Appl Intell (2009) 31: 283–291
DOI 10.1007/s10489-008-0126-y

Switching between type-2 fuzzy sets and intuitionistic fuzzy sets: an application in medical diagnosis

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Published online: 7 May 2008

Abstract When dealing with vagueness, there are situations when there is insufficient information available, making it impossible to satisfactorily evaluate membership. The intuitionistic fuzzy set theory is more suitable than fuzzy sets to deal with such problem. In 1996, Atanassov proposed the mapping from intuitionistic fuzzy sets to fuzzy sets. Furthermore, intuitionistic fuzzy sets are isomorphic to interval valued fuzzy sets, and interval valued fuzzy sets are regarded as the special cases of type-2 fuzzy sets in recently studies. However, their discussions are not only hardly comprehending but also lacking the reliable applications. In this study, the advantage of type-2 fuzzy sets is employed, and the switching relation between type-2 fuzzy sets and intuitionistic fuzzy sets is defined axiomatically. The switching results are applied to show the usefulness of the proposed method in pattern recognition and medical diagnosis reasoning.

Keywords Type-2 fuzzy sets · Type-2 similarity · Intuitionistic fuzzy set · Pattern recognition · Medical diagnosis

1 Introduction

The theory of fuzzy sets (FSs), proposed by Zadeh [1], is being successful applied in various fields. Through his research of FSs, Atanassov extended the well-known notion of FSs as intuitionistic fuzzy sets (IFSs) which are very effective in dealing with vagueness [2]. Taranli and Coker proposed several types of fuzzy connectedness in intuitionistic fuzzy topological spaces [6]. De et al. defined regular operations with IFSs as well as "Intuitionistic Medical Knowledge" to show and discuss the relationship between symptoms and diagnosis [7, 8]. Szmidt and Kacprzyk proposed the distance between IFSs for medical diagnosis and they also proposed a similarity measure based on the combination of parameters for elements and geometrical interpretation for medical diagnosis [9–11]. Based on this, Li and Cheng proposed similarity measures for IFSs and applied similarity measures to pattern recognition [12]. Liang and Shi proposed several new similarity measures and proved the relationships between similarity measures [13]. Li et al compared and summarized the counter-intuitive examples of IFSs and vague sets. Their method is based on single-element sets (or extend to multi-element sets), and demonstrates the drawbacks of some similarity measures [14]. Deschrijver and Kerre present intuitionistic fuzzy version of the triangular compositions given by theories proposed by others, such as De and Kerre [15].

In 1986, Atanassov proposed the mapping association from IFSs to FSs [2]. Although IFSs can be conclude as being an extension of FSs, there are deficiencies in this mapping. They are difficult to comprehend and lack axiom definition. In addition, when considering certain variations of FSs such Sugeno's or Yager's [17, 18], the mapping does not satisfy the elemental intuitionism condition given by Atanassov.

Because the standard FSs are not able to directly model uncertainties, IFSs are not identical to the FSs and have the earlier mentioned deficiencies. On the other hand, type-2 fuzzy sets (T2FSs) are very appropriate for our purpose. The advantage of T2FSs is the fact that their membership

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functions themselves are fuzzy. That is, T2FSs can provide a greater ability to handle imprecise and imperfect information [19]. Thus, in this study, the author defines and provides the mutual switch between IFSs and T2FSs, as well as the medical diagnosis with imperfect facts involving the type-2 similarity and our switching method.

The remainder of th is paper is organized as follows. Sections 2 and 3 briefly review essential concepts of T2FSs and IFSs. Section 4 defines and proves the proposed mutual-switch between two sets. Section 5 presents the empirical examples of pattern recognition and medical diagnosis to illustrate the utility of the proposed switch. Conclusions are drawn in Sect. 6.

2 Basic notions and definitions of type-2 fuzzy sets

In the section, we firstly discuss T2FSs as proposed by Mendel [20], and then the type-2 similarity is discussed in Sect. 2.2.

2.1 Basic definition of type-2 fuzzy sets

T2FSs were initially defined by Zadeh [1]. A T2FS is characterized by a fuzzy membership. The distinction between a T2FS and a FS is that the membership value for a T2FS is a FS in [0, 1], whereas the membership grade of a standard FS is a crisp value in [0, 1]. To clarify statement, the FS '*tall*' is represented as

$$tall = \frac{0.95}{Michael} + \frac{0.4}{Danny} + \frac{0.6}{Robert}.$$

Conversely, the interpretation of T2FS is

$$tall = \frac{High}{Michael} + \frac{Low}{Danny} + \frac{Medium}{Robert},$$

where membership functions of High, Low and Medium themselves are FSs. The former set is measured by one condition for one element, while the latter set is measured by several conditions for one element. T2FSs are useful when the exact membership function for a type-1 FS cannot be easily determined, and are thus advantageous for incorporating uncertainties.

According to Mendel in 2001 [20], T2FSs are defined as follows. For the sake of simplicity, the universe of discourse is assumed as a finite set, although the definition is applied for infinite sets. In addition, the T2FS in X is denoted as $FS_2(X)$.

Definition 2.1 A T2FS $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$, where X is a universal set, $x \in X$ and $u \in J_X \subseteq [0,1]$; that is,

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) \mid \forall x \in X, \forall u \in J_x\},$$

where $0 \le \mu_{\tilde{A}}(x,u) \le 1$. Thus, $\tilde{A}$ is expressed as

$$\tilde{A} = \sum_{x\in X}\sum_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u), \quad J_x \subseteq [0,1],$$

where $\sum\sum$ indicates the union over all admissibles x and u.

