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Recent advancement of flow-induced piezoelectric vibration energy harvesting techniques: principles, structures, and nonlinear designs*

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Abstract Energy harvesting induced from flowing fluids (e.g., air and water flows) is a well-known process, which can be regarded as a sustainable and renewable energy source. In addition to traditional high-efficiency devices (e.g., turbines and watermills), the micro-power extracting technologies based on the flow-induced vibration (FIV) effect have sparked great concerns by virtue of their prospective applications as a self-power source for the microelectronic devices in recent years. This article aims to conduct a

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comprehensive review for the FIV working principle and their potential applications for energy harvesting. First, various classifications of the FIV effect for energy harvesting are briefly introduced, such as vortex-induced vibration (VIV), galloping, flutter, and wake-induced vibration (WIV). Next, the development of FIV energy harvesting techniques is reviewed to discuss the research works in the past three years. The application of hybrid FIV energy harvesting techniques that can enhance the harvesting performance is also presented. Furthermore, the nonlinear designs of FIV-based energy harvesters are reported in this study, e.g., multi-stability and limit-cycle oscillation (LCO) phenomena. Moreover, advanced FIV-based energy harvesting studies for fluid engineering applications are briefly mentioned. Finally, conclusions and future outlook are summarized.

Key words vibration-driven energy harvesting, flow-induced vibration (FIV), piezoelectric approach, nonlinear design

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1 Introduction

Hitherto, numerous innovative technologies have been developing at a rapid pace, such as smart electronics and portable gadgets, wearable and implantable devices, and remote detecting and monitoring systems. They play crucial roles in the advancement of public safety, human healthcare, industrial automation, and engineering management to develop smart cities. In the present era of the Internet of Things (IoT), it is expected to transform the global development by connecting billions of devices in the near future based on the wireless sensor networks[1]. Currently, numerous smart electronics and wearable devices as well as other wireless network devices are resorted to electrochemical batteries as the primary power. However, as the conventional power, the batteries require periodic replacement or recharging due to their limited lifespan and energy capacity. Moreover, the use of electrochemical batteries may induce an environmental problem because of the existence of toxic chemicals and heavy metals contents.

The exploration of renewable and sustainable energy resources is an urgent demand for developing eco-friendly, clean, and alternative energy technologies. Macro-scale power resources (e.g., hydro-electricity, solar energy, wind energy, geothermal energy, hydrogen energy, and tidal energy[2]) and micro-/nano-scale creative technologies (e.g., vibration energy harvesting (VEH)[3]) are being investigated to supply electrical power for micro-electromechanical devices and wireless sensor networks. VEH is an attractive and direct technique to convert mechanical energy to electrical energy[4], where mechanical energy can be harnessed from various ambient sources, e.g., mechanical equipment, automotive, human motions, and fluid flows[5].

Fluid flow is the motion of a fluid subject to unbalanced forces, which is ubiquitous in nature. Flow-induced vibration (FIV) is a common physical phenomenon, by which the flow around bluff bodies creates forces exciting vibration. Steady and unsteady flows are two kinds of flow types. For the vibration in a steady flow, the mutual interaction between a flowing fluid and a structure leading to large-amplitude vibrations is the most commonly observed scenario. Under an unsteady flow, turbulence forces are the dominant factor causing structural vibration excitations. Such phenomena associated with FIV are widely used, including vortex-induced vibration (VIV), galloping, wake-induced vibration (WIV), flutter, buffeting, and fluid elastic instability. The FIVs arising from distinct fluid dynamic phenomena can be classified by the nature of flow and the behavior of structures[6–7], as shown in Fig. 1.

The influence of FIV has been experienced in numerous engineering fields, including aerospace industry, power generation (turbine blades and heat exchangers), civil structures (bridges and skyscrapers), and offshore technology. With continuously developing technologies, FIV can be used to harness energy from the sea/ocean currents, rivers, and pipe flows. Recently, FIV energy harvesting technologies have attracted dramatic research works[8–14]. It

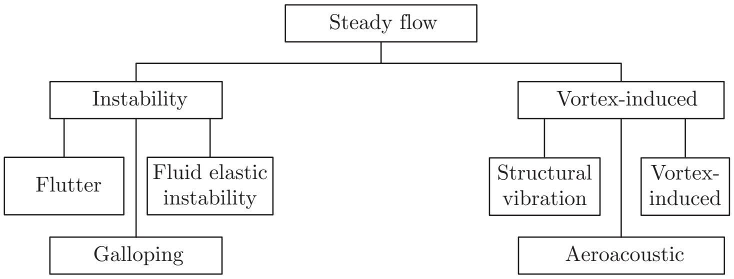


Fig. 1 Classification of FIV[6–7]

can be used as an alternative approach to power micro-electronic devices located in remote locations or inconveniently accessible places. Following the wide-range concepts of FIV, numerous studies have been conducted on scavenging flow energy using VIV galloping, WIV, and fluttering effects. To overcome the technical barrier of conventional linear VEH technologies having a narrow frequency bandwidth, various nonlinear oscillating systems have been proposed to extend the working efficiency of energy harvesters under low-frequency and low-amplitude excitation levels[15–17].

This work mainly targets on a systematic review of the FIV energy harvesting techniques in recent three years. First, various types of FIV mechanisms, such as VIV, galloping, and flutter, are introduced. In addition, the nonlinear design of FIV energy harvesting techniques, using limit-cycle oscillations (LCOs) and magnetic enhancement approaches, is reviewed and discussed. Practical engineering applications of using such techniques are also reported. Finally, the major research findings and outlook on the development of FIV energy harvesting techniques are summarized. It is expected that the present review can offer a better understanding and critical evaluation of developing these techniques in engineering fields.

2 FIV phenomena for energy harvesting and potential applications

In recent years, energy harvesting from ambient vibration sources has become a hot research area for supplying power to wireless electronics[8–11,18]. Various working mechanisms, i.e., piezoelectric, electromagnetic, electrostatic, and hybrid approaches, are designed and analyzed for energy harvesting. The FIV approach is a common phenomenon of fluid-structure interaction. Therefore, significant works have been done on the vibration-based energy harvesting by means of FIV. In this section, four kinds of FIV and the working principles on energy harvesting are comprehensively reviewed.

