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A fuzzy extended analytic network process-based approach for global supplier selection

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Abstract With increasing globalization, supplier selection has become more and more important than before. In the process of determining the best supplier, the expert judgements might be vague or incomplete due to the inherent uncertainty and imprecision of their perception. In addition to that, the sub-criteria are relevant to each other in the selection of right supplier. In this paper, a novel methodology based on fuzzy set theory and analytic network process (FEANP) is developed to address both the uncertain information involved and the interrelationships among the attributes. This paper concludes with a case study describing the implementation of this model for a real-world supplier selection scenario. We demonstrate the efficiency of the proposed model by comparing with existing method.

Keywords Supplier selection · Analytic network process · Fuzzy set theory

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1 Introduction

Nowadays, the companies or enterprises have to work with different suppliers to perform their activities. Strategic partnership with better suppliers can promote the companies' potential advantages over others: improved quality, flexibility, profitability and business performance, reduced lead time. As a result supplier selection problem has received considerable attention from both academia and industry. The major objective of this problem is to cut down the supply chain risk, optimize inventory levels and manufacturing process as well as maximizing customer satisfaction and revenue [1, 10].

From the perspective of decision making, supplier selection is a multi-criteria decision making (MCDM) problem in the presence of many criteria and sub-criteria. Up to date, a number of MCDM methods have been proposed in the literature [8, 24, 32]. These methods include analytic hierarchy process (AHP) [5, 6, 12, 27], analytic network process (ANP) [21, 41], strategy fuzzy simple multi-attribute rating technique (SMART) [10], grey relational analysis (GRA) [23, 37, 43], TOPSIS [4, 30] and others [11, 19, 42].

Among them, AHP is a structured technique for organizing and analyzing complex decisions, which was developed by Saaty in the 1970s. It has been extensively studied and refined for multi-attribute decision making problems since then. For example, [17] developed an AHP model to assess the different dimensions of supplier environmental performance. [16] built a structure framework for determining the key capabilities based on the AHP. In spite of its popularity and simple concepts, AHP is often criticized for its inability to deal with interactions and dependencies across the entities involved in the decision making problems [36]. Analytic network process (ANP) is an alternative approach in replace of AHP. It overcomes the limitations of AHP and provides the ability to handle the dependencies and interactions across the elements at various levels. It has been widely used to deal with supplier selection problems [15, 44]. For example, Kuo and Lin [26] combined ANP with data envelopment analysis (DEA) technique to provide a consistent and reasonable technique for high-tech industry evaluation. Gencer and Gürpinar [15] put forward an ANP-based method to address the supplier selection criteria in a feedback system and implemented it in an electronic company.

Another issue in supplier selection problems is how to deal with the uncertain information in supplier selection problems [20, 40]. How to handle imprecise information involved in this process effectively is still an open issue. Several theories have been proposed to deal with this problem [3, 9, 18, 22, 28, 45, 48, 49]. Among them, the fuzzy set theory, which was introduced by Zadeh [46] is an efficient tool to handle this problem. Due to its flexibility in handling uncertain information, the fuzzy sets theory is widely used in many decision under uncertain problems [29, 31, 33]. Deng et al. [13] combined Dempster-Shafer theory of evidence (DST) and fuzzy sets theory (FST) to deal with the supplier selection problem. Kuo [25] presented an effective approach based on combining VIKOR, GRA, and interval-valued fuzzy sets to evaluate service quality of Chinese cross-strait passenger airlines via customer surveys.

Although many approaches are available, some problems still exist. In the method proposed by [13], Dempster-Shafer theory is unable to address the dependencies among the attributes while in practical applications, it is very common for one criterion to be dependent on the other. For example, the quality of a car is in association with its price. Although some frameworks have been presented to address this problem [2, 14], they raise other problems [39], including expensive computation, and quesionable idempotency requirement. The method proposed by [25] requires the construction of interval-valued fuzzy sets and consumes large amounts of data. In practical applications, it is expensive or impossible to produce so much data. In the method proposed by [26], the DEA technique is insufficient to process the imprecise data or information involved in the process of determining the optimal decision alternative. Although the method proposed by [15] accounts for the relationships across supplier selection criteria, it overlooks the uncertain information involved in this process.

Considering the deficiencies in the existing approaches, we are motivated to propose a more general framework to address the supplier selection problem. Here, we propose a new MCDM method called generalized fuzzy extended analytic network process (FEANP). In our method, triangular fuzzy numbers are applied to build pairwise comparison matrices according to the linguistic comparisons provided by the experts. According to ANP, we formulate a supermatrix composed by the weights of the corresponding attribute. After its convergence, the weight associated with each attribute can be obtained. Based on the overall objective index, the right supplier is determined. Finally, the proposed method is implemented and its efficiency is demonstrated by comparing with existing method.

The contributions of the proposed method are two-fold. On the one hand, the proposed method can handle the epistemic uncertainty during the decision making process. For example, experts' judgements and preferences on the alternatives might be uncertain because the evaluation criteria are subjective and qualitative in nature. On the other hand, by accounting for the dependencies across the criteria, our method is quite general and applicable to real-world problems since dependencies are quite common in many real-world problems.