Definition 2.2 For each value of x, say $x = x'$,

$$\mu_{\tilde{A}}(x = x', u) \equiv \mu_{\tilde{A}}(x) = \sum_{u\in J_{x'}} f_{x'}(u)/u,$$
$$\text{for } u \in J_{x'} \subseteq [0,1] \text{ and } x \in X, \tag{1}$$

where $\mu_{\tilde{A}}(x)$ is represented as the secondary membership function, $f_{x'}(u)$ is the amplitude of a secondary membership function called a secondary grade.

Definition 2.3 The domain of a secondary membership function is called the primary membership of x; in (1), and J_x is the primary membership of x. Moreover, the amplitude of a secondary membership function is called a secondary grade, and $f_x(u)$ is a secondary grade in (1).

Additionally, the general form of the union of all secondary sets is represented by the concept of secondary FSs. That is,

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid \forall x \in X\},$$

or, as

$$\begin{aligned}\tilde{A} &= \sum_{x\in X} \mu_{\tilde{A}}(x)/x \\ &= \sum_{x\in X}\sum_{u\in J_x} [f_x(u)/u]/x, \quad J_x \subseteq [0,1]. \end{aligned} \tag{2}$$

To clearly illustrate the T2FS, Fig. 1 depicts an example of T2FS.

Moreover, the complement of a T2FS is defined as,

Definition 2.4 The complement of $\tilde{A}^c$ is another T2FS,

$$\begin{aligned}\tilde{A}^c &= \sum_{x\in X} \mu_{\tilde{A}^c}(x)/x \\ &= \sum_{x\in X}\sum_{u\in J_x} [f_x(u)/(1-u)]/x, \quad J_x \subseteq [0,1].\end{aligned}$$

In addition, according to [22], the subordinate operation are defined as:

Definition 2.5 Let $\tilde{A} \subset \tilde{B}$ if and only if $u \le v, \forall f_x(u) \le f_x(v)$ and $x \in X$. Note that and are secondary memberships of $\tilde{A}$, $\tilde{B}$, respectively.

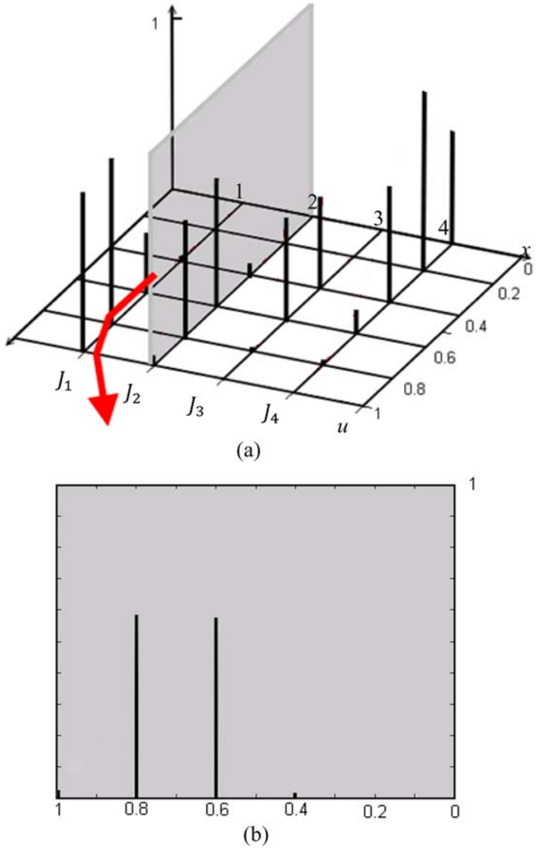

Fig. 1 (**a**) An example of T2FS; (**b**) the secondary membership function at $x = 2$

2.2 Type-2 fuzzy similarity

Much work has already been gone into defining the degree of similarity between two standard FSs. However, the similarity between T2FSs has received little attention, and the historical similarity measures cannot be directly applied to measure T2FSs. Hung and Yang proposed the axiom definition for a type-2 similarity measure based on Hausdorff distance [22]. They compute the similarities by treating the secondary grades as having two opposite elements consisting of coexistence on peer secondary membership functions. However, according to Fig. 1 and Mendel's definition,[1] elements in the primary memberships of x are not guaranteed to consist on the coexistence of these two T2FSs. Thus, Hung's method cannot compute the correctly answer. In Own [23], Own proposed the type-2 similarity based on the Swapan and Pappis method [24, 25] (assume that $\tilde{A}$, $\tilde{B}$ be two type-2 fuzzy sets defined on the universe of discourse X for all sets having N elements),

$$S(A, B) = \frac{\sum_{x \in X}\{\mu_A(x)\mu_B(x)\}}{\sum_{x \in X} \max\{\mu_A(x)\mu_B(x)\}^{2'}}$$

$S(A, B)$ is the measure of the fuzzy similarity, hence,

$$\tilde{S}(\tilde{A}, \tilde{B}) = \frac{1}{N}\sum S(\tilde{A}, \tilde{B}) = \frac{1}{N}\sum_{i=1}^{N} \frac{\sum_{u,v}\{u \cdot v\}}{\sum_{u,v} \max\{u \cdot v\}^2},$$

$$\text{for all } f_x(u) = g_x(v), \tag{3}$$

where $\tilde{S}(\tilde{A}, \tilde{B})$ is the measure of the type-2 similarity, $\tilde{A}, \tilde{B} \in FS_2(X)$, u and v are the primary memberships of $\tilde{A}$, and $\tilde{B}$, and $f_x(u)$ and $g_x(v)$ are secondary grades of $\tilde{A}$ and $\tilde{B}$. Please note that secondary grade is used as the indeterminacy index to select the corresponding memberships of each measured sets.

