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Effects of stretching/shrinking on the thermal performance of a fully wetted convective-radiative longitudinal fin of exponential profile*

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Abstract The present investigation focuses on the thermal performance of a fully wet stretching/shrinking longitudinal fin of exponential profile coated with a mechanism like a conveyer belt. The modeled equation is non-dimensionalized and solved by applying the Runge-Kutta-Fehlberg (RKF) method. The effects of parameters such as the wet parameter, the fin shape parameter, and the stretching/shrinking parameter on the heat transfer and thermal characteristics of the fin are graphically analyzed and discussed. It is inferred that the negative effects of motion and internal heat generation on the fin heat transfer rate can be lessened by setting a shrinking mechanism on the fin surface. The current examination is inclined towards practical applications and is beneficial to the design of fins.

Key words fully wet longitudinal fin, convection, exponential profile, internal heat generation, moving fin

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Nomenclature

A,	thermal conductivity parameter;
$A(x)$,	cross-sectional area of the fin, m^2;
A_{b},	area of the fin base, m^2;
T_{a},	ambient temperature, K;
b_2,	variable parameter, K^{-1};
c_p,	specific heat at constant pressure, J/(kg·K);
h_{D},	uniform mass transfer coefficient;
h_{a},	coefficient of convective heat transfer at T_{a}, $\mathrm{W/(m^2{\cdot}K)}$;
h,	coefficient of convective heat transfer, $\mathrm{W/(m^2{\cdot}K)}$;
G,	generation number;
k_{a},	thermal conductivity at T_{a}, W/(m·K);

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k,	thermal conductivity, W/(m·K);	q,	base heat transfer rate, W;
L,	fin length, m;	S,	stretching/shrinking parameter of the fin;
m_0, m_1,	constants;	s^*,	stretching/shrinking rate of the fin, m^{-1};
m_2,	wet parameter;	T,	local fin temperature, K;
N_{r},	radiative parameter;	T_{b},	base temperature, K;
Pe,	Peclet number;	$t(x)$,	fin thickness, m;
p,	exponential index of h;	t_{b},	base thickness of the fin, m;
Q,	non-dimensional heat transfer rate;	U,	constant velocity of the fin, m/s;
q_{a}^*,	rate of internal heat generation at T_{a}, W/m^3;	W,	width, m;
		X,	dimensionless length;
q^*,	rate of internal heat generation, W/m^3;	x,	axial coordinate of the fin, m.

Greek symbols

ω,	humidity ratio of the saturated air;	θ_{a},	nondimensional ambient temperature;
ν,	shape parameter of the fin;	θ,	dimensionless temperature;
α,	measure of thermal conductivity variation with temperature, K^{-1};	ε,	surface emissivity of the fin;
		i_{fg},	latent heat of water evaporation, J/kg;
ω_{a},	humidity ratio of the surrounding air;	σ,	Stefan-Boltzmann constant, W/(m^2·K^4);
ϵ_{G},	nondimensional internal heat generation parameter;	ρ,	density of the ambient fluid, kg/m^3.

Subscripts

a,	ambient;	b,	base.

1 Introduction

The fin of adaptable geometry serves as a handy tool for increasing the surface area and thereby increasing the heat transfer rate. Extended surface or fin has been applied in areas such as aviation engineering, automobile industries, nuclear power plants, microelectronics, and home electronic systems. Research in the field of heat transfer enhancement via fin is going on since decades. To date, the study on extended surface has taken various dimensions, making the fin problem practically oriented stage by stage. Kraus et al.[1] extensively documented the evolution of extended surface technology. Kiwan[2] implemented Darcy's formulation to model the natural convection porous fin problem, and conferred that the porous fin forfeited the conventional solid one with the enhancement in the surface area-to-volume ratio. Gorla and Bakier[3] studied the effect of radiation on the thermal performance of a fin, and deduced that the loss of heat via radiation further enhanced the heat transfer rate of the fin. Torabi and Aziz[4] studied the fin efficiency, considering the non-uniformity of convective, conductive, and radiative coefficients of heat transfer. In the work, a T-shaped fin was chosen for scrutiny, and it was derived that the temperature dependence of the three coefficients had a major effect on the fin efficiency. Vahabzadeh et al.[5] optimized a concave parabolic porous pin fin, and showed that it was more suitable than triangular, rectangular, and convex parabolic spines. Darvishi et al.[6] utilized the pseudo spectral method to scrutinize a hyperbolic profiled annular fin problem, and showed that the shorter fin was more efficient than the longer one.

