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A mathematical model for solving fully fuzzy linear programming problem with trapezoidal fuzzy numbers

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Abstract In this paper, an efficient method is introduced to solve fully fuzzy linear programming problems. The proposed method is derived from the multi-objective linear programming problem and lexicographic ordering method. Theoretical analysis for the proposed method has been provided. Moreover, some numerical experiments are given to show the preference of the proposed methods and are compared with some available methods.

Keywords Fully fuzzy linear programming · Trapezoidal fuzzy numbers · Multi-objective linear programming

1 Introduction

Linear programming (LP) problem play a major role in various field of science and engineering. In many applications, some of the LP parameters are represented by fuzzy numbers rather than crisp numbers. Therefore, developing mathematical models and numerical procedures for the fuzzy LP would be of interest. The concept of fuzzy set was first introduced by Zadeh [1]. After that, the use of fuzzy set theory has been making rapid progress in the field of optimization [2–6]. Some authors have considered fuzzy linear programming, in which not all parts of the problem were assumed to be fuzzy, e.g., only the right hand side and the objective function coefficients were fuzzy; or only the variables were fuzzy. For example, Maleki et al. [7] solved LP problems by the Rouben's method of comparison of fuzzy numbers in which all decision parameters are fuzzy numbers. Ramik [8] introduced a class of FLP problems and defined the concepts of feasible and α-efficient solutions. Ganesan and Veeramani [9] introduced a new type of fuzzy arithmetic for symmetric trapezoidal fuzzy numbers and proposed a method for solving symmetric trapezoidal FLP problems without converting them to crisp LP problems. Following the method of [9], Ebrahimnejad and Tavana [10], proposed a new method for solving FLP problems in which the coefficients of the objective function and the values of the right-hand-side are represented by symmetric trapezoidal fuzzy numbers while the elements of the coefficient matrix are represented by real numbers. They converted the FLP problem into an equivalent crisp LP problem and solve the crisp problem with the standard primal simplex method. Wan and Dong [11] solved linear programming with trapezoidal fuzzy numbers. These authors consider the multi-objective linear programming (MOLP) problem in to four objective functions and used membership functions.

The fuzzy linear programming problems in which all the parameters and variables are represented by fuzzy numbers are known as fully fuzzy linear programming (FFLP) problems. FFLP problem with inequality constraints studied in [12–14]. However, the main disadvantage of the solution

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obtained by the existing methods are that, it does not satisfy the constraints exactly i.e. it is not possible to obtain the fuzzy number of the right hand side of the constraint by putting the obtained solution in the left hand side of the constraint. Dehghan et al. [15] proposed some practical methods to solve fully fuzzy linear system (FFLS) that are comparable to the well known methods. Then they extended a new method employing Linear Programming (LP) for solving square and non-square fuzzy systems. Lotfi et al. [16] applied the concept of the symmetric triangular fuzzy number, obtained a new method for solving FFLP by converting a FFLP into two corresponding LPs. Kumar et al. [17–19] pointed out the shortcomings of the above methods. To overcome these shortcomings, they proposed a new method for finding the fuzzy optimal solution of FFLP problems with equality constraints. Saberi Najafi and Edalatpanah [20] pointed out the method of [17] needs some corrections to make the model well in general. Veeramani and Duraisamy [21] proposed a new approach of solving FFLP problem using the concept of nearest symmetric triangular fuzzy number approximation with preserve expected interval. Ezzati et al. [22] applied lexicographic ordering on triangular fuzzy numbers and MOLP problem introduced a new algorithm to solve FFLP.

In this paper, based on MOLP problems and lexicographic method we design a new strategy to solve fully fuzzy linear programming problem with trapezoidal fuzzy numbers. We show that our proposed method needs less computation cost than some existing methods.

The rest of this paper is organized as follows: some basic definitions and notations are present in Section 2. In Section 3, the general form of FFLP with new method is presented. In Section 4, advantages of the proposed method over some existing methods are discussed. To show the applications of the proposed method, some real life problem and comparison analysis are discussed in Section 5. Finally, the conclusion is been drawn in the last section.

2 Preliminaries

We begin with some basic and fundamental notations and preliminary results which we refer to later [18, 22, 23].

Definition 2.1 Let X denote a universal set. The fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}} : X \to [0, 1]$; which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval [0, 1], to each element $x \in X$, where the value of $\mu_{\tilde{A}}(x)$ at x shows the grade of membership of x in $\tilde{A}$. A fuzzy subset $\tilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\tilde{A}}(x)$ and is often written $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$.

The class of fuzzy sets on X is denoted with $\tau(X)$.

Definition 2.2 A trapezoidal fuzzy number $\tilde{A}$ = (a, b, c, d) is said to be non-negative (non-positive) fuzzy number if and only if $a \geq 0$ ($a \leq 0$). The set of non-negative (non-positive) fuzzy numbers may be represented by $F(R^+)$.

Definition 2.3 A fuzzy number $\tilde{A}$ = (a, b, c, d) is said to be a trapezoidal fuzzy number if its membership function is given as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b, \\ 1\,, & b \leq x \leq c \\ \frac{(x-d)}{(c-d)}, & c \leq x \leq d \\ 0\,, & else. \end{cases}$$

Definition 2.4 Let $\tilde{A} = (a, b, c, d)$ and $\tilde{B} = (e, f, g, h)$ be two non-negative trapezoidal fuzzy numbers then:

(i) $\tilde{A} \oplus \tilde{B} = (a, b, c, d) \oplus (e, f, g, h) = (a + e, b + f, c + g, d + h)$,
(ii) $\tilde{A} - \tilde{B} = (a, b, c, d) - (e, f, g, h) = (a - h, b - g, c - f, d - e)$,
(iii) Let $\tilde{A} = (a, b, c, d)$ and $\tilde{B} = (e, f, g, h)$ be two non-negative trapezoidal fuzzy numbers then:

$$\tilde{A} \otimes \tilde{B} = (\alpha, \beta, \gamma, \delta),$$

where $\alpha = \min(ae, ah, de, dh)$, $\beta = \min(bf, bg, cf, cg)$, $\gamma = \max(bf, bg, cf, cg)$, $\delta = \max(ae, ah, de, dh)$.

Definition 2.5 A ranking function is a function $\Re : F(R) \to R$ which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{A} = (a, b, c, d)$ is a trapezoidal fuzzy number then $\Re(\tilde{A}) = \frac{a+b+c+d}{4}$.

Definition 2.6 Two trapezoidal fuzzy numbers $\tilde{A} = (a, b, c, d)$ and $\tilde{B} = (e, f, g, h)$ are said to be equal if and only if a=e, b=f, c=g and d=h.

Definition 2.7 Let $\tilde{A} = (a, b, c, d)$ and $\tilde{B} = (e, f, g, h)$ be two trapezoidal fuzzy number then:

(i) $\tilde{A} \leq \tilde{B} \Leftrightarrow \Re(\tilde{A}) \leq \Re(\tilde{B})$,
(ii) $\tilde{A} < \tilde{B} \Leftrightarrow \Re(\tilde{A}) < \Re(\tilde{B})$.