2.1 VIV and application to energy harvesting

2.1.1 VIV

In fluid dynamics, the mechanism controlling VIV can be defined as a bluff body in a crossflow under vibrations when a fluid associates with the periodic shedding of vortices from structures working at a natural frequency[19]. Resonance can induce the large-amplitude vibration of structures, and the mutual interaction is highly complex due to the vortex effect. The vortices created downstream would detach periodically from either side of the bluff body, thereby exerting a dynamic force on the bluff body. Here, the bluff body can be defined as a body that, as a result of its configuration shapes, has separated flow over the surface of its structure. Circular cylinders are the common bluff body shape. The effects of VIV can generally induce inline and transverse vibrations, as shown in Fig. 2. It is generally affected by numerous system parameters, including the Reynolds number, the Strouhal number, the shape parameter, the structural stiffness, and the damping coefficient[7,20].

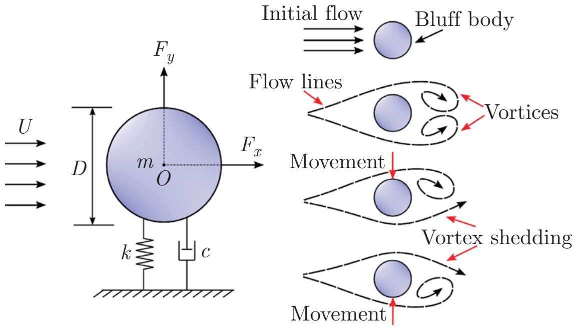


Fig. 2 Vibration of cylinder subject to vortex shedding[20] (color online)

2.1.2 Recent development of VIV-based energy harvesting techniques

A piezoelectric cantilever beam, having a circular cylinder attached to its free end, as an elastically mounted circular cylinder immersed in a two-dimensional flow is always regarded as a typical VIV-based energy harvester (see Fig. 3). For this kind of energy harvesters, Dai et al.[21] derived a nonlinear distributed-parameter model and studied the effects of varying the tip mass of the cylinder, the length of piezoelectric sheets, and the electrical load resistance on the synchronization region. The performance of this harvester and its power density were also investigated under various aero-electromechanical behaviors. Zhang et al.[20] discussed the influence of the Reynolds number on the power generation output. The results showed that the Reynolds number played a critical role in the dynamic behavior of the structure for energy harvesting.

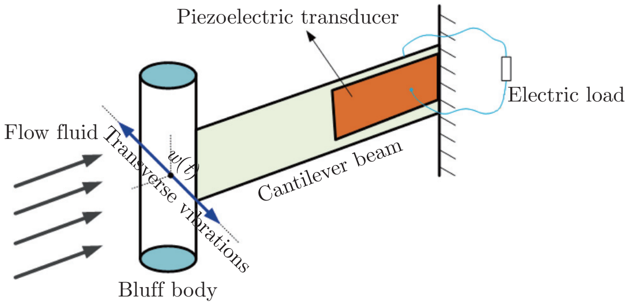


Fig. 3 Typical VIV-based energy harvester (color online)

In this design, the cantilever beam, as an elastic element, is a crucial factor for the performance of FIV energy harvesters due to its structural dynamics. Various kinds of beam structures were proposed and studied. For example, Wang et al.[22] modified the configuration shape of a cantilever beam to propose a cross-coupled dual-beam structure as a piezoelectric energy harvester (PEH) based on the VIV induced effect under wind flows. The results showed that the upper and bottom piezoelectric beams could generate the maximum power outputs of 6.77 μW and 56.64 μW, respectively. Wang et al.[23–24] further designed a bio-inspired leaf venation prototype for VIV-induced energy harvesting, as shown in Fig. 4. They confirmed that the bionic reinforced structure could significantly improve the working efficiency of piezoelectric energy harvesters under wind-induced vibrations. Wang et al.[25] also proposed a non-contact VIV-based piezoelectric wind energy harvester using an indirectly excited composite piezoelectric transducer to provide better environmental adaptability and improve working reliability.

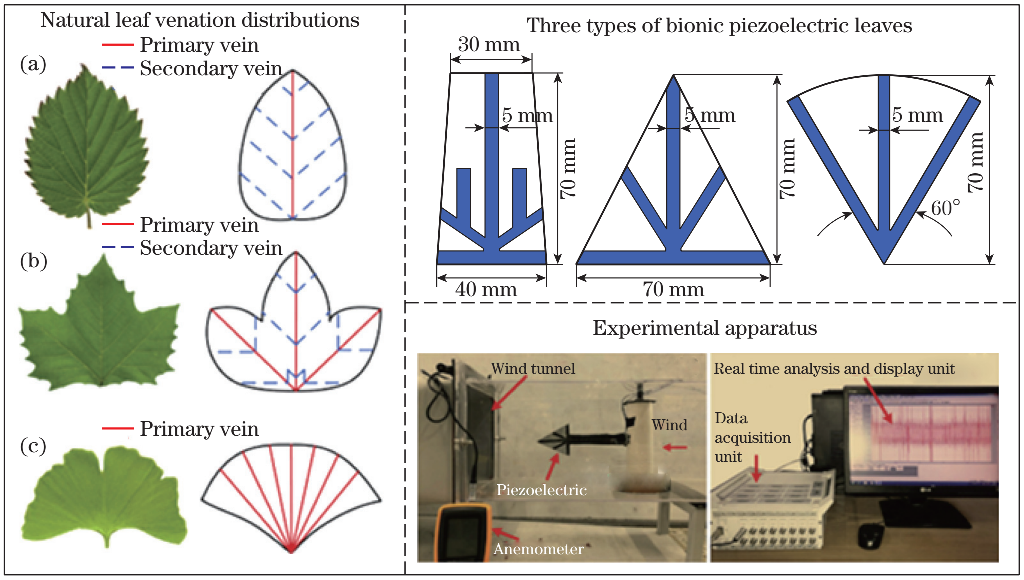


Fig. 4 Bio-inspired VIV-driven energy harvester with bionic leaves[24] (color online)

On the other hand, Su and Lin[26] proposed a bi-directional VIV-based energy harvester that can work well in two orthogonal directions. This design structure consisted of the U-shaped beam, a pair of piezoelectric patches, and a foam cylinder attached to the center of the beam.