The rest of the paper is structured as follows. Section 2 introduces basic theories including analytical hierarchy process, fuzzy sets theory, and analytic network process. Section 3 details the proposed method. A real-world case is used to illustrate the method and the results of the application are discussed in Section 4. Section 5 ends the paper with concluding remarks.

2 Preliminaries

In this section, basic concepts related to the analytical hierarchy process, fuzzy sets theory, and analytic network process are briefly introduced.

2.1 Analytical Hierarchy Process [35]

The first step of AHP is to establish a hierarchical structure of the decision problem. Then, in each hierarchical level, a nominal scale is used to construct a pairwise comparison judgement matrix.

Definition 1 Assuming $(E_1, \cdots, E_i, \cdots, E_n)$ are n decision elements, the pairwise comparison judgement matrix is denoted as $M_{n\times n} = [m_{ij}]$, which satisfies:

$$m_{ij} = \frac{1}{m_{ji}} \tag{1}$$

where each element m_{ij} represents the judgment concerning the relative importance of decision element E_i over E_j.

With the matrix constructed, the third step is to calculate the eigenvector of the matrix.

Definition 2 The eigenvectors of the $n \times n$ pairwise comparison judgement matrix can be denoted as: $\mathrm{w} = (w_1, \cdots, w_i, \cdots, w_n)^T$, and calculated as follows:

$$A\mathrm{w} = \lambda_{\max}\mathrm{w}, \quad \lambda_{\max} \geq n \tag{2}$$

where $\lambda_{\max}$ is the maximum eigenvalue in the eigenvector $\vec{w}$ of matrix $M_{n\times n}$.

Before we transform the eigenvector into the weights of elements, the consistency of the matrix should be checked.

Definition 3 A consistency index(CI) [35] is used to measure the inconsistency within each pairwise comparison judgement matrix, which is formulated as follows:

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{3}$$

Accordingly, the consistency ratio(CR) can be calculated as:

$$CR = \frac{CI}{RI} \tag{4}$$

where RI is the random consistency index. The value of RI is related to the dimension of the matrix, which is listed in Table 1.

If the value of CR is less than 0.1, the consistency of the pairwise comparison matrix M is acceptable. Moreover, the eigenvectors of the pairwise comparison judgement matrix can be normalized as final weights of decision elements. Otherwise, the consistency check is not successful and the elements in the matrix should be revised.

2.2 Fuzzy sets

In 1965, the notion of fuzzy sets was firstly introduced by Zadeh [47], providing a natural way of dealing with problems in which the source of imprecision is the absence of a sharply defined criterion of class membership.

A brief introduction of fuzzy sets is given as follows.

Definition 4 A fuzzy set A is defined on a universe X may be given as:

$$A = \{\langle x, \mu_A(x)\rangle \mid x \in X\}$$

Table 1 The value of RI (random consistency index)

dimension	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46	1.49

where $\mu_A : X \to [0, 1]$ is the membership function A. The membership value $\mu_A(x)$ describes the degree of belongingness of $x \in X$ in A.

For a finite set $A = \{x_1, \ldots, x_i, \ldots, x_n\}$, the fuzzy set (A, m) is often denoted by $\{^{\mu_A(x_1)}/_{x_1}, \ldots, ^{\mu_A(x_i)}/_{x_i}, \ldots, ^{\mu_A(x_n)}/_{x_n}\}$.

Definition 5 A fuzzy number is a special fuzzy set. A fuzzy number A on $\Re$ is defined to be a triangular fuzzy number if its membership function $\mu_{\tilde{A}}(x) : \Re \to [0, 1]$ is equal to

$$\mu_{\tilde{A}} = \begin{cases} (x-l)/(m-l), & l \leq x \leq m \\ (u-l)/(u-m), & m \leq x \leq u \\ 0, & otherwise \end{cases}$$

where l and u represent the lower and upper bounds of the fuzzy number $\tilde{A}$, respectively, and m is the median value. The triangular fuzzy number (TFN) can be denoted as $\tilde{A} = (l, m, u)$.

In Fig. 1, $N1$, $N3$, $N5$, $N7$ and $N9$ are used to represent the pairwise comparison of decision variables from " Equal" to "Absolute", and TFNs $N2$, $N4$, $N6$ and $N8$ represent the middle preference values between them.

2.3 Analytic network process

The analytic network process (ANP) is a generalization of the analytic hierarchy process (AHP) used in multi-criteria decision analysis. AHP structures a decision problem into a hierarchy with a goal, decision criteria, and alternatives, while the ANP structures it as a network. In this way, ANP can model complex decision problems, where a hierarchical model as used in AHP is not sufficient [34].

The process of ANP involves three substeps, which are shown as follows [38].