In the following, we propose a new definition to compare two arbitrary trapezoidal fuzzy numbers based on lexicographic method.

Definition 2.8 Let $\tilde{A} = (a, b, c, d)$ and $\tilde{B} = (e, f, g, h)$ be two arbitrary trapezoidal fuzzy numbers. We say that $\tilde{A}$ is relatively less than $\tilde{B}$, which is written by $\tilde{A} < \tilde{B}$ if and only if:

(i) $(b-a) > (f-e)$ or,
(ii) $b < f$ and $(b-a) = (f-e)$ or,
(iii) $b = f$, $(b-a) = (f-e)$ and $\left(\frac{(b+c)}{2} < \frac{(f+g)}{2}\right)$, or
(iv) $b = f$, $(b-a) = (f-e)$, $\left(\frac{(b+c)}{2} = \frac{(f+g)}{2}\right)$, and $(d-c) < (h-g)$.

Remark 2.1 It is clear from above definition that $b = f$, $(b-a) = (f-e)$, $\left(\frac{(b+c)}{2} = \frac{(f+g)}{2}\right)$ and $(d-c) = (h-g)$ if and only if $\tilde{A} = \tilde{B}$.

3 Proposed method

Consider the standard form of FFLP problem as follows:

$$\begin{aligned} &\text{Max} \quad (\text{Min})\tilde{c}^t\tilde{x} \\ &\qquad \text{s. t. } \tilde{A}\tilde{x} = \tilde{b}, \end{aligned} \tag{1}$$

$\tilde{x}$ is a non-negative fuzzy number.

Where $\tilde{A} = (a_{ij})_{m\times n}$ is the coefficient matrix, $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, ...\tilde{b}_m)^t$ is the available resource vector, $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, ..., \tilde{c}_n)^t$ is the objective coefficient vector, and $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n)^t$ is the decision variable vector.

Let $\tilde{s}, \tilde{x}^*$ be a feasible region and an exact optimal solution of problem (1), respectively. If there exist an $\tilde{x}' \in \tilde{s}$ such that $\tilde{c}^t\tilde{x}' = \tilde{c}^t\tilde{x}^*$, then $\tilde{x}'$ is also an exact optimal solution of problem (1) and is called an alternative exact optimal solution.

Next, we establish the new method. Let $\tilde{c}^t\tilde{x} = ((c^tx)^l, (c^tx)^m, (c^tx)^n, (c^tx)^r)$, $\tilde{A}\tilde{x} = ((Ax)^l, (Ax)^m, (Ax)^n, (Ax)^r)$, $\tilde{b} = ((b)^l, (b)^m, (b)^n, (b)^r)$, $\tilde{x} = ((x)^l, (x)^m, (x)^n, (x)^r)$, then the steps of new method are as follows:

Step 1: Using Definitions 2.2 and 2.3, (1) can be shown by the following multi-objective programming:

$$\begin{aligned} &\text{Max (Min)}\{(c^tx)^l, (c^tx)^m, (c^tx)^n, (c^tx)^r\}, \\ &\text{Subject to } ((Ax)^l, (Ax)^m, (Ax)^n, (Ax)^r) \\ &\qquad = ((b)^l, (b)^m, (b)^n, (b)^r), \\ &(x)^m - (x)^l \geq 0, \ (x)^n - (x)^m \geq 0, \ (x)^r \\ &\qquad -(x)^n \geq 0, \ (x)^l \geq 0. \end{aligned} \tag{2}$$

However, the above four objective functions $(c^tx)^l, (c^tx)^m, (c^tx)^n, (c^tx)^r$ should always preserve the form of the trapezoidal fuzzy numbers during the optimization process. Thus in order to keep the trapezoidal fuzzy numbers (convex and normal) shape of the possibility, we change the above four objective functions in an effective way.

Based on Definition 2.6, the (2) may be written as:

$$\begin{aligned} &\text{Max (Min)}\{(c^tx)^l, (c^tx)^m, (c^tx)^n, (c^tx)^r\}, \\ &\text{Subject to } (Ax)^l = (b)^l, (Ax)^m = (b)^m, (Ax)^n \\ &\qquad = (b)^n, (Ax)^r = (b)^r, \\ &(x)^m - (x)^l \geq 0, \ (x)^n - (x)^m \geq 0, \ (x)^r \\ &\qquad -(x)^n \geq 0, \ (x)^l \geq 0. \end{aligned} \tag{3}$$

Step 2: Based on Definition 2.8, (3) can be transformed into the following MOLP problem with four crisp objective functions model as:

$$\begin{aligned} &\text{Min (Max)}(c^tx)^m - (c^tx)^l, \\ &\text{Max (Min)}(c^tx)^m, \\ &\text{Max (Min)}\frac{1}{2}[(c^tx)^m + (c^tx)^n], \\ &\text{Max (Min)}(c^tx)^r - (c^tx)^n, \\ &\text{Subject to } (Ax)^l = (b)^l, (Ax)^m = (b)^m, (Ax)^n \\ &\qquad = (b)^n, (Ax)^r = (b)^r, \\ &(x)^m - (x)^l \geq 0, \ (x)^n - (x)^m \geq 0, \ (x)^r \\ &\qquad -(x)^n \geq 0, \ (x)^l \geq 0. \end{aligned} \tag{4}$$

Step 3: Although (4) is an also MOLP model, it can be effectively keep the trapezoidal fuzzy numbers shape of objective function $\tilde{z}$. There are few standards ways of defining a solution of multi objective programming model. Normally, the lexicographic method will be used to obtain an optimal solution of (4). So, we have:

$$\begin{aligned} &\text{Min (Max)}(c^tx)^m - (c^tx)^l, \\ &\text{Subject to } (Ax)^l = (b)^l, (Ax)^m = (b)^m, (Ax)^n \\ &\qquad = (b)^n, (Ax)^r = (b)^r, \\ &(x)^m - (x)^l \geq 0, \ (x)^n - (x)^m \geq 0, \ (x)^r \\ &\qquad -(x)^n \geq 0, \ (x)^l \geq 0. \end{aligned} \tag{5}$$

If (5) has a unique optimal solution, then it is an optimal solution of (2). Otherwise we proceed to next step.

Step 4: Solve the following problem over the optimal solutions that are found in Step 3 as follows:

$$\begin{aligned} &\text{Max (Min)}(c^tx)^m, \\ &\text{Subject to } (c^tx)^m - (c^tx)^l = Q, \\ &(Ax)^l = (b)^l, (Ax)^m = (b)^m, (Ax)^n \\ &\qquad = (b)^n, (Ax)^r = (b)^r, \\ &(x)^m - (x)^l \geq 0, \ (x)^n - (x)^m \geq 0, \ (x)^r \\ &\qquad -(x)^n \geq 0, \ (x)^l \geq 0. \end{aligned} \tag{6}$$

Where Q is the optimal value of (5). If (6) has a unique solution, then it is also an optimal solution of (2) and stop. Otherwise go to next step.

