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A Formalism for Representing and Reasoning with Temporal Information, Event and Change

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Abstract. In this paper we present a general formalism for representing and reasoning with temporal information, event and change. The temporal framework is a theory of time that takes both points and interval as temporal primitives and where the base logic is that of Kleene's three-valued logic. Thus, we can avoid the Divided Instant Problem (DIP). We present a three-valued based Temporal First-Order Nonmonotonic Logic (TFONL) that employs an explicit representation of time and events. We may embody default logic into TFONL, which takes into consideration the frame, qualification and ramification problems.

Keywords: three-valued, temporal, nonmonotonic, event, change

1. Introduction

Modelling the dynamic aspects of the world is one of the most challenging problems in Artificial Intelligence(AI). Many AI areas such as medical diagnosis and explanation, planning, condition monitoring and fault diagnosis, and dynamic system modeling require an explicit representation of time and causal relationships. The term modeling itself suggests incompleteness. We can identify three research areas: temporal reasoning, reasoning about events and change and dealing with incompleteness and uncertainty of information.

Many general theories of event and/or time have been proposed such as the situation calculus [10], the event calculus [9], McDermott's temporal logic [11], Allen's theory of action and time [2]. These proposals played an important role (1) in establishing two main competitors as temporal primitives, namely point and interval; (2) in pointing out the general problems such as reasoning by default, the interaction of actions, the frame, qualification and ramification problems, and (3) in showing that some ontological decisions need to be made before defining a general theory of time, change and event (cf. [22]).

There are two different types of approaches to temporal reasoning. The first type is concerned with reasoning about change, events, actions and causality. The goal is to determine the consequent state given that some events have occurred starting from some initial state. The second type deals with reasoning about temporal constraints on time-dependent entities. It aims to determine whether a set of temporal constraints is consistent or what consequences follow from a set of temporal constraints with no assumptions about properties of temporal facts. In this paper, we deal with the first of these approaches.

Most of the influential formalisms for reasoning about actions/events (e.g., [10] and [9]) seem to pay little attention to the temporal ontology as they focus on other problems such as the frame, qualification and quantification problems. One important issue in these approaches is that only points are taken as primitive temporal elements while intervals have to be constructed from points. This may lead to the so-called *Dividing Instant Problem* (DIP) [22], the problem of specifying whether time intervals are closed or open at their starting/ending points (cf. [2] and [7]). If two adjacent intervals include their ending-points, then they would have, at least, one ending-point in common. Hence, if a proposition A has a different truth-value in every interval, then at the common end-point, A will be both true and false. Similarly, if two intervals do not include their ending-points, there will be points at which the truth or falsity of some assertion is undefined. One solution would be to take point-based intervals as semi-open (e.g., all intervals include their left ending-points and exclude their right ones). However, This seems to be arbitrary.

Interval based-approaches have been shown to overcome the DIP(cf. [2] and [21]). However, this is at the expense of expressiveness because those approaches cannot deal with instantaneous events and fluents.

In this paper we present a formalism that employs an explicit temporal representation with a theory of time that takes both points and interval as temporal primitives and where the base logic is that of Kleene's three-valued logic. Thus, we can avoid the DIP Problem. We present a three-valued based Temporal First-Order Nonmonotonic Logic (TFONL) that employs an explicit representation of time and events. TFONL is an extension of the quantified version of the non-temporal system **T3**, which is a three-valued based nonmonotonic logic system (cf. [12]). We may embody default logic into TFONL, which takes into consideration the frame, qualification and ramification problems. Furthermore, it incorporates to a domain description the set of rules governing change.

In Section 2 we present a theory of time, PI, that takes both points and interval as primitives and where the base logic is Kleene three-valued logic. In Section 3 we present a three-valued based Temporal First-Order Nonmonotonic Logic (TFONL) that employs an explicit representation of time and events. In Section 4, we show how the frame, qualification and ramification problems are dealt with. In Section 5, we show how default logic can be embodied in TFONL. In Section 6, we incorporate a domain description and the rules governing change to ensure that change to a state resulting from the successful occurrence of an event is minimal. A comparison with other temporal theories is made in Section 7 and a presentation of the point-based and interval-based temporal theories is given in the Appendix.

2. A Time Theory Based on Points and Intervals (PI)

Classical logic does not seem to be well equipped to cope with statements containing explicit temporal reference. There are a number of issues that must be addressed when formalizing time. Among these are the ability to represent and reason with instantaneous and non-instantaneous fluents and events while avoiding the DIP problem.

In this section we provide a system PI where both points (P) and intervals (I) are primitives and where the base logic is a three-valued one. Let i, j, k, r, m, n denote intervals and p, p_1 denote points.

A time structure is a tuple:

$$M_T = \langle P, I, < p, \text{Meets}, \text{ Within}, \rangle$$

where

(1) P and I are non-empty sets of points and intervals respectively,

(2) $<P$ is a precedence relation on points of time. $<P$ has the following properties:

- (P1) $(p1 <_P p2)$ & $(p2 <_P p3) \rightarrow p1 <_P p3$ (Transitivity)
- (P2) $\neg(p_1 <_P p1)$ (Irreflexivity)
- (P3) $(p1 <_P p2) \vee (p1 = p2) \vee (p2 <_P p1)$ (linearity)
- (P4) $(\forall p)(\exists p1)(p <_P p1)$
- (P5) $(\forall p)(\exists p1)(p1 <_P p)$
- (P6) $(\forall p1)(\forall p2)(p1 <_P p2)(\exists p3)(p1 <_P p3$ & $p3 <_P p2)$ (Density)
- (P4) (resp. P5) states that for any time point p, there exist a point $p1$ that comes after it (resp. before it).