2.1.3 Effects of improved bluff bodies on VIV-based energy harvesting

In addition to the elastic beam structure, the bluff body shape is another key component of VIV-driven energy harvesting. Changing the bluff body's configurations or its arrangements can alter the dynamic characteristics. Hence, various bluff body shapes or multiple combined bluff bodies with different shapes have been analyzed in the literature. Conventional bluff body shape is often a smooth circular cylinder, Zheng et al.[27] and Wang et al.[28] provided comprehensive discussions on the effects of three combinations of the circular and square cross-sections, corresponding to different values of an attack angle. The dynamic characteristics of spindle-/butterfly-like[29–30], diamond[31], and square bluff bodies[32] were also studied. These studies showed that distinct dynamic responses could result by changing the cross-sectional configurations of bluff bodies, which is highly beneficial to the enhancement of voltage output.

The influence of surface morphology of bluff bodies has been studied and verified to improve the energy harvesting efficiency. Non-smooth cylinders, having protrusions and pits[33], meta-surface patterns[34], and grooves[35], are graphically presented in Fig. 5. They were proposed to decorate on the surface of ordinary bluff bodies for potential VIV-based energy harvesting. These structures could alter the direction and strength of flow fields around the bluff bodies, resulting in stronger dynamic forces to achieve a better energy harvesting performance. Besides, various attachments on smooth cylinders were designed for improving the structural dynamics of bluff bodies, such as Y-shaped attachments[16], two symmetric splitters in different relative angular designs[36], and small-size triangular- and rod-shaped protrusions[37–38] (see Fig. 6). The results demonstrated a better working performance of the proposed face configurations in energy harvesting.

It is worth noting that an array of two mutually interacting bluff bodies can widen the working frequency bandwidth of VIV-based energy harvesters[39]. Bluff body devices integrated with a tuned mass system were proposed to match the vortex shedding frequency under the random nature of wind effects. The results demonstrated that the novel design not only could

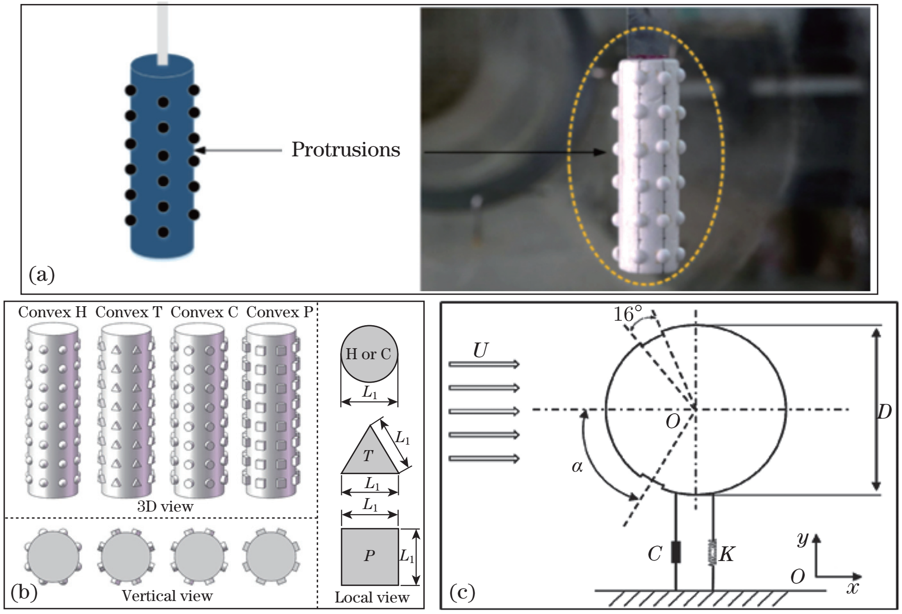


Fig. 5 VIV-based energy harvesters with different surface morphological designs[33–35] (color online)

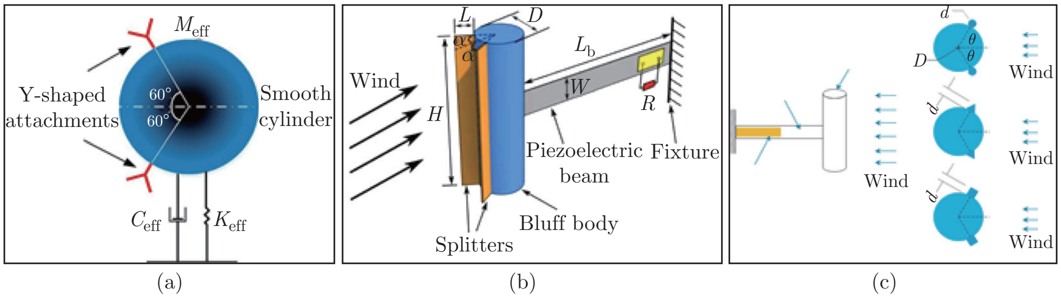


Fig. 6 VIV-based energy harvesters with different design attachments on smooth cylinders[16,36–38] (color online)

increase the velocity band, but also enhance the peak power output by 294%.

2.2 Galloping and application to energy harvesting

2.2.1 Galloping effect

Transverse galloping is an aero/hydro-elastic instability phenomenon of FIV. It can be induced by the nonlinear interaction between an unstable wake and a bluff body with high-frequency vortex shedding. The galloping effect is a typical self-excited vibration as the positive energy transferring from a flow fluid to a bluff body, in which a negative linear damping force can amplify the amplitude of transverse motions to a nonlinear behavior and lead to a stable limit cycle[40]. In nature, galloping is different from VIV as it does not have the lock-in region and cannot be self-limited. As a result, it will continue to force structures oscillating in one direction with flow velocity increasing to a critical value, leading to a catastrophic effect on structures[11]. This effect mostly occurs in prismatic and flexible structures, such as triangle, rectangular, and angular sections. Because the galloping effect is usually characterized by low-frequency and high-amplitude oscillations, the nature-induced phenomenon can be adopted for energy harvesting.

