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A novel similarity measure on intuitionistic fuzzy sets with its applications

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Abstract The intuitionistic fuzzy set, as a generation of Zadeh' fuzzy set, can express and process uncertainty much better, by introducing hesitation degree. Similarity measures between intuitionistic fuzzy sets (IFSs) are used to indicate the similarity degree between the information carried by IFSs. Although several similarity measures for intuitionistic fuzzy sets have been proposed in previous studies, some of those cannot satisfy the axioms of similarity, or provide counter-intuitive cases. In this paper, we first review several widely used similarity measures and then propose new similarity measures. As the consistency of two IFSs, the proposed similarity measure is defined by the direct operation on the membership function, non-membership function, hesitation function and the upper bound of membership function of two IFS, rather than based on the distance measure or the relationship of membership and non-membership functions. It proves that the proposed similarity measures satisfy the properties of the axiomatic definition for similarity measures. Comparison between the previous similarity measures and the proposed similarity measure indicates that the proposed similarity measure does not provide any counter-intuitive cases. Moreover, it is demonstrated that the proposed similarity measure is capable of discriminating the difference between patterns.

Keywords Intuitionistic fuzzy set · Distance measure · Similarity measure · Pattern recognition

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1 Introduction

Since it was proposed by Zadeh [1], the theory of fuzzy set (FS) has achieved a great success due to its capability of handling uncertainty. Therefore, over the last decades, several higher order fuzzy sets have been introduced in the literature. Intuitionistic fuzzy set (IFS), as one of the higher order fuzzy sets, was proposed by Atanassov [2] to deal with vagueness. The main advantage of the IFS is its property to cope with the uncertainty that may exist due to information impression. Because it assigns to each element a membership degree, a non-membership degree and a hesitation degree, and thus, IFS constitutes an extension of Zadeh's fuzzy set which only assigns to each element a membership degree [3]. So IFS is regarded as a more effective way to deal with vagueness than fuzzy set. Although Gau and Buehrer later presented vague set [4], it was pointed out by Bustince and Burillo that the notion of vague sets was the same as that of IFS [5].

The definition of similarity measure between two IFSs is one of the most interesting topics in IFSs theory. A similarity measure is defined to compare the information carried by IFSs. Measures of similarity between IFSs, as an important tool for decision making, pattern recognition, machine learning, and image processing, has received much attention in recent years [6–27]. Among the similarity measures proposed, a few of them come from the well-known distance measures [8–12], such as the Hamming distance, the Euclidian distance and the Hausdorff distance. Other similarity measures are defined based on the linear or non-linear relationship of the membership and non-membership functions of IFSs [13–20]. There are also other kinds of similarity measures, e.g., similarity defined by entropy measures for IFSs, similarity induced by interval comparison and cosine similarity [7, 21–27].

If taking a closer examination on the existing similarity measures between IFSs, we can find that some of those cannot fully satisfy the axiomatic definition of similarity by providing counter-intuitive cases; others are lack of definitude physical meaning with complicated expressions. Therefore, the definition of similarity measure is still an open problem achieving more interest. In this paper, we propose a new similarity measures with relative simple expression. The proposed similarity measure can be considered as the consistency of two IFSs. We define it by the direct operation on the membership function, non-membership function, hesitation function and the upper bound of membership function of two IFS, rather than defining it based on the distance measure or the relationship of membership and non-membership functions. The computation of our proposed similarity involves operations of multiplication and evolution without choosing other parameters, which is relatively simple and concise. Illustrative examples reveal that the proposed measures satisfy the properties of the axiomatic definition for similarity measures. In addition, several comparative examples are provided to show the performance of the proposed similarity measure.

The remainder of this paper is organized as follows. Section 2 presents the definitions related to the IFSs, similarity measure between IFSs, and existing similarity measures together. The new similarity measure, along with its interpretations is presented in Section 3. Comparison between the proposed similarity measure and the existing similarity measures is carried out in Section 4. The application of the proposed similarity measure to pattern recognition is presented in Section 5, followed by the conclusion of this paper in Section 6.

2 Intuitionistic fuzzy set and similarity measures

In this section, we firstly recall the basic definitions related to IFS and similarity measure to facilitate subsequent interpretation. Critical analyses on the existing similarity measures are then presented.

2.1 Basic definitions

Definition 1 Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universe of discourse, then a fuzzy set A in X is defined as follows:

$$A = \{\langle x, \mu_A(x)\rangle \,|x \in X\} \tag{1}$$

where $\mu_A(x) : X \to [0, 1]$ is the membership degree.

Definition 2 An IFS A in X defined by Atanassov can be written as:

$$A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle \,|x \in X\} \tag{2}$$

where $\mu_A(x) : X \to [0, 1]$ and $\upsilon_A(x) : X \to [0, 1]$ are membership degree and non-membership degree, respectively, with the condition:

$$0 \le \mu_A(x) + \upsilon_A(x) \le 1 \tag{3}$$

$\pi_A(x)$ determined by the following expression:

$$\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x) \tag{4}$$

is called the hesitancy degree of the element $x \in X$ to the set A, and $\pi_A(x) \in [0, 1], \forall x \in X$.

$\pi_A(x)$ is also called the intuitionistic index of x to A. Greater $\pi_A(x)$ indicates more vagueness on x. Obviously, when $\pi_A(x) = 0, \forall x \in X$, the IFS degenerates into an ordinary fuzzy set.

In the sequel, the couple $\langle\mu_A(x), \upsilon_A(x)\rangle$ is called an IFS or intuitionistic fuzzy value (IFV) for clarity. Let $IFSs(X)$ denote the set of all IFSs in X.