(3) Meets is axiomatized following Allen and Hayes in [3] as follows:

- (I1) $(\forall i, j)(\exists k)$ (Meets(i, k) & Meets$(j, k) \supset (\forall r)$ (Meets$(i, r) \equiv$ Meets(j, r))
- (I2) $(\forall i, j)(\exists k)$ (Meets(k, i) & Meets(k, j) $\supset$ $(\forall r)$ (Meets$(r, i) \equiv$ Meets(r, j)))
- (I3) $(\forall i, j, k, r)$ (Meets(i, j) & Meets$(k, r) \supset$
  Meets(i, r), XOR
  $(\exists m)$ (Meets(i, m) & Meets(m, r) XOR
  $(\exists n)$(Meets(k, n) & Meets(n, j)
- (I4) $(\forall i)$($(\exists j, k)$ (Meets(j, i) & Meets(i, k))
- (I5) $(\forall i, j)$ (Meets$(i, j) \supset$
  $(\exists k = i + j,)(\exists m, n)$ (Meets(m, i) &
  Meets(i, j) & Meets(j, n) &
  Meets(m, k) & Meets(k, n))

where XOR denotes exclusive OR. (I1) and (I2) state that every interval has a unique start point and a unique end point. (I3) define all the possible relations between any two meeting places. (I4) states that every interval has one interval the precedes and an interval that succeeds it. $k = i + j$ is only definable if Meets(i, j) holds and k contains exactly i, j and their meeting points p, i.e., $k = i \cup \{p\} \cup j$. (I5) states that for any two adjacent interval i and j, there exist an interval k such that $k = i + j$.

(4) Within is a point-interval relation that is governed by the following axiom:

(PI_1) $\forall i \exists p$ Within(p, i)

We may add the following definition:

Definition 2.1. Duration $(t) = 0$ if $t \in$ P and Duration $(t) > 0$ if $t \in$ I.

Given the above set of axioms we may define other interval-interval, point-interval relations. For instance: $L_B(p, i)$ which states that p is the lower limit (beginning) of i can be defined as:

$L_B(p, i)$ iff $(\forall p_1)$[(Within$(p_1, i) \supset p < p_1$) and $(\forall p_2)$ if ($p_2 <> p$ and Within$(p_1, i) \supset p_2 < p_1$) then $p2 < p$].

Similarly, we may define, $L_E(p, i)$, that p is the upper limit (end) of i.

From these definitions, we may derive the following axioms:

(PI_2) $(\forall i)(\forall p)(\forall p_1)(L_B(p, i) \ \& \ L_E(p_1, i) \supset p <_P p_1)$

(PI_3) $(\forall i)(\exists p)(\exists p_1)(L_B(p, i) \ \& \ L_E(p_1, i))$

(PI_4) $(\forall i)(L_B(p, i) \ \& \ L_B(p_1, i)) \supset p = p1$

(PI_5) $(\forall i)(L_E(p, i) \ \& \ L_E(p_1, i)) \supset p = p1$

(PI_6) $(\forall i)(\forall j)(L_B(p, i) \ \& \ L_E(p_1, i) \ \& \ L_B(p, j) \ \& \ L_E(p_1, j)) \supset i = j$

3. Temporal FONL (TFONL)

Our knowledge about real world is usually incomplete and uncertain. The notion of modeling itself suggests incompleteness and uncertainty. Temporal uncertainty could result in many contexts: If two events $e1$ and $e2$ occurs at some points p_1 and p_2 somewhere between p_k and p_r, then the relation between e_1 and e_2 is not determined. Event *e* occurs during interval I, but it is not known exactly when or for how long if it is a durative event. Event *e* occurs during interval *i* for a duration of 10. A fluent such as "the patient temperature is high" that was observed at a time point p within an interval during which it holds. An uncertain temporal relation is a disjunction of two or more exact relations.

Default reasoning [15] is appropriate in those situations where we have only partial knowledge of the actual state of affair. In such states, some things are known (to be true or false) but others are in doubt. Obeid in [12] proposes that partial states of knowledge are to be represented by partial models or information.

In this paper, we employ a Temporal First Order Nonmonotonic Logic (TFONL). The system is based on the quantified version of the non-temporal system **T3**, which is a three-valued based nonmonotonic logic system (cf. [12]). The language has a third value *undefined* which was used by Kleene to describe computations that may not terminate. We have incomplete knowledge and thus we cannot, as classical logic suggests, determine the truth or falsity of every sentence. One of the advantages of T3 is that *defaults* can be represented as sentences in the object language in the system. Obeid in [14] shows that there is a one-to-one correspondence between extensions of a default theory and appropriate minimal information states (Partial Models), which provide the semantic account (models) of the system **T3**.

The language, L_{T3}, of $T3$ is that of Kleene's three-valued logic extended with the modal operators "M" (Epistemic Possibility) and "P" (Plausibility). In T3, "L" is the dual of "M" and "N" be the dual of "P", i.e., LA $\equiv \sim$ M $\sim$ A and NA $\equiv\sim$P $\sim$A. Obeid in [12] defines a truth functional implication "$\supset$" , i.e., an implication that behaves exactly like the material implication of classical logic, as follows: (A $\supset$ B = M($\sim$A & B) V $\sim$A V B.

Nonmonotonic reasoning is represented via the *epistemic possibility operator* M and the *plausibility operator* P. Informally, MA states that A is not established as false. Using M, we may define the operators U (*undefined*), D (*defined*) and $\neg$ (*classical negation*) where UA is true if the truth value of A is undefined and DA is true if the truth value of A is not undefined.

Notational Definition 3.1.

UA $\equiv$ MA & M $\sim$ A
DA $\equiv\sim$ UA
$\neg$A $\equiv$ DA & $\sim$ A

3.1. Syntax of LFONL

We extend the language L_{T3} to allow for the expression of quantified temporal expressions and relations, and for the occurrence of events. We need four sorts P (for points), I (for intervals), E for events and $L3$ (for the three-valued base logic literals).