3 Intuitionistic fuzzy sets

In ordinary FS theory there is no means to incorporate that hesitation in the membership degrees. In 1986, Atanassov proposed IFSs and gave us the possibility to model hesitation and uncertainty by using an additional degree [2]. Each IFS $\check{A}$ allots a membership degree $\mu_{\check{A}}(x)$ and a non-membership degree $\nu_{\check{A}}(x)$ for each element x of the universe X, note that $\mu_{\check{A}}(x) \in [0, 1], \nu_{\check{A}}(x) \in [0, 1]$ and $\mu_{\check{A}}(x) + \nu_{\check{A}}(x) \leq 1$. The value $\pi_{\check{A}}(x) = 1 - (\mu_{\check{A}}(x) + \nu_{\check{A}}(x))$ which is called the hesitation part, is the degree of hesitancy whether x belongs to $\check{A}$. In addition, all IFSs in X are represented as $IFS(X)$.

Definition 3.1 When a universe of discourse X is discrete, an IFS $\check{A}$ is denoted as:

$$\check{A} = \sum [x, \nu_{\check{A}}(x), \mu_{\check{A}}(x)], \quad \forall x \in X. \tag{4}$$

For the sake of simplicity, the universe of discourse is assumed as a finite set too, although the definition can be applied for infinite sets.

The following properties are expressed for all $\check{A}$ and $\check{B}$ belonging to $IFS(X)$ [2, 3],

(P1) $\check{A} \leq \check{B}$ if and only if $\mu_{\check{A}}(x) \leq \mu_{\check{B}}(x)$ and $\nu_{\check{A}}(x) \geq \nu_{\check{B}}(x)$ for all $x \in X$.
(P2) $\check{A} = \check{B}$ if and only if $\check{A} \leq \check{B}$ and $\check{A} \geq \check{B}$.
(P3) $\check{A}_c = \sum [x, \nu_{\check{A}}(x), \mu_{\check{A}}(x)]$, $\forall x \in X$.

4 Mutual switch between a type-2 fuzzy set and an intuitionistic fuzzy set

Accordingly, Atanassov associated a mapping from $IFS(X)$ to $FS(X)$, and he defined the following operator [2, 5]:

$$\begin{aligned} \check{A} &= \{\langle x, \mu_{\check{A}}(x), \nu_{\check{A}}(x)\rangle \mid x \in X\} \\ &\rightarrow \quad f_\alpha(\check{A}) = \{\langle x, \mu_{\check{A}}(x) + \alpha\pi_{\check{A}}(x), \\ &\qquad 1 - \mu_{\check{A}}(x) - \alpha\pi_{\check{A}}(x)\rangle \mid x \in X\}, \end{aligned} \tag{5}$$

where $f_\alpha : IFS(X) \to FS(X),\ \alpha \in [0, 1]$. This operator f_α coincides with the operator D_α given in [2]. However, the deficiencies of (5) are as follows.

1. The equation only considers the membership degree and doesn't refer the non-membership degree directly.
2. This operator cannot handle the reverse switch from FS to IFS.
3. Considering the complement of FS such as Sugeno's or Yager's [17, 18], the sum of membership and non-membership from one, the result derived from (5) is a negative number, and thus not satisfy the elemental intuitionism condition given by Atanassov [2–5].

Besides, for each element x, a T2FS can model hesitation and more uncertainties by using the additional degrees. For instance, the polarizing concepts, i.e., more/less, optimistic/pessimistic, membership/non-membership can be inferred by the secondary grades. That is, the secondary grades are defined to measure the magnitudes, which allow us to weight the degree of intuitionism of an IFS. Thus, to conquer the above deficiencies, the mutual switch between T2FS and IFS is introduced as follows.

Accordingly, in our study, the mapping from $IFS(X)$ to $FS_2(X)$ is defined as follows,

Definition 4.1 Let $\check{A} \in IFS(X)$ define as:

$$\check{A} = \sum [x, \mu_{\check{A}}(x), \nu_{\check{A}}(x)], \tag{6}$$

where $x \in X$. Then, the mapping from $IFS(X)$ to $FS_2(X)$ is defined as

$$\tilde{A} = \sum [1/(\mu_{\check{A}}(x) + p\pi_{\check{A}}(x)) + 0/(1 - \nu_{\check{A}}(x) + p\pi_{\check{A}}(x))]/x, \tag{7}$$

where $p \in [0, 1]$. The secondary grade in (7) represents the indeterminacy index of the membership/non-membership degree, which models the unhesitancy of deciding degree to which an object satisfies a particular property. That is, 1/0 in secondary grades means the total certainty/uncertainty on membership/non-membership herein. Besides, according to (P1) (properties of IFSs), the definition of non-membership degree of an element x in IFS is $\nu_{\check{A}}(x) \geq \nu_{\check{B}}(x) \Leftrightarrow \check{A} \leq \check{B}$. This notion conflicts with the natural generalization of a standard FS of Zadeh's containment statement, $\nu_{\check{A}}(x) \leq \nu_{\check{B}}(x) \Leftrightarrow \check{A} \leq \check{B}$. Thus, the non-membership value in (7) is obtained by subtracting the $\nu_{\check{B}}(x)$ from one.