It is worth noting that besides **Definition 2** there are other possible representations of IF sets proposed in the literature. Hong and Choi [29] proposed to use an interval representation $[\mu_A(x), 1 - \upsilon_A(x)]$ of intuitionistic fuzzy set A in X instead of pair $\langle\mu_A(x), \upsilon_A(x)\rangle$. This approach is equivalent to the interval valued fuzzy sets interpretation of IF set, where $\mu_A(x)$ and $1 - \upsilon_A(x)$ represent the lower bound and upper bound of membership degree, respectively. Obviously, $[\mu_A(x), 1 - \upsilon_A(x)]$ is a valid interval, since $\mu_A(x) \le 1 - \upsilon_A(x)$ always holds for $\mu_A(x) + \upsilon_A(x) \le 1$.

Definition 3 For $A \in IFSs(X)$ and $B \in IFSs(X)$, some relations between them are defined as:

(R1) $A \subseteq B$ iff $\forall x \in X \mu_A(x) \le \mu_B(x), \upsilon_A(x) \ge \upsilon_B(x)$;
(R2) $A = B$ iff $\forall x \in X \mu_A(x) = \mu_B(x), \upsilon_A(x) = \upsilon_B(x)$;
(R3) $A^C = \{\langle x, \upsilon_A(x), \mu_A(x)\rangle \,|x \in X\}$, where A^C is the complement of A.

Definition 4 Let D denote a mapping $D : IFS \times IFS \to [0, 1]$, if $D(A, B)$ satisfies the following properties, $D(A, B)$ is called a distance between $A \in IFSs(X)$ and $B \in IFSs(X)$.

(DP1) $0 \le D(A, B) \le 1$;
(DP2) $D(A, B) = 0$, if and only if $A = B$;
(DP3) $D(A, B) = D(B, A)$;
(DP4) If $A \subseteq B \subseteq C$, then $D(A, B) \le D(A, C)$, and $D(B, C) \le D(A, C)$.

Definition 5 A mapping $S : IFS \times IFS \to [0, 1]$ is called a degree of similarity between $A \in IFSs(X)$ and $B \in IFSs(X)$, if $S(A, B)$ satisfies the following properties:

(SP1) $0 \le S(A, B) \le 1$;
(SP2) $S(A, B) = 1$, if and only if $A = B$;
(SP3) $S(A, B) = S(B, A)$;

(SP4) If $A \subseteq B \subseteq C$, then $S(A, B) \geq S(A, C)$, and $S(B, C) \geq S(A, C)$.

Because distance and similarity measures are complementary concepts, similarity measures can be used to define distance measures, and vice versa.

2.2 Existing similarity measures

Because of the relationship between distance measure and similarity measure for IFSs, they are always defined together. For the sake of convenience, we would not differentiate these two concepts in this subsection.

The first study was carried out by Szmidt and Kacprzyk [8] extending the well-known distances measures, such as the Hamming Distance, the Euclidian Distance, to IFS and comparing them with the approaches used for ordinary fuzzy sets. However, Wang and Xin [9] implied that the distance measure of Szmidt and Kacprzyk [8] were not effective in some cases. Therefore, several new distance measures were proposed and applied to pattern recognition. Grzegorzewski [10] also extended the Hamming distance, the Euclidean distance, and their normalized counterparts to IFS. Later, Chen [11] pointed out that some errors existed in Grzegorzewski [10] by showing some counter examples. Hung and Yang [12] extended the Hausdorff distance to IFSs and proposed three similarity measures. On the other hand, instead of extending the well-known measures, some studies defined new similarity measures for IFSs. Li and Cheng [13] suggested a new similarity measure for IFSs based on the membership degree and the non-membership degree. Afterwards, Li [26] defined another two dissimilarity measures between intuitionistic fuzzy sets of a finite set, and it was proved that both of the measures are metrical. Mitchell [14] showed that the similarity measure of Li and Cheng [13] had some counter-intuitive cases and modified that similarity measure based on statistical point of view. Moreover, Liang and Shi [15] presented some examples to show that the similarity measure of Li and Cheng [13] was not reasonable for some conditions, and therefore proposed several new similarity measures for IFSs. Li et al. [16] analyzed, compared and summarized the existing similarity measures between IFSs/vague sets by their counter-intuitive examples in pattern recognition. Ye [7] conducted a similar comparative study of the existing similarity measures between IFSs and proposed a cosine similarity measure and a weighted cosine similarity measure. Hwang et al. [17] proposed a similarity measure for IFSs in which Sugeno integral was used for aggregation. The proposed similarity measure was applied to clustering problem. Xu [18] introduced a series of similarity measures for IFSs and applied them to multiple attribute decision making problem based on intuitionistic fuzzy information. Xu and Chen [19] introduced a series of distance and similarity measures, which are various combinations and generalizations of the weighted Hamming distance, the weighted Euclidean distance and the weighted Hausdorff distance. Xu and Yager [20] developed a similarity measure between IFSs and applied the developed similarity measure for consensus analysis in group decision making based on intuitionistic fuzzy preference relations. Xia and Xu [6] proposed a series of distance measures based on the intuitionistic fuzzy point operators.

As an addition to aforementioned studies, some attempts have been done to define similarity measures based on the relationships between distance measure, similarity measure and entropy of IFSs. Zeng and Guo [21] investigated the relationship among the normalized distance, the similarity measure, the inclusion measure, and the entropy of interval-valued fuzzy sets. It was also showed that the similarity measure, the inclusion measure, and the entropy of interval-valued fuzzy sets could be induced by the normalized distance of interval-valued fuzzy sets based on their axiomatic definitions. Wei et al. [22] introduced an entropy measure generalizing the existing entropy measures for IFS. They also introduced an approach to construct similarity measures using entropy measures for IFS and IFSs.