Definition 3.2. The vocabulary of L_{FONL} consists of the following symbols:

$$\neg, \text{V}, \&, \rightarrow, \forall, \exists, <, \text{Meets}, \ \text{Within}\,(,),$$

and the following mutually disjoint countable sets of symbols:

CONST_{PI}, CONST_{E} (constants of the sorts $\text{P} \cup \text{I}$ and E),
VAR_{PI}, VAR_{E} , VAR_3 (variable of sorts $\text{P} \cup \text{I}$, E and the three-valued base logic),
FUNC_{PI} , FUNC_{E} (function symbols of arity $n > 0$ of sorts $\text{P} \cup \text{I}$ and E)
PRED (first order relation symbols of arity $n \geq 0$)
HOPRED (higher-order relation symbols of arity $n \geq 1$)

If $\text{S} \in \{\text{D, I, P}\}$ then TERM_{S} is the minimal set such that:

(1) $\text{CONST}_S \cup \text{V}_{\text{S}} \subseteq \text{TERM}_{\text{S}}$
(2) if f is an n-ary function symbol in FUNC_{P} and $u_1, \ldots, u_n$ are TERM_{S} then $f(u_1, \ldots, u_n)$ TERM_{S}.

Let $\text{TERM}_{\text{PI}} = \text{TERM}_{\text{P}} \cup \text{TERM}_{\text{I}}$
TERM_{E} is the minimal set such that

(1) $\text{CONST}_{\text{E}} \cup \text{VAR}_{\text{E}} \subseteq \text{TERM}_{\text{E}}$
(2) if f is an n-ary function symbol in FUNC_{E} and $u_1, \ldots, u_n$ are TERM_{D} then $f(u_1, \ldots, u_n)$ TERM_{E}.

Let $\Pi = r(u_1, \ldots, u_n) : r \in \text{PRED}$ and $u_1, \ldots, u_n \text{TERM}_{\text{D}}$ be the set of atom in the base logic and let $\Xi = \Pi \cup \neg \text{l} : \text{l} \in \Pi$. Then $\text{TERM}_3 = \Xi \cup \text{VAR}_3$

Definition 3.3. The language, L_{TFONL}, is the minimal set that satisfies the following conditions:

If $t, t' \in \text{TERM}_{\text{P}}$ then $t = t' \in L_{\text{TFONL}}$ and $t < t' \in L_{\text{TFONL}}$
If $i, i' \in \text{TERM}_{\text{i}}$ then $i = i' \in L_{\text{TFONL}}$ and $\text{Meets}(i, i') \in L_{\text{TFONL}}$
If $u_1, \cdots, u_n \in \text{TERM}_{\text{D}}$, $r \in \text{PRED}$ and $t \in \text{TERM}_{\text{PI}}$ then $A[t] \in L_{\text{TFONL}}$
If $l \in \text{VAR}_3$ and $t \in \text{TERM}_{\text{PI}}$ then $l[t] \in L_{\text{TFONL}}$
If $e_1, e_2, \cdots, e_{\text{n}} \in \text{TERM}_{\text{E}}$, $l_1,^{*} m \in \text{TERM}_3$, $\text{hr} \in \text{HOPRED}$ and $t \in \text{TERM}_{\text{PI}}$ then $\text{hr}(e_1, e_2, \cdots, e_n, l_1, \cdots, l_m)[t] \in L_{\text{TFONL}}$
If S is a sort and $u, u' \in S$ the $u = u' \in L_{\text{TFONL}}$
If A, B $\in L_{\text{TFONL}}$, then A & B, AVB, ¬ A, A → B $\in L_{\text{TFONL}}$
If S is of sort D, P, I or E, $x \in \text{V}_{\text{S}}$ and $A \in L_{\text{TFONL}}$ then $\forall x$ A and $\exists x$A $\in L_{\text{TFONL}}$

3.2. *Model Theory of LFONL*

Definition 3.4. A Model for L_{TFONL} is a stucture

$$M = \langle D, E, P, I, R_{\text{I}}, R_{\text{P}}, F, \text{PRED}, \ \text{HOPRED}, \Phi, \Psi, R, R', \rangle$$

where

(1) D, E, I, P are mutually disjoint non-empty sets
(2) R_I is a binary relation on I that satisfies the conditions (I1)–(I5) satisfied by Meets.
(3) R_P is a binary relation on P that is irreflexive, transitive, linear and unbounded.
(4) $F = \langle F_D, F_I, F_P, F_E \rangle$ such that for $S \in D, I, P, E$, F_S is a set of n-pace functions of type $S^n \rightarrow S$ where $n \geq 1$.
(5) PRED is a set of partial n-place functions of type $D^n \rightarrow$ true, false for $n \geq 0$
(6) HOPRED is a set of partial function of type $E^n \times \text{TERM}_3$ {true, false} for $n + m \geq 1$
(7) $\Phi =< \Phi_{\text{D}}, \Phi_{\text{P}}, \Phi_{\text{I}}, \Phi_{\text{E}}, \Phi_3, \Phi_{\text{FD}}, \Phi_{\text{FP}}, \Phi_{\text{FI}}, \Phi_{\text{FE}}, \Phi_{\text{PRED}}, \Phi_{\text{HOPRED}} >$ is an interpretation function such that:
for $S \in \{D, I, P, E\}$, $\Phi_S : S \rightarrow S$ and $\Phi_{\text{FS}} : F_{\text{S}} \rightarrow F_{\text{S}}$
Φ_3: $\text{TERM}_3 \rightarrow \text{TERM}_3$ is the identity function.
Φ_{PRED}: PRED × PI → PRED
Φ_{HOPRED}: HOPRED × PI → HOPRED
(8) Ψ is a non-empty set of information states
(9) R and R' are binary relation on Ψ.

Intuitively, D is a set of objects, E is a set of event types, I is a set of interval and P is a set of points. $<P, I, <P$, Meets, Within, $>$ constitutes the temporal framework. For each n-place relation symbol r and a $t \in P \cup I$, $\Phi_{\text{PRED}}(r, t)$ is the partial characteristic function of the relation denoted by r at/during t. Similarly, for each higher-order relation symbol hr and

$t \in P \cup I$, Φ_{HOPRED} (hr, t) is the partial characteristic function of the relation denoted by hr at/during t.

Terms are interpreted in the standard way with the exception of $T3$ literals which are interpreted as themselves.

The interpretation of R may be thought of *as epistemic possible* extension between states, and that of R' as *plausibility* between states. Given s, s_1 are members of Ψ, we shall write $s\ R\ s_1$ to mean that the information state s_1 is an *epistemic possible* extension of the information state s and $s\ R'\ s1$ to mean that $s1$ is a *plausible alternative* to s.