Step 5: Solve the following problem over the optimal solutions that are found in Step 4 as follows:

$$\begin{aligned}
&\text{Max (Min)}\frac{1}{2}[(c^t x)^m + (c^t x)^n],\\
&\text{Subject to } (c^t x)^m = W,\\
&(c^t x)^m - (c^t x)^l = Q, \qquad (7)\\
&(Ax)^l = (b)^l, (Ax)^m = (b)^m, (Ax)^n\\
&\quad = (b)^n, (Ax)^r = (b)^r,\\
&(x)^m - (x)^l \geq 0,\ (x)^n - (x)^m \geq 0,\ (x)^r\\
&\quad -(x)^n \geq 0,\ (x)^l \geq 0.
\end{aligned}$$

Where W is the optimal value of (6). If (7) has a unique solution, then it is also an optimal solution of Eq. (2) and stop. Otherwise go to Step 6.

Step 6: Solve the following problem over the optimal solutions that are found in Step 5 as follows:

$$\begin{aligned}
&\text{Max (Min)}(c^t x)^r - (c^t x)^n,\\
&\text{Subject to } \frac{1}{2}[(c^t x)^m + (c^t x)^n] = Y,\\
&(c^t x)^m = W,\\
&(c^t x)^m - (c^t x)^l = Q, \qquad (8)\\
&(Ax)^l = (b)^l, (Ax)^m = (b)^m, (Ax)^n\\
&\quad = (b)^n, (Ax)^r = (b)^r,\\
&(x)^m - (x)^l \geq 0,\ (x)^n - (x)^m \geq 0,\ (x)^r\\
&\quad -(x)^n \geq 0,\ (x)^l \geq 0,
\end{aligned}$$

where, Y is the optimal value of (7).

In Step 6, we get an exact optimal solution which is equivalent to (2).

Theorem 3.1 *If $\tilde{x}^* = ((x^*)^l, (x^*)^m, (x^*)^n, (x^*)^r)$ be an optimal solution of (5–8), then it is also an exact optimal solution of (2).*

Proof We only prove the case of maximization, as the case of minimization can be similarly verified. By the method of contradiction, let $\tilde{x}^* = ((x^*)^l, (x^*)^m, (x^*)^n, (x^*)^r)$ be an optimal solution of (5–8), but it is not the exact optimal solution of problem (2). Let us consider $\tilde{x}^0 = (x^0)^l, (x^0)^m, (x^0)^n, (x^0)^r$, then in the case of maximization:

$$(c^t x^*)^l, (c^t x^*)^m, (c^t x^*)^n, (c^t x^*)^r \prec (c^t x^0)^l, (c^t x^0)^m, (c^t x^0)^n, (c^t x^0)^r.$$

Based on Definition 2.8, we have three conditions as follows:

Case (i) Let $(c^t x^*)^m - (c^t x^*)^l \prec (c^t x^0)^m - (c^t x^0)^l$. Furthermore, with respect to the assumption we have:

$$(Ax^0)^l = (\tilde{b})^l,$$
$$(Ax^0)^m = (\tilde{b})^m,$$
$$(Ax^0)^n = (\tilde{b})^n,$$
$$(Ax^0)^r = (\tilde{b})^r,$$

$$(x^0)^m - (x^0)^l \geq 0,\ (x^0)^n - (x^0)^m \geq 0,\ (x^0)^r - (x^0)^n \geq 0,\ (x^0)^l \geq 0.$$

Therefore, $((x^0)^l, (x^0)^m, (x^0)^n, (x^0)^r)$ is a feasible solution of (5) in which the objective value in $((x^0)^l, (x^0)^m, (x^0)^n, (x^0)^r)$ is greater than the objective value in $((x^*)^l, (x^*)^m, (x^*)^n, (x^*)^r)$. But it is contradiction.

Case (ii) Let consider $(c^t x^*)^m - (c^t x^*)^l = (c^t x^0)^m - (c^t x^0)^l$ and $(c^t x^*)^m < (c^t x^0)^m$. Furthermore, with respect to the assumption we have:

$$(Ax^0)^l = (\tilde{b})^l,$$
$$(Ax^0)^m = (\tilde{b})^m,$$
$$(Ax^0)^n = (\tilde{b})^n,$$
$$(Ax^0)^r = (\tilde{b})^r,$$

$$(x^0)^m - (x^0)^l \geq 0,\ (x^0)^n - (x^0)^m \geq 0,\ (x^0)^r - (x^0)^n \geq 0,\ (x^0)^l \geq 0.$$

Therefore, $((x^0)^l, (x^0)^m, (x^0)^n, (x^0)^r)$ is a feasible solution of (6) in which the objective value in $((x^0)^l, (x^0)^m, (x^0)^n, (x^0)^r)$ is greater than the objective value in $((x^*)^l, (x^*)^m, (x^*)^n, (x^*)^r)$. But it is contradiction.

Case (iii) Consider $(c^t x^*)^m - (c^t x^*)^l = (c^t x^0)^m - (c^t x^0)^l$., $(c^t x^*)^m = (c^t x^0)^m$ and $\frac{1}{2}((c^t x^*)^m + (c^t x^*)^n) < \frac{1}{2}((c^t x^0)^m + (c^t x^0)^n)$. Furthermore, with respect to the assumption we have:

$$(Ax^0)^l = (\tilde{b})^l,$$
$$(Ax^0)^m = (\tilde{b})^m,$$
$$(Ax^0)^n = (\tilde{b})^n,$$
$$(Ax^0)^r = (\tilde{b})^r,$$

$$(x^0)^m - (x^0)^l \geq 0,\ (x^0)^n - (x^0)^m \geq 0,\ (x^0)^r - (x^0)^n \geq 0,\ (x^0)^l \geq 0.$$

Therefore, $((x^0)^l, (x^0)^m, (x^0)^n, (x^0)^r)$ is a feasible solution of (7) in which the objective value in $((x^0)^l, (x^0)^m, (x^0)^n, (x^0)^r)$ is greater than the objective value in $((x^*)^l, (x^*)^m, (x^*)^n, (x^*)^r)$. But it is contradiction.

Case (iv) Let $(c^t x^*)^m - (c^t x^*)^l = (c^t x^0)^m - (c^t x^0)^l$., $(c^t x^*)^m = (c^t x^0)^m$, $\frac{1}{2}((c^t x^*)^m + (c^t x^*)^n) = \frac{1}{2}((c^t x^0)^m + (c^t x^0)^n)$ and $(c^t x^*)^r - (c^t x^*)^n < (c^t x^0)^r - (c^t x^0)^n$. Furthermore, with respect to the assumption we have:

$$(Ax^0)^l = (\tilde{b})^l,$$

$$(Ax^0)^m = (\tilde{b})^m,$$
$$(Ax^0)^n = (\tilde{b})^n,$$
$$(Ax^0)^r = (\tilde{b})^r,$$

$$(x^0)^m - (x^0)^l \geq 0,\ (x^0)^n - (x^0)^m \geq 0,\ (x^0)^r - (x^0)^n \geq 0,\ (x^0)^l \geq 0.$$

Therefore, $((x^0)^l, (x^0)^m, (x^0)^n, (x^0)^r)$ is a feasible solution of (8) in which the objective value in $((x^0)^l, (x^0)^m, (x^0)^n, (x^0)^r)$ is greater than the objective value in $((x^*)^l, (x^*)^m, (x^*)^n, (x^*)^r)$. But it is contradiction.

