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Design of an intelligent prediction-based neural network controller for multi-scroll chaotic systems

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Abstract An indispensable part of the precise control of multi-scroll chaotic systems, model identification has received increasing attention in recent years. Because of plant uncertainty and unmodeled dynamics, conventional control methods cannot guarantee a sufficiently high-performance for stabilizing multi-scroll chaotic systems. In an effort to tackle the matter better, we propose an intelligent controller called the adaptive neural network prediction-based controller (NN-PbC). The specified neural network is trained with the system model, which is extracted from a time series. In actual practice, the data are divided into two sets. One set is used for training and the other set for testing. In fact, a generalized NN will perform well for both training and testing data. The prediction-based control method is then applied to the obtained neural network model to stabilize the multiple equilibrium points. The stability of the closed-loop system is proven. In addition, simulation examples on two typical multi-scroll chaotic systems are presented to demonstrate the effectiveness of the proposed controller.

Keywords Adaptive neural network · Multi-scroll chaotic systems · Prediction-based control · System identification

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1 Introduction

Chaotic systems, which possess a dense set of unstable periodic orbits, have been widely studied and applied in many real-world applications and laboratory experiments in areas such as electronic circuits, power system protection, secure communication, and smart grids [1–4]. Furthermore, the circuit design and implementation of multi-scroll chaotic systems has been a subject of increasing interest because of their potential applications in various chaos-based technologies and information systems [5–17]. Accordingly, a good number of researchers and practitioners globally have paid close attention to the control of multi-scroll chaotic systems. Recent investigations have considered the control problem of multi-scroll chaotic systems [18–21]. However, few of them have considered the modeling task using neural network (NN) systems. In fact, the main challenges of controlling multi-scroll chaotic systems are sensitivity to initial conditions, unpredictability, and the fact that the mathematical models for almost real-world multi-scroll chaotic systems are not available.

NNs have received much attention because they are able to approximate complex nonlinear functions and learn well [22, 23]. A number of studies have considered NNs for modeling and controlling chaotic systems [24–32]. Currently, the usual methods of modeling and controlling chaotic systems may be categorized into offline and online methods, where the goal is to enable prediction and decision-making. Offline methods explore stationary data. However, the online methods mainly depend on intelligent algorithms and optimization theories to estimate the parameters of the NN. Some examples are the Levenberg-Marquardt, descent gradient, and other intelligent algorithms. Poznyak

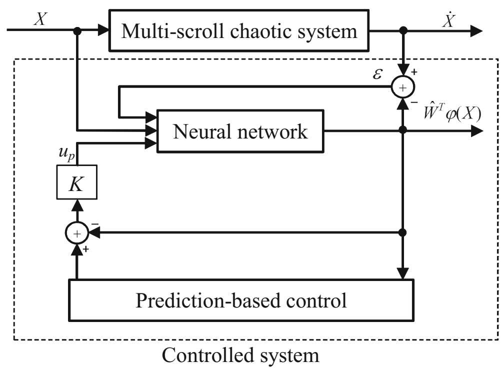


Fig. 1 Block diagram of the NN prediction-based control scheme

et al. developed a sliding mode technique to learn the weights of dynamic NNs [26]. A recurrent high-order NN was developed by Lu et al. for both identifying and controlling unknown chaotic systems, in which the feedback linearization technique is used in an adaptive manner [28]. Qin et al. [32] introduced a control scheme based on the back-propagation NN. The scheme can control the chaotic response to a prospective external signal, which can be a periodic, nonlinear, or even non-analytical discontinuous function. However, the use of NNs has mostly focused on classical chaotic systems, and very few results have been obtained for the NN modeling of multi-scroll chaotic systems. Much of the literature on this problem has addressed the automatic classification of multi-scroll chaotic systems using discrete wavelet NNs and the modeling process using bond-graph theory [33, 34]. In fact, developing models for multi-scroll chaotic systems is needed for accurate prediction, control, diagnostics, and design purposes in real-world applications, where the system models are subject to parameter uncertainties. In contrast to conventional chaos control methods, the modern intelligent algorithms using NNs and fuzzy logic are able to promptly find solutions to chaos control problems in real-world applications.

Recent contributions to the prediction-based control of chaotic systems may be divided into two main categories: 1) improving prediction-based control to stabilize continuous-time chaotic systems [35] and 2) combining prediction-based methods with intelligent algorithms such as NN and fuzzy logic to stabilize a wide range of chaotic systems subject to plant uncertainty and unmodeled dynamics [36, 37]. Although many techniques are devoted to the control of chaotic systems, there is, to the best of our knowledge, no contribution that concerns intelligent chaos control for stabilizing multi-scroll chaotic systems subject to plant uncertainty and unmodeled dynamics. As a result, we address in this paper the design of an NN for constructing models that incorporate a priori knowledge in the form of differential equations for multi-scroll chaotic systems. Using this framework, the multi-layer perceptron (MLP) feed-forward back-propagation NN is used to model some well-known multi-scroll chaotic systems. The specified NN is trained using the obtained system model extracted from the forecasting time series. The Levenberg-Marquardt and gradient descent algorithms are used for the training and learning processes, respectively. Note that during the training process, the NN tries to match the outputs with the desired target values. Slightly different from the training process, the learning process changes or refines weights and decides how they should be manipulated. Consequently, the Levenberg-Marquardt algorithm is used for the training process because global convergence can be established without requiring the existence of an accumulation point. Furthermore, it is considered to be the fastest back-propagation algorithm. However, it does consume more memory than other methods. Hence, we make use of the descent gradient algorithm in the learning process to address this shortcoming. The prediction-based control method is then applied to the obtained NN model in order to stabilize multiple equilibrium points. The stability of the closed-loop system is guaranteed using the Lyapunov stability theorem. Numerical simulations are provided to confirm the ability of the proposed NN to model and stabilize multi-scroll chaotic systems .

The rest of the paper is organized as follows. Descriptions of the dynamical properties of multi-scroll chaotic systems and the NN structure are presented in Section 2.

Table 1 Performance of the NN modeling of a 3-D n-scroll Chua's circuit

Results	1 hidden layer		2 hidden layers		3 hidden layers	
	60	160	[30 10]	[72 12]	[30 20 10]	[42 12 6]
Time T (sec)	69	155	77	343	259	180
R_test	1	1	0.998	0.999	0.998	0.999
MSE	0.0461	0.0002	0.2010	0.0027	0.2210	0.0645

Table 2 Performance of the NN modeling of a 3-D Chen multi-scroll system

Results	1 hidden layer		2 hidden layers		3 hidden layers	
	160	280	[60 20]	[90 12]	[26 14 6]	[40 20 10]
Time T (sec)	267	362	531	853	144	392
R_test	0.9970	0.999	0.980	0.995	0.902	0.985
MSE	1.050	0.004	4.880	1.970	38	5.79

An improved variant of the prediction-based control method and problem formulation are provided in Section 3. Section 4 details the design of the intelligent prediction-based controller. Section 5 presents an experimental study on two typical multi-scroll chaotic systems. Numerical simulations are presented in Section 6. Finally, the conclusions are drawn in Section 7.

2 System description

2.1 Multi-scroll chaotic systems

A multi-scroll chaotic system is a chaotic system modified so that it generates multiple scrolls. Because its prototype model was first introduced by Suykens et al. in 1993 [5], extensive investigations have been conducted to develop new models of multi-directional multi-scroll chaotic systems.

Multi-scroll chaotic systems can be designed using different approaches such as pulse width modulation functions [5], nonlinear modulating functions [6], step circuits [7], positive-type second generation current conveyors, negative-type second generation current conveyors [8], and field-programmable gate array circuits [17]. In particular, the design of multi-scroll chaotic systems via nonlinear modulating functions has attracted much interest because of their simple circuit implementations. One of the main results of this design approach is the use of the sine function.

Consider the multi-scroll chaotic system expressed as

$$\dot{X}(t) = f(t, X(t)), \tag{1}$$

where $X = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ is the state vector and $f(t, X(t))$ is a continuous function.

Two typical examples of multi-scroll chaotic systems that are based on nonlinear modulating functions are shown below.