Oguntala et al.[7] used an iterative scheme to analyze the fin problem exposed to the magnetic environment and verified the obtained results numerically. The study discovered that the magnetic field had a favorable effect on the thermal attributes of the fin. Hoseinzadeh et al.[8] scrutinized the thermal performance of a uniformly thick and porous straight fin subjected to a laminar flow with the homotopy analysis method (HAM). Sowmya et al.[9] examined the fins made up of functionally graded materials (FGMs). Kundu and Yook[10] studied radial, longitudinal, and pin fins subjected to certain limitations. The unified investigation upholds the importance of porous fin structures by conducting a superior analysis compared with the earlier works in the direction. Turkyilmazoglu[11] solved the fin problem involving a parabolic profiled spine, and underlined the prominence of the Peclet number in determining the profile's ideal dimensions. Turkyilmazoglu[12] looked at a novel notion of fin length expansion and contraction, and made an attempt to provide similarity solutions for the fin problem.

The fin geometry can be broadly classified into longitudinal (straight), radial, and pin fin structures. The present work concentrates on the longitudinal fin structures. Torabi et al.[13] investigated the efficiency of straight fin structures of constant volume and variable thickness, considering all thermo-physical properties. Adopting an advanced technique of volume adjustment, they found that the concave parabolic fin profile was more efficient than the trapezoidal and rectangular fin profiles. Further, the inverse relationship of the fin temperature with the fin taper ratio was discovered. A similar study was conducted by Torabi and Qiao[14]. Hatami and Ganji[15] considered longitudinal fins of various geometries and materials, and developed an accurate analytical solution with the method of least squares. It was inferred that exponential fin was favorable in supporting heat transfer. Kundu and Lee[16] implemented the calculus of variation-based analytics in an optimal study of porous fin structures, demonstrated the dependence of the fin shape on its porosity, and established the inclination of the optimum fin profile towards a non-zero tip thickness. Kundu et al.[17] analyzed the mass and heat transfer in a fin of exponential profile with the differential transformation method (DTM), and concluded that the humidity had little discernible influence on the efficiency of any of the considered fin profiles. Turkyilmazoglu[18] investigated the exponential fin of growing and decaying profiles subjected to movement and non-uniform heat generation analytically. The exponential fin of the growing profile was found to be more efficient than the other profiles, and was portrayed as a possible substitute for rectangular fin profiles.

Sharqawy and Zubair[19] developed an analytical solution for the efficiency of a wetted straight fin by introducing a more appropriate relationship involving fin temperature and humidity ratio, and presented an alternative fin parameter which did not require the fin tip conditions to compute the problem. The study revealed that the fin efficiency was directly proportional to the atmospheric pressure. Hatami et al.[20] used the modified fin parameter suggested by Ref. [19] to investigate the porous fins in totally wet circumstances, considering the semi-spherical fin structure. The method of least squares was applied, and the results were validated via numerical calculations. The main outcome was that mass transfer should be limited to achieve high fin efficiency. Khani et al.[21] numerically examined the radial fin problem under wet and porous conditions with the method of spectral collocation, and recorded the positive effect of the fully wet nature of a fin on the heat transfer rate. In a recent investigation, Sowmya et al.[22] considered a longitudinal fin of different profiles under wet conditions, and demonstrated their transient thermal responses. The study was performed by employing the finite difference method (FDM). It was revealed that the wet porous parameter had a negative effect on the fin efficiency.

Since fin is under continuous motion in a variety of applications, there is a necessity to analyze the effect of movement on its thermal profile. Ma et al.[23] adopted a numerical method to simulate the thermal profile of a moving porous straight fin, and compared the efficiency of different fin geometries based on the volume adjustment technique. It was reported that the local fin temperature and efficiency of a fin are directly proportional to the Peclet number. Gireesha et al.[24] scrutinized an annular porous fin wetted in a fluid and subjected to motion with the finite element method (FEM), and concluded that an upsurge in the Peclet number caused an increase in the fin surface temperature. The study of extended surface is incomplete without examining the heat generation inside the fin. Concerned to the above aspects, Mosayebidorcheh et al.[25] studied the thermal profiles and thermal gradients of various geometries of a straight fin under unsteady conditions. In the study, the hybrid DTM-FDM was applied for the space and time analyses of fins, and the parameter of internal heat generation varied linearly with temperature. The hybrid block method was enforced by Alkasassbeh et al.[26] to scrutinize the energy equation of a convective porous fin, and the negative effect of heat generation on fin cooling was demonstrated.