Therefore $\tilde{x}^* = ((x^*)^l, (x^*)^m, (x^*)^n, (x^*)^r)$ is an exact optimal solution of problem (2). □

4 Advantages of the proposed method over the existing methods

The FFLP problem with inequality constraints, in which decision variables are unrestricted fuzzy coefficients or fuzzy variables and the other parameters are represented by trapezoidal fuzzy numbers cannot be solved by any of the existing methods [17, 19] while the proposed method can be used to these situations.

Consider the following examples.

Example 4.1

Maximize $((2, 3, 4, 5) \otimes \tilde{x}_1 \oplus (2, 4, 6, 8) \otimes \tilde{x}_2)$

Subject to

$$(2, 4, 6, 8) \otimes \tilde{x}_1 \oplus (2, 5, 7, 8) \otimes \tilde{x}_2 = (-20, 2, 25, 48),$$
$$(2, 3, 5, 6) \otimes \tilde{x}_1 \oplus (6, 7, 8, 9) \otimes \tilde{x}_2 = (-23, -4, 18, 36),$$

where $\tilde{x}_1$ is non-negative trapezoidal fuzzy number and $\tilde{x}_2$is an unrestricted trapezoidal fuzzy number.

Example 4.2

Maximize $((1, 2, 3, 4) \otimes \tilde{x}_1 \oplus (2, 4, 6, 8) \otimes \tilde{x}_2)$

Subject to

$$(0, 1, 2, 3) \otimes \tilde{x}_1 \oplus (1, 3, 5, 7) \otimes \tilde{x}_2 = (-8, 2, 27, 57),$$
$$(2, 4, 7, 9) \otimes \tilde{x}_1 \oplus (2, 3, 5, 6) \otimes \tilde{x}_2 = (-25, -8, 34, 81),$$

where $\tilde{x}_1$ and $\tilde{x}_2$are unrestricted trapezoidal fuzzy numbers.

In Table 1, we obtain the solution of above examples with three different methods.

We can see that due to limitation of the existing methods of [17, 19] these methods cannot be applied for solving fuzzy optimal solution of the mentioned FFLP problems.

Here, we implement the solution of Example 4.2 by our method.

Table 1 Results of the chosen FFLP problems

Example	Exact fuzzy optimal solution	
	Methods of [17, 19]	Proposed method
4.1	Not applicable	$\tilde{x}_1 = (2, 4, 5, 6)$, $\tilde{x}_2 = (-3, -2, -1, 0)$
4.2	Not applicable	$\tilde{x}_1 = (-3, -2, 1.4, 5)$, $\tilde{x}_2 = (1, 2, 4.84, 6)$

Step 1: Assuming $\tilde{x}_1 = ((x_1)^l, (x_1)^m, (x_1)^n, (x_1)^r)$ and $\tilde{x}_2 = ((x_2)^l, (x_2)^m, (x_2)^n, (x_2)^r)$ the FFLP problems can be written as follows:

$$\text{Max } \tilde{z} = (1, 2, 3, 4)\tilde{x}_1 + (2, 4, 6, 8)\tilde{x}_2 \tag{9}$$

Subject to

$$(0, 1, 2, 3)\tilde{x}_1 + (1, 3, 5, 7)\tilde{x}_2 = (-8, 2, 27, 57),$$
$$(2, 4, 7, 9)\tilde{x}_1 + (2, 3, 5, 6)\tilde{x}_2 = (-25, -8, 34, 81),$$

$\tilde{x}_1, \tilde{x}_2$ are unrestricted trapezoidal fuzzy numbers.

Step 2: Using Step 1, the FFLP problem may be written as follows:

$$\text{Max } \tilde{z} = (1(x_1)^l + 2(x_2)^l,\ 2(x_1)^m + 4(x_2)^m,\ 3(x_1)^n + 6(x_2)^n,\ 4(x_1)^r + 8(x_2)^r$$

Subject to

$$\begin{aligned}
&0(x_1)^l + 1(x_2)^l = -8,\\
&1(x_1)^m + 3(x_2)^m = 2,\\
&2(x_1)^n + 5(x_2)^n = 27,\\
&3(x_1)^r + 7(x_2)^r = 57,\\
&2(x_1)^l + 2(x_2)^l = -25,\\
&4(x_1)^m + 3(x_2)^m = -8,\\
&7(x_1)^n + 5(x_2)^n = 34,\\
&9(x_1)^r + 6(x_2)^r = 81,
\end{aligned} \tag{10}$$

where $((x_1)^l, (x_1)^m, (x_1)^n, (x_1)^r)$ and $((x_2)^l, (x_2)^m, (x_2)^n, (x_2)^r)$ are unrestricted trapezoidal fuzzy umbers.

Step 3: Based on Step 2, the above problem in Step 1 is converted to the MOLP problem as follows:

$$\text{Min } z_1 = 2(x_1)^m + 4(x_2)^m - (1(x_1)^l + 2(x_2)^l),$$
$$\text{Max } z_2 = 2(x_1)^m + 4(x_2)^m,$$
$$\text{Max } z_3 = \frac{1}{2}\left(2(x_1)^m + 4(x_2)^m + 3(x_1)^n + 6(x_2)^n\right),$$
$$\text{Max } z_4 = 4(x_1)^r + 8(x_2)^r - \left(3(x_1)^n + 6(x_2)^n\right),$$

Subject to

$$0(x_1)^l + 1(x_2)^l = -8,$$

$$1(x_1)^m + 3(x_2)^m = 2,$$

$$\begin{aligned} &2(x_1)^n + 5(x_2)^n = 27, \\ &3(x_1)^r + 7(x_2)^r = 57, \\ &2(x_1)^l + 2(x_2)^l = -25, \\ &4(x_1)^m + 3(x_2)^m = -8, \\ &7(x_1)^n + 5(x_2)^n = 34, \\ &9(x_1)^r + 6(x_2)^r = 81, \end{aligned} \tag{11}$$

where $((x_1)^l, (x_1)^m, (x_1)^n, (x_1)^r)$ and $((x_2)^l, (x_2)^m, (x_2)^n, (x_2)^r)$ are unrestricted trapezoidal fuzzy umbers.

Using steps 3, 4, 5 and 6 , the optimal solution of problem (4) is achieved as follows:

$$\tilde{x}^* = \begin{cases} \tilde{x}_1^* = \left((x_1^*)^l,\ (x_1^*)^m,\ (x_1^*)^n,\ (x_1^*)^r\right) = (-3, -2, 1.4, 5), \\ \tilde{x}_2^* = \left((x_2^*)^l,\ (x_2^*)^m,\ (x_2^*)^n,\ (x_2^*)^r\right) = (1, 2, 4.84, 6), \end{cases}$$

The optimal value of the objective function is obtained. Therefore, the optimal value of problem (2) may be written as follows:

$$\tilde{z} = (z_1,\ z_2,\ z_3,\ z_4) = (-1, 4, 33.24, 68).$$

For the case of FFLP with inequality constraints, non-negative variables and trapezoidal fuzzy numbers, we consider the Example 7.3 of [17] and obtain three different solutions from three methods. The results are obtained in Table 2.