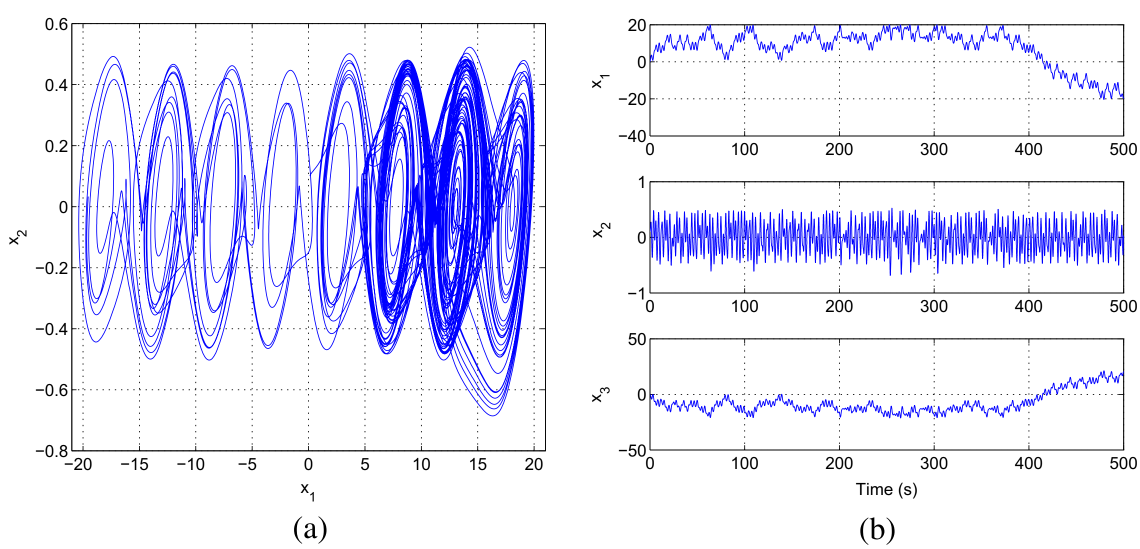


Fig. 2 Results of NN modeling of an n-scroll Chua's circuit. **a** Phase plane. **b** Time responses

Example 1 Consider the n-scroll Chua's circuit, given by the following equations [6]:

$$\begin{cases} \dot{x}_1 = \alpha(x_2 - h(x_1)) \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -\beta x_2 \end{cases} \quad (2)$$

where x_1, x_2, and x_3 are the state variables, α and β are the system parameters, and $h(x_1)$ is a nonlinear function such that:

$$h(x_1) = \begin{cases} \frac{b\pi}{2a}(x_1 - 2ac) & \text{if } x_1 \geq 2ac \\ -b\sin\left(\frac{\pi x_1}{2a} + d\right) & \text{if } -2ac < x_1 < 2ac \\ \frac{b\pi}{2a}(x_1 + 2ac) & \text{if } x_1 \leq -2ac \end{cases} \quad (3)$$

When $\alpha = 10.814$, $\beta = 14$, $a = 1.3$, $b = 0.11$, $c = 7$, and $d = 0$, an 8-scroll attractor is produced. This multi-scroll chaotic system admits the origin $\bar{X}_0 = (0, 0, 0)$ as an unstable equilibrium point and has the following unstable equilibrium points $\bar{X}_{\pm 1} = (\pm 2.6, 0, \mp 2.6)$, $\bar{X}_{\pm 2} = (\pm 5.2, 0, \mp 5.2)$, $\bar{X}_{\pm 3} = (\pm 7.8, 0, \mp 7.8)$, $\bar{X}_{\pm 4} = (\pm 10.4, 0, \mp 10.4)$, $\bar{X}_{\pm 5} = (\pm 13, 0, \mp 13)$, $\bar{X}_{\pm 6} = (\pm 15.6, 0, \mp 15.6)$, and $\bar{X}_{\pm 7} = (\pm 18.2, 0, \mp 18.2)$.

Example 2 A typical multi-scroll Chen system is described by [15]:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = (c - a - x_3 + d\sin x_3)\,x_1 + cx_2 \\ \dot{x}_3 = x_1 x_2 - bx_3 \end{cases} \quad (4)$$

where x_1, x_2, and x_3 are the state variables, a, b, c, and d are the system parameters. When $a = 35$, $b = 3$, $c = 28$, and $d = 8$, a 6-scroll attractor is produced. This multi-scroll chaotic system has multiple unstable equilibrium points given by $\bar{X}_0 = (0, 0, 0)$, $\bar{X}_{\pm 1} = (\pm 6.999, \pm 6.999, 16.331)$, $\bar{X}_{\pm 2} = (\pm 7.457, \pm 7.457, 18.536)$, $\bar{X}_{\pm 3} = (\pm 8.102, \pm 8.102, 21.880)$, $\bar{X}_{\pm 4} = (\pm 8.792, \pm 8.792, 25.771)$, $\bar{X}_{\pm 5} = (\pm 9.059$, and $\pm 9.059, 27.356)$.

Because a multi-scroll chaotic system is highly nonlinear and subject to plant uncertainty, some effective approaches like NNs and fuzzy logic are often used in its modeling and controlling processes.

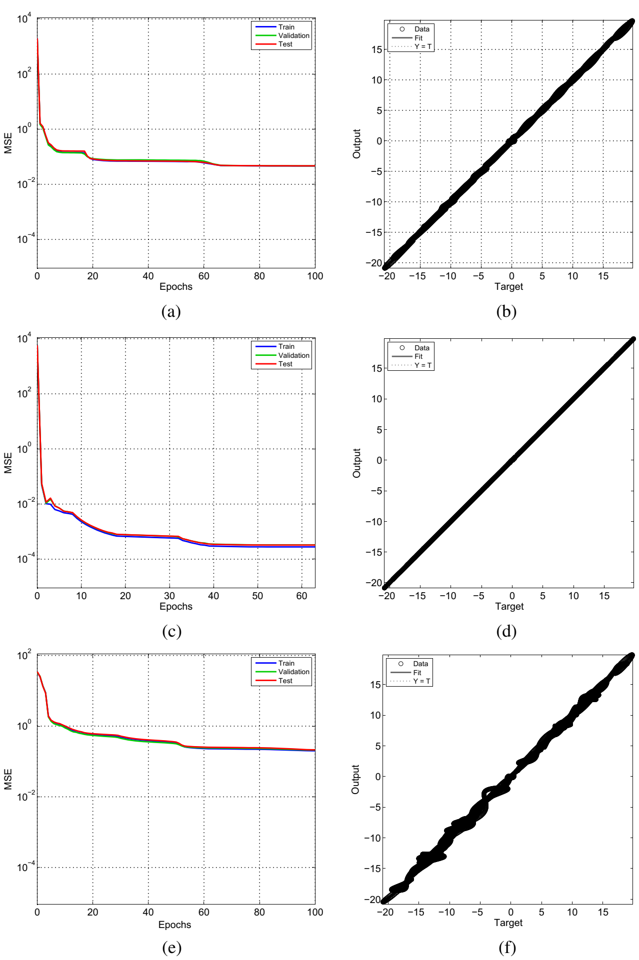


Fig. 3 Results for NN modeling of an n-scroll Chua's circuit

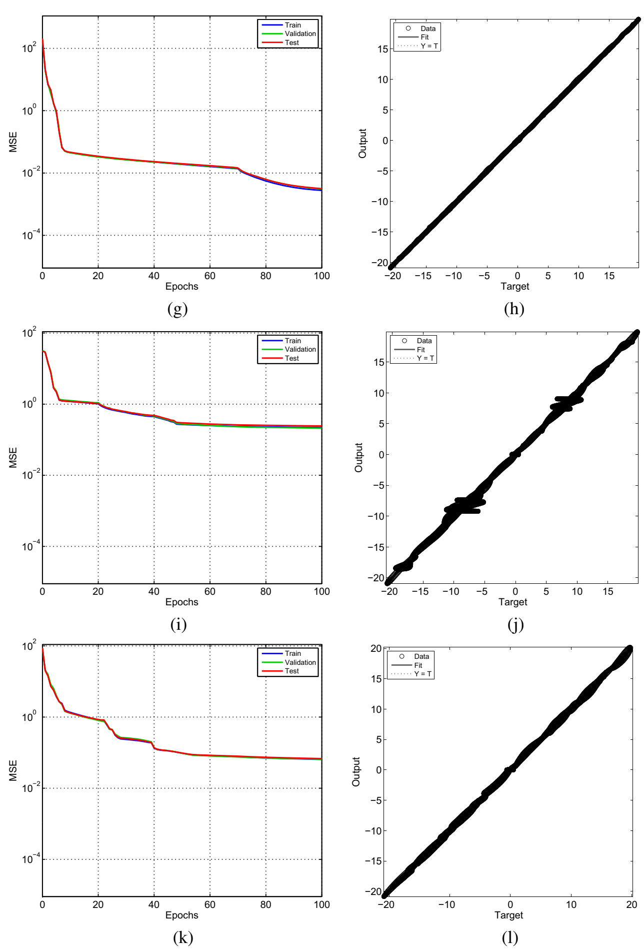


Fig. 3 (continued)

2.2 NN structure

A variety of NN structures have been proposed in the literature for system prediction, control, and diagnostics to meet various kinds of modeling requirements [22, 23]. The feed-forward back-propagation NN is one of the most well-known NN structures, capable of approximating complex nonlinear functions. Two important classes of feed-forward NNs are the MLP and the radial basis function networks. In common with other NN-based methods, the hidden layers and number of neurons are selected via a trial-and-error method.