Accordingly, the following proposition is proved to validate the mapping from T2FS and IFS. To simply the proof, IFS is defined as the extension of the FSs, that is, the membership and non-membership degrees are added and are equal to one.

Proposition 4.1 Let $\check{A}, \check{B} \in IFS(X)$, denote that the switching from IFS to T2FS are validated, then $\check{A} \leq \check{B}$ if and only if $\tilde{A} \leq \tilde{B}$.

Proof Assume that two IFSs $\check{A}$ and $\check{B}$ are defined as,

$$\check{A} = \sum [x, \mu_{\check{A}}(x), \nu_{\check{A}}(x)],$$

and

$$\check{B} = \sum [x, \mu_{\check{B}}(x), \nu_{\check{B}}(x)],$$

where $\forall x \in X$.

Thus, two corresponding T2FSs are defined as

$$\tilde{A} = \sum [1/(\mu_{\check{A}}(x) + p\pi_{\check{A}}(x)) + 0/(1 - \nu_{\check{A}}(x) + p\pi_{\check{A}}(x))]/x,$$

and

$$\tilde{B} = \sum [1/(\mu_{\check{B}}(x) + p\pi_{\check{B}}(x)) + 0/(1 - \nu_{\check{B}}(x) + p\pi_{\check{B}}(x))]/x.$$

Furthermore, the hesitation part in IFS is the indicator of the hesitancy whether x belongs to IFS. Hence, the author defined $\pi_{\check{A}}(x) \leq \pi_{\check{B}}(x)$, if $\check{A} \leq \check{B}$ exists.

Hence, assume that $\check{A} \leq \check{B}$, then

$$\mu_{\check{A}}(x) \leq \mu_B(x) \quad \text{and} \quad \nu_{\check{A}}(x) \geq \nu_B(x).$$

Accordingly, we obtain as follows,

$$\mu_{\check{A}}(x) \leq \mu_{\check{B}}(x) \Rightarrow \mu_{\check{A}}(x) + p\pi_{\check{A}}(x) \leq \mu_{\check{B}}(x) + p\pi_{\check{B}}(x),$$

and

$$\nu_{\check{A}}(x) \geq \nu_{\check{B}}(x) \Rightarrow 1 - \nu_{\check{A}}(x) \leq 1 - \nu_{\check{B}}(x) \Rightarrow 1 - \nu_{\check{A}}(x) + p\pi_{\check{A}}(x) \leq 1 - \nu_{\check{B}}(x) + p\pi_{\check{B}}(x).$$

Note that $p \in [0, 1]$.

Hence, we obtain $\tilde{A} \leq \tilde{B}$, the proposition is proven. □

Furthermore, it is known that if A is a FS on a referential X and $c : [0, 1] \to [0, 1]$ is a fuzzy complement, the set

$$A = \sum [x, \mu_A(x), c(\mu_A(x))] \tag{8}$$

is treated as an IFS [7]. However, according to our previous statement, non-membership value is not a natural generalization of the standard FS. Besides, if we take Sugeno's negation [17]

$$c_\lambda(x) = \frac{1 - x}{1 + \lambda x}, \quad \text{with } -1 < \lambda < 0,$$

or Yager's negation [18]

$$c_\lambda(x) = (1 - x^\omega)^{1/\omega}, \quad \text{with } 1 < \omega,$$

as a fuzzy complement, then (8) is not an IFS, because $\mu_A(x) + c(\mu_A(x)) > 1$ and therefore $\pi_A(x) < 0$. Hence, in our study, for the purpose of clarifying the relationship between T2FS and IFS, the reverse mapping from $FS_2(X)$ to $IFS(X)$ is defined as follows,

Definition 4.2 Let $\tilde{A} \in FS_2(X)$ define as,

$$\tilde{A} = \sum [1/\mu_1(x) + 0/\mu_2(x)]/x, \tag{9}$$

where $x_i \in X$. Then, the mapping from $FS_2(X)$ to $IFS(X)$ is defined as

$$\check{A} = \sum [x, \mu_1(x) - p\pi_{\check{A}}(x), 1 - \mu_2(x) - p\pi_{\check{A}}(x)], \tag{10}$$

where

$$\pi_{\check{A}}(x) = \begin{cases} \frac{|\mu_1(x) - \mu_2(x)|}{2p-1}, & \text{if } \mu_1(x) > \mu_2(x), \\ \frac{|\mu_1(x) - \mu_2(x)|}{1-2p}, & \text{if } \mu_1(x) \le \mu_2(x), \end{cases} \tag{11}$$

and $p \in [0, 1]$.

Proposition 4.2 According to (10), denote that the membership degree, non-membership degree and hesitation part are summed to one.