Besides, many other kinds of similarity measure between IFSs are emerging. Boran and Akay [23] proposed a new general type of similarity measure for IFS with two parameters, expressing L_p norm and the level of uncertainty, respectively. This similarity measure can also make sense in terms of counter-intuitive cases. Zhang and Yu [24] presented a new distance measure based on interval comparison, where the IFSs were respectively transformed into the symmetric triangular fuzzy numbers. Comparison with the widely used methods indicated that the proposed method contained more information, with much less loss of information. Li et al. [25] introduced an axiomatic definition of the similarity measure of IFSs. The relationship between the entropy and the similarity measure of IFS was investigated in detail. It was proved that the similarity measure and the entropy of IFS can be transformed into each other based on their axiomatic definitions. G.A. Papakostas et al. [27] investigated the main theoretical and computational properties of the measures, as well as the relationships between them. A comparison of the distance and similarity measures was carried out by them, from a pattern recognition point of view.

As a summation, Table 1 presents several well-known similarity measures that will be analyzed in this paper. In this table, we let $X = \{x_1, x_2, \cdots, x_n\}$ be a universe of discourse, $A \in IFSs(X)$ and $B \in IFSs(X)$ be two IFSs in X, denoted by $A = \{\langle x, \mu_A(x), v_A(x)\rangle \,|x \in X\}$ and $B = \{\langle x, \mu_B(x), v_B(x)\rangle \,|x \in X\}$, respectively. For clarity, we only give the expressions of similarity measures, with

Table 1 Similarity measures proposed by other authors

Author(s)	Similarity measure
Li and Cheng [13]	$S_{DC}(A,B)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}\lvert\varphi_A(x_i)-\varphi_B(x_i)\rvert^p}{n}}$
Mitchell [14]	$S_{HB}(A,B)=\frac{1}{2}\left(\rho_\mu(A,B)+\rho_v(A,B)\right)$
Hong and Kim [29]	$S_H(A,B)=1-\frac{\sum_{i=1}^{n}\lvert(\mu_A(x_i)-\mu_B(x_i))-(v_A(x_i)-v_B(x_i))\rvert}{2n}$
Liang and Shi [15]	$S_e^p(A,B)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}(\phi_\mu(x_i)+\phi_v(x_i))^p}{n}}$
Liang and Shi [15]	$S_s^p(A,B)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}\lvert\psi_{s1}(x_i)+\psi_{s2}(x_i)\rvert^p}{n}}$
Liang and Shi [15]	$S_h^p(A,B)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}(\eta_1(x_i)+\eta_2(x_i)+\eta_3(x_i))^p}{3n}}$
Chen [28]	$S_C(A,B)=1-\frac{\sum_{i=1}^{n}\lvert(\mu_A(x_i)-v_A(x_i))-(\mu_B(x_i)-v_B(x_i))\rvert}{2n}$
Ye [7]	$C_{IFS}(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{\mu_A(x_i)\mu_B(x_i)+v_A(x_i)v_B(x_i)}{\sqrt{(\mu_A(x_i))^2+(v_A(x_i))^2}\sqrt{(\mu_B(x_i))^2+(v_B(x_i))^2}}$
Li [31]	$S_O(A,B)=1-\sqrt{\frac{\sum_{i=1}^{n}\left((\mu_A(x_i)-\mu_B(x_i))^2+(v_A(x_i)-v_B(x_i))^2\right)}{2n}}$
Hung and Yang[12]	$S_{HY}^1(A,B)=1-d_H(A,B)S_{HY}^2(A,B)=\frac{e^{-d_H(A,B)}-e^{-1}}{1-e^{-1}}S_{HY}^3(A,B)=\frac{1-d_H(A,B)}{1+d_H(A,B)}$
Li and Xu [30]	$S_L(A,B)=1-\frac{\sum_{i=1}^{n}\lvert(\mu_A(x_i)-v_A(x_i))-(\mu_B(x_i)-v_B(x_i))\rvert}{4n}-\frac{\sum_{i=1}^{n}(\lvert\mu_A(x_i)-\mu_B(x_i)\rvert+\lvert v_A(x_i)-v_B(x_i)\rvert)}{4n}$
Boran and Akay [23]	$S_t^p(A,B)$ $=1-\sqrt[p]{\sum_{i=1}^{n}\frac{1}{2n(1+p)}\left\{\lvert t(\mu_A(x_i)-\mu_B(x_i))-(v_A(x_i)-v_B(x_i))\rvert^p+\lvert t(v_A(x_i)-v_B(x_i))-(u_A(x_i)-u_B(x_i))\rvert^p\right\}}$

an absence of the interpretations of other intermediate variables, which can be found in related references. Since a comprehensive analysis on most of these similarity measures has been carried out by Li et al. in [16], it is trivial to present such analysis repeatedly. More discussion about the drawbacks of these similarity measures will be detailed in Section 4.

3 A new similarity measure

Let $A=\{\langle x,\mu_A(x),v_A(x)\rangle\,|x\in X\}$ and $B=\{\langle x,\mu_B(x),v_B(x)\rangle\,|x\in X\}$ be two IFSs in X. We propose a new similarity measure. If we consider A and B as intervals representation, the information carried by them is determined by not only the lower and upper bounds, but also the span of the interval. So we can define a similarity measure between A and B as:

$$S_Y(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\left(\sqrt{\mu_A(x_i)\mu_B(x_i)}+2\sqrt{v_A(x_i)v_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}+\sqrt{(1-v_A(x_i))(1-v_B(x_i))}\right) \quad (5)$$

Theorem 1 *$S_Y(A,B)$ is a similarity measure between two IFSs A and B in X.*

Proof For the sake of simplicity, IFSs A and B are denoted by $A=\{\langle\mu_A(x_i),v_A(x_i)\rangle\}$ and $B=\{\langle\mu_B(x_i),v_B(x_i)\rangle\}$, respectively.