In a novel approach, Turkyilmazoglu[27] analytically studied the stretching/shrinking moving fin problem, considering the rectangular profiled longitudinal fin exposed to convective environment. The study implied that the mechanism of shrinking enhanced the fin efficiency. Mosavat et al.[28] investigated the effect of the stretching/shrinking mechanism on the energy profile of a moving fin of distinct geometries. The current investigation intends to numerically examine the effects of the fully wet nature, the stretching/shrinking mechanism, and the heat generation on the heat transfer characteristics of an exponential profiled longitudinal fin with a varying fin shape parameter. The study complies with the discussions embedded with the physical interpretations based on the graphical depictions of the thermal profile of the fin and the base heat transfer rate. The present work theoretically depicts the role of shrinking in aiding the fin's purpose, and emphasizes further research in this arena.

2 Mathematical model

A longitudinal fin of exponential profile with the length L, the base thickness t_{b}, and the width W is mounted on a surface kept at the temperature T_{b}. The solid fin matrix is entirely wetted in a fluid, and loses heat to the surroundings, which is kept at the temperature T_{a}, by convection and radiation. The fin temperature is assumed to affect the thermal conductivity k and the coefficient of convective heat transfer h. Further, the fin moves horizontally along the x-axis at a constant speed U, implying that either the base surface is moving along the x-direction at a constant speed U and the surrounding fluid is stagnant or the base surface is at rest and the fluid is moving with a constant velocity U. But the interaction between the fin surface and the fluid varies with the fin shape. Thus, the velocity U is constant, but the velocity distribution is relative to the fin shape[18,23,29]. Further, the fin surface is set with a mechanism that undergoes stretching/shrinking at the rate of s^* in the motion direction. To begin with a linear relationship has been assumed between the fin velocity and the stretching/shrinking parameter s^*[27]. The other assumptions made while modeling the fin problem are that the fin's temperature distribution is one-dimensional, the fin tip is adiabatic, and the analysis is under the steady-state condition.

The defined fin problem has a geometry as depicted in Fig. 1. Here, the thickness of the fin is defined by $t(x)$, which is dependent on the fin shape parameter ν and given by[14]

$$t(x) = t_{\mathrm{b}} \mathrm{e}^{\nu(\frac{x}{L}-1)}. \tag{1}$$

As pictured in Fig. 1, $\nu > 0$ corresponds to an exponential fin with a thin tip (e.g., tapered exponential fin), $\nu = 0$ corresponds to a rectangular fin, and $\nu < 0$ corresponds to an exponential fin with a thick tip (e.g., inverted exponential fin).

Thus, for the above defined impermeable fin problem, the energy balance equation per unit width is given by[13,27]

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}x}\Big(k(T)t(x)\frac{\mathrm{d}T}{\mathrm{d}x}\Big) + q^*(T)t(x) - 2\varepsilon\sigma(T^4 - T_{\mathrm{a}}^4) - 2h(T)(T - T_{\mathrm{a}}) \\
&- 2h_{\mathrm{D}} i_{\mathrm{fg}}(\omega - \omega_{\mathrm{a}}) + \rho c_p t(x) U(1 + s^* x)\frac{\mathrm{d}T}{\mathrm{d}x} = 0.
\end{aligned} \tag{2}$$

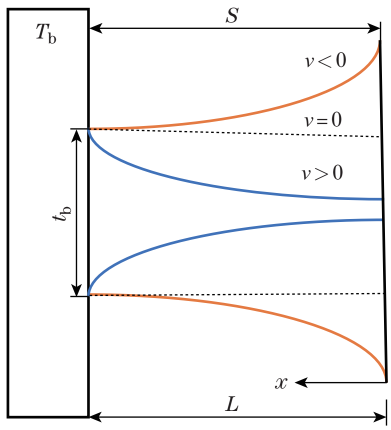

Fig. 1 Stretching/shrinking exponential profiled longitudinal fin with the varying fin shape parameter ν (color online)

The required boundary conditions for the adiabatic tip fin are

$$\begin{cases} T = T_{\mathrm{b}} \quad \text{at} \quad x = L, \\ \dfrac{\mathrm{d}T}{\mathrm{d}x} = 0 \quad \text{at} \quad x = 0. \end{cases} \tag{3}$$