Proof According to (10), we want to prove

$$\mu_1(x) - p\pi_{\check{A}}(x) + 1 - \mu_2(x) - p\pi_{\check{A}}(x) + \pi_{\check{A}}(x) = 1.$$

That is,

$$\mu_1(x) - \mu_2(x) + 1 + (1 - 2p)\pi_{\check{A}}(x)$$
$$\Rightarrow \quad \mu_1(x) - \mu_2(x) + 1 + (1 - 2p)\frac{|\mu_1(x) - \mu_2(x)}{2p - 1}.$$

Note that, substitute (11) into the above derivation.

Assume that $\mu_1(x) > \mu_2(x)$, and then the above equation is obtained as

$$\mu_1(x) - \mu_2(x) + 1 + (1 - 2p)\frac{\mu_1(x) - \mu_2(x)}{2p - 1}$$
$$\Rightarrow \quad \mu_1(x) - \mu_2(x) + 1 - (\mu_1(x) - \mu_2(x)) = 1.$$

Thus, the proposition is proven. $\mu_1(x) \le \mu_2(x)$ is derived as the same. □

Proposition 4.3 Let $\tilde{A}$ is defined as follows,

$$\tilde{A} = \sum [1/\mu_{\tilde{A}}(x) + 0/c(\mu_{\tilde{A}}(x))]/x,$$

and the fuzzy complement is Sugeno's negation. Denote that the conversation from $FS_2(X)$ to $IFS(X)$ is validated, then

$$\mu_{\check{A}}(x) - p\pi_{\check{A}}(x) + (1 - c(\mu_{\check{A}}(x))) - p\pi_{\check{A}}(x) \le 1.$$

Proof Herein, we assume that $p = 1$. Hence,

$$\pi_{\check{A}}(x) = \mu_{\check{A}}(x) - \frac{1 - \mu_{\check{A}}(x)}{1 + \lambda\mu_{\check{A}}(x)}.$$

Thus,

$$\begin{aligned} &\mu_{\check{A}}(x) - \pi_{\check{A}}(x) + (1 - c(\mu_{\check{A}}(x))) - \pi_{\check{A}}(x) \\ &\quad = 1 - \mu_{\check{A}}(x) - 3(1 - \mu_{\check{A}}(x))/(1 + \lambda\mu_{\check{A}}(x)) \\ &\quad = \frac{1 + \lambda\mu_{\check{A}}(x) - \mu_{\check{A}}(x) - \lambda\mu_{\check{A}}(x)^2 - 3 + 3\mu_{\check{A}}(x)}{1 + \lambda\mu_{\check{A}}(x)} \\ &\quad = \frac{(2\mu_{\check{A}}(x) - \lambda\mu_{\check{A}}(x)^2 - 2) + \lambda\mu_{\check{A}}(x)}{1 + \lambda\mu_{\check{A}}(x)} \end{aligned} \tag{12}$$

$$\because -1 < \lambda < 0 \quad \text{and} \quad 0 \le \mu_{\check{A}}(x) \le 1$$
$$\Rightarrow \quad 2\mu_{\check{A}}(x) - \lambda\mu_{\check{A}}(x)^2 - 2 < 1.$$

$\because$ (11) can be derived as

$$\mu_{\check{A}}(x) - \pi_{\check{A}}(x) + (1 - c(\mu_{\check{A}}(x))) - \pi_{\check{A}}(x) < 1. \tag{13}$$

Conversely, assume that $\mu_{\check{A}}(x) + (1 - c(\mu_{\check{A}}(x))) \le 1$, then we apply the Sugeno's negation,

$$\mu_{\check{A}}(x) + \left(1 - \frac{1 - \mu_{\check{A}}(x)}{1 + \lambda\mu_{\check{A}}(x)}\right) \le 1$$
$$\Rightarrow \quad \frac{\mu_{\check{A}}(x) + \lambda(\mu_{\check{A}}(x))^2 - 1 + \mu_{\check{A}}(x)}{1 + \lambda\mu_{\check{A}}(x)}$$

$\because 1 + \lambda\mu_{\check{A}}(x) > 0$, then

$$2\mu_{\check{A}}(x) + \lambda(\mu_{\check{A}}(x))^2 - 1 \le 0$$
$$\Rightarrow \quad 2\mu_{\check{A}}(x) + \lambda(\mu_{\check{A}}(x))^2 - 1 \le 0. \tag{14}$$

$\because$ (13) is equal to (14), thus, the proposition is proven. □

5 Applications

5.1 Pattern recognition

In order to demonstrate the application of the introduced switching method to pattern recognition, the author considers the problem discussed in Dengfeng [26], Mitchell [27], and Vlachos [28]. There are three patterns P_1, P_2

Table 1 Symmetric discrimination measures ('*' marks as the recognizing result)

	P_1	P_2	P_3
Dengfeng's method [26]	0.78	0.8	0.85*
Mitchell's method [27]	0.54	0.54	0.61*
Vlachos's method [28]	0.4492	0.3487	0.2480*
Our proposed method	2.0169	2.0585	2.1253*