Based on the MLP approximation property, a nonlinear function $f(t, X(t))$ can be approximated by [22]

$$f(t, X(t)) = W^T\varphi(X) + \varepsilon(X) \tag{5}$$

where W is the weight vector, $\varphi(X)$ is the activation function, $\varepsilon(X)$ is the approximation error of MLP, and $X \in \mathbb{R}^q$ is the input vector, where q is the number of input nodes. A sigmoid function is chosen as the activation function of the hidden layer to help the network learn the nonlinear dynamics of the multi-scroll chaotic system, i.e., the relationships between the input and output layers of the neural network. The activation function of the output layer is assigned a linear (purelin) function. Moreover, the following assumptions are made throughout this paper.

A1 The weight vector W is bounded, i.e., $\|W\| \le W_M \in \mathbb{R}^+$.

A2 The approximation error $\varepsilon(X)$ is bounded, i.e., $\|\varepsilon(X)\| \le \varepsilon_M \in \mathbb{R}^+$.

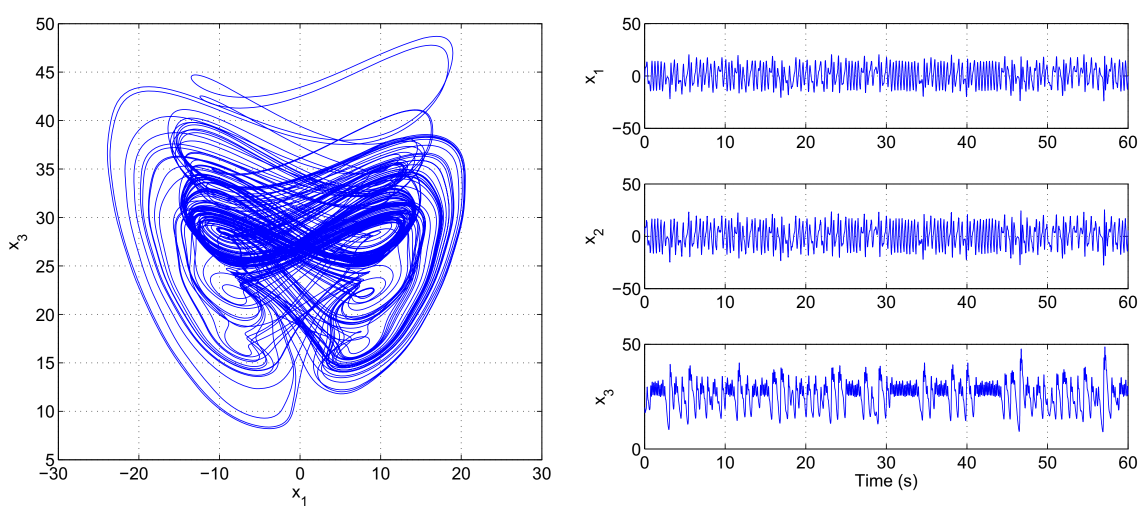
It is necessary to have accurate and sufficient training data for good NN model development. Thus, the NN modeling is performed in two phases:

1 Training phase: the NN system memorizes the patterns of the learning data set. It selects neuron characteristics and topology, minimizes error, and stops according to the stopping criteria.
2 Testing phase: the NN system predicts and tests data sets. The performance of the NN system on the testing data set represents its generalization ability.

In actual practice, the data are divided into two sets. One set is used for training and the other set for testing. In fact, a generalized NN will perform well for both training and testing data.

3 Improvement of the prediction-based control method and problem formulation

The prediction-based control method may be considered as a kind of adaptive control strategy [38]. It was originally introduced by Ushio and Yamamoto in 1999 [39] and has succeeded in controlling discrete-time chaotic systems. Many studies have proposed extensions to deal with different kinds of chaotic systems. Among them, Boukabou et al. [35] proposed a chaos control method for continuous-time systems. It was shown that this method guarantees the stability of the obtained controlled system. It has also been shown that the number of prediction steps has very little effect on the tracking performance of the controller, meaning that even one-step-ahead predictive control is sufficiently effective.



(a) (b)

Fig. 4 Results of NN modeling for a multi-scroll Chen system. **a** Phase plane. **b** Time responses

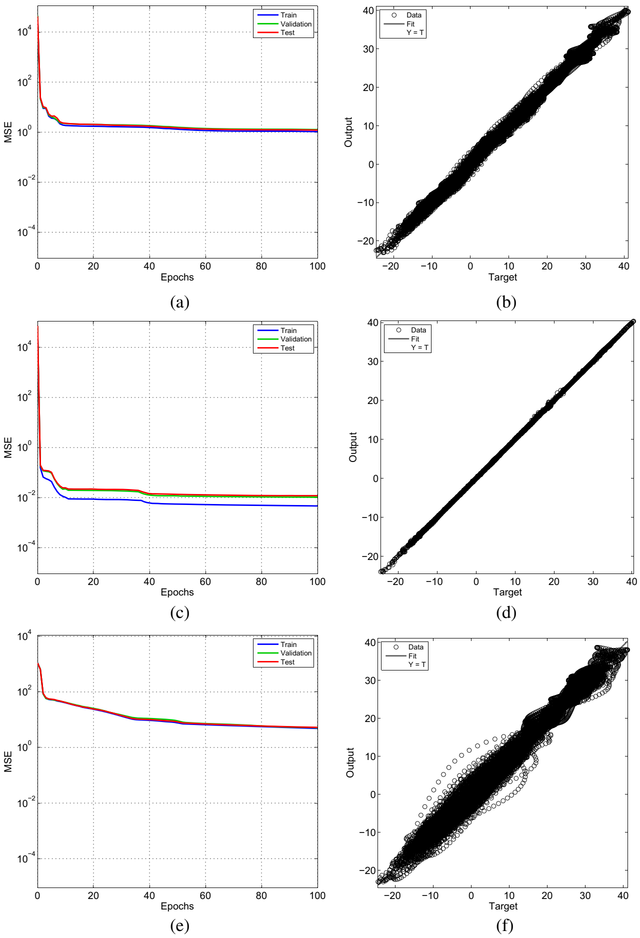


Fig. 5 Results of NN modeling of the multi-scroll Chen system

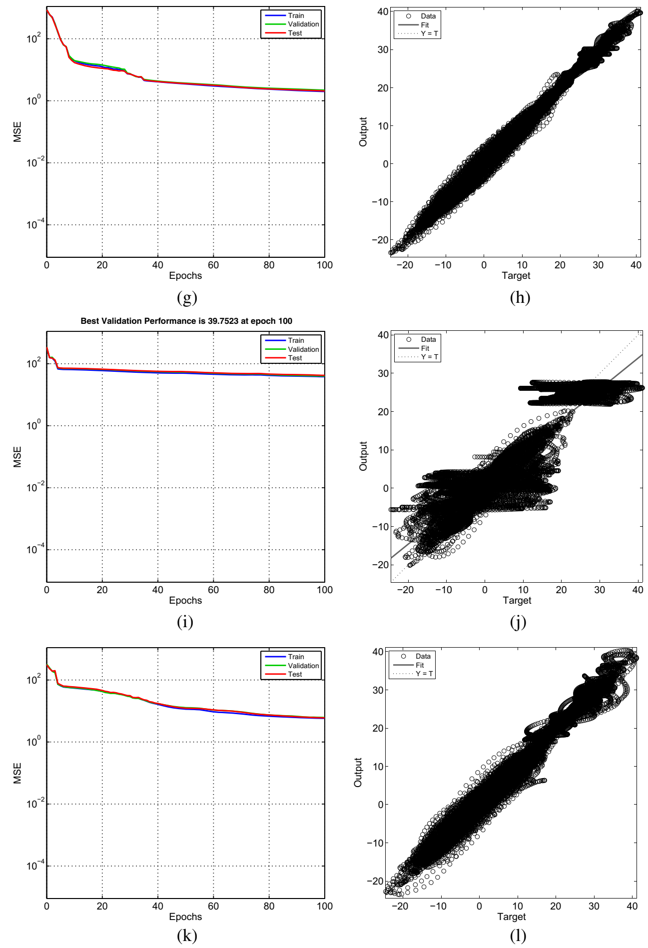


Fig. 5 (continued)

3.1 Prediction-based control principles

Consider a chaotic system with a state equation under prediction-based control input u_p of the form

$$\dot{X}(t) = f(t, X(t)) + u_p(t) \quad (6)$$

where $X \in \mathbb{R}^n$ is the state vector, $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function, and $u_p \in \mathbb{R}^n$ is the control action. We assume that $f(\cdot)$ is differentiable and the system generates chaos when $u_p(t) = 0$. The task is to find a control $u_p(t)$ such that the controlled system trajectory tracks the target

$$\lim_{t\to\infty} \left\| X(t) - \bar{X} \right\| = 0, \quad (7)$$

where $\bar{X}$ is the unstable equilibrium point to be stabilized.