- Without assuming the interdependence among criteria, the decision makers are asked to evaluate the proposed criteria pairwise. They responded to questions such as:" which criteria should be emphasized more in determining the supplier, how much more?". The responses were presented numerically and scaled on the basis of Saaty's 1-9 scale. A reciprocal value will be automatically assigned to the reverse comparison. Once the pairwise comparisons are completed, we can get the the local weight vector $w1$ according to the following equation.

  $$\mathbf{A}w_1 = \lambda_{\max} w_1$$

  where $\lambda_{\max}$ is the largest eigenvalue of the pairwise comparison matrix $\mathbf{A}$. The obtained weight vector will

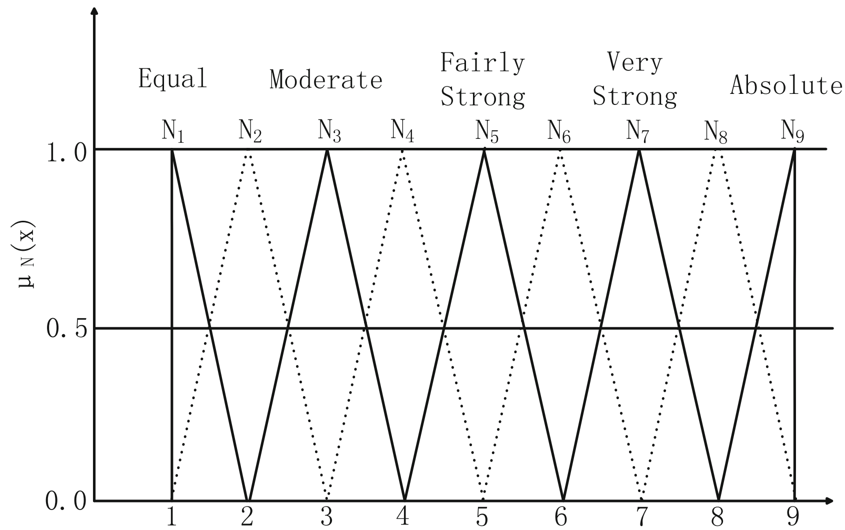

Fig. 1 Nine fuzzy numbers

be normalized by dividing each value by its column total to obtain the normalized local weight vector $w2$.

– In what follows, we need to resolve the effects of the interdependence that exists between the evaluation criteria. The decision makers are asked to examine the impact of all the criteria on each other by using pairwise comparisons as well. They responded to questions such as: "which criteria affect criterion 1 more: criterion 2 or criterion 3? how much more?" Various pairwise comparison matrices are formulated for each criterion. These pairwise comparison matrices are used to identify the relative impacts of criteria