2.2.2 Recent development of galloping-based energy harvesting

In terms of galloping-based energy harvesters (GEHs), many research works have been reported to investigate various configurations of bluff bodies for energy harvesting. Triangular and rectangular cylinders are two common shapes which have been identified as an ideal bluff body for GEHs. In the case of rectangular cylinder bluff bodies, Javed and Abdelkefi[41] studied the impact of aerodynamic force on the instability performance of the GEH. Yu and Zhang[42] focused on the analysis for the efficacy of the side ratio, i.e., between the cylinder width and the height on energy harvesting. To improve the dynamic characteristics of square bluff bodies, Sun et al.[43] designed a nested bluff-body that comprised the outer and inner square cross-sectional bluff bodies in tandem, as shown in Fig. 7(a), increasing the power density of 27.8%. Liao et al.[44] proposed a complex GEH constructed by a flexible frame, a square bluff body, a piezoelectric cantilever beam, a spindle, and a deflector (see Fig. 7(b)). The results of these studies provided guidance for the design of high-efficiency energy harvesters using square-type flexible aerodynamic bluff bodies. In the case of triangular types and its design variants, Wang et al.[45] discussed the impact of different vertex angles to achieve an efficient GEH. Tan et al.[46–47] further studied the working performance of a GEH with various cross-sectional configurations, such as regular triangle, semi-circle, and trapezoidal bluff bodies. Furthermore, a piezoelectromagnetic synergy GEH was also proposed[48].

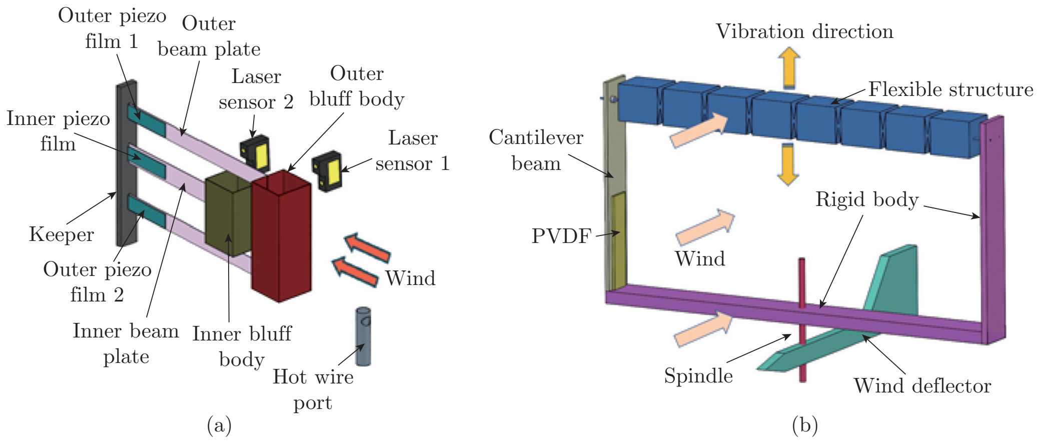


Fig. 7 Schematics of GEHs with (a) nested bluff body[43] and (b) flexible frame[44] (color online)

Considering the working principle of GEHs, Zhao[49] proposed a compact bistable GEH, in which a D-shaped bluff body attached to a piezoelectric cantilever was used, and a magnetic interaction approach was introduced to enhance the power generation. Petrini and Gkoumas[50] designed a customizable aerodynamic fin based on the VIV and galloping effects, in which the working efficiencies of both circular and T sections were compared for data analysis. Sobhanirad and Afsharfard[51] embedded a clamp-guided piezoelectric beam design with a tip mass in the bluff body to perform as an oscillator of the GEH. The results showed that optimized structural parameters of this design could work well under normal wind speeds ($3\,\mathrm{m\cdot s^{-1}}$–$6\,\mathrm{m\cdot s^{-1}}$). To design a GEH, the advantages of a stepped piezoelectric beam were also evaluated for the enhancement of power generation[52]. Furthermore, a GEH comprised of a piezo-magneto-elastic coupled double-beam was also analyzed[53], where the magnet-induced bistable nonlinearity was introduced in the system to enhance the working performance of energy harvesting.

2.3 Fluttering effect and application to energy harvesting

2.3.1 Flutter

Flutter is a self-excited and potentially destructive aeroelastic phenomenon that normally occurs in flexible bodies with relatively flat shapes under a fluid flow, e.g., airplane wings and bridge decks. Although flutter is accompanied by vortex shedding with frequency equal to the flutter frequency, such an effect differs from the phenomenon of VIV. For the VIV phenomenon, the formation of alternating shed vortices can generate a coupling oscillatory effect with structural vibration. Under a specific flow velocity, the phenomenon of lock-in occurs. While for the fluttering effect, the strength of vibration increases monotonically with velocity, such that it is easy to destroy structures. Flutter always involves two degrees of freedom, i.e., a flap-wise motion and a torsional motion. This effect can be analyzed by both linear and nonlinear models according to the stability of bluff bodies[54–55]. As it is generally self-excited to produce large-amplitude motions, it is conducive to carrying out structure-borne vibration energy harvesting. Understanding the subtle behavior of this dynamic phenomenon is able to design a robust FIV-based energy harvesting technique under time-varying and broadband frequencies.