From Table 2, we can see that the ranking result of the proposed method is higher than the other existing methods.

5 Application of proposed method in real life problems

In this section, to show the application of proposed method, we test this method for two real life problems. One is production problem and other is diet problem. Furthermore, we compare our method with the existing methods of [9, 18].

5.1 A production problem [9]

A company produces three products p_1, p_2 and p_3. These products are processed on three different machines M_1, M_2 and M_3. The time required manufacturing one unit of each product and the daily capacity of the machines are given below:

Data of each product and daily capacity of machines:

Time per unit (minutes)				
Machines	p_1	p_2	p_3	Machine capacity (min/day)
M_1	12	13	12	490
M_2	14	–	13	470
M_3	12	15	–	480

Note that the time availability can vary from day to day due to break down of machines, overtime work etc. Finally the profit for each product can also vary due to variations in price. At the same time the company wants to keep the profit somewhat close to Rs. 14 for p_1, Rs. 13 for p_2, and Rs. 16 for p_3. The company wants to determine the range of each product per day to maximize its profit. It is assumed that all the amounts produced are consumed in the market.

Since the profit from each product and the time availability on each machine is uncertain, the number of units to be produced on each product will also be uncertain. So we will model the problem as a fully fuzzy linear programming problem. We use a trapezoidal fuzzy number for each uncertain value.

The parameters and variables are also modeled as trapezoidal fuzzy numbers taking into the nature of the problem and other requirements. So we formulate the given fully fuzzy linear programming problem as:

$$\begin{aligned} &\text{Max } \tilde{z} = (13, 15, 2, 2)\tilde{x}_1 + (12, 14, 3, 3) \\ &\tilde{x}_2 + (15, 17, 2, 2)\tilde{x}_3 \\ &\text{Subject to } (11, 13, 2, 2)\tilde{x}_1 + (12, 14, 1, 1) \\ &\tilde{x}_2 + (11, 13, 22)\tilde{x}_3 \le (475, 505, 6, 6), \\ &(12, 16, 1, 1)\tilde{x}_1 + (12, 14, 1, 1)\tilde{x}_3 \le (460, 480, 8, 8), \\ &(11, 13, 2, 2)\tilde{x}_1 + (14, 16, 3, 3)\tilde{x}_2 \le (465, 495, 5, 5), \\ &\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \ge 0. \end{aligned} \tag{12}$$

Using Step 1, the FFLP problem may be written as follows:

$$\begin{aligned} \text{Max } \tilde{z} = (&13(x_1)^l + 12(x_2)^l + 15(x_3)^l,\ 15(x_1)^m + 14(x_2)^m \\ &+17(x_3)^m,\ 2(x_1)^n + 3(x_2)^n + 2(x_3)^n,\ 2(x_1)^r \\ &+3(x_2)^r + 2(x_3)^r \end{aligned}$$

subject to

$$11(x_1)^l + 12(x_2)^l + 11(x_3)^l \le 475,$$

$$13(x_1)^m + 14(x_2)^m + 13(x_3)^m \le 505,$$

$$\begin{aligned} &2(x_1)^n + (x_2)^n + 2(x_3)^n \le 6, \\ &2(x_1)^r + (x_2)^r + 2(x_3)^r \le 6, \\ &12(x_1)^l + 12(x_3)^l \le 460, \\ &16(x_1)^m + 14(x_3)^m \le 480, \end{aligned} \tag{13}$$

Table 2 Results of the chosen FFLP problems

Fuzzy optimal solution	Method of [18]	Method of [14]	Proposed method
$\tilde{x}_1^*$	(0, 0, 0, 3.2)	(0, 0, 0, 3.2)	(0, 0, 0, 3)
$\tilde{x}_2^*$	(0.94, 0.94, 0, 0)	(0.92, 0.95, 0, 0)	(1.5, 1.5, 0, 0.5)
$\Re$(x)	6.8	6.7	7.2

$(x_1)^n + (x_3)^n \leq 8,$
$(x_1)^r + (x_3)^r \leq 8,$
$11(x_1)^l + 14(x_2)^l \leq 465,$
$13(x_1)^m + 16(x_2)^m \leq 495,$
$2(x_1)^n + 3(x_2)^n \leq 5,$
$2(x_1)^r + 3(x_2)^r \leq 5,$
$(x)^m - (x)^l \geq 0,\ \ (x)^n - (x)^m \geq 0,\ (x)^r - (x)^n \geq 0,\ (x)^l \geq 0.$

Based on Step 2, the above problem in Step 1 is converted to the MOLP problem as follows:

$$\begin{aligned}
\text{Min } z_1 &= 15(x_1)^m + 14(x_2)^m + 17(x_3)^m \\
&\quad - \left(13(x_1)^l + 12(x_2)^l + 15(x_3)^l\right), \\
\text{Max } z_2 &= 15(x_1)^m + 14(x_2)^m + 17(x_3)^m, \\
\text{Max } z_3 &= \frac{1}{2}\left(15(x_1)^m + 14(x_2)^m + 17(x_3)^m + 2(x_1)^n \right.\\
&\quad \left. +3(x_2)^n + 2(x_3)^n\right), \\
\text{Max } z_4 &= 2(x_1)^r + 3(x_2)^r + 2(x_3)^r - (2(x_1)^n + 3(x_2)^n \\
&\quad +2(x_3)^n),
\end{aligned}$$

Subject to

$$\begin{aligned}
&11(x_1)^l + 12(x_2)^l + 11(x_3)^l \leq 475, \\
&13(x_1)^m + 14(x_2)^m + 13(x_3)^m \leq 505, \\
&2(x_1)^n + (x_2)^n + 2(x_3)^n \leq 6, \qquad (14)\\
&2(x_1)^r + (x_2)^r + 2(x_3)^r \leq 6, \\
&12(x_1)^l + 12(x_3)^l \leq 460, \\
&16(x_1)^m + 14(x_3)^m \leq 480, \\
&(x_1)^n + (x_3)^n \leq 8, \\
&(x_1)^r + (x_3)^r \leq 8, \\
&11(x_1)^l + 14(x_2)^l \leq 465, \\
&13(x_1)^m + 16(x_2)^m \leq 495, \\
&2(x_1)^n + 3(x_2)^n \leq 5, \\
&2(x_1)^r + 3(x_2)^r \leq 5, \\
&(x)^m - (x)^l \geq 0,\ \ (x)^n - (x)^m \geq 0,\ (x)^r - (x)^n \geq 0,\ (x)^l \geq 0.
\end{aligned}$$

Using Step 3, 4, 5 and 6, the optimal solution of problem (14) is achieved as follows:

$$\tilde{x}^* = \begin{cases} \tilde{x}_1^* = \left((x_1^*)^l,\ (x_1^*)^m,\ (x_1^*)^n,\ (x_1^*)^r\right) = (0,\ 0,\ 0,\ 0), \\ \tilde{x}_2^* = \left((x_2^*)^l,\ (x_2^*)^m,\ (x_2^*)^n,\ (x_2^*)^r\right) = (4.23,\ 4.23,\ 1.66,\ 1.66), \\ \tilde{x}_3^* = \left((x_3^*)^l,\ (x_3^*)^m,\ (x_3^*)^n,\ (x_3^*)^r\right) = (34.28,\ 34.28,\ 2,\ 2), \end{cases}$$