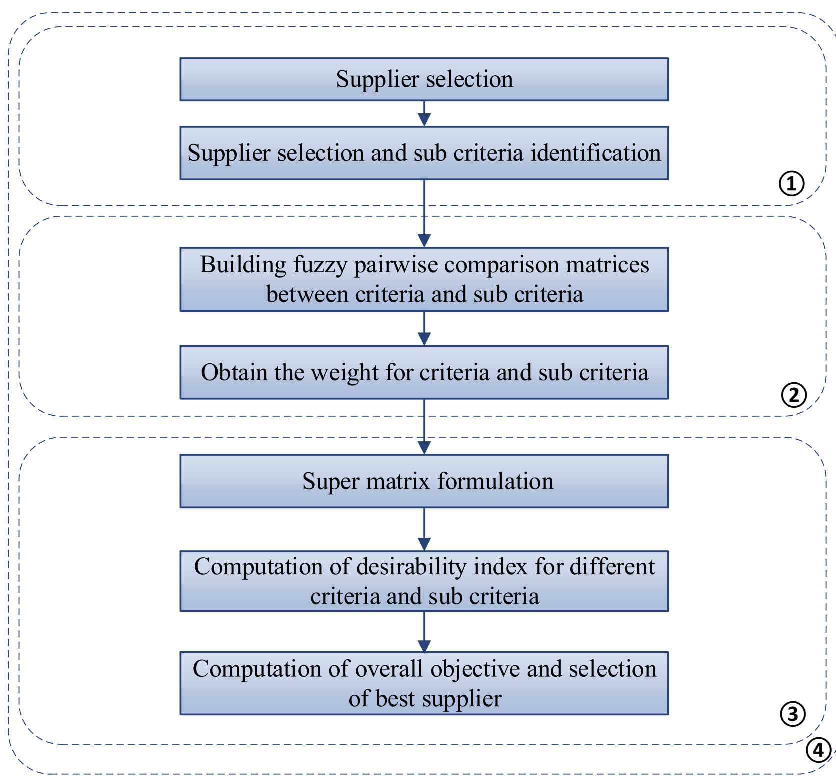

Fig. 2 The flowchart of the proposed method

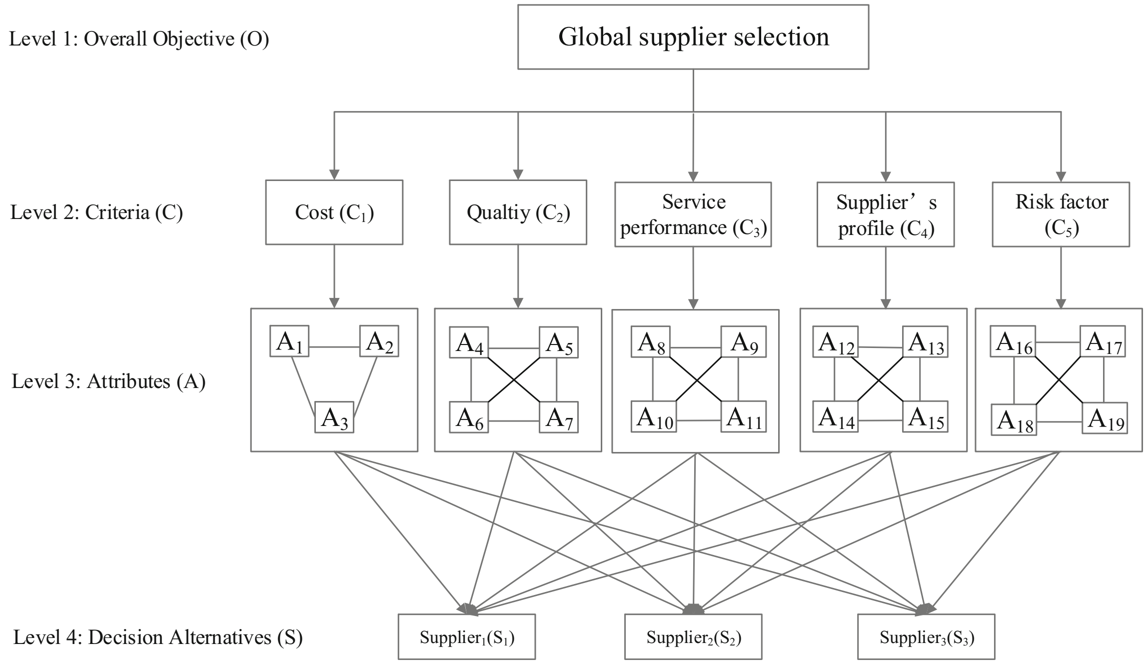

Fig. 3 Hierarchy for the global supplier selection

interdependent relationships. The normalized principal eigenvectors for these matrices are calculated and shown as column component in the interdependence weight matrix of criteria B, where zeros are assigned to the eigenvector weights of the criteria from which a given criterion is given.

– Now we can obtain the interdependence priorities of the criteria by synthesizing the results from the previous two steps as follows:

$$w_c = Bw_2^T$$

3 FEANP: A new multi-criterion decision making methodology

Generally speaking, there is one issue that needs to be solved when implementing FEANP methodology. It is how to determine the weights according to the fuzzy pairwise comparison judgement matrix. In this section, the general description of the new FEANP is briefly introduced first. Then, the new proposed methodology is detailed step by step.

3.1 Framework of FEANP

As shown in Fig. 2, the method begins with identifying criteria and sub-criteria. Then, the dependencies and interactions across the criteria are constructed. In what follows, the fuzzy pairwise matrices from each expert are built according to the expertise. After constructing these preference values, the weights of each criteria and sub criteria can be obtained. The super matrix is formulated by the weights of the criteria. Then, the desirability index for different criteria and sub criteria can be obtained. Finally, the best supplier in the decision alternatives can be selected.

Table 2 The fuzzy evaluation matrix with respect to the overall objective

	C1	C2	C3	C4	C5	W_o
C1	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)	(5/2,3,7/2)	(5/2,3,7/2)	0.45
C2	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)	(5/2,3,7/2)	(5/2,3,7/2)	0.35
C3	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)	0.09
C4	(2/7,1/3,2/5)	(2/7,1/3,2/5)	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)	0.06
C5	(2/7,1/3,2/5)	(2/7,1/3,2/5)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1,1,1)	0.05

Table 3 The fuzzy evaluation matrix with respect to criterion C_1

C_1	A_1	A_2	A_3	W_{C_1}
A_1	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)	0.58
A_2	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)	0.30
A_3	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1,1,1)	0.12

3.2 Supplier selection and sub-criteria identification

To evaluate supplier selection problem, first it is required to carefully identify all possible elements and factors that need to be inspected and assessed. For example, the supplier selection process can be characterized in terms of the suppliers' quality and services, which can be further assessed in terms of more concrete factors. It is very costly to examine and assess all of these elements on a regular basis. The key issue in this step is to ensure that all supplier factors that need to be considered are included, while those requiring extensive identification and evaluation effort but having little impact on supplier selection are excluded. Furthermore, the interrelationships among decision levels and attributes should be constructed during the process.

3.3 Priority calculation at different levels

Suppose $X = \{x_1, x_2, \cdots, x_n\}$ is an object set and $O = \{o_1, o_2, \cdots, o_n\}$ is an objective set. According to the method proposed by Chang [7], each object is taken and extent analysis for each goal is performed respectively. Therefore, m extent analysis values for each object can be obtained with the following signs:

$$M_{g_i}^1, M_{g_i}^2, \cdots, M_{g_i}^m, i = 1, 2, \cdots, n \tag{5}$$

where all the $M_{g_i}^j$ $(j = 1, 2, \cdots, n)$ are triangular fuzzy numbers.