The proposed prediction-based control method is based on the prediction of a p-time future state and the control law is calculated from the difference between the current state and the future state of the uncontrolled multi-scroll chaotic system. We choose the feedback control input as follows

$$u_p(t) = K(A - I)X(t) \quad (8)$$

where $A = \left.\frac{\partial f}{\partial X}\right|_{X=\bar{X}}$ represents the system's Jacobian matrix, evaluated at the desired unstable equilibrium point

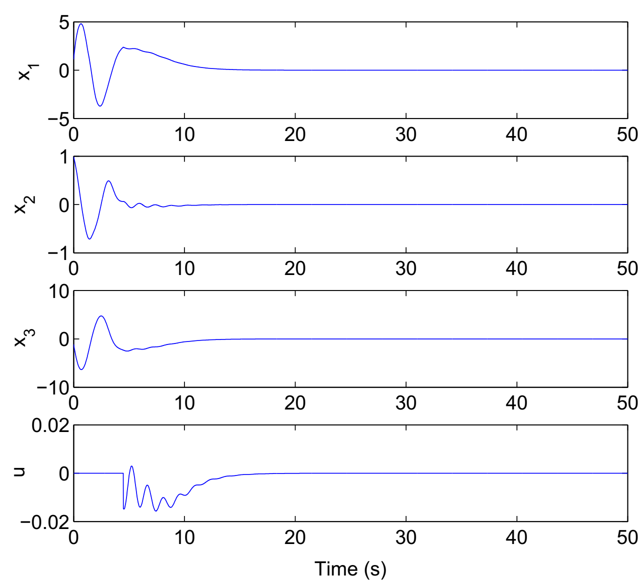


Fig. 6 Stabilizing the NN model of the n-scroll Chua's circuit on $\bar{X}_0$

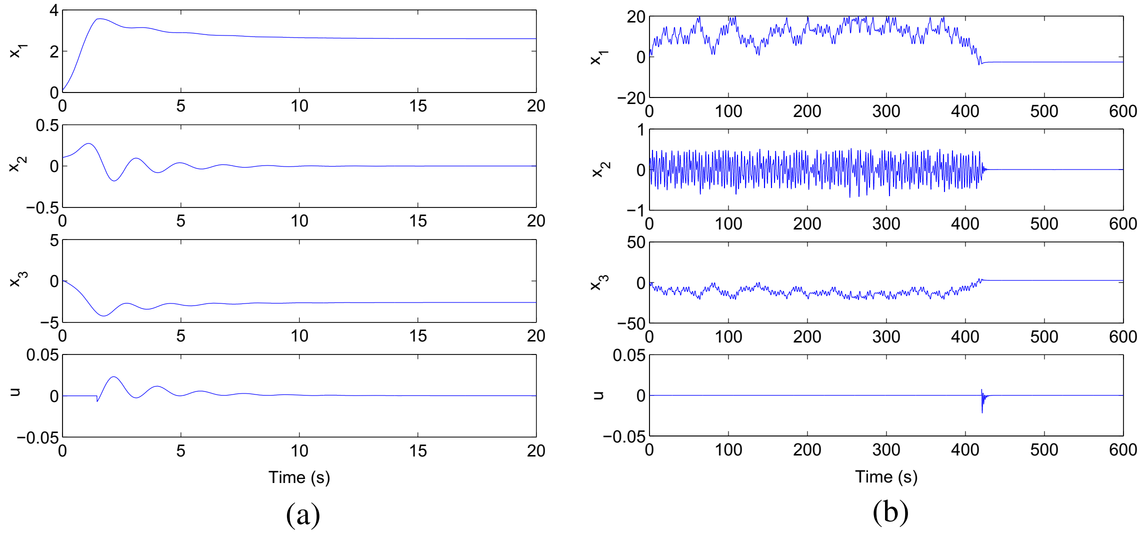


Fig. 7 Stabilizing the NN model of the n-scroll Chua's circuit on **a** $\bar{X}_{+1}$ and **b** $\bar{X}_{-1}$

to be stabilized, I is the identity matrix, and K is a feedback gain matrix.

The linearized multi-scroll chaotic system at the desired unstable equilibrium point is under prediction-based control, which is described as follows [35]

$$\dot{X}(t) = AX(t) + K(A - I)X(t) = -\bar{A}X(t) \tag{9}$$

Remark 1 The prediction-based control method stabilizes the chaotic system (6) for gain K, satisfying the sufficient conditions $\bar{A} > 0$ and $\det(A - I) \neq 0$ [35].

3.2 Problem formulation

Because of plant uncertainty and unmodeled dynamics, conventional control methods, including prediction-based control, cannot guarantee a sufficiently high-performance for stabilizing multi-scroll chaotic systems. To solve this problem, we propose an intelligent control design based on the prediction-based control method and NN system. The multi-scroll chaotic system (1) is under control input $U(t)$ as follows:

$$\dot{X}(t) = f(t, X(t)) + U(t). \tag{10}$$

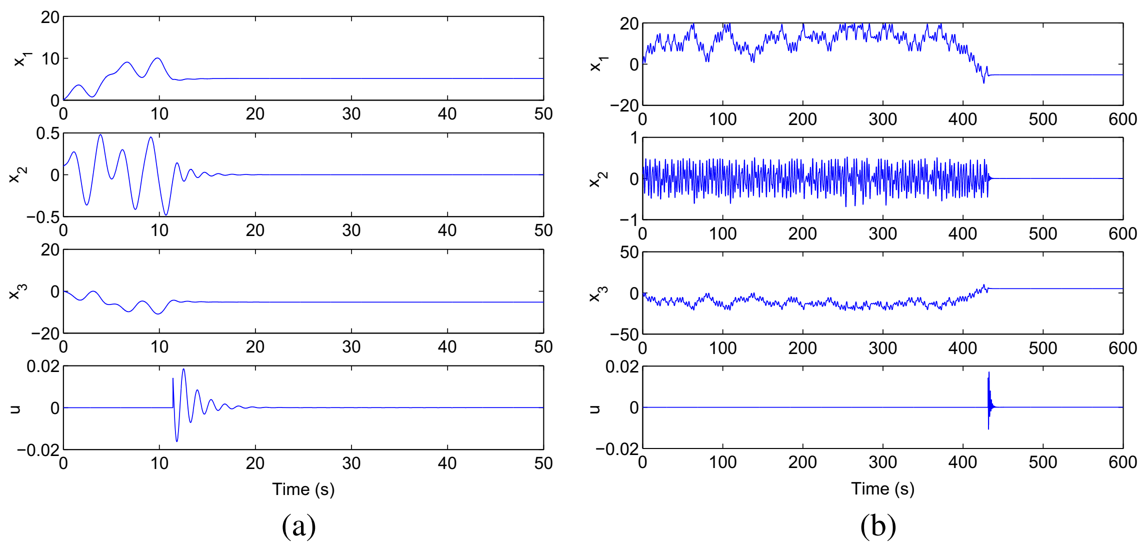
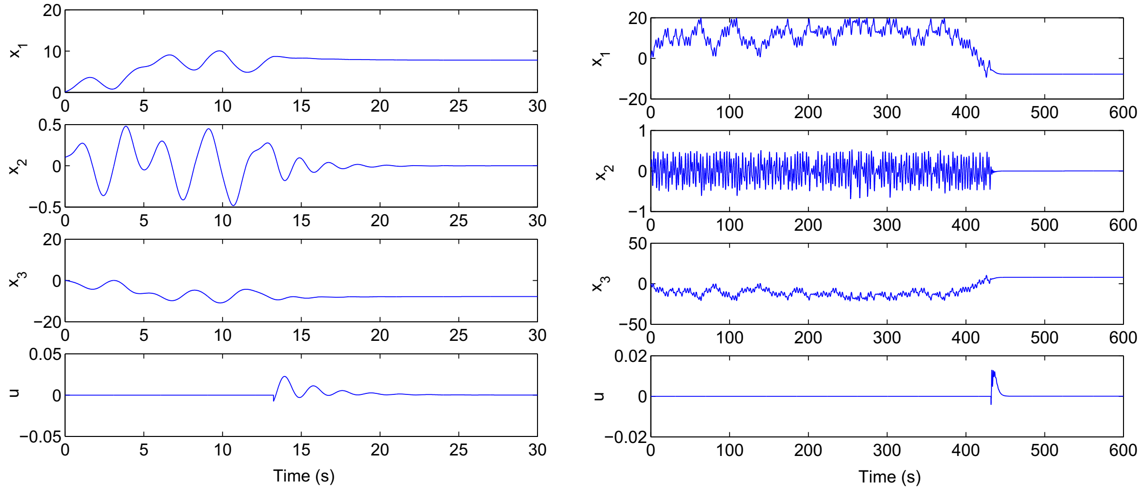