The optimal value of the objective function is obtained. Therefore, the optimal value of problem (2) may be written as follows:

$$\tilde{z} = (z_1,\ z_2,\ z_3,\ z_4) = (564.96, 680.98, 80.98, 80.98).$$

Now, using the Kumar's method [18] the fuzzy value of the objective function is:

$$\tilde{z} = (z_1,\ z_2,\ z_3,\ z_4) = (594.1, 675.08, 85.26, 85.26).$$

Using the Ganesan's method [9] the fuzzy value of the objective function is:

$$\tilde{z} = (z_1,\ z_2,\ z_3,\ z_4) = (557.6, 711.6, 110.27, 110.27).$$

Based on ranking number, by comparing the results of fuzzy value of optimal solution of proposed method with the existing method [9, 18], we conclude that our proposed method is more effective because:

$$\begin{aligned} 622.97 &= (\tilde{z})_{\text{proposed method}} < (\tilde{z})_{\text{Kumar's method}} = 634.59 \\ &< (\tilde{z})_{\text{Ganesan's method}} = 634.6. \end{aligned}$$

In the proposed methodology the FFLP problem is solved by using LINGO Version 11.0. Accordingly obtained optimal solution by both the methods are also depicted in Fig. 1.

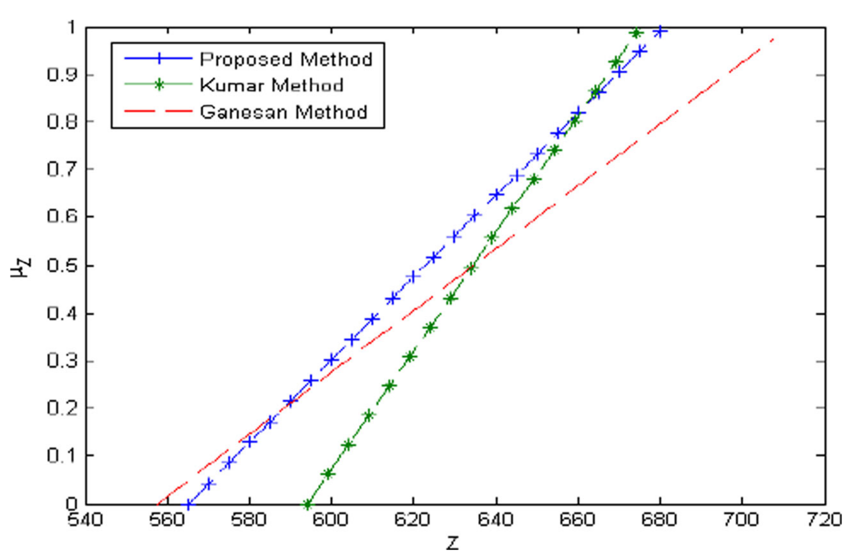

Fig. 1 Membership function of the optimal solution by the present method and existing methods [9, 18]

Table 3 Costs, nutrients and predetermined nutritional restrictions

Nutrient	Milk (glass)	Salad (500 mg)	Minimum nutrient required
Vitamin A	(13, 15, 2, 2)	(11, 13, 4, 4)	(100, 102, 6, 6)
Calcium	(14, 16, 2, 2)	(12, 14, 4, 4)	(112,114, 8, 8)
Cost/unit	(10, 14, 3, 3)	(13, 15, 5, 5)	

5.2 Diet problem

In TATA Main Hospital Jamshedpur, India have two nutritional experiments (Vitamin A and Calcium) with two products Milk (glass) and Salad (500 mg). Table 3 summarizes the quantity of each nutrient available in foods and their daily requirement for good health conditions of an individual, as well as the unitary cost of these foods. The objective is to maximize the total diet cost and comply with nutritional restrictions. Since the cost/ unit from each nutrient is uncertain, the number of units to be produced on each product will also be uncertain. So we will model the problem as a fully fuzzy linear programming problem. We use a trapezoidal fuzzy number for each uncertain value.

The parameters and variables are also modeled as trapezoidal fuzzy numbers taking into the nature of the problem and other requirements.

Decision-making variables:

$\tilde{x}_1$ = Quantity of milk (in glass)
$\tilde{x}_2$ = Quantity of salad (in 500 mg)

So we formulate the given fully fuzzy linear programming problem as:

$$\begin{aligned} \text{Max } \tilde{z} = \;& (10, 14, 3, 3)\tilde{x}_1 + (13, 15, 5, 5)\tilde{x}_2 \\ \text{Subject to } \;& (13, 15, 2, 2)\tilde{x}_1 + (11, 13, 4, 4)\tilde{x}_2 \leq (100, 102, 6, 6), \\ & (14, 16, 2, 2)\tilde{x}_1 + (12, 14, 4, 4)\tilde{x}_2 \leq (112, 114, 8, 8), \\ & \tilde{x}_1, \quad \tilde{x}_2 \geq 0. \end{aligned} \tag{15}$$

Using Step 1, the FFLP problem may be written as follows:

$$\begin{aligned} \text{Max } \tilde{z} = \;& (10(x_1)^l + 13(x_2)^l,\ 14(x_1)^m + 15(x_2)^m,\ 3(x_1)^n \\ & +5(x_2)^n,\ 3(x_1)^r + 5(x_2)^r \end{aligned}$$

Subject to

$$\begin{aligned} & 13(x_1)^l + 11(x_2)^l \leq 100, \\ & 15(x_1)^m + 13(x_2)^m \leq 102, \\ & 2(x_1)^n + 4(x_2)^n \leq 6, \\ & 2(x_1)^r + 4(x_2)^r \leq 6, \\ & 14(x_1)^l + 12(x_2)^l \leq 112, \\ & 16(x_1)^m + 14(x_2)^m \leq 114, \\ & 2(x_1)^n + 4(x_2)^n \leq 8, \\ & 2(x_1)^r + 4(x_2)^r \leq 8, \\ & (x)^m - (x)^l \geq 0,\ \ (x)^n - (x)^m \geq 0,\ (x)^r - (x)^n \geq 0,\ (x)^l \geq 0. \end{aligned} \tag{16}$$

Based on Step 2, the above problem in Step 1 is converted to the MOLP problem as follows:

$$\begin{aligned} \text{Min } z_1 &= 14(x_1)^m + 15(x_2)^m - \left(10(x_1)^l + 13(x_2)^l\right), \\ \text{Max } z_2 &= 14(x_1)^m + 15(x_2)^m, \\ \text{Max } z_3 &= \frac{1}{2}\left(14(x_1)^m + 15(x_2)^m + 3(x_1)^n + 5(x_2)^n\right), \\ \text{Max } z_4 &= 3(x_1)^r + 5(x_2)^r - \left(3(x_1)^n + 5(x_2)^n\right), \end{aligned}$$