In fact, we employ three notions of extension defined on partial states: *Refinement* ($\leq_{REF}$), *epistemic possibility* (R) and *plausibility* (R'). Epistemic possibility and plausibility are formally defined above. Refinement, however, simply reflects an informational order of states (partial models) based on that of the truth values of sentences belonging to these states where an informational order of truth values is as follows: $U \leq_{REF} F$, $U \leq_{REF} T$, $U \leq_{REF} U$, $F \leq_{REF} F$, $T \leq_{REF} T$.

Definition 3.5. A variable assignment for a TFONL model is a function $g = \langle g_D, g_P, g_I, g_E, g_3\rangle$ where $g_S : V_S :\rightarrow S$ for any of the sorts.

Let $g \approx g'$ indicates that variable assignments g and g' differ at most on the assignment to variable x.

Definition 3.6. Let M be a TFONL model with interpretation function Φ, and let g be a variable assignment for M, then the term evaluation function Φ_g is defined as follows:

For any of the sorts $S \in D, I, P, E$

If $c \in C_S$ then $\Phi_g(c) = \Phi_S(c)$
If $x \in V_S$ then $\Phi_g(x) = g_S(c)$
If $u = f(u_1, \ldots, u_n) \in \text{TERM}_S$ then $\Phi_g(u) = \Phi_{FS}(f)(\Phi_g(u_1), \ldots, \Phi_g(u_n))$
If $A \in \text{TERM}_3$ then $\Phi_g(c) = \Phi_3(A)$

Definition 3.7. Let M be a TFONL model with interpretation function Φ, and let g be a variable assignment for M. Let $p, p' \in P$, $I, I' \in I$, A, B be wffs then, the truth "$\models g$" and the falsity "$\dashv g$" notions are recursively defined as follows:

(i) $\mathbf{M}, s \models g$ true
(ii) $\mathbf{M}, s \models g\ p\langle p'$ iff $\langle \Phi_I(s, p), \Phi_I(s, p')\rangle \in R_P$
(iii) $\mathbf{M}, s \models g$ Meets(i, i') iff $\langle \Phi_I(s, i), \Phi_I(s, i')\rangle \in R_I$
(iv) $\mathbf{M}, s \models g\ u = u'$ iff $\Phi_g(u)$ is $\Phi_g(u')$
(v) $\mathbf{M}, s \models g\ r(u_1, \ldots, u_n)[t]$ iff $\Phi_{PRED}(r, \Phi_g(t))(\Phi_g(u_1), \ldots, \Phi_g(u_n)) = \text{true}$
(vi) $\mathbf{M}, s \models g\ l[t]$ iff $l \in \text{VAR}_3$ and $\mathbf{M}, s \models g \Phi_g(l)[t]$
(vii) $\mathbf{M}, s \models g$ hr$(e_1, \ldots, e_n, l_1, \ldots, l_m)[t]$ iff

$\Phi_{HHOPRED}$ (hr, $\Phi_g(t))(\Phi_g(e_1), \ldots, \Phi_g(e_n)\Phi_g(l_1), \ldots, \Phi_g(1_m)) = \text{true}$

(viii) $\mathbf{M}, s \models g\ A \,\&\, B$ iff $\mathbf{M}, s \models g\ A$ and $\mathbf{M}, s \models g\ B$
(ix) $\mathbf{M}, s \models g \sim A$ iff $\mathbf{M}, s \models g\ A$
(x) $\mathbf{M}, s \models g\ \forall x A$ iff $M, s' \models g' A$ for all g' such that $g \approx g'$

where $g \approx g'$ indicates that variable assignments g and g' differ at most on the assignment to variable x.

(xi) $\mathbf{M}, s \models g$ MA iff $(\exists\ s_1 S)(sRs_1$ and $\mathbf{M}, s1 \not\models g \sim A)$
(xii) $\mathbf{M}, s \models g$ PA iff $(\exists s_1 S)(sR's_1$ and $\mathbf{M}, s_1 \models g\ A)$

(i′) $\mathbf{M}, s \dashv g$ false
(ii′) $\mathbf{M}, s \dashv g p\langle p'$ iff $\langle \Psi_I(s, p), \Psi_I(s, p')\rangle \notin R_P$
(iii′) $\mathbf{M}, s \dashv g$ Meets(i, i') iff $\langle \Psi_I(s, i), \Psi_I(s, i')\rangle \notin R_I$
(iv′) $\mathbf{M}, s \dashv g u = u'$ iff $\Psi_g(u)$ is not $\Psi_g(u')$
(v′) $\mathbf{M}, s \dashv g\ r(u_1, \ldots, u_n)[t]$ iff $\Psi_{Pr}(r, \Psi_g(t))(\Psi_g(u_1), \ldots, \Psi_g(u_n)) = \text{false}$
(vi′) $\mathbf{M}, s \dashv g l[t]$ iff $l \in \text{VAR}_3$ and $\mathbf{M}, s \dashv g \Psi_g(l)[t]$
(vii′) $\mathbf{M}, s \dashv g$hr$(e_1, \cdots, e_n, l_1 \cdots, l_m)[t]$ iff

$\Phi_{HHOPRED}$ (hr, $\Phi_g(t))(\Phi_g(e_1), \ldots, \Phi_g(e_n)\Phi_g(l_1), \ldots, \Phi_g(l_m)) = \text{false}$

(viii′) $\mathbf{M}, s \dashv g A \,\&\, B$ iff $\mathbf{M}, s \dashv g A$ or $\mathbf{M}, s \dashv g B$
(ix′) $\mathbf{M}, s \dashv g \sim A$ iff $\mathbf{M}, s \models g\ A$
(x′) $\mathbf{M}, s \dashv g \forall x A$ iff $\mathbf{M}, s \dashv g' A$ for all g' such that $g \approx g'$

where $g \approx g'$ indicates that variable assignments g and g' differ at most on the assignment to variable x.