2.3.2 Recent development of flutter-based energy harvesting

As mentioned previously, the principle of energy recovery schemes using aeroelasticity has been well documented and refined. However, piezoelectric patches are often located at the constraints of an airfoil section to restrict the amplitude of oscillation, resulting in low voltage output and working efficiency[56–57]. A proper strategy should optimize the flutter onset speed to maintain the power output. Airfoil sections are the most common bluff body shape. Considering aeroelastic behavior, Hafezi and Mirdamadi[55] designed an adaptive flutter-based energy harvester (FEH), while a sliding mass on a piezoelectric beam was introduced to accommodate the flutter onset speed with the floating speed. The results demonstrated a wider operational range and a better energy harvesting performance. Bao et al.[56] fixed a piezoelectric beam to the trailing edge of the airfoil section to control the self-excitation and vibration effects. Abdehvand et al.[58–59] further studied a magneto-electro-elastic coupled FEH with nonlinear aeroelastic characteristics. These models are illustrated in Fig. 8.

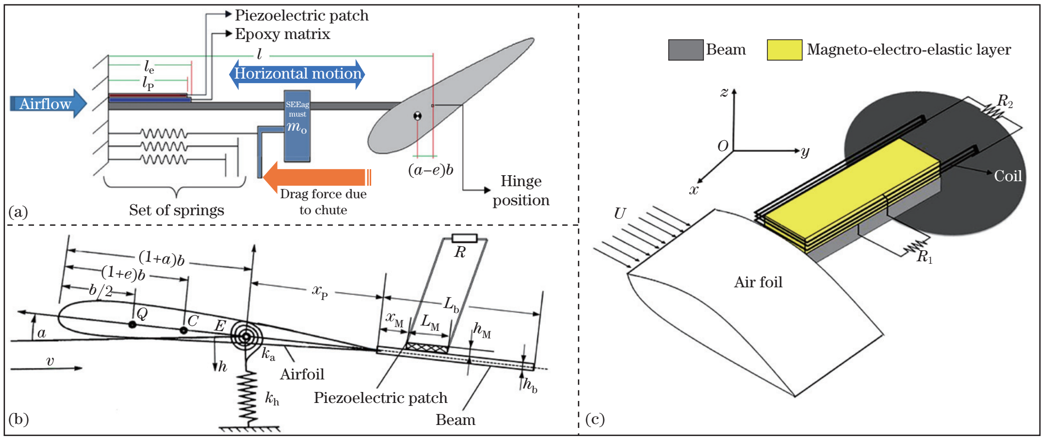


Fig. 8 Various FEH models: (a) adaptive type[55], (b) trailing edge type[56], and (c) magneto-electro-elastic coupled type[58–59] (color online)

Flags can easily be triggered to generate flutter vibration when subject to an axial airflow in subsonic wind, and a single flag-type FEH[60–63] is presented in Fig. 9(a). Sun et al.[64] further proposed a fluttering double-flag type triboelectric nanogenerator for wind energy harvesting, as shown in Fig. 9(b). Here, the flutter contact between the two films was investigated and the results showed a good power generation performance. In these works, the flag-type devices, fixed at the leading edge located upstream and free at the trailing edge located downstream, were designed. In addition, an inverted flag design (i.e., free to flap at the upstream leading edge and fixed at the downstream trailing edge), was also analyzed[65–67]. In Ref. [66], the inverted flag design may exhibit four main dynamic modes: (i) a static or small-amplitude vibration aligned with the incoming flow; (ii) a large-amplitude limit-cycle flapping oscillation; (iii) a relatively small-amplitude vibration around a fully deflected configuration; and (iv) a full deflection mode, as shown in Fig. 9(c). It is clear that a large-amplitude limit-cycle flapping oscillation mode is most applicable for energy harvesting. During a limit-cycle flapping motion, the instantaneous configuration of the inverted flag is a cylindrical surface that is either entirely concave or entirely convex, thereby ensuring no electrical charge cancellation during operation with piezoelectric materials.

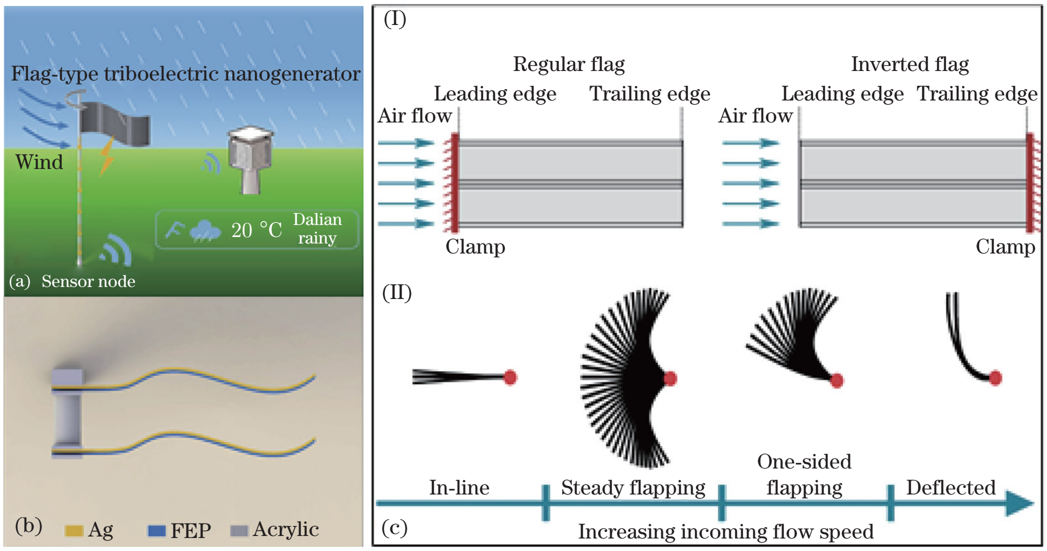


Fig. 9 Various FEH desgins: (a) one-flag[61], (b) two-flag[64], and (c) inverted mode[66] (color online)

2.4 WIV and application to energy harvesting

2.4.1 WIV

When two or more bluff bodies are aligned into an array pattern under an air flow, complex dynamic responses will result due to the interactive action between the upstream and downstream bodies. Wakes of the upstream body may impact the downstream body and induce vibrations, a phenomenon that is called the WIV[68]. As shown in Fig. 10(a), a traditional arrangement of bluff bodies of WIV consists of two cylinders, the upstream cylinder being static while the downstream cylinder being elastically mounted[69–70]. A typical WIV effect can be strengthened by a gradual accumulation of amplitude persisting to high reduced velocities, which is different from a typical VIV response that occurs in a bounded resonance range. Unlike the VIV phenomenon, the WIV mechanism is caused by the unsteady vortex-structure interaction as it vibrates across the upstream wake. Using WIV-based energy harvesting techniques, a bluff body is normally attached to the free end of a piezoelectric cantilever beam and another bluff body is located at its front position[71], as shown in Fig. 10(b). It is known that the performance of this system depends on the position of the upstream cylinder and the tandem separation between two bluff bodies.