Since the fin is assumed to be fully wet, we have

$$\omega - \omega_{\mathrm{a}} = b_2(T - T_{\mathrm{a}}). \tag{4}$$

The fin thermal conductivity, convective heat transfer coefficient, and internal heat generation are assumed to be temperature dependent, and are given by[20,30–31]

$$k(T) = k_{\mathrm{a}}(1 + \alpha(T - T_{\mathrm{a}})), \tag{5}$$

$$h(T) = h_{\mathrm{a}}\Big(\frac{T - T_{\mathrm{a}}}{T_{\mathrm{b}} - T_{\mathrm{a}}}\Big)^p = h_{\mathrm{D}} c_p Le^{\frac{2}{3}}, \tag{6}$$

$$q^*(T) = q_{\mathrm{a}}^*(1 + \epsilon_{\mathrm{g}}(T - T_{\mathrm{a}})). \tag{7}$$

Implementing Eqs. (1), (4)–(7) in Eq. (2) yields

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}x}\Big(k_{\mathrm{a}}(1 + \alpha(T - T_{\mathrm{a}}))t_{\mathrm{b}}\mathrm{e}^{\nu(\frac{x}{L}-1)}\frac{\mathrm{d}T}{\mathrm{d}x}\Big) + q_{\mathrm{a}}^*(1 + \epsilon_{\mathrm{g}}(T - T_{\mathrm{a}}))t_{\mathrm{b}}\mathrm{e}^{\nu(\frac{x}{L}-1)} \\ &- 2\varepsilon\sigma(T^4 - T_{\mathrm{a}}^4) - 2h_{\mathrm{a}}\Big(\frac{T - T_{\mathrm{a}}}{T_{\mathrm{b}} - T_{\mathrm{a}}}\Big)^p(T - T_{\mathrm{a}}) \\ &- 2\frac{h_{\mathrm{a}} i_{\mathrm{fg}} b_2}{c_p Le^{\frac{2}{3}}}\Big(\frac{T - T_{\mathrm{a}}}{T_{\mathrm{b}} - T_{\mathrm{a}}}\Big)^p(T - T_{\mathrm{a}}) \\ &+ \rho c_p t_{\mathrm{b}}\mathrm{e}^{\nu(\frac{x}{L}-1)}U(1 + s^* x)\frac{\mathrm{d}T}{\mathrm{d}x} = 0. \end{aligned} \tag{8}$$

The dimensionless form of Eq. (8) is a nonlinear ordinary differential equation of second order, and is given by

$$\begin{aligned}&(1+A(\theta-\theta_{\rm a}))\mathrm{e}^{\nu(X-1)}\frac{\mathrm{d}^2\theta}{\mathrm{d}X^2}+\nu(1+A(\theta-\theta_{\rm a}))\mathrm{e}^{\nu(X-1)}\frac{\mathrm{d}\theta}{\mathrm{d}X}\\&+A\mathrm{e}^{\nu(X-1)}\Big(\frac{\mathrm{d}\theta}{\mathrm{d}X}\Big)^2+\mathrm{e}^{\nu(X-1)}G(1+\epsilon_{\rm G}(\theta-\theta_{\rm a}))\\&-N_{\rm r}(\theta^4-\theta_{\rm a}^4)-m_2\frac{(\theta-\theta_{\rm a})^{p+1}}{(1-\theta_{\rm a})^p}\\&+\mathrm{e}^{\nu(X-1)}Pe(1+SX)\frac{\mathrm{d}\theta}{\mathrm{d}X}=0.\end{aligned}\tag{9}$$

The respective dimensionless boundary conditions are

$$\begin{cases}\theta=1 \quad \text{at} \quad X=1,\\ \dfrac{\mathrm{d}\theta}{\mathrm{d}X}=0 \quad \text{at} \quad X=0.\end{cases}\tag{10}$$

This is achieved by utilizing the following parameters:

$$\begin{cases}\theta=\dfrac{T}{T_{\rm b}},\quad \theta_{\rm a}=\dfrac{T_{\rm a}}{T_{\rm b}},\quad X=\dfrac{x}{L},\quad A=\alpha T_{\rm b},\quad N_{\rm r}=\dfrac{2\sigma\epsilon T_{\rm b}^3L^2}{k_{\rm a}t_{\rm b}},\\ \epsilon_{\rm G}=\epsilon_{\rm g}T_{\rm b},\quad m_0=\dfrac{2h_{\rm a}L^2}{k_{\rm a}t_{\rm b}},\quad m_1=\dfrac{2h_{\rm a}i_{\rm fg}b_2L^2}{k_{\rm a}t_{\rm b}c_pLe^{\frac{2}{3}}},\\ m_2=m_0+m_1,\quad Pe=\dfrac{\rho c_pUL}{k_{\rm a}},\quad S=s^*L,\quad G=\dfrac{q_{\rm a}^*L^2}{k_{\rm a}T_{\rm b}},\end{cases}\tag{11}$$

