternative in the TOPSIS approach is defined to rank the alternatives. The approach has been validated through a case study and comparison with the other existing techniques. From the results and their corresponding comparative studies, it has been observed that the proposed TOPSIS approach, based on SPA, can solve the DM problem with better efficiency and is more suitable in handling the real situations based on IFS. In the future, we shall extend this approach to other domains.

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