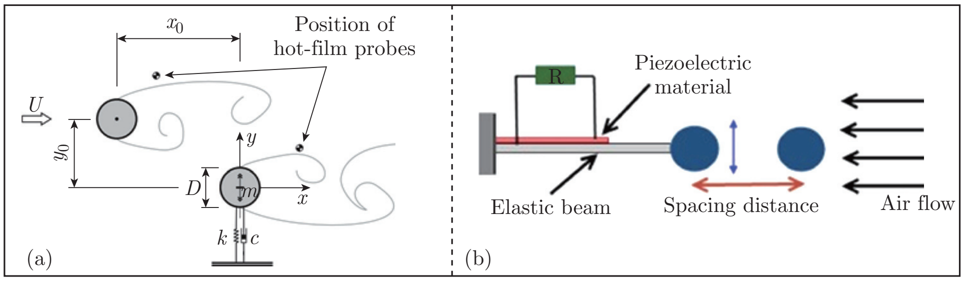


Fig. 10 Schematics of (a) WIV with typical arrangement of two cylinders[69] and (b) typical regime of WIV energy harvester[71] (color online)

2.4.2 Recent development of WIV-based energy harvesting

By considering the WIV effect, various configuration shapes and arrangements of the static and dynamic bluff bodies can produce different dynamic response levels. In general, the configuration shapes of static and dynamic bluff bodies are often designed in circular cylinders[72], rectangular cylinders[73], or circular bluff bodies combined with other special cross-sections (e.g., D-shape[74], C-shape[75–76], diamond-shape[77]). Cao et al.[78–79] studied a WIV-based energy harvester with variable-width piezoelectric beams, where the effect of the magnet force on the power generation was examined. In addition, double-circle cylinder[80] and double-plate[81] upstream bluff bodies were also presented for energy harvesting. Such designs are able to enhance the power output.

2.5 Hybrid FIV-based energy harvesting

Many works have been done for FIV-based energy harvesting techniques by using a single approach of the aforementioned mechanisms. To go beyond the limitations of these approaches, recent research studies have shifted to focus on integrating different working mechanisms together into a single module. These hybrid designs allow to take advantages of each working mechanism to enhance the working performance of energy harvesting. For example, Wang et al.[28] and Yang et al.[29] presented two different types of hybrid FIV-based energy harvesters with various bluff body shapes by coupling the VIV and galloping effects. The former study discussed the influence of different cross-sectioned bluff bodies, and the latter one analyzed the effect of the genetic algorithm on the structural optimization. Sun et al.[82] further studied a low-velocity water flow energy harvester using the VIV and galloping effects, and discussed the vortex contour of three kinds of bluff bodies. Shan et al.[83] proposed a complex piezoelectric energy harvester having a cantilever beam attached to the trailing edge of the mobile airfoil, including a flexible spring energy harvester and a cantilever beam energy harvester, to harness energy concurrently.

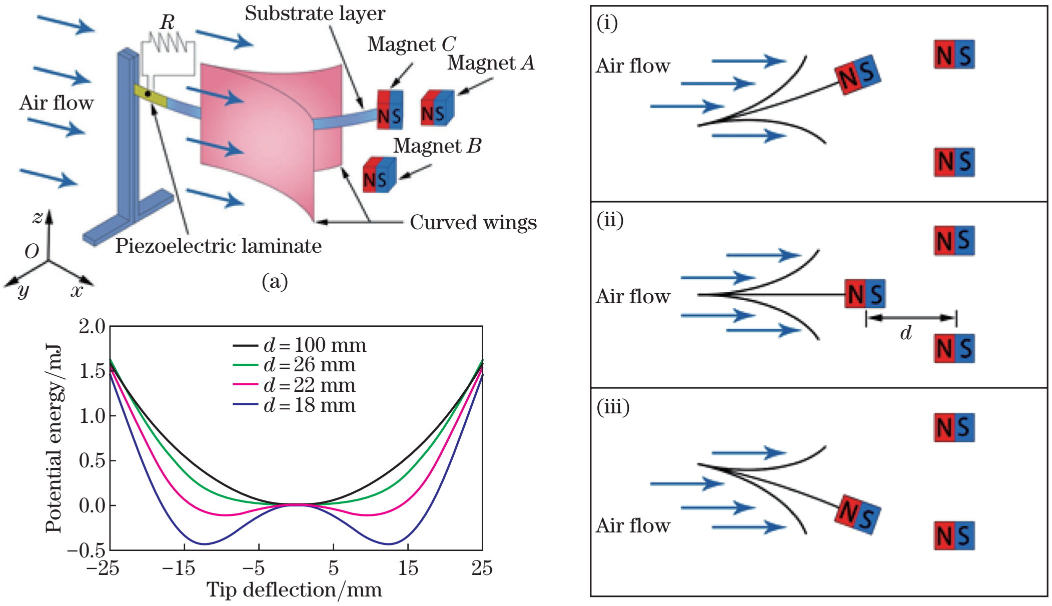
3 Nonlinear design of FIV-based energy harvesters

In general, earlier designs and structures of vibration-based energy harvesters were mainly investigated by using the linear vibration theory. It is known that these designs are only able to work at a very narrow frequency bandwidth due to resonance, resulting in very poor environmental adaptability under random ambient sources. On the contrary, exploring mechanical nonlinearities in the system design can offer an excellent advantage of broadening the working frequency bandwidths of vibration-based energy harvesters under complex ambient excitations[3,15,84]. Therefore, there has been a growing research interest in developing nonlinear FIV-based energy harvesting techniques.