Subject to

$$\begin{aligned} & 13(x_1)^l + 11(x_2)^l \leq 100, \\ & 15(x_1)^m + 13(x_2)^m \leq 102, \\ & 2(x_1)^n + 4(x_2)^n \leq 6, \\ & 2(x_1)^r + 4(x_2)^r \leq 6, \\ & 14(x_1)^l + 12(x_2)^l \leq 112, \\ & 16(x_1)^m + 14(x_2)^m \leq 114, \\ & 2(x_1)^n + 4(x_2)^n \leq 8, \\ & 2(x_1)^r + 4(x_2)^r \leq 8, \\ & (x)^m - (x)^l \geq 0,\ \ (x)^n - (x)^m \geq 0,\ (x)^r - (x)^n \geq 0,\ (x)^l \geq 0. \end{aligned} \tag{17}$$

Using Step 3, 4, 5 and 6, the optimal solution of problem (17) is achieved as follows:

$$\tilde{x}^* = \begin{cases} \tilde{x}_1^* = \left((x_1^*)^l,\ (x_1^*)^m,\ (x_1^*)^n,\ (x_1^*)^r\right) = (3,\ 3,\ 3,\ 3), \\ \tilde{x}_2^* = \left((x_2^*)^l,\ (x_2^*)^m,\ (x_2^*)^n,\ (x_2^*)^r\right) = (4.38,\ 4.38,\ 0,\ 0), \end{cases}$$

The optimal value of the objective function is obtained. Therefore, the optimal value of problem (2) may be written as follows:

$$\tilde{z} = (z_1,\ z_2,\ z_3,\ z_4) = (86.94, 95.7, 15, 15).$$

Now, using the Kumar's method [18] the fuzzy value of the objective function is:

$$\tilde{z} = (z_1,\ z_2,\ z_3,\ z_4) = (64.34, 76.7, 7.5, 7.5).$$

Now, using the Ganesan's method [8] the fuzzy value of the objective function is:

$$\tilde{z} = (z_1,\ z_2,\ z_3,\ z_4) = (85, 103, 8, 8).$$

Based on ranking number, by comparing the results of fuzzy value of optimal solution of proposed method with the existing method [9, 18], we conclude that our proposed method is more effective because:

$$\begin{aligned} 70.53 \;=\; & (\tilde{z})_{\text{Kumar's method}} < 91.32 = (\tilde{z})_{\text{proposed method}} \\ & < (\tilde{z})_{\text{Ganesan's method}} = 94. \end{aligned}$$

In the proposed methodology the FFLP problem is solved by using LINGO Version 11.0. Accordingly obtained optimal solution by both the methods are also depicted in Fig. 2

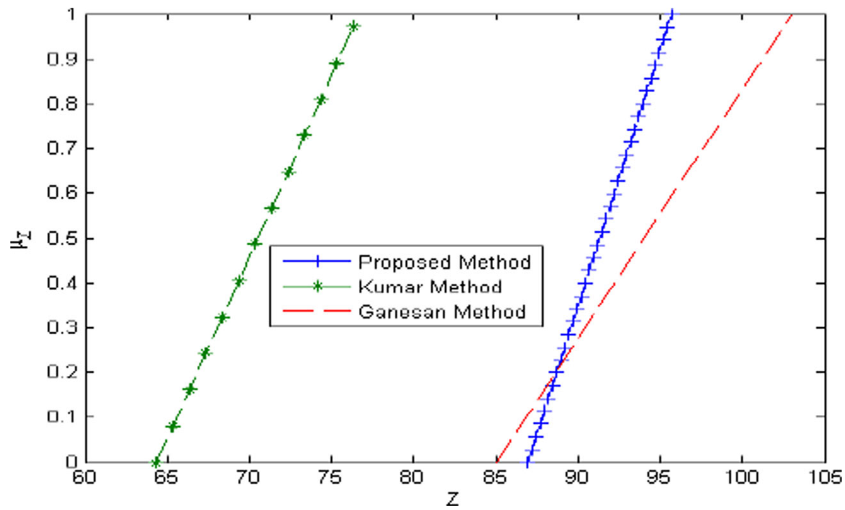

Fig. 2 Membership function of the optimal solution by the present method and existing methods [9, 18]

5.3 A company production planning

The data collected from an owner of a regional Alloy company (data is provided with a legal agreement that the name of the company will not be disclosed) situated in Balasore (Odisha, India) is shown in Table 4.

An alloy producer, produces 2 types of alloys P_1 and P_2. These alloys consists of Zinc and Tin (F_1, F_2) used in per unit of alloys. The availability of alloys are depend on its production but production depends on men, machine etc. of alloys are not known exactly due to long power cut, labour's over time work, unexpected failures in machine etc. The transportation cost of daily supplies Zinc and Tin is not known exactly due to variations in rates of fuel, traffic problems etc. So, all the parameters of the production company are uncertain quantities with hesitation. According to past experience of the owner, the daily supplies of metal is represented by trapezoidal fuzzy numbers in Table 4. The average cost of per kilogram of P_1 and P_2 are (8, 12, 2, 2) and (5, 7, 1, 1) units, respectively. The maximum daily supplyes metals F_1 and F_2 are approximately (46, 52, 2, 2) and (42, 48, 4, 4) units, respectively. The producer wants to know, in order to minimize the cost of Zinc and Tin, how many kilograms of alloys P_1 and P_2 he must produce daily?

Table 4 The data of daily supplies of metals and alloys

	Products	
Metal	P1	P2
F_1	(1,1,1,1)	(5,7,1,1)
F_2	(3,5,2,2)	(1,3,2,2)

Now if we assume that the fuzzy variables $\tilde{x}_1$ and $\tilde{x}_2$ are the daily supplyes metals of F_1 and F_2 respectively, then this can be formulated as follows:

$$\text{Max } \tilde{z} = (8, 12, 2, 2)\tilde{x}_1 + (5, 7, 1, 1)\tilde{x}_2$$

$$\text{Subject to } (1, 1, 1, 1)\ \tilde{x}_1 + (5,7, 1, 1)\ \tilde{x}_2 \leq (46, 52, 2, 2),$$

$$(3, 5, 2, 2)\tilde{x}_1 + (1, 3, 2, 2)\tilde{x}_2 \leq (42, 48, 4, 4), \tag{18}$$

$$\tilde{x}_1, \tilde{x}_2 \geq 0.$$

Similarly as above, using the proposed method, the fuzzy optimal solution of this considered FFLP problem can be obtained as:

$$\tilde{x}^* = \begin{cases} \tilde{x}_1^* = \left((x_1^*)^l,\ (x_1^*)^m,\ (x_1^*)^n,\ (x_1^*)^r\right) = (6.72, 8.9, 1.27, 1.27), \\ \tilde{x}_2^* = \left((x_2^*)^l,\ (x_2^*)^m,\ (x_2^*)^n,\ (x_2^*)^r\right) = (6.18,\ 7.54,\ 0.54,\ 0.54), \end{cases}$$