where $\theta_{\rm a}$ is the dimensionless ambient temperature, A is the thermal conductivity parameter, $N_{\rm r}$ is the radiation parameter, Pe is the Peclet number associated with the fin motion, $\epsilon_{\rm G}$ is the internal heat generation parameter, S is the stretching/shrinking parameter, m_2 is the wet parameter, and G is the generation number.

The amount of heat that enters the fin via base is another parameter to assess the fin thermal performance. It can be calculated by applying Fourier's law at the base of the fin as[25]

$$q=k_{\rm a}(1+\alpha(T_{\rm b}-T_{\rm a}))A_{\rm b}\frac{\mathrm{d}T(L)}{\mathrm{d}x}.\tag{12}$$

The dimensionless fin heat transfer rate is

$$Q=\frac{qL}{k_{\rm a}A_{\rm b}T_{\rm b}}=(1+A(1-\theta_{\rm a}))\theta'(1).\tag{13}$$

3 Method of solution

The Runge-Kutta-Fehlberg (RKF) 4th- and 5th-order schemes in MAPLE software are used to solve the 2nd-order non-linear ordinary differential equation (9) along with the boundary conditions in Eq. (10). Initially, the given boundary value problem (BVP) is converted into an initial value problem (IVP). Then, the obtained IVP is solved via the RKF 4th- and 5th-order formulae, respectively, and the error involved is calculated. The viable option to minimize the error is by increasing the step size. The accuracy of the obtained solution is in the order of 10^{-6} with a step size of 0.001. The present model and code are validated in Table 1 by comparing the obtained numerical solutions with the DTM results from Torabi et al.[32].

Table 1 Comparison of numerical results with the DTM results for the fin tip temperature when $N_{\mathrm{r}} = 0.25$, $Pe = 0.25$, $\theta_{\mathrm{a}} = 0$, $m_1 = 0$, $\epsilon_{\mathrm{G}} = 0$, $G = 0$, $S = 0$, $p = 0$, and $\nu = 0$

A	$m_0 = 0.25$		$m_0 = 0.5$	
	Torabi et al.[32]	Present result	Torabi et al.[32]	Present result
0	0.835 8	0.835 8	0.765 2	0.765 2
0.25	0.858 4	0.858 4	0.795 5	0.795 5
0.5	0.875 6	0.875 6	0.819 2	0.819 2
0.75	0.889 2	0.889 2	0.838 1	0.838 1
1	0.900 1	0.900 1	0.853 6	0.853 6

4 Results and discussion

Figures 2–11 show the effects of important factors on the energy field and base heat transfer rate of the shrinking, stagnant, and stretching fins with tapered exponential, rectangular, and inverted exponential profiles by altering each parameter while keeping the others constant. The constant values used throughout the analysis are as follows:

$$\begin{cases} N_{\mathrm{r}} = 1, \quad m_2 = 1, \quad p = 2, \quad A = 0.2, \quad G = 0.1, \\ \epsilon_{\mathrm{G}} = 0.2, \quad \theta_{\mathrm{a}} = 0.4, \quad Pe = 1. \end{cases}$$

The temperature distributions in the shrinking, stagnant, and stretching fins of the tapered exponential, rectangular, and inverted exponential profiles upon the variation of the Peclet number Pe are pictured in Fig. 2. It is deciphered that the Peclet number decreases the fin thermal drop rate. This is owing to the decrease in the exposure time of the fins to the ambient fluid for the process of convective heat loss. Moreover, the effects are more pronounced in the stretching fin cases than in the shrinking fin cases. This is because the stretching mechanism added to the fin motion further decreases the exposure time of the fin whereas the shrinking mechanism increases the fin exposure time, leading to the in crease in heat loss through convection. Further, the thermal drop rates of the tapered exponential, rectangular, and inverted exponential fin profiles are in a descending order.