Fig. 8 Stabilizing the NN model of n-scroll Chua's circuit on **a** $\bar{X}_{+2}$ and **b** $\bar{X}_{-2}$



(a) (b)

Fig. 9 Stabilizing the NN model of the n-scroll Chua's circuit on **a** $\bar{X}_{+3}$ and **b** $\bar{X}_{-3}$

In the following section, we address the problem of designing a NN for constructing models that incorporate a priori knowledge in the form of differential equations for multi-scroll chaotic systems. The prediction-based controller is then applied to the obtained NN models in order to track the system trajectory towards the desired equilibrium point.

4 NN-PbC design

Figure 1 shows the block diagram of the NN-PbC control scheme. Clearly, the implementation of the NN basis functions depends only on the desired reference information.

In order to stabilize the multi-scroll chaotic system effectively, we propose an intelligent prediction-based controller. We define the control input $U(t)$ as follows:

$$U(t) = -\hat{f}(t, X(t)) + u_p(t), \tag{11}$$

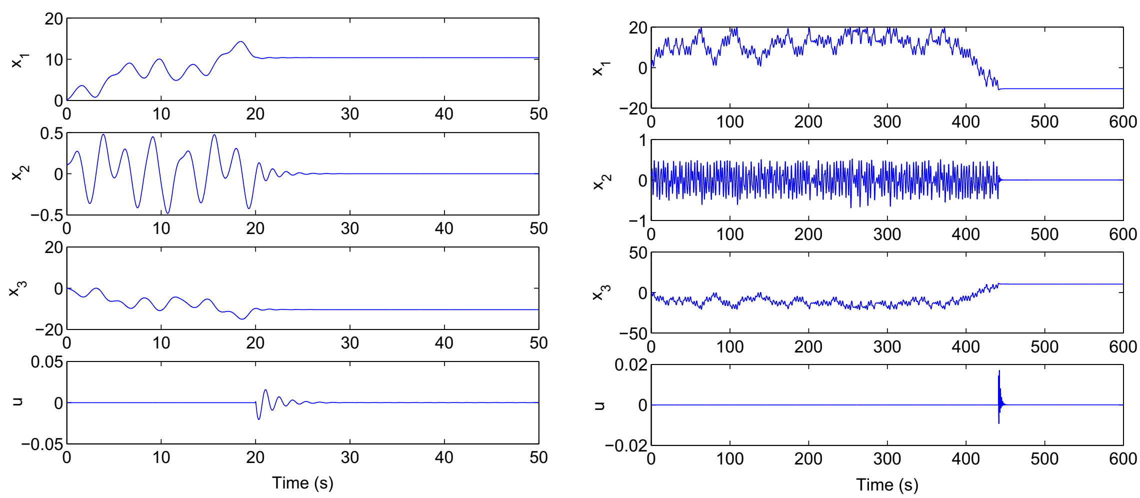
where $u_p(t)$ is the prediction-based control law given in (8) and $\hat{f}(t, X(t))$ is the adaptive NN system such that

$$\hat{f}(t, X(t)) = \hat{W}^T \varphi(X), \tag{12}$$

where $\hat{W}$ is the estimated value of weight vector W.

Substituting (5) and (11) into (10) gives

$$\begin{aligned} \dot{X}(t) &= -\bar{A}X(t) + W^T\varphi(X) - \hat{W}^T\varphi(X) + \varepsilon(X) \\ &= -\bar{A}X(t) + \tilde{W}^T\varphi(X) + \varepsilon(X) \end{aligned} \tag{13}$$



(a) (b)

Fig. 10 Stabilizing the NN model of the n-scroll Chua's circuit on **a** $\bar{X}_{+4}$ and **b** $\bar{X}_{-4}$

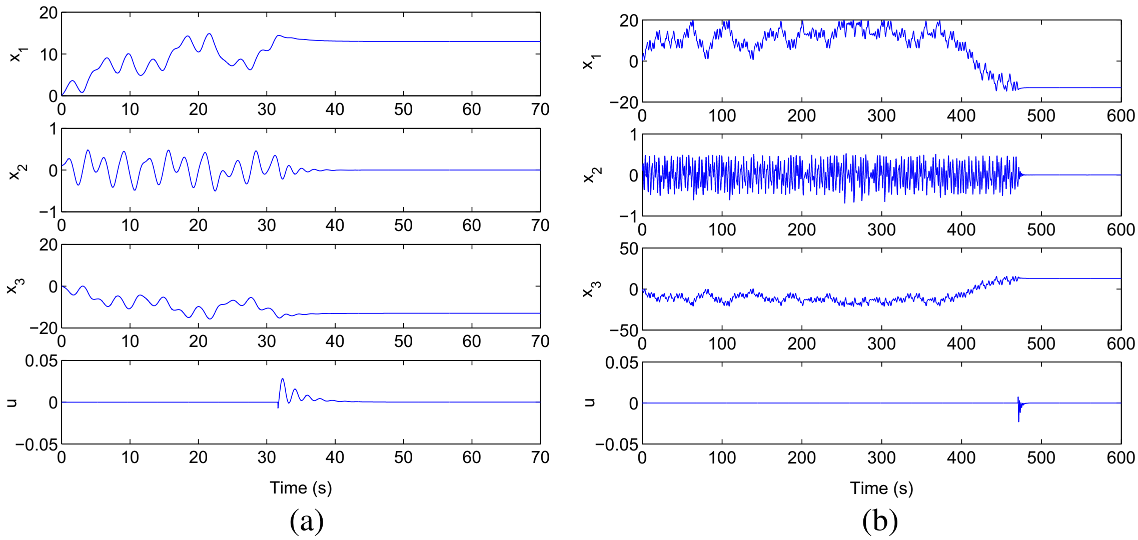


Fig. 11 Stabilizing the NN model of the n-scroll Chua's circuit on **a** $\bar{X}_{+5}$ and **b** $\bar{X}_{-5}$

where $\tilde{W} = W - \hat{W}$ represents the weight approximation error.

Theorem 1 *Let the control action be provided by the prediction-based controller (8). The adaptation law of the NN weight is given by*

$$\dot{\hat{W}} = F\left(\varphi(X)\, X^T - \gamma \, \|X\| \, \hat{W}\right), \tag{14}$$

where F is a positive definite matrix and γ is a positive constant. The stability of the closed-loop system (13) can then be guaranteed.