The optimal value of the objective function is obtained. Therefore, the optimal value of problem (2) may be written as follows:

$$\tilde{z} = (z_1,\ z_2,\ z_3,\ z_4) = (104.36, 134.36, 16, 16).$$

Now, using the Kumar's method [18] the fuzzy value of the objective function is:

$$\tilde{z} = (z_1,\ z_2,\ z_3,\ z_4) = (102, 131.95, 11, 11).$$

Now, using the Ganesan's method [8] the fuzzy value of the objective function is:

$$\tilde{z} = (z_1,\ z_2,\ z_3,\ z_4) = (102, 131.95, 11, 11).$$

Based on ranking number, by comparing the results of fuzzy value of optimal solution of proposed method with the existing methods [9, 18], we have the following results:

$$\begin{aligned} 119.36 &= \Re(\tilde{z})_{\text{proposed method}} > \Re(\tilde{z})_{\text{Ganesan's method}} \\ &= \Re(\tilde{z})_{\text{ Kumar's method}} = 116.975. \end{aligned}$$

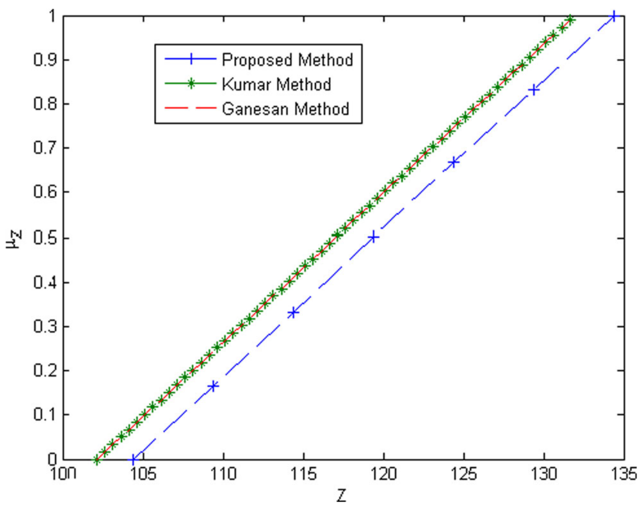

Fig. 3 Membership function of the optimal solution by the present method and existing methods [9, 18]

In the proposed methodology the FFLP problem is solved by using LINGO Version 11.0. Accordingly obtained optimal solution by both the methods are also depicted in Fig. 3.

Now if we analyze optimal solution of the considered above three real life problems, we note that our proposed method was 100 % successful, numerically. But, the method proposed in [9, 18], failed in 15 % of the problems numerically. Therefore, our proposed method and solution are also more suitable than the method and the solution proposed in [9, 18]. The iteration and Elapsed times of the proposed method in [9, 18] are greater than our proposed method. All the problems are solved by LINGO. Therefore, from the above real life problem and the above discussion, we can conclude that our proposed method is more robust than the method proposed in [9, 18]. Also, our proposed approximate solution is significantly more suitable than the approximate solution proposed in [9, 18].

The difference between proposed method and existing methods [9, 18] membership function is depicted in Figs. 1, 2 and 3. If so, the decision maker may consider prosed method membership function as a more acceptable one most of the time. The difference between these two models is caused from the shape of membership functions.

6 Conclusions

In this paper, a new model has been designed to solve the FFLP problem. Based on new lexicographic ordering on trapezoidal fuzzy numbers, we proposed the auxiliary MOLP model to solve the corresponding linear programming. The proposed scheme presented promising results from the aspects of both computing efficiency and performance. It is our belief that the proposed method for solution of FFLP problem in real life problems as well as simple problem may be considerable for mathematician working in this field.

Although the developed method was illustrated using an production problem, diet problem and company production planning problem, it will be expected to be applicable to real-life decision problems in many areas, such as risk investment, engineering management, supply chain management, transportation problem.

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References

1. Zadeh LA (1965) Fuzzy sets. Inf Control 8:338–353
2. Bellman RE, Zadeh LA (1970) Decision making in a fuzzy environment. Manage Sci 17:141–164
3. Zimmerman HJ (1978) Fuzzy programming and linear programming with several objective functions. Fuz Sets and syst 1:45–55
4. Campos L, Verdegay JL (1989) Linear programming problems and ranking of fuzzy numbers. Fuz Sets and Syst 32:1–11
5. We HC (2008) Using the technique of scalarization to solve the multi objective programming problems with fuzzy coefficients. Math Comput Model 48:232–248
6. Das S, Mandal T (2015) A single stage single constraints linear fractional programming problem. Oper Res and Appl: An Intern J 2:1–5
7. Maleki HR, Tata M, Mashinchi M (2000) Linear programming with fuzzy variables. Fuzzy Sets Syst 109:21–33
8. Ramik J (2005) Duality in fuzzy linear programming: some new concepts and results. Fuzzy Optim Decis Making 4:25–39
9. Ganesan K, Veeramani P (2006) Fuzzy linear programs with trapezoidal fuzzy numbers. Ann Oper Res 143:305–315
10. Ebrahimnejad A, Tavana M (2014) A novel method for solving linear programming problems with symmetric trapezoidal fuzzy numbers. Appl Math Modell 38:4388–4395
11. Wan SP, Dong JY (2014) Possibility linear programming with trapezoidal fuzzy numbers. Appl Math Model 38:1660–1672
12. Buckley JJ, Feuring T (2000) Evalutionary algorithm solution to fuzzy problems: fuzzy linear programming. Fuzzy Sets Syst 109:35–53
13. Hashemi SM, Modarres M, Nasrabadi E, Nasrabadi MM (2006) Fully fuzzified linear programming, solution and duality. J Intell Fuzzy Syst 17:253–261
14. Allahviranloo T, Lotfi FH, Kiasary MK, Kiani NA, Alizadeh L (2008) Solving full fuzzy linear programming problem by the ranking function. Appl Math Sci 2:19–32
15. Dehghan M, Hashemi B, Ghatee M (2006) Computational methods for solving fully fuzzy linear systems. Appl Math Comput 179:328–343
16. Lotfi FH, Allahviranloo T, Jondabeha MA, Alizadeh L (2009) Solving a fully fuzzy linear programming using lexicography method and fuzzy approximate solution. Appl Math Model 33:3151–3156
17. Kumar A, Kaur J, Singh P (2011) A new method for solving fully fuzzy linear programming problems. Appl Math Model 35:817–823

18. Kumar A, Kaur J (2011) A new method for fuzzy linear programming programs with trapezoidal fuzzy numbers. J Fuzzy Valued Syst Annal. doi:10.5899/2011/jfsva-00102
19. Kumar A, Singh P (2012) A new method for solving fully fuzzy linear programming problems. Annal of Fuzzy Math and Inform 3:103–118
20. Saberi Najafi H, Edalatpanah SA (2013) A note on "A new method for solving fully fuzzy linear programming problems". Appl Math Model 37:7865–7867
21. Veeramani C, Duraisamy C (2012) Solving fuzzy linear programming problem using symmetric fuzzy number approximation. Inter J Oper Res 15:321–336
22. Ezzati R, khorram E, Enayati R (2015) A new algorithm to solve fully fuzzy linear programming problems using the MOLP problem. Appl Math Model 39:3183–3193
23. Kaufmann A, Gupta MM (1985) Introduction to fuzzy arithmetic theory and applications. Van Nostrand Reinhold, New York

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