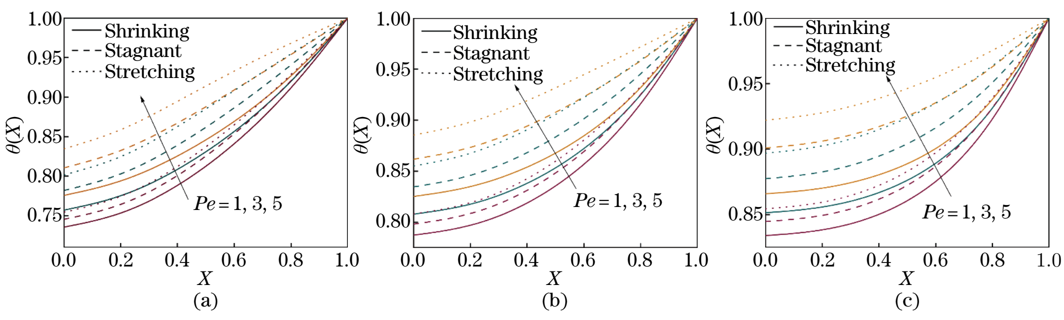

Fig. 2 Thermal profiles of the shrinking, stagnant, and stretching fins for diverse values of Pe when (a) $\nu = 1$, (b) $\nu = 0$, and (c) $\nu = -1$ (color online)

Figure 3 depicts the consequences of the thermal conductivity parameter A on the energy fields of the shrinking, stagnant, and stretching fins of various exponential profiles. It is noted that, when the value of A moves from negative to positive, the temperature of the fin surface

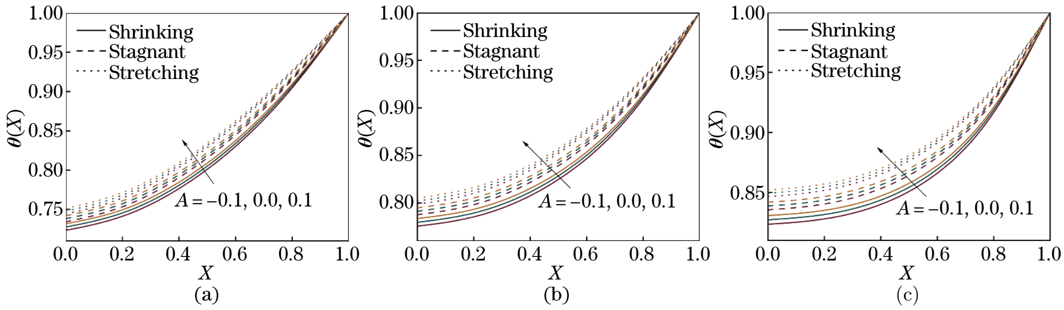

Fig. 3 Thermal profiles of the shrinking, stagnant, and stretching fins for diverse values of A when (a) $\nu = 1$, (b) $\nu = 0$, and (c) $\nu = -1$ (color online)

rises. Here, positive and negative values of A correspond to the direct and inverse relationships of thermal conductivity with the fin surface temperature. Hence, the increase in the parameter A ascends the average thermal conductivity of the fin material and thus the increase in the fin surface temperature, resulting in higher thermal profiles. Further, the shrinking mechanism decreases the fin temperature, whereas the stretching mechanism increases the fin temperature. Also, the energy profile of the tapered exponential fin stays below the rectangular fin profile, which is further preceded by the inverted exponential fin.

The significance of the exponential index p on the energy attribute of the shrinking, stagnant, and stretching fins of various exponential profiles is exhibited in Fig. 4. The temperature distribution in the fin grows as the value of p rises. Here, the ascending values of p correspond to the increase in the sensitivity of the heat transfer coefficient h with the fin temperature. As can be seen from Eq. (6), the increased sensitivity of h with the local fin temperature results in lower values of h as compared with $h_{\rm a}$. This negativity affects the heat loss via convection, resulting in an increase in the fin surface temperature. Further, the energy profile of the shrinking fin is below the energy profile of the stretching one. It can also be noticed that the same trend is followed by all the three considered profiles.