Proof Choose the Lyapunov function as follows:

$$V(v) = \frac{1}{2} X^T(t) X(t) + \frac{1}{2} tr(\tilde{W}^T F^{-1} \tilde{W}) \tag{15}$$

The time derivative of V is given by

$$\begin{aligned} \dot{V} &= X^T(t)\dot{X}(t) + tr(\tilde{W}^T F^{-1} \dot{\tilde{W}}) \\ &= X^T(t)\dot{X}(t) - tr(\tilde{W}^T F^{-1} \dot{\hat{W}}). \end{aligned} \tag{16}$$

Applying (13) and (14) to (16), we get

$$\begin{aligned} \dot{V} &= -X^T \bar{A} X + X^T \tilde{W}^T \varphi(X) + X^T \varepsilon(X) \\ &\quad -tr\left(\tilde{W}^T \varphi(X)\, X^T - \gamma \, \|X\| \, \hat{W}\right) \end{aligned} \tag{17}$$

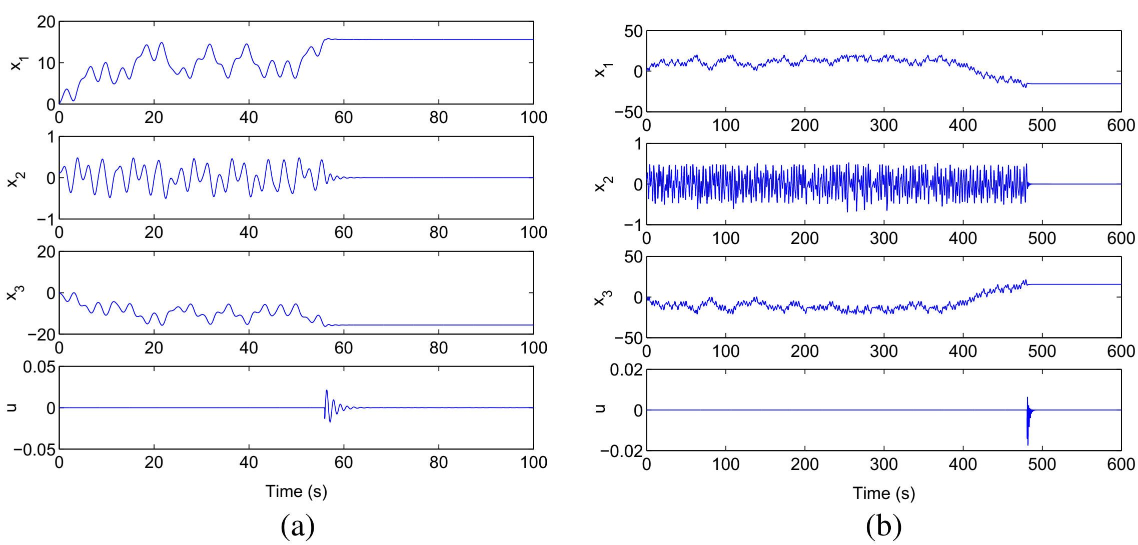


Fig. 12 Stabilizing the NN model of the n-scroll Chua's circuit on **a** $\bar{X}_{+6}$ and **b** $\bar{X}_{-6}$

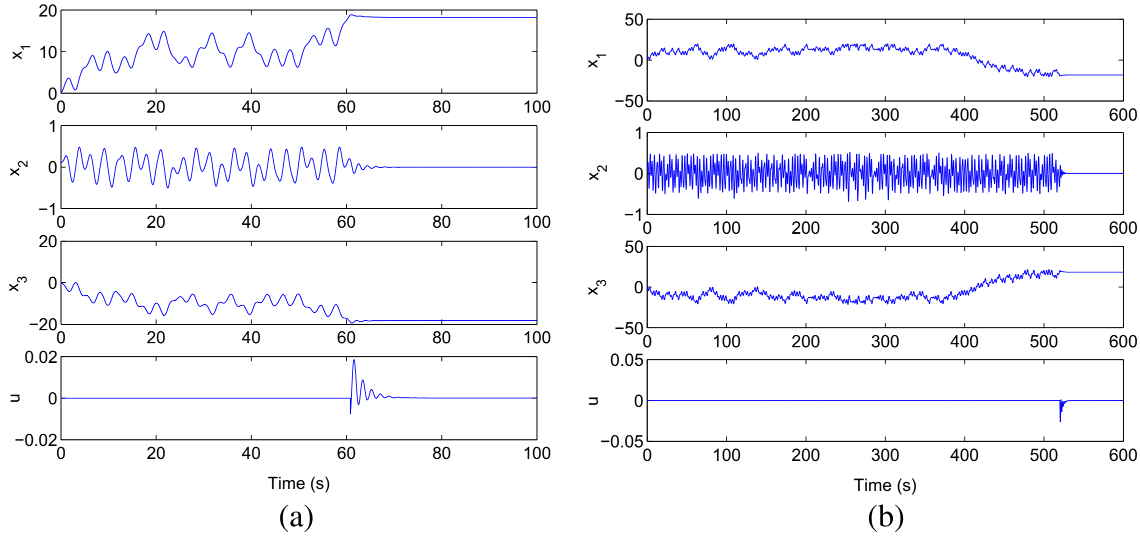


Fig. 13 Stabilizing the NN model of the n-scroll Chua's circuit on **a** $\bar{X}_{+7}$ and **b** $\bar{X}_{-7}$

Because $\alpha^T\beta = tr\left(\beta\alpha^T\right)$, we have

$$X^T\tilde{W}^T\varphi(X) = tr\left(\tilde{W}^T\varphi(X)X^T\right) \tag{18}$$

which, substituted in (17) yields

$$\dot{V} = -X^T\bar{A}X + X^T\varepsilon(X) + \gamma\|X\|\, tr\left(\tilde{W}^T\hat{W}\right) \tag{19}$$

Using the Frobenius norm $\|\cdot\|_F$, we obtain

$$\begin{aligned} tr\left(\tilde{W}^T\hat{W}\right) &= tr\left[\tilde{W}^T\left(W-\tilde{W}\right)\right] = \left\langle\tilde{W}, W\right\rangle_F - \left\|\tilde{W}\right\|_F^2 \\ &= \left\|\tilde{W}\right\|_F\|W\|_F - \left\|\tilde{W}\right\|_F^2 \end{aligned} \tag{20}$$

According to assumption *A1*, we have

$$\left\|\tilde{W}\right\|_F\|W\|_F - \left\|\tilde{W}\right\|_F^2 \le \left\|\tilde{W}\right\|_F W_M - \left\|\tilde{W}\right\|_F^2 \tag{21}$$

Completing the squares yields

$$\left\|\tilde{W}\right\|_F W_M - \left\|\tilde{W}\right\|_F^2 = -\left(\left\|\tilde{W}\right\|_F^2 - \tfrac{1}{2}W_M\right)^2 + \frac{1}{4}W_M^2 \tag{22}$$

Rewriting (19), we obtain

$$\begin{aligned} \dot{V} \le -\|X\|\Big\{&\lambda_{\min}(\bar{A})\|X\| + \alpha\left(\left\|\tilde{W}\right\|_F - \tfrac{1}{2}W_M\right)^2 \\ &-\varepsilon_M - \tfrac{1}{4}\alpha W_M^2\Big\} \end{aligned} \tag{23}$$

by virtue of assumption *A2*, where $\lambda_{\min}\left(\bar{A}\right)$ is the smallest eigenvalue of matrix $\bar{A}$. Inequality (23) is guaranteed to be negative as long as either (24) or (25) holds:

$$\|X\| \ge \left(\varepsilon_M + \frac{1}{4}\alpha W_M^2\right)/\lambda_{\min}\left(\bar{A}\right) = \Omega_X \tag{24}$$

$$\left\|\tilde{W}\right\|_F \ge \sqrt{\varepsilon_M + \frac{1}{4}\alpha W_M^2} + \frac{1}{2}W_M = \Omega_{\tilde{W}}, \tag{25}$$

where Ω_X and $\Omega_{\tilde{W}}$ are convergence regions. According to Lyapunov stability theory [40], the closed-loop system is stable. This completes the proof. □

Remark 2 Note that there is no clear-cut methodology to decide parameters, topologies, or the method of training. The NN architecture is composed of one input layer, one output layer, and one or two hidden layers. The data set is extracted from a real system and then divided into three sets: one set (60 %) is for training (these data are presented to the network during training, and the network is adjusted according to its error), another set (20 %) is for testing (these data are not used during training and so provide an independent measure of network performance during and after training),