(SP1) For each $x,y\in[0,+\infty]$, we have $0\leq\sqrt{xy}\leq\frac{x+y}{2}$.

For $0\leq\mu(x_i)\leq 1$, $0\leq v(x_i)\leq 1$, $0\leq\pi(x_i)\leq 1$ and $0\leq 1-v(x_i)\leq 1$, we can get:

(i) $\leq\sqrt{\mu_A(x_i)\mu_B(x_i)}+2\sqrt{v_A(x_i)v_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}+\sqrt{(1-v_A(x_i))(1-v_B(x_i))}$
$\leq\frac{\mu_A(x_i)+\mu_B(x_i)}{2}+2\cdot\frac{v_A(x_i)+v_B(x_i)}{2}+\frac{\pi_A(x_i)+\pi_B(x_i)}{2}+\frac{1-v_A(x_i)+1-v_B(x_i)}{2},$
$=1+\frac{\mu_A(x_i)+v_A(x_i)+\pi_A(x_i)}{2}+\frac{\mu_B(x_i)+v_B(x_i)+\pi_B(x_i)}{2}$
$=2$

and

$$0\leq\sum_{i=1}^{n}\left(\sqrt{\mu_A(x_i)\mu_B(x_i)}+2\sqrt{v_A(x_i)v_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}+\sqrt{(1-v_A(x_i))(1-v_B(x_i))}\right)\leq 2n.$$

So we have $0\leq S_Y(A,B)\leq 1$.

(SP2) We know that $\sqrt{xy}$ achieves its maximum value $\frac{x+y}{2}$ when $x=y$. Therefore, we have:

(ii) $S_Y(A,B)=1\Leftrightarrow\sqrt{\mu_A(x_i)\mu_B(x_i)}+2\sqrt{v_A(x_i)v_B(x_i)}+\sqrt{\pi_A(x_i)\pi_B(x_i)}+\sqrt{(1-v_A(x_i))(1-v_B(x_i))}=2$
$\Leftrightarrow\mu_A(x_i)=\mu_B(x_i),v_A(x_i)=v_B(x_i),\pi_A(x_i)=\pi_B(x_i),1-v_A(x_i)=1-v_B(x_i)$
$\Leftrightarrow A=B$

Thus, $S_Y(A,B)=1$, if and only if $A=B$.

(SP3) It is easy to note that the expression of $S_Y(A, B)$ is commutative. So we have $S_Y(A, B) = S_Y(B, A)$.

(SP4) Let $C = \{\langle \mu_C(x_i), \upsilon_C(x_i)\rangle\}$ be another IFS in X, satisfying $A \subseteq B \subseteq C$. We have $0 \le \mu_A(x_i) \le \mu_B(x_i) \le \mu_C(x_i) \le 1$ and $0 \le \upsilon_C(x_i) \le \upsilon_B(x_i) \le \upsilon_A(x_i) \le 1$, for $\forall x \in X$. Based on (5), the similarity measures between (B, C) and (A, C), can be written as:

$$S_Y(B,C) = \frac{1}{2n}\sum\nolimits_{i=1}^{n}\left(\sqrt{\mu_B(x_i)\mu_C(x_i)} + \sqrt{\upsilon_B(x_i)\upsilon_C(x_i)} + \sqrt{\pi_B(x_i)\pi_C(x_i)} + \sqrt{(1-\upsilon_B(x_i))\,(1-\upsilon_C(x_i))}\right)$$

$$S_Y(A,C) = \frac{1}{2n}\sum\nolimits_{i=1}^{n}\left(\sqrt{\mu_A(x_i)\mu_C(x_i)} + \sqrt{\upsilon_A(x_i)\upsilon_C(x_i)} + \sqrt{\pi_A(x_i)\pi_C(x_i)} + \sqrt{(1-\upsilon_A(x_i))\,(1-\upsilon_C(x_i))}\right)$$

For $a, b \in [0, 1]$, $a + b \le 1$, we define a function f as:

$$f(x,y) = \sqrt{ax} + 2\sqrt{by} + \sqrt{(1-a-b)(1-x-y)} + \sqrt{(1-b)(1-y)}$$

where $x, y \in [0, 1]$, $x + y \in [0, 1]$.
Then we have:

$$\frac{\partial f}{\partial x} = \frac{\sqrt{a}}{2\sqrt{x}} - \frac{\sqrt{1-a-b}}{2\sqrt{1-x-y}} = \frac{(a-x)(1-b)}{2\sqrt{x(1-x-y)}\left(\sqrt{a(1-x-y)} + \sqrt{(1-a-b)x}\right)},$$

$$\frac{\partial f}{\partial y} = \frac{\sqrt{b}}{\sqrt{y}} - \frac{\sqrt{1-a-b}}{2\sqrt{1-x-y}} - \frac{\sqrt{1-b}}{2\sqrt{1-y}} = \frac{(b-y)(1-a)}{2\sqrt{y(1-x-y)}\left(\sqrt{b(1-x-y)} + \sqrt{(1-a-b)y}\right)} + \frac{b-y}{2\sqrt{y(1-y)}\left(\sqrt{b(1-y)} + \sqrt{(1-b)y}\right)}.$$

Given $a \le x \le 1, b \le 1$, we have $\frac{\partial f}{\partial x} \le 0$, which means that f is a decreasing function of x, when $x \ge a$.

For $0 \le x \le a, b \le 1$, we can get $\frac{\partial f}{\partial x} \ge 0$, which means that f is an increasing function of x, when, $x \le a$.

Similarly, we can also get $\frac{\partial f}{\partial y} \ge 0$ for $0 \le y \le b, a \le 1$ and $\frac{\partial f}{\partial y} \le 0$ for $b \le y \le 1, a \le 1$. These indicate that f is an increasing function of y for $y \le b$, but a decreasing function when $y \le b$.