(xi′) $\mathbf{M}, s \dashv g$MA iff $(\forall \in s_1 S)$ (if $s\ R\ s_1$ then $\mathbf{M}, s_1 \models g \sim A)$

(xii′) $\mathbf{M}, s \dashv g$ PA iff $(\forall \in s_1 S)$ (if $s\ R'\ s_1$ then $\mathbf{M}, s_1 \not\models g \sim A)$

Definition 3.8. A WFF A is true in a TFONL-model M (written $M) \models A)$ if $\mathbf{M}, s \models g\ A$ for all information states s and variable assignments g. A formula A is false in M (written M $\dashv A)$ if M, $s \dashv g\ A$ for all information states s and all variable assignments g.

4. Inertia, Qualification and Ramification

A theory of time, change and event has to address the problem of representing the outcome of a sequence of events given some state s. Some of events which may fail because their preconditions are not satisfied. Thus, in order to represent events, it is necessary to represent changes in state. Using TFONL, a state can be taken as the set of facts (together with rules default that form a default theory) which are true at a particular point or during a particular interval. Changes in the state are then represented by changes in the truth-values of fluents. The effect of an event may be represented by an axiom of change:

$$\begin{array}{l}\text{PRECOND}(e)[t_1]\ \&\ \text{OCC}(e)[t_2]\ \&\ C(t_1, t_2, t_3)\\ \text{IMPLIESPOSTCOND}(e)[t_3]\end{array} \quad (1)$$

where $t_1, t_2, t_3 \in$ P $\cup$ I and IMPLIES $\in \{\supset, \Rightarrow\}$ as needed and C

Propositions of form (1) describe the direct effects of events on the state. They can be read as the occurrence of e at t_2 causes POSTCOND(e) at t_3 if PRECOND(e) holds at t_1 where $C(t_1, t_2, t_3)$ reflects the constraints on t_1, t_2 and t_3.

It is important to note that our view of events is causal. That is, if the preconditions of an event are satisfied and the event occurs, then its postconditions/consequents must be true. However, this is not always the case as there may be an infinite number of reasons why an event can fail despite the fact that its preconditions are true when it occurs. This in fact explains why we have employed two types of implication: $\supset$ (material) and $\Rightarrow$ (defeasible). The material implication is only used if we are absolutely certain that the effect will take place if the preconditions hold and the event occurs.

Example 4.1. A fire alarm starts ringing immediately after some smoke has been detected and keeps ringing as long as smoke remains. The alarm is electrical, so may be subject to a power cut. This situation can be described as follows:

Power-on$[t_{\text{po}}]$ & Occ(Smoke)$[t_s]$ & During(t_{po}, t_s) & Start$(t_s, t_a) \Rightarrow$ alarm$[t_a]$
Occ(Power-cut)$[t_{\text{pc}}]$ & Finishes$(t_{\text{po}}, t_{\text{pc}})$ & Starts$(t_{\text{pc}}, t_{\text{na}})$ & Meets$(t_{\text{a}}, t_{\text{na}}) \Rightarrow$ alarm$[t_{\text{no}}]$

t_{pc} starts when t_{po} (power-on) finishes and t_{na} ($\neg$ alarm) begins. Notice that we have not stated that t_{s} finishes as smoke could still be present after the alarm has stopped due to power cut.

It is important to note (1) the distinction between power-on as a fluent and power-cut as an event occurrence, and (2) the use of $\Rightarrow$ as we may not know all the preconditions required for a successful event.

It is also necessary to represent what persists, i.e., does not change from a time point to another or when it is realized that an event has occurred. This can be achieved by a common sense law of inertia which simply state that a fluent remains in their truth value status unless there is reason fo it not to do so. This can be stated as follows:

$$A[t] \Rightarrow A[t'] \quad \text{where } t < t' \qquad \text{(Inertia)}$$

Relations between fluents in a particular world (information state) are expressed as:

$$\begin{array}{l}A_1(X_1)(t_1), \ldots A_m(X_m)(t_m)\ \&\ C(t_1, \ldots t_m, t'1, \ldots .t'_n)\\ \qquad \text{IMPLIESB}_1(\text{Y}_1)(\text{t}'_1), \cdots .B_n(Y_n)(\text{t}'_n)\end{array} \quad (2)$$

where IMPLIES $\in \{\supset, \Rightarrow\}$ as needed;

We shall refer to propositions of form (2) as static formulae (for examples cf. [13]).

5. Default Reasoning within TFONL

One of the advantages of T3 is that *defaults* can be represented as sentences in the object language in the system. Obeid in [14] shows that there is a one-to-one correspondence between extensions of a default theory and appropriate minimal information states (Partial Models), which provide the semantic account (models) of the system **T3**. This is very useful because it has been shown that causal reasoning can be embedded in default logic and normal defaults (defaults of the form $A : B / B$) provide a solution to the frame problem (Cf. [19]) provided the rest of the default theory is set up properly.

First, we define a mapping from the First Order Predicate Calculus (FOPC) into **TFONL** as follows:

Definition 5.1. The mapping is defined by

$\mu\ (p) = p'$ (for atomic p)
$\mu(A \wedge B) = \mu(A) \,\&\, \mu(B)$
$\mu(A \vee B) = \mu(A) V \mu(B)$
$\mu(A \supset B) = \mu(A) \rightarrow \mu(B)$
$\mu(\neg A) = \sim \mu(A)$
$\mu(\forall x A) = \forall x \mu(A)$

The connectives $\neg$, $\wedge$, $\vee$, $\supset$ are those of FOPC and the connectives $\sim$, &, V and $\rightarrow$ belong to TFONL. We shall omit reference to when the context is clear. Time units are simply indexes.

Definition 5.2. Let $\Delta = \langle \text{DEF}, W \rangle$ be a default theory where DEF is a set of defaults and W is a set of FOPC formulae. A translation of Δ into TFONL is performed using τ which is defined as follows:

$$\tau(A) = L(\mu(A)) \quad \text{If } A \in W$$
$$\tau(A : B/C) = L(\mu(A) \,\&\, N(\mu(\text{B})) \supset L(\mu(C))$$
$$\text{If } A : B/C \in \text{DEF}.$$

We write $\tau(W) = \tau(A) : A \in W$ and τ (DEF) $= \tau(d) : d \in$ DEF to denote the translations of W and DEF respectively. Let $A \Rightarrow B$ denote $(A : B/B)$. Reference to τ and μ will be omitted, throughout the paper, when the context is clear.