3.1 Application of multi-stable characteristics

In recent years, intensive investigations of mono-stable, bi-stable, tri-stable, and even higher-stable nonlinear characteristics have been reported on vibratory energy harvesters[85–88]. The distinct merit of such design systems has a wider range of operating frequencies, thereby improving the energy conversion efficiency. This design strategy can be generalized to incorporate to FIV-based energy harvesting techniques. For instance, Naseer et al.[89–90] introduced nonlinear attractive magnetic forces to design a nonlinear VIV-based energy harvester, where both the mono- and bi-stable regimes were analyzed. The complex hardening and synchronization nonlinear behaviors can be controlled by adjusting the magnetic force, so that two stable configurations can be switched. In addition, many researchers (e.g., Javed and Abdelkefi[91], Hou et al.[92], and Yang et al.[93]) proposed different types of magnet arrangements and bluff body shapes to incorporate into magnetic-induced nonlinear multi-stable VIV-based energy harvesters.

Zhou et al.[94] studied a Y-shaped bi-stable energy harvester that consisted of a cantilever beam with a tip magnet, two curved wings, a piezoelectric laminate, and two fixed magnets, as shown in Fig. 11. The results demonstrated a snap-through process to achieve coherent resonance at a wide range of airflow speeds. Zou et al.[95] further introduced a pair of wings to facilitate the FIV effect by coupling a magnetic flextensional transducer. The nonlinear bistable characteristics showed that the performance of the energy harvester could be enhanced.



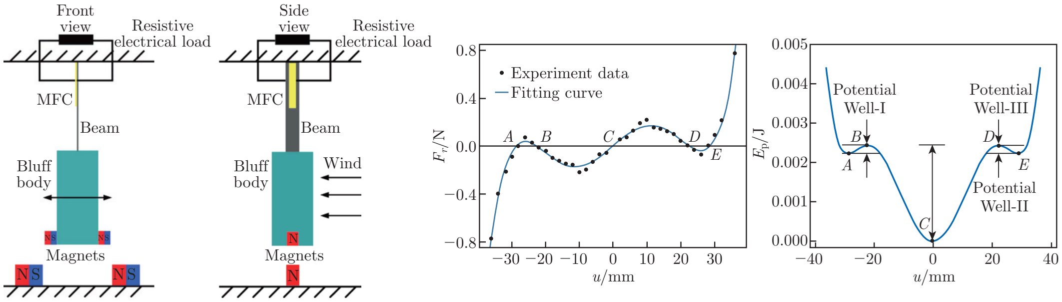
(a)

(b)

(c)

Fig. 11 (a) Schematic diagram of Y-shaped bi-stable flow-induced energy harvester; (b) potential energy functions; and (c) three equilibrium positions: (i) stable state, (ii) unstable state, and (iii) stable state[94] (color online)

Moreover, Wang et al.[96] considered the use of nonlinear magnetic force on a traditional galloping-based energy harvester with a square bluff body, as shown in Fig. 12. Using the nonlinear tri-stability mechanism, although technically, we require designing a more complicated structure for energy harvesting, the formation of a shallower potential well in the dynamic system can favor the process of energy harvesting under low-threshold excitation levels. Besides, Qin et al.[97], Wang et al.[98], and Zhou et al.[99] have also reported several works on the design



(a) (b) (c)

Fig. 12 Tri-stable galloping-based energy harvester[96]: (a) system design, (b) fitting curve of restoring force, and (c) potential energy function (color online)

of multi-stable FIV-driven energy harvesters using the combined VIV and galloping effects.

3.2 Application of LCOs

LCOs are a critical phenomenon of aeroelastic nonlinearities that can lead to large-amplitude and self-excited motions at a flow velocity above or occasionally below the linear flutter speed. Dunnmon et al.[100] introduced LCOs to design a nonlinear FIV-based energy harvester, in which a piezoelectric cantilever beam under an axial flow direction was analyzed. Not only the extremely large-amplitude characteristics of LCOs can sufficiently increase the energy harvesting efficiency, but also the self-excitation ability that does not require an external force to trigger large-amplitude motions can greatly improve the effectiveness and practicality of the energy harvesting paradigm.

De-Marqui[15] considered a criterion for the segmentation of piezoelectric layers to enhance the performance of energy harvesting subjected to aeroelastic LCOs in a flexible cantilever under an axial flow. Park et al.[101] presented an optimization technique that allows designers to implement LCOs for an aeroelastic energy harvester under nonlinear dynamic motions. Wu et al.[102] further proposed an airfoil-based piezo-aeroelastic energy harvester with an additional supporting device to harness mechanical energy from lead-lag motions. The bifurcation analysis showed a mutual transition process between LCOs and multi-periodic/chaotic motions. Andrianne et al.[103] experimentally investigated the energy harvesting performance of a square cylinder in an airflow, and found that a higher branch of the VIV-galloping curve could result in better energy harvesting performance. Due to the existence of LCOs in the FIV phenomenon, many research works on FIV-based energy harvesters due to LCOs have been discussed in the previous sections, and thus it would not be repeated here.

3.3 Other applications of mechanical nonlinearities

Mechanical nonlinearities can be inherently presented in a variety of forms in dynamic systems due to its geometric or material properties. There are many kinds of nonlinear FIV-based energy harvesters. For example, Sun et al.[104] applied a piecewise-linear spring-mass system as a nonlinear oscillator to design an FIV-based energy harvester using the VIV and galloping effects. The results demonstrated that the proposed structure could adapt to different flow regions. Seyed-Aghazadeh et al.[105] studied an FIV-based energy harvester consisting of an inherent nonlinear elastic L-shaped beam. Mcneil and Abdelkefi[17] also derived the nonlinear reduced-order model of an energy harvesting absorber under the effects of mechanical and VIV excitations. Lai et al.[106–107] further combined piezoelectric ceramic sheets and a vibro-impact dielectric elastomer generator together to design a hybrid piezo-dielectric VIV-driven energy harvester, as shown in Fig. 13. The results indicated that using a vibro-impact force could achieve a higher power generation performance in the lock-in region of the VIV effect.