Table 2 Symptoms characteristic for the patients considered

	Temperature	*Headache*	*Stomach pain*	*Cough*	*Chest pain*
Al	(0.8,0.1)	(0.6,0.1)	(0.2,0.8)	(0.6,0.1)	(0.1,0.6)
Bob	(0.0,0.8)	(0.4,0.4)	(0.6,0.1)	(0.1,0.7)	(0.1,0.8)
Joe	(0.8,0.1)	(0.8,0.1)	(0.0,0.6)	(0.2,0.7)	(0.0,0.5)
Ted	(0.6,0.1)	(0.5,0.4)	(0.3,0.4)	(0.7,0.2)	(0.3,0.4)

and P_3, which have classifications C_1, C_2 and C_3, respectively. The patterns are represented by the following IFSs in $X = \{x_1, x_2, x_3\}$:

$$P_1 = \{(x_1, 1.0, 0.0), (x_2, 0.8, 0.0), (x_3, 0.7, 0.1)\},$$

$$P_2 = \{(x_1, 0.8, 0.1), (x_2, 1.0, 0.0), (x_3, 0.9, 0.0)\},$$

$$P_2 = \{(x_1, 0.6, 0.2), (x_2, 0.8, 0.0), (x_3, 1.0, 0.0)\}.$$

Given an unknown pattern Q, represented by the IFS as

$$Q = \{(x_1, 0.5, 0.3), (x_2, 0.6, 0.2), (x_3, 0.8, 0.1)\}.$$

Herein, the purpose of this application is to classify Q into three classes C_1, C_2 and C_3. Accordingly to the principle of minimum discrimination information between IFSs, the classified process is derived as

$$k = \arg\min_k \{D_{\mathrm{IFS}}(P_k, Q)\},$$

where $k \in \{1, 2, 3\}$ [28]. Table 1 presents the classified results, and Q has correctly being classified to C_3.

On the contrary, according to (10), four patterns P_1, P_2, P_3 and Q are mapped to T2FSs as

$$P_1 = \frac{\frac{1}{1.0}}{x_1} + \frac{\frac{1}{0.8} + \frac{0}{1}}{x_2} + \frac{\frac{1}{0.7} + \frac{0}{0.9}}{x_3},$$

$$P_2 = \frac{\frac{1}{0.8} + \frac{0}{0.9}}{x_1} + \frac{\frac{1}{1.0}}{x_2} + \frac{\frac{1}{0.9} + \frac{0}{1.0}}{x_3},$$

$$P_3 = \frac{\frac{1}{0.6} + \frac{0}{0.8}}{x_1} + \frac{\frac{1}{0.8} + \frac{0}{1}}{x_2} + \frac{\frac{1}{1.0}}{x_3},$$

$$Q = \frac{\frac{1}{0.5} + \frac{0}{0.7}}{x_1} + \frac{\frac{1}{0.6} + \frac{0}{0.8}}{x_2} + \frac{\frac{1}{0.8} + \frac{0}{0.9}}{x_3},$$

where we regard the membership values $(1.0, 0)$ of IFS as a general fuzzy set, and $p = 0$ for simplification. After our derivation, Q has correctly being classified to C_3. A result is shown in Table 1 too, and the classification is in agreement with the ones obtained in [26–28].

5.2 Medical diagnosis

A medical knowledge base is focus on how to make a proper diagnosis D for a patient T with given values of symptoms S. Thus, in this section, the technique for handling medical diagnostic problems based on type-2 similarity is presented. In a given pathology, suppose that S is a set of symptoms, D is a set of diagnoses, and T is a set of patients. Analogous to De's notation of "Intuitionistic Medical Knowledge", the author defines "Type-2 Similarity Medical Knowledge" is defined as a reasoning process from the set of symptoms S to the set of diagnoses D.

Let there be four patients Al, Bob, Joe and Ted, i.e. $T = \{Al, Bob, Joe, Ted\}$. Their symptoms are temperature, headache, stomach pain, cough and chest pain, i.e. $S = \{$*Temperature*, *Headache*, *Stomach pain*, *Cough*, *Chest pain*$\}$. The set of Diagnosis is defined, i.e. $D = \{$*Viral Fever*, *Malaria*, *Typhoid*, *Stomach problem*, *Heart problem*$\}$. According to [10, 11], the intuitionistic fuzzy relations $T \to S$ and $S \to D$ are given in Table 2 and Table 3.

Hence, the purpose is to calculate for each patient t_j of his symptoms from a set of symptoms s_i characteristic for each diagnosis d_k. The reasoning process is as follows. (i) switch the obtained medical knowledge base from $IFS(X)$ to $FS_2(X)$, (ii) to calculate the similarity of symptoms s_i between each patient t_j and each diagnosis d_k, where $i = 1, \ldots, 5$, $j = 1, \ldots, 4$ and $k = 1, \ldots, 5$, (iii) to determine the higher similarity, pointing to a proper diagnosis. According to (7), the relationships of $T \to S$ and $D \to S$, the mapping

Table 3 Symptoms characteristic for the diagnoses considered

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Temperature	(0.4,0.0)	(0.7,0.0)	(0.3,0.3)	(0.1,0.7)	(0.1,0.8)
Headache	(0.3,0.5)	(0.2,0.6)	(0.6,0.1)	(0.2,0.4)	(0.0,0.8)
Stomach pain	(0.1,0.7)	(0.0,0.9)	(0.2,0.7)	(0.8,0.0)	(0.2,0.8)
Cough	(0.4,0.3)	(0.7,0.0)	(0.2,0.6)	(0.2,0.7)	(0.2,0.8)
Chest pain	(0.1,0.7)	(0.1,0.8)	(0.1,0.9)	(0.2,0.7)	(0.8,0.1)

Table 4 Result is measured by type-2 similarity of $p = 0$ ('*' marks as the diagnosis result)