Given $a = \mu_A(x_i), b = \upsilon_A(x_i)$ and two couples $(\mu_B(x_i), \upsilon_B(x_i))$, $(\mu_C(x_i), \upsilon_C(x_i))$, satisfying $a = \mu_A(x_i) \le \mu_B(x_i) \le \mu_C(x_i)$ and $\upsilon_C(x_i) \le \upsilon_B(x_i) \le \upsilon_A(x_i) = b$, we can get:

$$f(\mu_C(x_i), \upsilon_C(x_i)) \le f(\mu_B(x_i), \upsilon_C(x_i)) \le f(\mu_B(x_i), \upsilon_B(x_i)).$$

And then

(iii) $$\sqrt{\mu_A(x_i)\mu_C(x_i)} + \sqrt{\upsilon_A(x_i)\upsilon_C(x_i)} + \sqrt{\pi_A(x_i)\pi_C(x_i)} + \sqrt{(1-\upsilon_A(x_i))\,(1-\upsilon_C(x_i))} \le \sqrt{\mu_A(x_i)\mu_B(x_i)} + 2\sqrt{\upsilon_A(x_i)\upsilon_B(x_i)} + \sqrt{\pi_A(x_i)\pi_B(x_i)} + \sqrt{(1-\upsilon_A(x_i))\,(1-\upsilon_B(x_i))}$$

Therefore, $S_Y(A, B) \ge S_Y(A, C)$.

In such a way, if we suppose $a = \mu_C(x_i) b = \upsilon_C(x_i)$, considering another two couples $(\mu_B(x_i), \upsilon_B(x_i))$ and $(\mu_A(x_i), \upsilon_A(x_i))$, we have: $\mu_A(x_i) \le \mu_B(x_i) \le \mu_C(x_i) = a, b = \upsilon_C(x_i) \le \upsilon_B(x_i) \le \upsilon_A(x_i)$.

Hence, it follows that $f(\mu_A(x_i), \upsilon_A(x_i)) \le f(\mu_B(x_i), \upsilon_A(x_i)) \le f(\mu_B(x_i), \upsilon_B(x_i))$, which can be written as:

(iv) $$\sqrt{\mu_A(x_i)\mu_C(x_i)} + \sqrt{\upsilon_A(x_i)\upsilon_C(x_i)} + \sqrt{\pi_A(x_i)\pi_C(x_i)} + \sqrt{(1-\upsilon_A(x_i))\,(1-\upsilon_C(x_i))} \le \sqrt{\mu_B(x_i)\mu_C(x_i)} + 2\sqrt{\upsilon_B(x_i)\upsilon_C(x_i)} + \sqrt{\pi_B(x_i)\pi_C(x_i)} + \sqrt{(1-\upsilon_B(x_i))\,(1-\upsilon_C(x_i))}.$$

Then we have $S_Y(B, C) \ge S_Y(A, C)$.

So the similarity measure $S_Y(A, B)$ satisfies all properties in Definition 5. It is a similarity measure between IFSs. □

Considering the weights of x_i, we can define the weighted similarity between two IFSs as:

$$S_{WY}(A,B) = \frac{1}{2}\sum\nolimits_{i=1}^{n} w_i\left(\sqrt{\mu_A(x_i)\mu_B(x_i)} + 2\sqrt{\upsilon_A(x_i)\upsilon_B(x_i)} + \sqrt{\pi_A(x_i)\pi_B(x_i)} + \sqrt{(1-\upsilon_A(x_i))\,(1-\upsilon_B(x_i))}\right) \tag{6}$$

where w_i is the weights factor of the features x_i, $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$

Theorem 2 *$S_{WY}(A, B)$ is the similarity measure between two IFSs A and B in X.*

Proof (SP1) Considering the expression (i), we can get:

$$0 \le \sum\nolimits_{i=1}^{n} w_i\left(\sqrt{\mu_A(x_i)\mu_B(x_i)} + 2\sqrt{\upsilon_A(x_i)\upsilon_B(x_i)} + \sqrt{\pi_A(x_i)\pi_B(x_i)} + \sqrt{(1-\upsilon_A(x_i))\,(1-\upsilon_B(x_i))}\right) \le \sum\nolimits_{i=1}^{n} 2w_i = 2\cdot\sum\nolimits_{i=1}^{n} w_i = 2$$

Therefore, $0 \le S_{WY}(A, B) \le 1$.

(SP2) Given the implication (ii), we have:

$$S_{WY}(A,B)=1 \Leftrightarrow w_i\left(\sqrt{\mu_A(x_i)\mu_B(x_i)}+2\sqrt{v_A(x_i)v_B(x_i)}\right.$$
$$\left.+\sqrt{\pi_A(x_i)\pi_B(x_i)}+\sqrt{(1-v_A(x_i))\,(1-v_B(x_i))}\right)=2w_i$$
$$\Leftrightarrow \sqrt{\mu_A(x_i)\mu_B(x_i)}+2\sqrt{v_A(x_i)v_B(x_i)}$$
$$+\sqrt{\pi_A(x_i)\pi_B(x_i)}+\sqrt{(1-v_A(x_i))\,(1-v_B(x_i))}=2$$
$$\Leftrightarrow \mu_A(x_i)=\mu_B(x_i),\, v_A(x_i)=v_B(x_i),\, \pi_A(x_i)=\pi_B(x_i),\, 1-v_A(x_i)=1-v_B(x_i)$$
$$\Leftrightarrow A=B$$

So we get $S_{WY}(A,B)=1 \ \Leftrightarrow A=B$.

(SP3) It is obvious that $S_{WY}(A,B)$ satisfies SP3.