Moreover, Da-Silva and Marques[108] connected linear and nonlinear springs in series as a

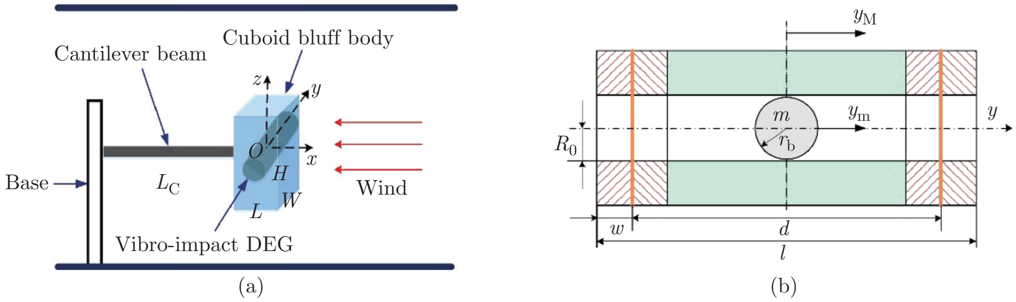


Fig. 13 (a) Hybrid piezo-dielectric energy harvester and (b) bluff body embedded in energy harvester[106] (color online)

multi-degree-of-freedom nonlinear energy sink in a sprung cylinder to construct a VIV-based energy harvester. The nonlinear bifurcation and LCO behavior of such a system were analyzed to show how the design could enhance the working efficiency of energy harvesting. Unlike the previous conventional oscillators, Sun and Seok[109] designed a movable bluff body which is an elastic constraint in an axial direction on a piezoelectric beam structure. It can be adaptively moved at a position, which can be determined by the balance of forces exerting on the bluff body under a given wind velocity.

4 Miscellaneous design approaches for FIV-based energy harvesting

In this section, few advanced FIV-based energy harvesting studies conducive to fluid engineering applications are briefly introduced. For instance, Tan et al.[110] experimentally studied a trout-inspired multi-functional robotic fish as an underwater swimmer and energy harvester. For the purpose of energy harvesting, the designed bionic fish was modelled as a WIV-based energy harvester with different diameters of cylindrical bluff bodies in a water tunnel. An average electrical power of 120 μW was yielded under a resonant condition. This design has shown potential to power up small sensors in the scenario of ecological monitoring. Chen et al.[111] designed a shape-adaptive and bionic-jellyfish triboelectric nanogenerator with polymeric thin films. The results indicated that the generated power could serve as a self-powered light emitting diode (LED) system. In addition, bionic-tree and bionic-fin structures with triboelectric nanogenerators were analyzed by Bian et al.[112] and Zhang et al.[113], respectively. These works are able to realize long-term and real-time measurements in fluid environments.

Skow et al.[114] and Cao et al.[115] proposed to convert the dynamic pressure of a hydraulic piping system into electrical power. It can be used to power sensor nodes or other low-power devices for intelligent monitoring and detection. In addition, Cho et al.[116] designed a coupling hydro-electromagnetic-piezoelectric energy harvester to construct a self-powered smart water meter. All of these advanced designs for FIV-based energy harvesting are thriving for remote wireless monitoring of water, oil, and gas pipeline systems.

5 Concluding remarks and future outlook

This paper presents an overview of the technologies for energy harvesting using flow-induced phenomena through the VIV, galloping, flutter, and WIV effects. The working principles of the four phenomena are introduced. The recent development of FIV-based energy harvesting techniques is discussed to show their basic mechanisms and fluid-structure interactions. In addition, various hybrid FIV-based design structures for energy harvesting are also presented. Moreover, the applications of LCOs and multi-stable nonlinearities to improve the energy harvesting performance are reviewed. Several advanced FIV-based energy harvesting studies are briefly reported. Since there are many relevant works in the literature, we mainly focus on the works in the recent three years.

In the literature, we know that great efforts have been made in understanding the mechanisms of various FIV-based energy harvesters. However, most of the works focused on the theoretical analysis, and an in-depth interdisciplinary study is still required. We are also aware of the promising application prospects for low-power electronic equipment and self-powered wireless sensor networks in engineering fields, such as mechanical engineering, aerospace engineering, pipeline engineering, biomedical engineering, and healthcare discipline. Unfortunately, so far, mature industrial applications are still limited, and there is ample room for exploring practical engineering applications. For the future outlook, Wang et al.[9] and Yang et al.[117] discussed the perspectives of nonlinear FIV-based vibration energy harvesting technologies. Through the present literature review, we recommend the following technical issues to be addressed. First, how to extend the frequency response range of such energy harvesters and to improve their power generation for adapting complex ambient environments are critical topics. In this regard, exploiting nonlinear and hybrid designs are effective ways. Second, advanced dielectric materials and man-made structures, including piezoelectric and dielectric materials, metamaterials, chiral materials, and bionic structures, can also be introduced to the design of vibration-based energy harvesters. From an engineering perspective, this is a multi-disciplinary design problem. Third, interfacing an optimal energy management strategy[118] for electronic circuit design is another technical challenge due to the energy conversion of harvesting performance. Finally, the miniaturization, durability, and robustness of FIV-based energy harvesting systems are also important design factors.

After the acceptance of the present work, it is worth noting that we recently found the review article by Ma and Zhou[119] on a similar subject, which was available online on 22 January, 2022. Before the appearance of this review paper[119], we have submitted our revised paper to *Applied Mathematics and Mechanics* (*English Edition*) on 4 January, 2022, and it was finally accepted by this Journal on 13 January, 2022. Both review papers were prepared and completed independently at about the same time. We are very pleased to point out that this work[119] is another important reference to this subject for researchers, as a holistic review of the relevant works in this field is provided in that review paper.

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