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Al	0.68*	0.59	0.47	0.41	0.38
Bob	0.47	0.41	0.51	0.75*	0.42
Joe	0.59	0.44	0.62*	0.50	0.36
Ted	0.65*	0.56	0.51	0.48	0.32

Table 5 Result is measured by type-2 similarity of $p = 1$ ('*' marks as the diagnosis result)

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Al	0.71*	0.69	0.46	0.47	0.46
Bob	0.54	0.51	0.53	0.83*	0.44
Joe	0.63	0.48	0.65	0.66*	0.45
Ted	0.77*	0.62	0.55	0.56	0.4

Table 6 Result is measured by Szmidt et al. in [10] ('*' marks as the diagnosis result)

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Al	0.29	0.25*	0.32	0.53	0.58
Bob	0.43	0.56	0.33	0.14*	0.46
Joe	0.36	0.41	0.32*	0.52	0.57
Ted	0.25*	0.29	0.35	0.43	0.5

Table 7 Result is measured by Szmidt et al. in [11] ('*' marks as the diagnosis result)

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Al	0.75*	1.19	1.31	3.27	∞
Bob	2.1	3.73	1.1	0.35*	∞
Joe	0.87	1.52	0.46*	2.61	∞
Ted	0.95	0.77*	1.67	∞	2.56

from $IFS(X)$ to $FS_2(X)$ is switched and listed as follows (for the sake of simplicity, take *Al* and *Temperature* for example),

$$Al = \frac{\frac{1}{0.8+p\cdot 0.1} + \frac{0}{0.9+p\cdot 0.1}}{Temperature} + \frac{\frac{1}{0.6+p\cdot 0.3} + \frac{0}{0.9+p\cdot 0.3}}{Headache} + \frac{\frac{1}{0.2} + \frac{0}{0.2}}{Stomach\ pain} + \frac{\frac{1}{0.6+p\cdot 0.3} + \frac{0}{0.9+p\cdot 0.3}}{Cough} + \frac{\frac{1}{0.1+p\cdot 0.3} + \frac{0}{0.4+p\cdot 0.3}}{Chest\ pain}$$

Table 8 Results measured by De et al. in [8] ('*' marks as the diagnosis result)

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Al	0.35	0.68*	0.57	0.04	0.08
Bob	0.2	0.08	0.32	0.57*	0.04
Joe	0.35	0.68*	0.57	0.04	0.05
Ted	0.32	0.68*	0.44	0.18	0.18

Table 9 Results measured by the formal fuzzy similarity ('*' marks as the diagnosis result)

	Viral fever	*Malaria*	*Typhoid*	*Stomach problem*	*Chest problem*
Al	0.8	0.84*	0.82	0.56	0.52
Bob	0.74	0.58	0.8	0.86*	0.66
Joe	0.74	0.74	0.8*	0.54	0.5
Ted	0.78	0.82*	0.76	0.62	0.58

Table 10 All the considered results

	$p=0$	$p=1$	Szmidt in (2001)	Szmidt in (2004)	De in (2001)	Fuzzy similarity
Al	*Viral fever*	*Viral fever*	*Malaria*	*Viral fever*	*Malaria*	*Malaria*
Bob	*Stomach problem*	*Stomach problem*	*Stomach problem*	*Stomach problem*	*Stomach problem*	*Stomach problem*
Joe	*Typhoid*	*Typhoid*	*Malaria*	*Typhoid*	*Typhoid*	*Typhoid*
Ted	*Viral fever*	*Viral fever*	*Malaria*	*Malaria*	*Malaria*	*Malaria*

and

$$\begin{aligned}
&\textit{Viral fever} \\
&= \frac{\frac{1}{0.4+p\cdot 0.6}+\frac{0}{1+p\cdot 0.6}}{\textit{Temperature}} + \frac{\frac{1}{0.3+p\cdot 0.2}+\frac{0}{0.5+p\cdot 0.2}}{\textit{Headache}} \\
&\quad + \frac{\frac{1}{0.1+p\cdot 0.2}+\frac{0}{0.3+p\cdot 0.2}}{\textit{Stomach pain}} + \frac{\frac{1}{0.4+p\cdot 0.3}+\frac{0}{0.7+p\cdot 0.3}}{\textit{Cough}} \\
&\quad + \frac{\frac{1}{0.1+p\cdot 0.2}+\frac{0}{0.3+p\cdot 0.2}}{\textit{Chest problem}}.
\end{aligned}$$

Note that $p \in [0, 1]$. The similarity measure (3) for each patient form the considered set of possible diagnoses are given in Tables 4 and 5 for $p = 0, 1$, respectively. Besides, in Table 6, Szmidt and Kacprzyk made their diagnoses by counting the distances of three parameters: membership function, non-membership and the hesitation margin for all the symptoms of each patient [10]. They also proposed the new geometrical interpretation to measure the similarity between IFSs, and to make a proper diagnosis in Table 7 [11]. In addition, De et al. defined the "Intuitionistic Medical Knowledge" to show and discuss the relationship between symptoms and diagnosis, the result of which is given in Table 8 [8]. The formal fuzzy similarity

$$S(A, B) = \frac{\sum_n \{1 - |\mu_A(x) - \mu_B(x)|\}}{n}$$

applied in the medical diagnosis is given in Table 9 [25].