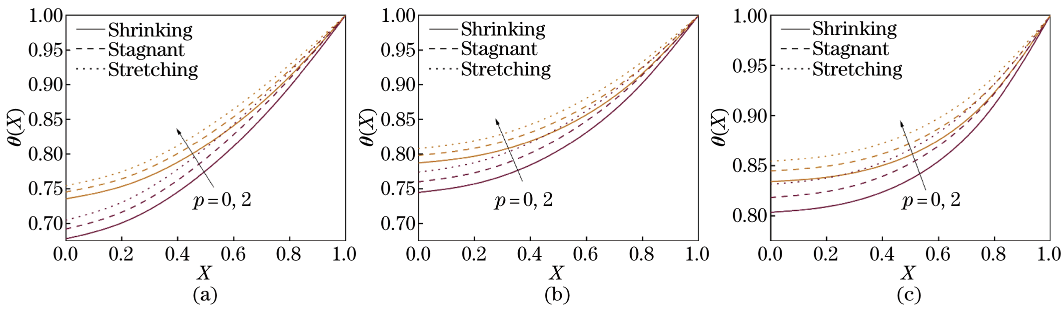

Fig. 4 Thermal profiles of the shrinking, stagnant, and stretching fins for diverse values of p when (a) $\nu = 1$, (b) $\nu = 0$, and (c) $\nu = -1$ (color online)

The negative effects of the generation number G on the thermal field of the shrinking, stagnant, and stretching fins of the tapered exponential, rectangular, and inverted exponential profiles are shown in Fig. 5. The reason for the negative effects is that an increase in the heat generation within the fin causes an upsurge in the fin temperature. This effect is more striking in the cases of rectangular and inverted exponential fin profiles because of the relatively higher fin volumes than the tapered exponential fin profile. It is further observed that, in all the cases,

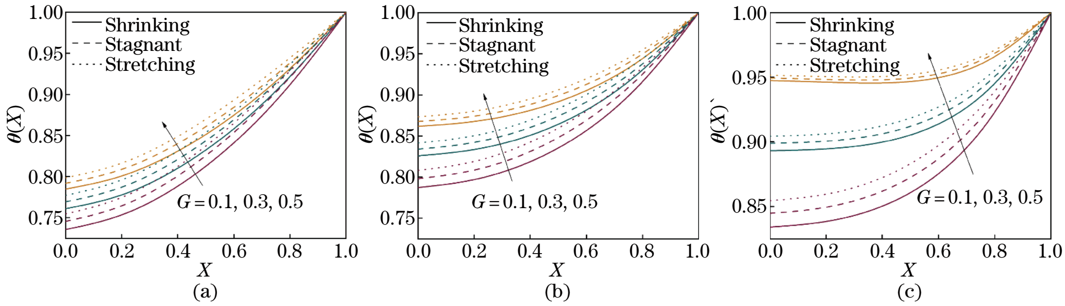

Fig. 5 Thermal profiles of the shrinking, stagnant, and stretching fins for diverse values of G when (a) $\nu = 1$, (b) $\nu = 0$, and (c) $\nu = -1$ (color online)

compared with the stagnant fin, the shrinking fin has a lower fin surface temperature, while the stretching fin has a higher fin surface temperature.

The repercussions of the wet parameter m_2 on an impermeable fully wet exponential fin of three different profiles subjected to the shrinking, stagnant, and stretching mechanisms are captured in Fig. 6. It is explored that, the increasing values of m_2 enhance the process of fin cooling by decreasing the fin surface temperature. The parameter m_2 accounts for both the wet nature of the fin and the convection from the fin surface. The wet nature around the fin enhances the heat absorption by the ambient fluid contributing to the convective heat loss. Moreover, the shrinking mechanism is observed to be in support of the fin cooling process rather than the stretching mechanism. Further, the fins of tapered exponential, rectangular, and inverted exponential profiles are affected in a similar way.

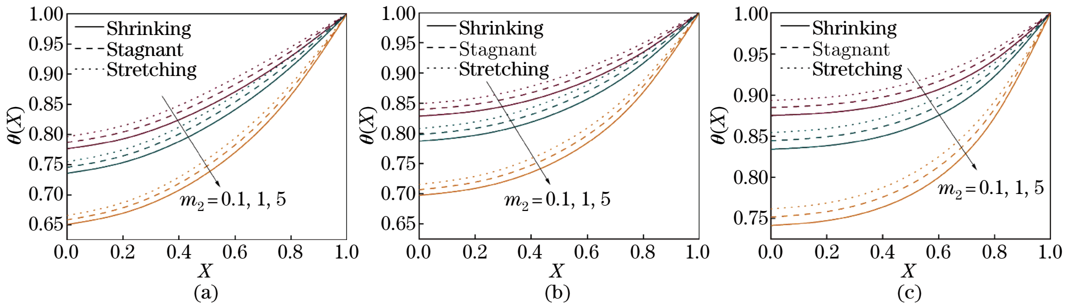

Fig. 6 Thermal profiles of the shrinking, stagnant, and stretching fins for diverse values of m_2 when (a) $\nu = 1$, (b) $\nu = 0$, and (c) $\nu = -1$ (color online)

The variations in the temperature field of the shrinking, stagnant, and stretching fins of the tapered exponential, rectangular, and inverted exponential profiles with the radiation parameter N_{r} are expressed in Fig. 7. It is seen that, with an increase in N_{r}, there is a sharp decrease in the fin tip temperature. This is because of the enhanced heat loss via the mode of radiative heat transfer. Further, the shrinking fin loses more heat than the stagnant one, which further loses more heat than the stretching one. Moreover, all the three profiles of the exponential fin are affected in the same trend.