The value of fuzzy synthetic extent with respect to the ith object is defined as:

$$S_i = \sum_{j=1}^{m} M_{g_i}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{g_i}^j \right]^{-1} \tag{6}$$

The degree of possibility of $M_1 \geq M_2$ is defied as:

$$V(M_1 \geq M_2) = \sup_{x \geq y} \left\lfloor \min\left(\mu_{M_1}(x), \mu_{M_2}(y)\right) \right\rfloor \tag{7}$$

When a pair (x, y) exists such that $x \geq y$ and $\mu_{M_1}(x) = \mu_{M_2}(y)$, then we have $V(M_1 \geq M_2) = 1$. Since N_1 and N_2 are convex fuzzy numbers we have:

$$\begin{aligned} &V(M_1 \geq M_2) = 1 \quad iff\ m_1 \geq m_2, \\ &V(M_1 \geq M_2) = hgt(M_1 \cap M_2) = \mu_{M_1}(d) \end{aligned} \tag{8}$$

where d is the ordinate of the highest intersection point D between μ_{M_1} and μ_{M_2}.

When $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$, the ordinate of D is given by (9):

$$\begin{aligned} V(M_1 \geq M_2) &= hgt(M_1 \cap M_2) \\ &= \frac{l_1 - u_1}{(m_2 - u_2) - (m_1 - l_1)} \end{aligned} \tag{9}$$

For the comparision of $M - 1$ and M_2, both the values of $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$ are required.

The degree possibility for a convex fuzzy number to be greater than k convex fuzzy numbers M_i $(i = 1, 2, \cdots, k)$ can be defined by:

$$\begin{aligned} &V(M \geq M_1, M_2, \cdots, M_k) = V[(M \geq M_1) \\ &\ and\ (M \geq M_2\ and \ldots and\ (M \geq M_k))] \\ &= \min V(M \geq M_i), i = 1, 2, \cdots, k. \end{aligned} \tag{10}$$

Assume that:

$$d'(A_i) = \min V(S_i \geq S_k) \tag{11}$$

For $k = 1, 2, \ldots, n; k \neq i$. Then the weight vector is given by:

$$W' = \left(d'(A_1), d'(A_2), \ldots, d'(A_n)\right)^T \tag{12}$$

Table 4 The fuzzy evaluation matrix with respect to criterion C_2

C_2	A_4	A_5	A_6	A_7	W_{C_2}
A4	(1,1,1)	(3/2,2,5/2)	(2/3,1,3/2)	(5/2,3,7/2)	0.3633
A5	(2/5,1/2,2/3)	(1,1,1)	(2/3,1,3/2)	(3/2,2,5/2)	0.2355
A6	(2/3,1,3/2)	(2/3,1,3/2)	(1,1,1)	(3/2,2,5/2)	0.2753
A7	(2/7,1/3,2/5)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1,1,1)	0.1259

Table 5 The fuzzy evaluation matrix with respect to criterion C_3

C_3	A_8	A_9	A_{10}	A_{11}	W_{C_3}
A_8	(1,1,1)	(3/2,2,5/2)	(5/2,3,7/2)	(7/2,4,9/2)	0.4503
A_9	(2/5,1/2,2/3)	(1,1,1)	(5/2,3,7/2)	(5/2,3,7/2)	0.3061
A_{10}	(2/7,1/3,2/5)	(2/7,1/3,2/5)	(1,1,1)	(3/2,2,5/2)	0.1377
A_{11}	(2/9,1/4,2/7)	(2/7,1/3,2/5)	(2/5,1/2,2/3)	(1,1,1)	0.1059

Table 6 The fuzzy evaluation matrix with respect to criterion C_4

C_4	A_{12}	A_{13}	A_{14}	A_{15}	W_{C_4}
A_{12}	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)	(7/2,4,9/2)	0.4873
A_{13}	(2/5,1/2,2/3)	(1,1,1)	(2/5,1/2,2/3)	(3/2,2,5/2)	0.2076
A_{14}	(2/5,1/2,2/3)	(2/7,1/3,2/5)	(1,1,1)	(3/2,2,5/2)	0.2
A_{15}	(2/9,1/4,2/7)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1,1,1)	0.1051

where A_i $(i = 1, 2, \ldots, n)$ are n elements.

After normalizing W', the normalized weight vectors are:

$$W = (d(A_1), d(A_2), \ldots, d(A_n))^T \quad (13)$$

where W is a nonfuzzy number and this gives the priority weights of one alternative over another.

In this way, the weights of the criteria at different levels can be obtained even though they are represented by triangular fuzzy numbers.

3.4 Super matrix formulation

The super matrix represents the interdependencies that exist among the elements of a system. The super matrix is constructed from the pairwise comparison matrices of interdependencies. According to the super matrix after convergence, the final weight of each criterion considering the interdependencies can be acquired. The super matrix after convergence reflects the relative importance measures of every criterion when taking into consideration the interdependencies.