(SP4) Since all $w_i \geq 0$, we can multiply nequality (iii) and (iv) by w_i as:

$$w_i\left(\sqrt{\mu_A(x_i)\mu_C(x_i)}+\sqrt{v_A(x_i)v_C(x_i)}\right.$$
$$\left.+\sqrt{\pi_A(x_i)\pi_C(x_i)}+\sqrt{(1-v_A(x_i))\,(1-v_C(x_i))}\right)$$
$$\leq w_i\left(\sqrt{\mu_A(x_i)\mu_B(x_i)}+2\sqrt{v_A(x_i)v_B(x_i)}\right.$$
$$\left.+\sqrt{\pi_A(x_i)\pi_B(x_i)}+\sqrt{(1-v_A(x_i))\,(1-v_B(x_i))}\right)$$

$$w_i\left(\sqrt{\mu_A(x_i)\mu_C(x_i)}+\sqrt{v_A(x_i)v_C(x_i)}\right.$$
$$\left.+\sqrt{\pi_A(x_i)\pi_C(x_i)}+\sqrt{(1-v_A(x_i))\,(1-v_C(x_i))}\right)$$
$$\leq w_i\left(\sqrt{\mu_B(x_i)\mu_C(x_i)}+2\sqrt{v_B(x_i)v_C(x_i)}\right.$$
$$\left.+\sqrt{\pi_B(x_i)\pi_C(x_i)}+\sqrt{(1-v_B(x_i))\,(1-v_C(x_i))}\right)$$

We obtain $S_{WY}(A,B) \geq S_{WY}(A,C)$ and $S_{WY}(B,C) \geq S_{WY}(A,C)$.

Therefore, $S_{WY}(A,B)$ is a similarity measure between IFSs A and B. □

4 Numerical comparisons

In order to illustrate the superiority of the proposed similarity measure, a comparison between the proposed similarity measure and all the existing similarity measures is conducted based on the numerical cases in [23], which are widely used as counter-intuitive examples. Table 2 presents the result with $p=1$ for S_{HB}, S_e^p, S_s^p, S_h^p and $p=1$ $t=2$ for S_t^p.

We can see that $S_C(A,B) = S_{DC}(A,B) = C_{IFS}(A,B) = 1$ for two different IF sets $A=\langle 0.3, 0.3\rangle$ and $B = \langle 0.4, 0.4\rangle$. This indicates that the second

Table 2 The comparison of similarity measures (counter-intuitive cases are in bold type)

	1	2	3	4	5	6
A	$\langle 0.3, 0.3\rangle$	$\langle 0.3, 0.4\rangle$	$\langle 1, 0\rangle$	$\langle 0.5, 0.5\rangle$	$\langle 0.4, 0.2\rangle$	$\langle 0.4, 0.2\rangle$
B	$\langle 0.4, 0.4\rangle$	$\langle 0.4, 0.3\rangle$	$\langle 0, 0\rangle$	$\langle 0, 0\rangle$	$\langle 0.5, 0.3\rangle$	$\langle 0.5, 0.2\rangle$
S_C	1	0.9	0.5	1	**1**	0.95
S_H	0.9	**0.9**	0.5	**0.5**	0.9	0.95
S_L	0.95	0.9	0.5	0.75	0.95	0.95
S_O	0.9	**0.9**	0.3	0.5	0.9	0.93
S_{DC}	1	0.9	0.5	1	**1**	0.95
S_{HB}	0.9	**0.9**	0.5	**0.5**	0.9	0.95
S_e^p	0.9	**0.9**	0.5	**0.5**	0.9	0.95
S_s^p	0.95	0.9	0.5	0.75	0.95	0.95
S_h^p	0.933	**0.933**	0.5	0.67	0.933	0.95
S_{HY}^1	0.9	0.9	0	0.5	0.9	**0.9**
S_{HY}^2	0.85	0.85	0	0.38	0.85	**0.85**
S_{HY}^3	0.82	0.82	0	0.33	0.82	**0.82**
C_{IFS}	1	0.96	0	**0**	0.9971	0.9965
S_t^p	0.967	0.9	0.5	0.833	0.967	0.95
S_Y	0.985	0.994	0.5	0.354	0.984	0.997

Table 3 The similarity measures between the known patterns and the unknown pattern in **Example 2** (Patterns not discriminated are in bold type). ($p = 1$ for S_{HB}, S_e^p, S_s^p, S_h^p and $p = 1, t = 2$ for S_t^p)

	$S(A_1, B)$	$S(A_2, B)$	$S(A_3, B)$		$S(A_1, B)$	$S(A_2, B)$	$S(A_3, B)$
S_C	1	**1**	1	S_s^p	1	0.967	0.900
S_H	1	**1**	1	S_h^p	1	0.956	0.867
S_L	1	0.967	0.9	S_{HY}^1	1	0.967	0.8
S_O	1	0.918	0.784	S_{HY}^2	1	0.898	0.713
S_{DC}	1	**1**	1	S_{HY}^3	1	0.875	0.667
S_{HB}	1	0.933	0.8	C_{IFS}	1	1	**1**
S_e^p	1	0.933	0.8	S_t^p	1	0.978	0.933