6. Domain Description and Minimal Change

A domain description Ω is a set of propositions of the form (1)–(2). The main difference between the language of domain description employed here and the one that was developed by Baral, Gelfond and Provetti in [5, 6] in that we allow events to fail to have their effects when IMPLIES is the defeasible implication $\Rightarrow$.

A domain description defines a transition function from events and a time unit (point) (defining an information state) to a set of information states. Intuitively, given an event e and a time unit t, the transition function κ (e, t) defines the set of states that may be reached after the occurrence of the event e is successful at t. If the occurrence of e is not successful, then κ (e, t) is an empty set.

An information state of Ω is an interpretation that is closed under the set of static formulae of Ω. The effect of an event e at time unit t is the set of formulae

$$\text{eff}_e(t) = \{\text{Conc}(e) : \Omega \text{ contains a formula of the form } \text{Pre}(e)[t_1] \,\&\, \text{Occ}(e)[t] \,\&\, C(t_1, t, t_3) \text{ IMPLIES Conc}(e)[t_3] \text{ and Pre}(e) \text{ holds at } t_1\}$$

Given the domain description Ω containing a set of static formulae STAT, we formally define κ (e, t), the set of states that may be reached after the event e has occurred successfully at a time t as follows:

$$\text{(S1)} \quad \kappa(e, t') = \{s' : \text{Conseq}(s') = \text{Conseq}((s \cap s') \cup \text{eff}_e(t) \cup \text{STAT})\} \quad \text{where } t < t'$$

The intuition behind the above formulation is as follows: the direct effects of the occurrence of an event e, if successful, at time unit t are given by eff_e(t), and all formulae in eff_e(s) must hold at any time unit after t. In addition, the static formulae, STAT, determine additional formulae that must hold (cf. [18]).

When IMPLIES is the material implication $\supset$, the case is straightforward. For the other case, let d be the formula representing the occurrence and effect of e. (S1) can be semantically expressed as

$$\text{(S}'\text{1)} \quad \kappa(e, t') = \{s' : s' = \text{EXTEND}(s, d) \text{ where EXTEND is defined below}\}$$

Definition 6.1. Let $\Delta = \langle$DEF,W$\rangle$ be a default theory, $\mathbf{M} = \langle$D, E, P, I, R_I, R_P, F, PRED, HOPRED, Φ, Ψ, R, $R'\rangle$ be a model of L_{TFONL}, $s \in \Psi$ is a minimal partial model of W, $s_1 \in \Psi$ is a partial model of W and let $\delta = d_1, \cdots, d_n \subseteq$ DEF be a set of defaults. We define, EXTEND$(s, \delta) = s_1$, that d extends s to yield an information state $s_1 \in S$ as follows:

EXTEND$(s, \delta) = s_1$ iff

(i) (for every i) $(1 \leq i \leq n)$ (APP(s, d_i)) and
(ii) s_1 is a minimal partial model of W that satisfies (a), (b) and (c):
  (a) $s R s_1$
  (b) $s R' s_1$
  (c) (for every i) $(1 \leq i \leq n)$ (SATISFY(s_1, d_i))

where if $d = A : B/C$ then
APP(s, d) iff $s \models \tau(A)$ and $s \models N\mu(B)$ and SATISFY(s, d) iff APP(s, d) and $s \models \tau(C)$

The intuition is that s' is a minimal information state that is both epistemically possible and plausible from the perspective of s and it holds the (direct and indirect) effects of the event e if its occurrence is successful.

We assume the existence of two time points t_0 and t_c which identify two information states s_0 and s_c representing the initial information state and the current information state. While s_c at t_c denotes the current information state (the truth value of fluents at t_c), we shall employ the term **situation** s_c to denote a history of successful events from the information state s_0 at t_0. to s_c at t_c.

Example 6.2. To illustrate the idea and the potential of the formalism, we present an example which is a variant of the Yale Shooting problem taken from [18]. There is a pilgrim and a turkey. The pilgrim has two guns. If the pilgrim fires a loaded gun, the turkey will not be alive in the resulting situation. Furthermore, one can make the turkey be not trotting by making it not alive, because any causal explanation for the turkey being not alive is also a causal explanation for the turkey not trotting. Initially the turkey is trotting and at least one of the two guns is loaded.

Based on this informal description, we can conclude, for instance, that the turkey is not trotting at any point after the pilgrim shoots his two guns, one after the other.

This is an example of a "temporal projection" action domain, in which we are told only about the values of fluents at the initial time point t_0. Furthermore, this is an "incomplete" temporal projection domain, since the information we are given about the initial situation does not completely describe it.

This action domain includes a "static causal law": whenever not alive is caused to be true, not trotting is also caused to be true. It follows from this static causal law that one can make the turkey be not trotting by making it be not alive. Therefore, shooting a loaded gun when the turkey is trotting has not only the "direct effect" of killing the turkey, but also the "indirect effect," or "ramification," of making it stop trotting.

(1) $L \neg$ Trotting$[t_0] \supset L$(false)
(2) $L \neg$ (Loaded(Gun1)$[t_0]$ V Loaded(Gun2) $[t_0]$) $\supset$ L(false)
(3) $L \neg$ Alive$[t] \supset L \neg$ Trotting$[t]$
(4) $t < t'$ & L(Loaded(x))$[t]$ & OCC(Shoot(x))$[t]$ $\supset L\neg$ Alive$[t']$
(5) True $\Rightarrow A[t_0]$
(6) True $\Rightarrow \neg A[t_0]$
(7) $t < t'$ & $A[t] \Rightarrow A[t']$
(8) $t < t'$ & $A[t] \Rightarrow \neg A[t']$

In the action language AC [5, 6], this action domain can be formalized as follows.

initially Trotting
initially Loaded(Gun1) V Loaded(Gun2)
¬ Alive **suffices** for ¬ Trotting
Shoot(x) **causes** ¬ Alive if Loaded(x)

This AC domain description entails, for instance, the AC proposition

¬ Trotting **after** Shoot(Gun1), Shoot(Gun2)

which says that ¬Trotting holds at any time point after performing the action sequence Shoot(Gun1); Shoot(Gun2) in the initial situation.