3.5 Computation of desirability index

The desirability index can be calculated as:

$$D_i = \sum_{j=1}^{j} \sum_{k=1}^{k} P_j A_{kj}^D A_{kj}^l S_{ikj} \quad (14)$$

where P_j denotes the relative importance of criterion j; A_{kj}^D represents the relative importance of sub criterion k of the criteria j for the dependency (D) relationships. A_{kj}^l expresses the stabilized importance weight for sub criterion k of the criterion j for interdependency (I) relationships; S_{ikj} denotes the relative impact of the supplier alternative i on sub criterion k of the criterion j in the system. In this way, the desirability index for every supplier can be constructed.

Table 7 The fuzzy evaluation matrix with respect to criterion C_5

C_5	A_{16}	A_{17}	A_{18}	A_{19}	W_{C_5}
A_{16}	(1,1,1)	(2/3,1,3/2)	(2/3,1,3/2)	(2/3,1,3/2)	0.2450
A_{17}	(2/3,1,3/2)	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)	0.3420
A_{18}	(2/3,1,3/2)	(2/5,1/2,2/3)	(1,1,1)	(3/2,2,5/2)	0.2493
A_{19}	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1,1,1)	0.1637

Table 8 Summary combination of priority weights: attributes of criterion C_1

	A_1	A_2	A_3	Alternative
Weight	0.58	0.30	0.12	priority weight
Alternatives				
S_1	0.71	0.44	0.69	0.63
S_2	0.13	0.36	0.08	0.19
S_3	0.16	0.20	0.23	0.18

3.6 Selection of suppliers

The overall objective index can be calculated by normalising the total desirability index for different suppliers. Based on the overall objective index, the best supplier in the supplier alternatives can be selected.

4 Application of FEANP

In this section, a numerical example originated from [5] is presented to illustrate the efficiency of the proposed method.

Owing to the large number of factors affecting the supplier selection decision, an orderly sequence of steps should be required to tackle it. The framework considered in this paper consists of five supplier selection criteria, namely cost, quality, service performance, suppliers profile, and risk factor. The various criteria and sub criteria are shown in Fig. 3.

In the model, the criterion cost (C_1) has three attributes: product price (A_1), freight cost (A_2) and tariff and custom xsduties (A_3). The criterion quality (C_2) has four

Table 9 Summary combination of priority weights: attributes of criterion C_2

	A_4	A_5	A_6	A_7	Alternative
Weight	0.41	0.21	0.26	0.12	priority weight
Alternatives					
S_1	0.51	0.51	0.69	0.87	0.60
S_2	0.23	0.23	0.08	0.00	0.16
S_3	0.26	0.26	0.23	0.13	0.24

Table 10 Summary combination of priority weights: attributes of criterion C_3

	A_8	A_9	A_{10}	A_{11}	Alternative priority
Weight	0.43	0.23	0.29	0.05	weight
Alternatives					
S_1	0.27	0.69	0.05	0.49	0.31
S_2	0.18	0.08	0.64	0.32	0.30
S_3	0.55	0.23	0.31	0.19	0.39

Table 12 Summary combination of priority weights: attributes of criterion C_5

	A_{16}	A_{17}	A_{18}	A_{19}	Alternative priority
Weight	0.27	0.43	0.30	0.00	weight
Alternatives					
S_1	0.83	0.45	0.69	0.33	0.65
S_2	0.17	0.45	0.08	0.33	0.19
S_3	0.00	0.10	0.23	0.34	0.16

factors: rejection rate of the product (A_4), increased lead time (A_5), quality assessment (A_6) and remedy for quality problems (A_7). The service performance criterion (C_3) has four attributes: delivery schedule (A_8), technological and R&D support (A_9), response to changes (A_{10}) and ease of communication (A_{11}). The supplier's profile criterion (C_4) consist of four criteria: financial status (A_{12}), customer base (A_{13}), performance history (A_{14}) and production facility and capacity (A_{15}). The risk factor (C_5) has four attributes: geographical location (A_{16}), political stability (A_{17}), economy (A_{18}) and terrorism (A_{19}).

In the original model used in [5], the authors do not consider the interdependencies among the attributes shown in level 3 of Fig. 3. In this paper, it is assumed that these attributes are related with each other. For example, product price (A_1) is related to the freight cost (A_2) and tariff and custom duties (A_3). Political stability (A_{17}) is influenced by the geographical location (A_{16}), economy (A_{18}), and terrorism (A_{19}).

Next, the fuzzy pairwise comparison of elements at each level is conducted with respect to their relative influence towards their control criterion using triangular fuzzy numbers. The fuzzy evaluation matrix with respect to the overall objective is shown in Table 2. The final weight vector in Table 2 is calculated according to the method presented in Section 3.3. It is observed from Table 2 that the fuzzy relative importance of cost (C_1) when compared to quality (C_2) in achieving the overall objective is (3/2,2,5/2). It is also observed that the criterion cost (C_1) has the maximum influence (0.45) on the overall objective. On the contrary, risk factor (C_5) has the minimum effect (0.05) on the overall objective. Similarly, the fuzzy pairwise comparison matrices and the weight vectors of each attribute in level 3 are shown in Tables 3, 4, 5, 6 and 7.

In a similar way, the fuzzy assessment matrices of decision alternatives with respect to corresponding attributes can be constructed. For criterion C_1, the summary combination of weights is listed in Table 8 by adding the weights per supplier multiplied by weights of the corresponding attributes. The results for the other attributes are shown in Tables 9, 10, 11 and 12.