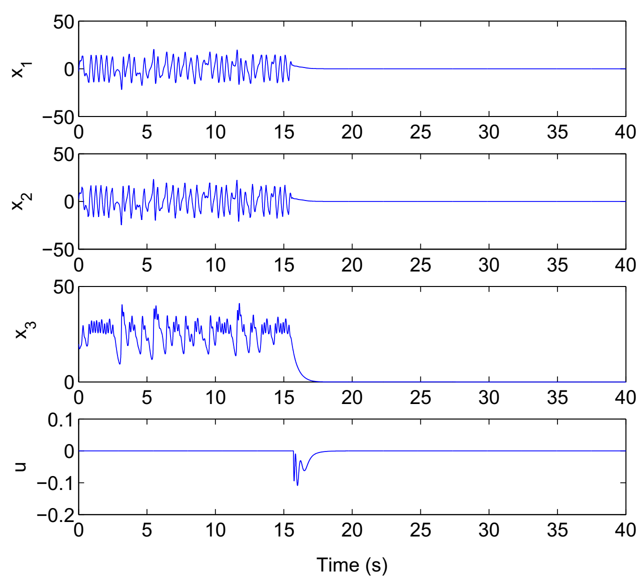


Fig. 14 Stabilizing the NN model of a multi-scroll Chen on $\bar{X}_0$

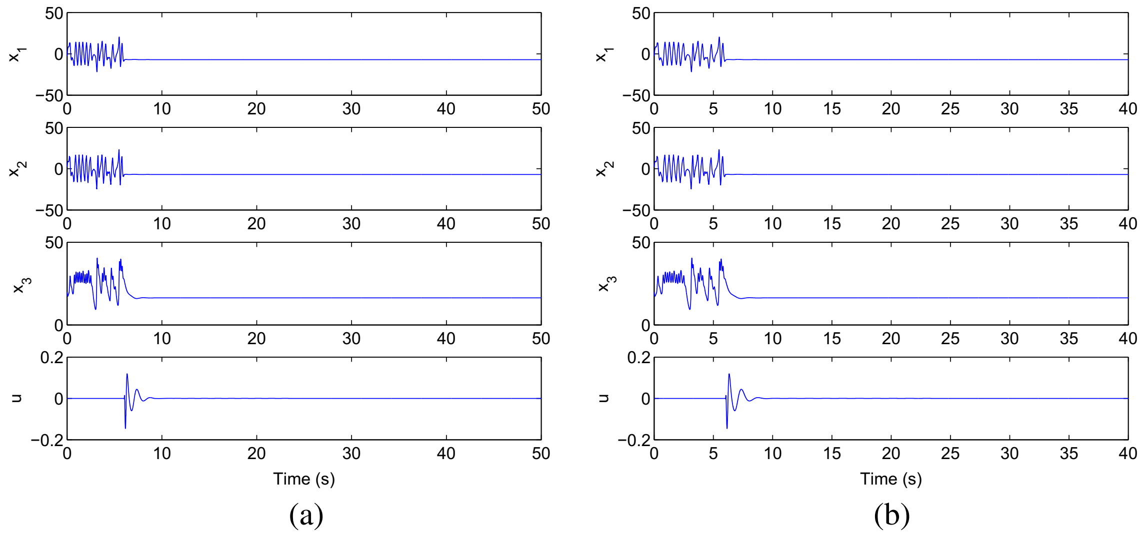


Fig. 15 Stabilizing the NN model of a multi-scroll Chen on **a** $\bar{X}_{+1}$ and **b** $\bar{X}_{-1}$

and the final set (20 %) is for validation (these data are used to measure network generalization and halt training when generalization stops improving). The process of adjusting the weights and biases of the NN system is repeated until one of the termination conditions is met. The training process may be terminated if (i) the mean squared error (MSE) goes below a specific value, i.e., MSE $\leq \varepsilon_M$, (ii) the magnitude of the gradient falls below a certain value, or (iii) a specified number of iterations have been completed.

Remark 3 In addition, note that the approximation error ε_M has some influence over the learning process and controlled loop performance in different ways. During the learning process, it is a measure of similarity between the real system and the NN model. Further, when ε_M is relatively large, a different NN model is generated from the real system, and consequently, its control is useless because the aim is to control the originally modeled real system. Barron [41] found that there exist lower bounds of order $(1/N_A)^{2/n}$ on approximation error ε_M if only the parameters of a linear combination of basis functions are adjusted. The stability proof of the proposed NN-PbC shows that the effect of the bounds on the approximation error can be alleviated by the judicious choice of gain matrix K.

Remark 4 Note that the stabilization process of multi-scroll chaotic systems on desired unstable fixed points is a standard procedure. Likewise, the NN models with additive

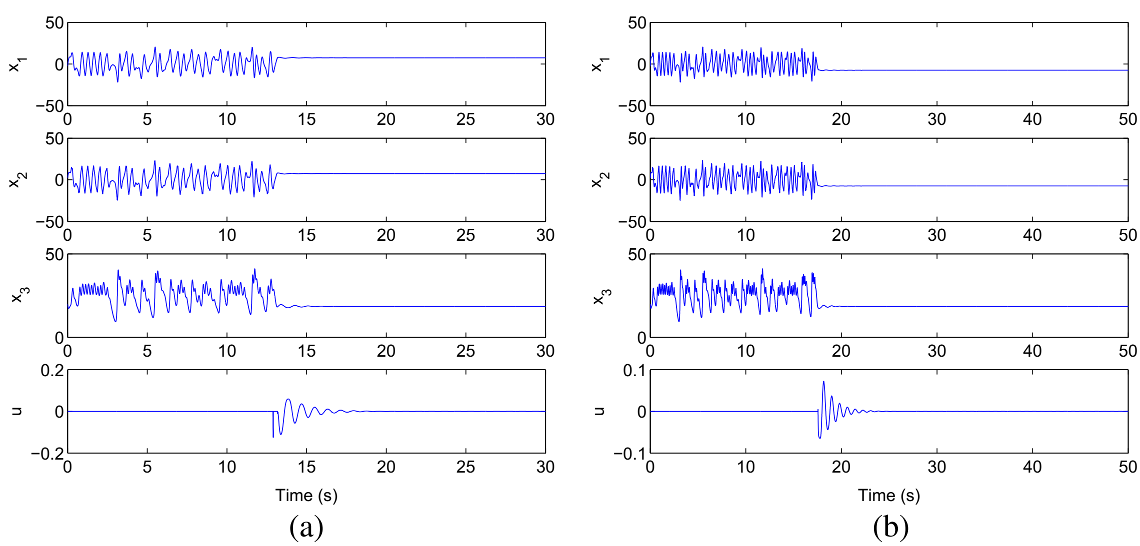


Fig. 16 Stabilizing the NN model of a multi-scroll Chen on **a** $\bar{X}_{+2}$ and **b** $\bar{X}_{-2}$

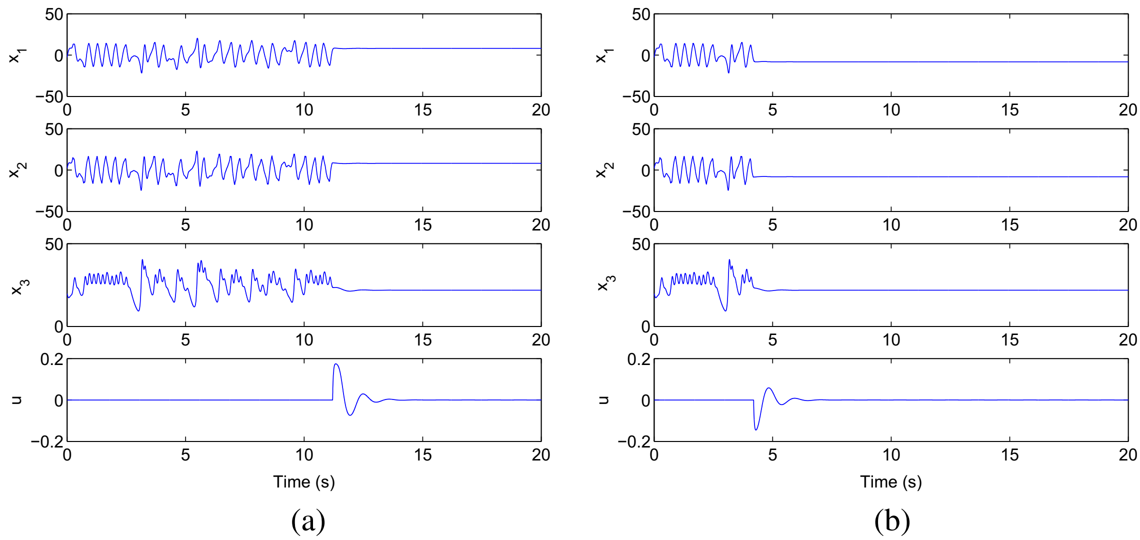


Fig. 17 Stabilizing the NN model of a multi-scroll Chen on **a** $\bar{X}_{+3}$ and **b** $\bar{X}_{-3}$

feedback control can also be stabilized, where the parameters of the NN-based controller are modified such that they satisfy the stability requirement.

5 Experiments

In this section, we present the results of our experiments. Here, NN system performance is quantitatively measured by the mean CPU time needed to train the NN to approximate each multi-scroll chaotic system, MSE, and linear regression index [42]. The results are summarized in Tables 1 and 2. All experiments were run 10 times to ensure the validity and accuracy of the experimental measurements.