axiom of similarity measure (S2) is not satisfied by $S_C(A, B)$, $S_{DC}(A, B)$ and $C_{IFS}(A, B)$. This also can be illustrated by $S_C(A, B) = S_{DC}(A, B) = 1$ when $A = \langle 0.5, 0.5\rangle$, $B = \langle 0, 0\rangle$ and $A = \langle 0.4, 0.2\rangle$, $B = \langle 0.5, 0.3\rangle$. As for S_H, S_O, S_{HB}, S_e^p, S_s^p and S_h^p, different pairs of A, B may provide the identical results, which cannot satisfy the application of pattern recognition. It can be read from Table 2 that $S_{HB} = 0.9$ for both $A = \langle 0.3, 0.3\rangle$, $B = \langle 0.4, 0.4\rangle$ and $A = \langle 0.3, 0.4\rangle$, $B = \langle 0.4, 0.3\rangle$. Such situation seems to be worse for S_{HY}^1, S_{HY}^2 and S_{HY}^3, where all the cases take the same similarity degree except case.3 and case.4. S_t^p seems to be reasonable without any counter-intuitive results, but it bring new problem with the choice of parameters p and t, which is still an open problem. Moreover, we can notice an interesting situation when comparing case.3 and case.4. For three IF sets $A = \langle 1, 0\rangle$, $B = \langle 0.5, 0.5\rangle$ and $C = \langle 0, 0\rangle$, intuitively, it is more reasonable to take the similarity degree between them as: $S_F(A, C) = 0.15$, $S_F(B, C) = 0.25$ than taking $S_t^p(A, C) = 0.5$ and $S_t^p(B, C) = 0.833$. In such a sense, the proposed similarity measure is the most reasonable one with a relative simple expression, and has none of the counter-intuitive cases. Three IF sets $A = \langle 0.4, 0.2\rangle$, $B = \langle 0.5, 0.3\rangle$ and $C = \langle 0.5, 0.2\rangle$ can be written in forms of interval values as: $A = [0.4, 0.8]$, $B = [0.5, 0.7]$ and $C = [0.5, 0.8]$, respectively. In such a sense, we can say that the similarity degree between A and C should not be less than the similarity degree between A and B, which is also illustrated by other similarity measures except S_C, S_{DC} and S_t^p (underlined cases). Therefore, our proposed similarity measure is in agreement with this analysis. The proposed similarity measure is the most reasonable similarity measure without any counter-intuitive cases. We must note that, among the measures listed in Table 2, S_t^p seems to be another metric measure without any counter-intuitive cases. However, it brings a new problem with the choice of the parameter p and t, which is also an important open problem facing by similarity measures S_{HB}, S_e^p, S_s^p and S_h^p. Therefore, we can say that our proposal is a satisfactory similarity measure satisfying all axiomatic properties, without any counter-intuitive cases and the problem of choosing other parameters.

In order to study the effectiveness of the proposed similarity measure for IFS in the application of pattern recognition, we consider the pattern recognition problem discussed in [7, 13].

Suppose there are m patterns, which can be represented by IFSs $A_j = \left\{\langle x_i, \mu_{A_j}(x_i), v_{A_j}(x_i)\rangle | x_i \in X\right\}$, $A_j \in IFSs(X)$, $j = 1, 2, \ldots, m$. Let the sample to be recognized be denoted as $B = \{\langle x_i, \mu_B(x_i), v_B(x_i)\rangle | x_i \in X\}$. According to the recognition principle of maximum degree of similarity between IFSs, the process of assigning B to A_k is described by:

$$k = \arg \max_{j=1,2,\ldots,m} \{S(A_j, B)\} \tag{7}$$

Example 1 Assume that there exists three known patterns A_1, A_2 and A_3, with class labels C_1, C_2 and C_3, respectively. Each pattern can be expressed by IFS in $X = \{x_1, x_2, x_3\}$ as:

Table 4 Symptoms characteristic for the patients

	Temperature	Headache	Stomach pain	Cough	Chest pain
Al	(0.8,0.1)	(0.6,0.1)	(0.2,0.8)	(0.6,0.1)	(0.1,0.6)
Bob	(0,0.8)	(0.4,0.4)	(0.6,0.1)	(0.1,0.7)	(0.1,0.8)
Joe	(0.8, 0.1)	(0.8, 0.1)	(0.0, 0.6)	(0.2, 0.7)	(0.0, 0.5)
Ted	(0.6, 0.1)	(0.5, 0.4)	(0.3, 0.4)	(0.7, 0.2)	(0.3, 0.4)

Table 5 Symptoms characteristic for the diagnoses

	Viral fever	Malaria	Typhoid	Stomach problem	Chest pain problem
Temperature	(0.4, 0.0)	(0.7, 0.0)	(0.3, 0.3)	(0.1, 0.7)	(0.1, 0.8)
Headache	(0.3, 0.5)	(0.2, 0.6)	(0.6, 0.1)	(0.2, 0.4)	(0, 0.8)
Stomach pain	(0.1, 0.7)	(0.0, 0.9)	(0.2, 0.7)	(0.8, 0.0)	(0.2, 0.8)
Cough	(0.4, 0.3)	(0.7, 0.0)	(0.2, 0.6)	(0.2, 0.7)	(0.2, 0.8)
Chest pain	(0.1, 0.7)	(0.1, 0.8)	(0.1, 0.9)	(0.2, 0.7)	(0.8, 0.1)

$$A_1 = \{\langle x_1, 1, 0\rangle, \langle x_2, 0.8, 0\rangle, \langle x_3, 0.7, 0.1\rangle\},$$

$$A_2 = \{\langle x_1, 0.8, 0.1\rangle, \langle x_2, 1, 0\rangle, \langle x_3, 0.9, 0\rangle\},$$

$$A_3 = \{\langle x_1, 0.6, 0.2\rangle, \langle x_2, 0.8, 0\rangle, \langle x_3, 1, 0\rangle\}.$$

The sample B need to be recognized is:

$$B = \{\langle x_1, 0.5, 0.3\rangle, \langle x_2, 0.6, 0.2\rangle, \langle x_3, 0.8, 0.1\rangle\}.$$

The similarity degree between A_i $(i = 1, 2, 3)$ and B calculated by (5) is:

$$S_Y(A_1, B)=0.887,\ S_Y(A_2, B)=0.913,\ S_Y(A_3, B)=0.936.$$

It can be observed that the pattern B should be classified to A_3 with a class label C_3. According to the recognition principle of maximum degree of similarity between IFSs. This result is in agreement with the one obtained in [7, 13].