Then, in order to capture the interdependencies existing in the attributes, AHP is used to construct pairwise comparisons. One such comparison is presented in Table 13. It shows that the importance of each attribute over other attributes when product price (A_1) is regarded as the controlling criterion, which is different from the application in [5]. These values are used in the formulation of the super matrix shown in column A_1 in Table 14. The super matrix allows to conduct a systematic analysis on all the attributes. The data in the super matrix is imported from the pairwise comparison matrices of interdependencies (Table 13). There are 19 pairwise comparison matrices of interdependencies in total. Each of the non-zero column in Table 14 shows the relative importance weight associated with the interdependent pairwise comparison matrices.

During the next stage, the super matrix is made to converge to obtain a stable set of weights. By raising the power of the super matrix to $2k + 1$, where k is an arbitrary number, the super matrix will converge to a stable value. The super matrix after convergence is shown in Table 15. Next, the desirability index is calculated. In

Table 11 Summary combination of priority weights: attributes of criterion C_4

	A_{12}	A_{13}	A_{14}	A_{15}	Alternative priority
Weight	0.61	0.06	0.21	0.12	weight
Alternatives					
S_1	0.83	0.45	0.69	0.33	0.72
S_2	0.17	0.45	0.08	0.33	0.19
S_3	0.00	0.10	0.23	0.34	0.09

Table 13 Pair wise comparison matrix for enablers under cost (C_1) and product price (A_1)

	A_2	A_3	Weight
A_2	1	2	0.6667
A_3	2	1	0.3333

Table 14 Super matrix before convergence

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	A_{18}	A_{19}
A_1	0	0.0170	0.0506																
A_2	0.6667	0	0.2920																
A_3	0.3333	0.0541	0																
A_4				0	0.2542	0.7112	0.7211												
A_5				0.0167	0	0.0323	0.2699												
A_6				0.6596	0.6962	0	0.0080												
A_7				0.3236	0.0496	0.2564	0												
A_8								0	0.5584	0.7221	0.6589								
A_9								0.1413	0	0.2699	0.0376								
A_{10}								0.5732	0.0970	0	0.3034								
A_{11}								0.2855	0.3446	0.0080	0								
A_{12}												0	0.2689	0.0190	0.3333				
A_{13}												0.3333	0	0.3119	0.3333				
A_{14}												0.3333	0.6681	0	0.3333				
A_{15}												0.3333	0.0630	0.6691	0				
A_{16}																0	0.0190	0.3333	0.0190
A_{17}																0.3767	0	0.3333	0.3119
A_{18}																0.0768	0.3119	0	0.6691
A_{19}																0.5465	0.6691	0.3333	0

Table 15 Super matrix after convergence

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	A_{18}	A_{19}
A_1	0.1640	0.1640	0.1640																
A_2	0.4437	0.4437	0.4437																
A_3	0.3923	0.3923	0.3923																
A_4				0.3971	0.3971	0.3971	0.3971												
A_5				0.0744	0.0744	0.0744	0.0744												
A_6				0.3154	0.3154	0.3154	0.3154												
A_7				0.2131	0.2131	0.2131	0.2131												
A_8								0.3998	0.3998	0.3998	0.3998								
A_9								0.1418	0.1418	0.1418	0.1418								
A_{10}								0.2931	0.2931	0.2931	0.2931								
A_{11}								0.1653	0.1653	0.1653	0.1653								
A_{12}												0.1647	0.1647	0.1647	0.1647				
A_{13}												0.2450	0.2450	0.2450	0.2450				
A_{14}												0.3115	0.3115	0.3115	0.3115				
A_{15}												0.2788	0.2788	0.2788	0.2788				
A_{16}																0.1135	0.1135	0.1135	0.1135
A_{17}																0.2484	0.2484	0.2484	0.2484
A_{18}																0.3074	0.3074	0.3074	0.3074
A_{19}																0.3307	0.3307	0.3307	0.3307

Table 16 Desirability index computed for different suppliers based on the five criteria