All the results from the considered methods are given in Table 10. The purpose of medical software or computer systems is to assist doctors in patient care and to help diagnosing complex medical conditions. It's difficult to tell which method can assist in making an exact diagnosis. From Table 10, it is obvious that Bob suffers from a stomach problem (all methods agree), Joe from typhoid (five out of the six methods agree). The above results show that our proposed method can assist with the diagnosing. For the diagnoses viral fever and malaria, the results indicate these two diagnoses are difficult to diagnose (almost half of the methods approve one or the other diagnosis), and that these two symptoms are involved with each other. It is here that the diagnosing result of our proposed method differs from other methods. In addition our method can be used as an assister. Distinct diagnosing results can be obtained by means of T2FSs, including a better ability to handle imprecise and imperfect information compared to IFSs.

6 Conclusions

T2FSs are significant for imprecise and imperfect information handling. However, their extensive applications are getting less attention. In this paper, a mutual switch between T2FSs and IFSs is defined and the medical diagnosis is generalized by the switching and reasoning with type-2 similarity. As a consequence, easy comprehension and axiom definition are provided during the switch. In addition, our method makes it possible to extend the usage of T2FSs and renews the relationship between T2FSs and IFSs.

Acknowledgements The author would like to thank the National Science Council of the Republic of China, Taiwan for financially supporting this research under Contract No. NSC 96-2218-E-129-001.

References

1. Zadeh LA (1965) Fuzzy sets. Inf Control 8:338–353
2. Atanassov K (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20:87–96
3. Atanassov K (1989) More on intuitionistic fuzzy sets. Fuzzy Sets Syst 33:37–46
4. Atanassov K (1994) Operators over interval valued intuitionistic fuzzy sets. Fuzzy Sets Syst 64:159–174
5. Atanassov K (1994) New operations defined over the intuitionistic fuzzy sets. Fuzzy Sets Syst 33:37–46
6. Taranli N, Coker D (2001) Fuzzy connectedness in intuitionistic fuzzy topological spaces. Fuzzy Sets Syst 116:369–375
7. De SK, Biswas A, Roy R (2000) Some operations on intuitionistic fuzzy sets. Fuzzy Sets Syst 114:477–484
8. De SK, Biswas A, Roy R (2001) An application of intuitionistic fuzzy sets in medical diagnosis. Fuzzy Sets Syst 117:209–213
9. Szmidt E, Kacprzyk J (2000) Distances between intuitionistic fuzzy sets. Fuzzy Sets Syst 114:505–518
10. Szmidt E, Kacprzyk J (2001) Intuitionistic fuzzy sets in intelligent data: analysis for medical diagnosis. In: ICCS, pp 263–271
11. Szmidt E, Kacprzyk J (2004) A similarity measure for intuitionistic fuzzy sets and its application in supporting medical diagnostic reasoning. In: ICAISC 2004, pp 388–393
12. Li D, Cheng C (2002) New similarity measures of Intuitionistic fuzzy sets and application to pattern recognition. Pattern Recognit Lett 23:221–225
13. Liang Z, Shi P (2003) Similarity measures on intuitionistic fuzzy sets. Pattern Recognit Lett 24:2687–2963
14. Yanhong L, Olson OL, Zhen Q (2007) Similarity measures between intuitionistic fuzzy (vague) sets: a comparative analysis. Pattern Recognit Lett 28:278–285
15. Deschrijver G, Kerre EE (2003) On the composition of intuitionistic fuzzy relations. Fuzzy Sets Syst 136:333–361
16. Burillo P, BUstince H (1992) Estructuras algebraicas en conjuntos IFS. In: II congreso nacional de lógicaly tecnología fuzzy, Boadilla del Monte, Madria, Spain, pp 135–147
17. Sugeno M, Terano T (1977) A model of learning based on fuzzy information. Kybernetes 6:157–166
18. Yager RR (1979) On the measure of fuzziness and negation, part II: lattices. Inf Control 44:236–260
19. Mendel JM (1999) Computing with words when words can mean different things to different people. In: International ICSC congress computation intelligence: methods application. 3rd annual symposium on fuzzy logic application, New York
20. Mendel JM (2001) Uncertain rule-based fuzzy logic systems: introduction and new direction. Prentice Hall, New York
21. Zadeh LA (1975) The concept of a linguistic variable and its application to approximate reasoning, I. Inf Sci 8:199–249
22. Hung W-L, Yang M-S (2004) Similarity measures between type-2 fuzzy sets. Int J Uncertain Fuzziness Knowl-Based Syst 12:827–841
23. Own C-M, Yu P-T (2006) Reasoning partially true knowledge by type-2 similarity. In: Proceedings of the 17th international conference on information management (ICIM 2006), I-Shou University, Kaohsiung
24. Pappis CP, Karacapilidis NI (1993) A comparative assessment of measures of similarity of fuzzy values. Fuzzy Sets Syst 56:171–174
25. Raha S, Pal NR, Ray KS (2002) Similarity-based approximate reasoning: methodology and application. IEEE Trans Syst Man Cybern 32:41–47
26. Dengfeng L, Chuntian C (2002) New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions. Pattern Recognit Lett 23:221–225
27. Mitchell HB (2003) On the Dengfeng-Chuntian similarity measure and its application to pattern recognition. Pattern Recognit Lett 24:3101–3104
28. Vlachos IK, Sergiadis GD (2007) Intuitionistic fuzzy information—applications to pattern recognition. Pattern Recognit Lett 28:197–206