5.1 NN modeling of an n-scroll Chua's circuit

In order to model an n-scroll Chua's circuit, the data set was extracted from a real system (2). Figure 2 depicts the results of ANN training and testing with the data points extracted from a time series of the real system. We can see that the ANN approximates the n-scroll Chua's circuit accurately with a small error norm.

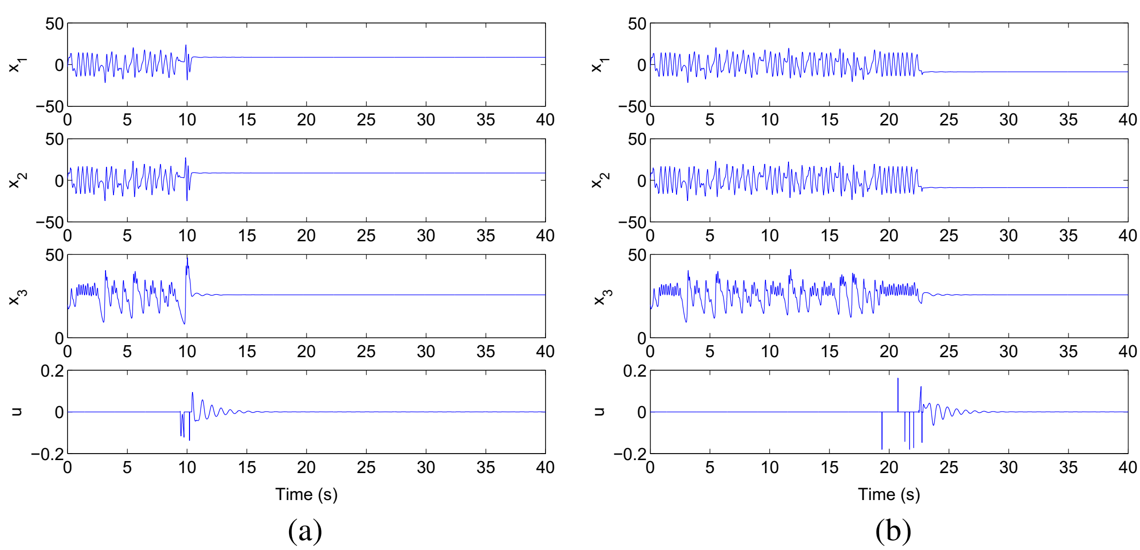


Fig. 18 Stabilizing the NN model of a multi-scroll Chen on **a** $\bar{X}_{+4}$ and **b** $\bar{X}_{-4}$

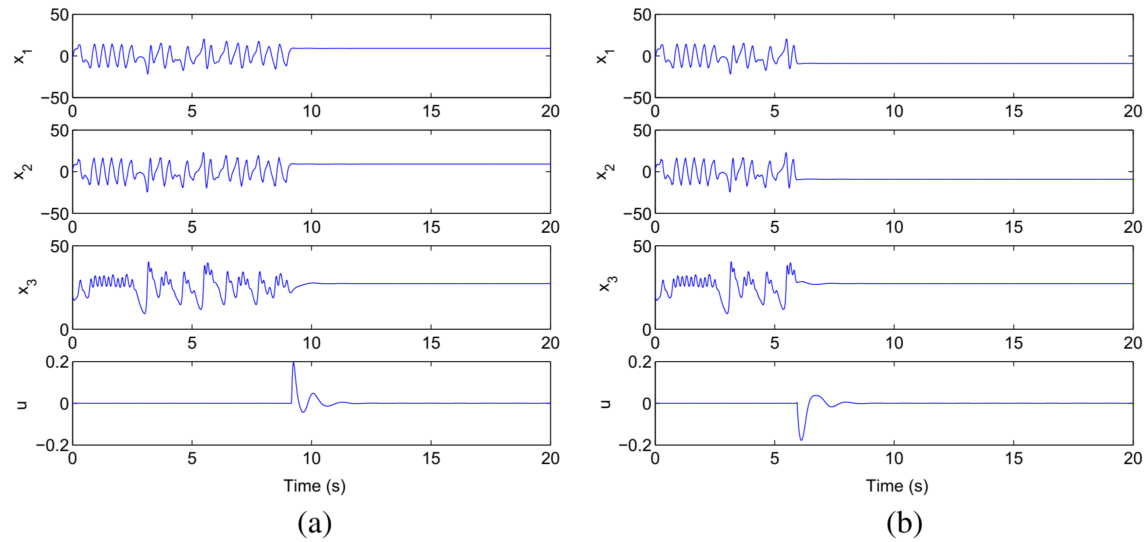


Fig. 19 Stabilizing the NN model of a multi-scroll Chen on **a** $\bar{X}_{+5}$ and **b** $\bar{X}_{-5}$

Table 1 and Fig. 3a–l present the results of diverse NN architectures for 3-D multi-scroll Chua system identification. From the results, we can clearly see that the NN with one hidden layer performs better than the others.

5.2 NN modeling of a multi-scroll Chen system

Figure 4 shows the results of ANN training and testing on data points extracted from the time series of a real system. We can see that the ANN approximates the multi-scroll Chen system accurately with a small error norm.

Table 2 and Fig. 5a–l present the results of diverse NN architectures for 3-D multi-scroll Chen system identification. From the results, we can clearly observe that the NN with one hidden layer performs better than the others.

6 Simulation results

In this section, we present results for stabilizing the obtained NN models of the two multi-scroll chaotic systems.

6.1 Stabilization of the *n*-scroll Chua NN model

The intelligent prediction-based adaptive NN control law was constructed in the form of (8) and (14) with gains $F = 10I_{3\times3}$ and $\gamma = 2$. The initial states were set to $X(0) = (0.1, 0.1, 0.1)$. For $K = diag(-1.5, 0, 0)$ and different time values, the ANN model of the *n*-scroll Chua's circuit converges towards unstable equilibrium points $\bar{X}_0$, $\bar{X}_{\pm2}$, $\bar{X}_{\pm4}$, and $\bar{X}_{\pm6}$. For $K = diag(-0.8, 0, 0)$, the controlled system states converges towards equilibrium points $\bar{X}_{\pm1}$, $\bar{X}_{\pm3}$, $\bar{X}_{\pm5}$, and $\bar{X}_{\pm7}$. The simulation results are shown in Figs. 6, 7, 8, 9, 10, 11, 12 and 13. Obviously, satisfactory tracking performance can be guaranteed, as the system states converges towards the desired equilibrium point with a small applied force.

6.2 Stabilization process of the multi-scroll Chen NN model

The intelligent prediction-based adaptive NN control law was designed in the form of (8) and (14) with gains $F = 10I_{3\times3}$ and $\gamma = 2$. The initial states were set as $X(0) = (-3, 2, 20)$. For $K = diag(0, -1.09, 0)$ and different time values, the ANN model of the multi-scroll Chen system converged towards the unstable equilibrium points $\bar{X}_0$, $\bar{X}_{\pm2}$, and $\bar{X}_{\pm4}$. For $K = diag(0, -0.97, 0)$, the controlled system states converged towards the equilibrium points $\bar{X}_{\pm1}$, $\bar{X}_{\pm3}$, and $\bar{X}_{\pm5}$. The simulation results are shown in Figs. 14, 15, 16, 17, 18 and 19. Consequently, satisfactory performance is achieved and the effects caused by the ANN modeling have been diminished.

7 Conclusion

A prediction-based tracking control design was proposed and solved for a class of multi-scroll chaotic systems. An NN system was employed to approximate the behaviors of multi-scroll chaotic dynamics that include plant uncertainty and unmodeled dynamics. Consequently, a prediction-based adaptive NN controller called NN-PbC was developed such that all the states of the closed-loop system converge towards the desired equilibrium point and the control input is as small as possible. The proposed control method is an optimal control strategy since future control inputs and future system responses are predicted using a NN system model and optimized at regular intervals with respect to a performance index. The salient feature of the NN-PbC design is that the control objective is obtained with nonlinearities in the multi-scroll chaotic system that are completely unknown. The proposed neural-adaptive learning shows both robustness and adaptation to changing system dynamics. To this end, a control signal is incorporated into the adaptive-learning scheme such that the obtained controlled system converges towards the desired unstable equilibrium point. Therefore, for practical applications, the intelligent control scheme developed here can be employed to handle a broader class of multi-directional multi-scroll systems in the presence of high-degree time-varying uncertainties. Finally, from the simulation results for various situations, it can be concluded that the proposed design achieves the desired results.

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