Criteria	$Sub-Criteria$	P_j	A_{kj}^D	A_{kj}^I	S_{1kj}	S_{2kj}	S_{3kj}	$Supplier_1(S_1)$	$Supplier_2(S_2)$	$Supplier_3(S_3)$
C_1	A_1	0.45	0.58	0.1640	0.63	0.19	0.18	0.0270	0.0081	0.0077
	A_2	0.45	0.30	0.4437	0.63	0.19	0.18	0.0377	0.0114	0.0108
	A_3	0.45	0.12	0.3923	0.63	0.19	0.18	0.0133	0.0040	0.0038
C_2	A_4	0.35	0.41	0.3971	0.60	0.16	0.24	0.0342	0.0091	0.0137
	A_5	0.35	0.21	0.0744	0.60	0.16	0.24	0.0033	0.0009	0.0013
	A_6	0.35	0.26	0.3154	0.60	0.16	0.24	0.0172	0.0046	0.0069
	A_7	0.35	0.12	0.2131	0.60	0.16	0.24	0.0054	0.0014	0.0021
C_3	A_8	0.09	0.43	0.3998	0.31	0.30	0.39	0.0048	0.0046	0.0060
	A_9	0.09	0.23	0.1418	0.31	0.30	0.39	0.0009	0.0009	0.0011
	A_{10}	0.09	0.29	0.2931	0.31	0.30	0.39	0.0024	0.0023	0.0030
	A_{11}	0.09	0.05	0.1653	0.31	0.30	0.39	0.0002	0.0002	0.0003
C_4	A_{12}	0.06	0.61	0.1647	0.72	0.19	0.09	0.0043	0.0011	0.0005
	A_{13}	0.06	0.06	0.2450	0.72	0.19	0.09	0.0006	0.0002	0
	A_{14}	0.06	0.21	0.3115	0.72	0.19	0.09	0.0028	0.0007	0.0004
	A_{15}	0.06	0.12	0.2788	0.72	0.19	0.09	0.0014	0.0004	0.0002
C_5	A_{16}	0.05	0.27	0.1135	0.65	0.19	0.16	0.0010	0.0003	0.0002
	A_{17}	0.05	0.43	0.2484	0.65	0.19	0.16	0.0035	0.0010	0.0009
	A_{18}	0.05	0.30	0.3074	0.65	0.19	0.16	0.0030	0.0009	0.0008
	A_{19}	0.05	0.00	0.3307	0.65	0.19	0.16	0	0	0
Total								0.1631	0.0522	0.0598

Table 16, the values of the third column are the relative importance of the five criteria with respect to the overall objective. These values have been imported from Table 2. The values in the fourth column are the relative importance of each attribute in influencing the five criteria (C_1, C_2, C_3, C_4, C_5). Also, these values have been imported from Tables 3–7. The values in the fifth column is the stable weight of the super matrix after convergence shown in Table 15. The next three columns are the summary combination of priority weights of the three suppliers, which are imported from Tables 8–12. The last three columns give the relative impact of each supplier in the supplier selection network. For the sake of illustration, the value corresponding to $Supplier_1(S_1)$ for the sub criteria A_1 is $0.0270\ (0.45 * 0.58 * 0.1640 * 0.63)$.

In order to make a final decision, the overall objective is calculated, which is obtained by normalizing the total desirability index for different suppliers. The result is shown Table 17. Figure 4 compares the presented approach with Chan and Kumar's method [5]. As we can see, the best supplier is S_1, which is the same as the result in [5]. On the other hand, the interdependencies play an important role in selecting the best supplier. The priority weights for supplier S_1 is 0.5929, which is 0.0071 less than the result of Chan and Kumar's method. Also, the values for supplier S_2 and S_3 are also different from that of Chan and Kumar's. The differences of the results between the proposed method and Felix's approach reflect the role that the interdependencies are playing in the model. Specifically here, it can be seen that there is a 7.1 % level of combined interdependencies in the priority weights for supplier S_1. For

Table 17 Overall objective for various suppliers

	Total desirability index	Overall objective
$Supplier_1(S_1)$	0.1631	0.5929
$Supplier_2(S_2)$	0.0522	0.1897
$Supplier_3(S_3)$	0.0598	0.2174

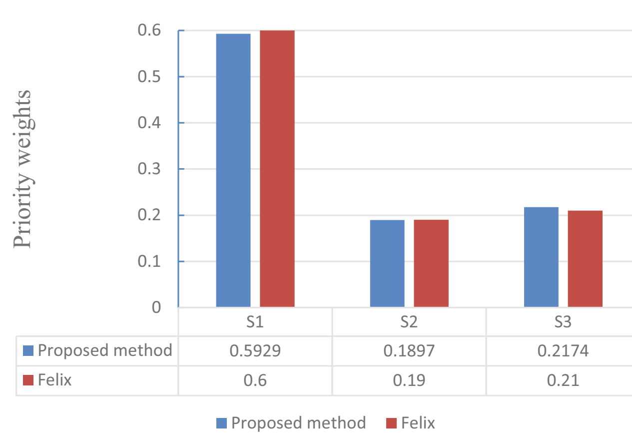

Fig. 4 Priority weights for various suppliers

supplier S_2, there is a 0.3 % level of combined interdependencies on the final result. For supplier S_3, the level of combined interdependencies is 7.4 %.

The advantage of the proposed approach is the control given to the decision makers. For example, they are able to give their preferences according to the knowledge they have. The triangular fuzzy numbers express the uncertain information efficiently. By applying FEANP, both the uncertain information and the interdependencies are taken into consideration in the decision model.

5 Conclusions

In this paper, a fuzzy extended analytic network process (FEANP) methodology is developed to deal with the supplier selection problem. Our approach has two features: one is that it can address the epistemic uncertainty in the available information. The other one is that it is capable of processing the interrelationships across the evaluation criteria. These two characteristics promote its future application to real-world problems. This method provides a new insight into solving the supplier problem in a more generalized way. In the near future, we will investigate its application in more complicated environments, such as considering demand uncertainty effect on the supplier selection process, supplier selection with the consideration of dynamic environment.

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