The domain description includes the proposition

¬ Alive **suffices** for ¬ Trotting

which describes the static causal law: it says that, in the action domain we are describing, whenever ¬ Alive is caused, ¬ Trotting is also caused. Because of this static causal law, it is impossible in this action domain for trotting to be true when alive is false. It is important to note here that contraposition fails.

The action domain in this example is correctly formalized in TFONL. (1) reflects the assertion that the turkey is initially trotting, by ensuring that there can be no consistent extension of the default theory in which the turkey is initially not trotting. Similarly, (2) states that at least one of the pilgrim's guns is initially loaded. (3) states that the turkey can be made to stop trotting by making it not alive. Notice that this default rule does not allow one to derive Alive at t_0 from Trotting at t_0, for instance. This reflects the fact that It is important to note here that contraposition fails for static causal laws. Nonetheless, in the context of the default theory as a whole, this default rule guarantees, for instance, that no consistent extension can include both Trotting at t_0 and ¬ Alive at t_0. (4) states that the turkey is not alive after the pilgrim fires a gun, if that gun is loaded. (5) and (6) reflect the obvious fact that each fluent is either true or false in the initial situation, by forcing each consistent extension of the default theory to include, for each fluent F, either the literal $F[t_0]$ or the literal $\neg F[t_0]$ is true. It is important to note that this need not be a requirement in TFONL. Furthermore, these two rules guarantee that the default theory takes into account every possible initial situation. (7) and (8) the inertia law, i.e., the principle that remain in the same truth-value status change unless they are made to change. For instance, since we have the literal Trotting[t_0], one of the persistence/inertia rules allows us to derive the literal Trotting[t'] where $t < t'$, as long as it is consistent to do so.

7. A Comparison with Previous Approaches

Van Benthem in [20] gives an insightful and thorough survey of theories of time. One of the choices which he explores is that of considering both points and intervals as a basis of a theory of time. However, he does not offer a final proposal for a logic.

The interval-based logic by Allen in [2] is one of the most influential approach in temporal AI. However, some details about the temporal structure itself are not very clear. There is no specification whether time is discrete, dense or continuous. For instance, once density is allowed something must be said about DIP. One of the weaknesses of the system is that it cannot deal with instantaneous events and fluents.

Galton in [7] provides a system G where both points and interval are primitive. Time is defined as the structure

$$\langle \text{I, P, Within, Limits, Allen's relations} \rangle$$

where I and P are non-empty sets of points and intervals respectively, Within and Limits are point-interval relations. Let p denotes a point, and i, j, r denote intervals, then G is axiomatized as follows:

(G1) $\forall i \exists p$ Within(p, i)
(G2) Within(p, i) & In$(i, j) \supset$ Within(p, j)
(G3) Within(p, i) & Within$(p, j) \supset \exists\, r(In(r, i)$ & In$(r, j))$
(G4) Within(p, i) & Limits$(p, j) \supset \exists\, r(In(r, i)$ & In$(r, j))$

where In is defined as the disjunction of the temporal relations (cf. [1]) During, Starts and Finishes.

G avoids the DIP problem. Some of the weaknesses in the system G are:

(1) Limits (p, i) leaves vague whether the point p is a start or an end of the interval i.
(2) No explicit ordering over points.

Vila's system [22] is one of the latest approaches that employs points and intervals as a part of the temporal ontology. The proposal is based on a dense conception of time. Event/actions and causality are not considered.

IP has a clear ordering $<$ of points and it is stronger than G. $<$ is irreflexive, asymmetric, transitive, unbounded and linear. IP imposes an order on the limits of an interval which rules out interval with zero duration. However, it is too strong as both points and interval are interdefinable which defies the main objective of both being primitives. Axioms $IP_{7.1}$, $IP_{7.2}$ and IP_9) state that any two points define an interval and any interval is limited by two points. One major concern of IP is the Dividing Instant Problem which results from a dense temporal structure. Vila claims that his theory solves the DIP problem as one does not get an answer for a query whether a fluent A is true or false at a meeting point between two adjacent interval. This is not a solution to DIP as not getting an answer could be equated with undefined which is problematic in a classical setting.

8. Concluding Remarks

Many applications need to handle the notion of time and change to some extent. Typically we need to consider instantaneous or durative events and fluents and both quantitative or qualitative temporal information. It is important to emphasize that temporal reasoning is not an isolated issue. and many other kinds of reasoning need to be considered in relation with it. Some of the important topics that need to be considered in relation with it are: deeper considerations of qualitative reasoning, the frame, qualification and ramification problems and non-monotonic reasoning.

In this paper a general formalism for representing and reasoning with temporal information, event and change is provided. The temporal framework is based on a theory of time that takes both points and interval as temporal primitives and where the base logic is that of Kleene's three-valued logic. The temporal framework takes into consideration the need to deal with instantaneous or durative events and fluents and both quantitative or qualitative temporal information and avoids the DIP problem. We have also presented a three-valued based Temporal First-Order Nonmonotonic Logic (TFONL) that employs an explicit representation of time and events. We have shown that default logic can be embodied into TFONL, which takes into consideration the frame, qualification and ramification problems.

Appendix

Several theories of time have been proposed which can be classified as point-, interval- or event-based theories according to whether the primitive time units are points, intervals or events.

Event-based theories ([8, 17]) take it that "time is no more than the totality of temporal relations between the events and processes which constitute the history of our world. Then defining time is a question about the actual relations between these events and processes." It is important to note that in these theories, an event denotes any temporal proposition, thus it includes both events and fluents.

Since interval-based theories are based on the intuition that intervals are the chunk of time occupied by events, the properties of intervals and events are very similar. (cf. [17]).