Let's assume that the weights of x_1, x_2 and x_3 are 0.5, 0.3, and 0.2, respectively, as they were assumed in [7]. Considering (6), we can get:

$$S_{WY}(A_1, B) = 0.853,\ S_{WY}(A_2, B) = 0.919,\ S_{WY}(A_3, B) = 0.949.$$

According to (7), B can be recognized as A_3, which is identical to the result obtained in [7, 13].

To make our similarity measure more transparent and comparable with the measures proposed earlier by other authors, the example analyzed in [17] will be discussed next.

Example 2 Assume that there are three IFS patterns in $X = \{x_1, x_2, x_3\}$. The three patterns are denoted as follows:

$$A_1 = \{\langle x_1, 0.3, 0.3\rangle, \langle x_2, 0.2, 0.2\rangle, \langle x_3, 0.1, 0.1\rangle\},$$

$$A_2 = \{\langle x_1, 0.2, 0.2\rangle, \langle x_2, 0.2, 0.2\rangle, \langle x_3, 0.2, 0.2\rangle\}$$

$$A_3 = \{\langle x_1, 0.4, 0.4\rangle, \langle x_2, 0.4, 0.4\rangle, \langle x_3, 0.4, 0.4\rangle\}$$

Assume that a sample $B = \{\langle x_1, 0.3, 0.3\rangle, \langle x_2, 0.2, 0.2\rangle, \langle x_3, 0.1, 0.1\rangle\}$ is to be classified.

The similarity degrees of $S(A_1, B)$, $S(A_2, B)$ and $S(A_3, B)$ calculated for all similarity measures listed in Table 1 are shown in Table 3.

The proposed similarity measure S_Y can be calculated by (5) as:

$$S_Y(A_1, B) = 1,\ S_Y(A_2, B) = 0.990,\ S_Y(A_3, B) = 0.932$$

It is obvious that B is equal to A_1. This indicates that sample B should be classified to A_1. However, the similarity degrees of $S(A_1, B)$, $S(A_2, B)$ and $S(A_3, B)$ are equal to each other when S_C, S_H, S_{DC} and C_{IFS} are employed. These four similarity measures are not capable of discriminating the difference between the three patterns. Fortunately, the results of $S_Y(A_i, B)$ $(i = 1, 2, 3)$ can be used to make correct classification conclusion. This means that the proposed similarity measure shows an identical performance with majority of the existing measures.

5 Applications in pattern recognition

Along with the previous investigation of classification capabilities of the proposed measure, an additional experiment discussed in [7, 22, 23, 32–36] will be presented as an application in pattern recognition. In this paper, we propose an alternative approach to medical diagnosis using the newly defined similarity measure.

Table 6 The proposed similarity measure S_Y between each patient's symptoms and the considered set of possible diagnoses

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Al	0.9347	0.9228	0.9223	0.7673	0.7490
Bob	0.8124	0.6775	0.8997	0.9760	0.8211
Joe	0.9152	0.8271	0.9188	0.7917	0.7456
Ted	0.9576	0.9034	0.9060	0.8577	0.8122

Suppose that there are four patients Al, Bob, Joe, Ted, represented as $P = \{$ Al, Bob, Joe, Ted$\}$. Their symptoms are $S = \{$ Temperature, Headache, Stomach pain, Cough, Chest pain$\}$. The set of diagnoses is defined as $D = \{$ Viral fever, Malaria, Typhoid, Stomach problem, Chest problem$\}$. The intuitionistic fuzzy relation $P \rightarrow S$ is presented in Table 4. Table 5 gives the intuitionistic fuzzy relation $S \rightarrow D$. Each element of the tables is given in the form of IFV, which is a pair of numbers corresponding to the membership and non-membership values, respectively.

In order to make a proper diagnosis for each patient, we calculate the similarity degree between each patient and each diagnose. According to the principle of maximum similarity degree, the higher similarity degree indicates a proper diagnosis. In Table 6, the similarity degree S_Y between patients and diagnoses are presented. According to the similarity degrees in Table 6, a conclusion can be made that Al suffers from Viral Fever, Bob suffers from Stomach problem, Joe suffers from Typhoid, and Ted suffers from Viral Fever. The diagnosis results for this case obtained in previous study have been presented in [23]. It is clear that our proposed method provides the same results obtained by Vlachos in [33], Own in [36] and Boran in [23]. Moreover, our proposed similarity measure is calculated based on the IFNs, without any other parameters such as p, t in [23]. So it can reduce the computation complexity.

6 Conclusion

Even though several similarity measures between IFSs have been proposed to cope with uncertainty in information systems, most of them have provided counter-intuitive results. In this study, a new similarity measure and weighted similarity measure between IFSs are proposed. The new similarity measure is calculated based on the operations on the membership degree $\mu_A(x)$, non-membership degree $\upsilon_A(x)$, hesitancy degree $\pi_A(x)$, as well as the upper bound of membership $1 - \upsilon_A(x)$. In some special cases where some of the existing similarity measure cannot provide reasonable results, the proposed similarity measure shows great capacity for discriminating IFSs. Moreover, investigation of the new measure's classification capability is carried out based on two numerical examples and medical diagnosis. It has been illustrated that the proposed similarity measure performs as well as or better than previous measures. However, our proposed similarity is not an absolute perfect one. It is stuck with the lack of definitude physical meaning. Efforts are continuing to look for a more excellent similarity measure for much better exploration and exploitation on IFS.

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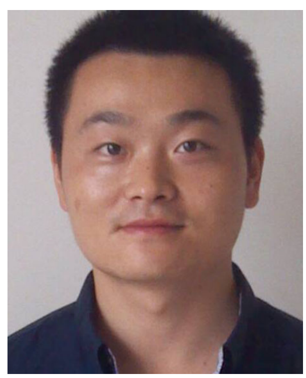

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