A1. Point-Based Time Structures

Point is the temporal primitive used in many AI systems (cf., [10, 11] and [16]). These theories are defined over a set of points and an ordering relation. A point-based structure is

$$\pi = < \mathrm{T}, <_{\mathrm{T}} >$$

where T is a non-empty set and $<_{\mathrm{T}}$ is a precedence relation on points of time. Varying the properties of $<_{\mathrm{T}}$ results in different systems. Van Benthem in [20] suggests that in a minimal point structure, $<_{\mathrm{T}}$ should have the following properties:

(T1) $(p1 <_T p2) \,\&\, (p2 <_T p3) \rightarrow p1 <_T p3$ (Transitivity)

(T2) $\neg(p1 <_T p1)$ (Irreflexivity)

(T3) $(p1 <_T p2) \vee (p1 = p2) V (p2 <_T p1)$ (linearity)

(T4) $(\forall p)(\exists p1)(p <_T p1)$

(T5) $(\forall p)(\exists p1)(p1 <_T p)$

(T4) (resp. T5) states that for any time point p, there exist a point $p1$ that comes after it (resp. before it).

In the structure given above, points are ordered and mapped on to a time line. We may choose to have the time line discrete or dense. In a dense model, points can be seen as rational or real numbers on a number line. The density axiom is:

(T6) $(\forall p1)(\forall p2)(p1 <_T p2)(\exists p3)(p1 <_T p3 \,\&\, p3 <_T p2)$

which simply states that, given any two points, there exists at least one point between them.

In a discrete structure, points can be seen as integer numbers on a number line. The corresponding axioms are:

(T7) $(\forall p)(\exists q)(p <_T q) \rightarrow (\exists r)(p <_T r \,\&\, \neg(\exists s)(p <_T s \,\&\, s <_T r))$

(T8) $(\forall p)(\exists q)(q <_T p) \rightarrow (\exists r)(r <_T p \,\&\, \neg(\exists s)(r <_T s \,\&\, s <_T p))$

The axiom (T7) (resp. T8) states that, given any point which is not the last (resp. first), there exist a point which is *just* after (resp. before) it.

A2 Interval-Based Theories

Interval-based theories in AI (cf. [2, 4, 17]) are based on the intuition that intervals are the chunk of time occupied by events. Allen in [1] presents a logic of time in which there were 13 possible primitive relations between intervals. Allen and Hayes in [3] show that all the other 12 relations can be define in terms of Meets which can be axiomatized as follows:

(I1) $(\forall i, j)(\exists k)(\text{Meets}(i, k) \,\&\, \text{Meets}(j, k) \supset (\forall l)(\text{Meets}(i, l) \equiv \text{Meets}(j, l))$

(I2) $(\forall i, j)\ (\exists k)(\text{Meets}(k, i) \,\&\, \text{Meets}(k, j) \supset (\forall l)(\text{Meets}(l, i) \equiv \text{Meets}(l, j))$

(I3) $(\forall i, j, k, l)(\text{Meets}(i, j) \,\&\, \text{Meets}(k, l)$
$\supset \text{Meets}\ (i, l)$, XOR
$(\exists m)(\text{Meets}(i, m) \,\&\, \text{Meets}(m, l)$ XOR
$(\exists n)(\text{Meets}(k, n) \,\&\, \text{Meets}(n, j)$

(I4) $(\forall i)(\ (\exists j, k)(\text{Meets}(j, i) \,\&\, \text{Meets}(i, k))$

(I5) $(\forall i, j)(\text{Meets}(i, j) \supset (\exists k = i + j,)(\exists m, n)(\text{Meets}(m, i) \,\&\, \text{Meets(i, j)} \,\&\, \text{Meets(j, n)} \,\&\, \text{Meets}(m, k) \,\&\, \text{Meets}(k, n))$

where XOR denotes exclusive OR. (I1) and (I2) state that every interval has a unique start point and a unique end point.(I3) define all the possible relations between any two meeting places. (I4) states that every interval has one interval the precedes and an interval that succeeds it. $k = i + j$ is only definable if $\text{Meets}(i, j)$ holds and k contains exactly i and j. (I5) states that for any two adjacent interval i and j, there exist an interval k such that $k = i + j$.

A3. Event-Based Structures

Event-based theories ([8, 17]) take it that "time is no more than the totality of temporal relations between the events and processes which constitute the history of our world. Then defining time is a question about the actual relations between these events and processes." It is important to note that in these theories, an event denotes any temporal proposition, thus it includes both events and fluents.

Since interval-based theories are based on the intuition that intervals are the chunk of time occupied by events, the properties of intervals and events are very similar. (cf. [17]).

An event-based structure is:

$$\varepsilon = \langle E, <_E, O \rangle$$

where E is a non-empty of events, and $<_E$ and O are binary operators on events. $<_E$ is the precedence operator and O is the overlapping operator. The axioms of ε are as follows:

(E1) $(e1 <_E e2) \rightarrow \neg(e2 <_E e1)$ (Irreflexivity $<_E$)

(E2) $(e1 <_E e2) \,\&\, (e2 <_E e3) \rightarrow (e1 <_E e)$ (Transitivity $<_E$)

(E3) $(e1 O e2) \rightarrow (e2 O e1)$ (Symmetry O)

(E4) $(e O e)$ (Reflexivity O)

(E5) $(e1 <_E e2) \rightarrow \neg(e1 O e2)$ (Separation)

(E6) $(e1 <_E e2) \,\&\, (e2 O e3) \,\&\, (e3 <_E e4) \rightarrow (e1 <_E e4)$

(E7) $(e1 <_E e2) V (e1 O e2) V (e2 <_E e1)$ (Linearity)

To capture the fact that there is no first event or no last event, we need to add one of the following axioms:

(E8) $(\forall e)(\exists e1)(e1 <_E e)$
$(\forall e)(\exists e1)(e <_E e1)$

For further analysis, readers are referred to [8, 19] and [20].

Since interval-based theories are based on the intuition that intervals are the chunk of time occupied by events, the properties of intervals and events are very similar. (cf[17]).

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