Figure 8 portrays the prominence of S and Pe on the heat transfer rate Q of the tapered exponential, rectangular, and inverted exponential fin profiles. As can be seen, increasing Pe decreases the fin heat loss rate. This effect becomes more prominent when the stretching/shrinking parameter S slides from $S = -1$ towards $S = 1$. It is further explored that the

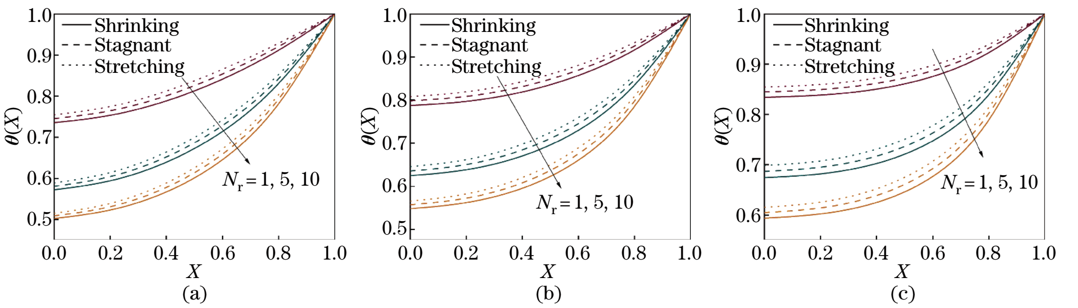

Fig. 7 Thermal profiles of the shrinking, stagnant, and stretching fins for diverse values of N_r when (a) $\nu = 1$, (b) $\nu = 0$, and (c) $\nu = -1$ (color online)

shrinking system aids in the cooling process of the moving fin and the stretching system further deteriorates the fin rate of heat loss. Besides, the heat loss rate in an inverted exponential fin profile is commendable as compared with the rectangular and tapered exponential fin profiles. This is because as per Newton's law of cooling, the heat transfer rate increases with the increase in the available surface area, and the surface area of the inverted exponential fin profile is greater than the other two profiles.

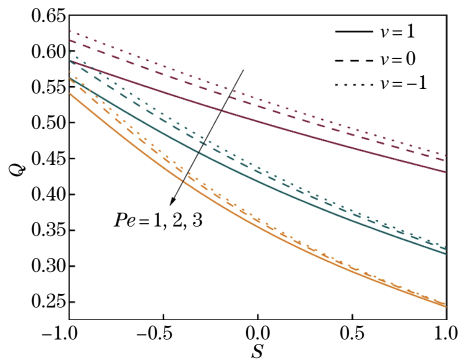

Fig. 8 Base heat transfer rate Q versus S for diverse values of Pe when $\nu = 1, 0, -1$ (color online)

The effects of the simultaneous variations of G and S on the heat transfer rate Q for the three distinct profiles of an exponential fin are pictured in Fig. 9. It is spotted that the generation number G negatively affects the fin heat dissipation. Also, increasing S causes a decline in the heat loss rate. Thus, the shrinking fin helps in the fin cooling process. Further, the inverted exponential fin profile assists the heat loss at smaller values of the generation number G whereas the tapered exponential profile enhances the heat loss at higher values of G.

The ascending values of the wet parameter m_2 and the descending values of S positively affect the heat transfer rate Q for all the three exponential fin profiles as depicted in Fig. 10. Thus, the wet nature, the convective heat loss, and the shrinking system amplify the fin cooling rate. Besides, it is noticed that, at lower values of m_2, the rectangular fin profile outperforms both the inverted and the tapered exponential profiles. On the contrary, at higher values of m_2, the inverted exponential fin profile intensifies the heat transfer process as compared with the rectangular and tapered exponential profiles.

Figure 11 depicts the implication of N_r on Q of the shrinking, stagnant, and stretching fins of the tapered exponential, rectangular, and inverted exponential profiles. The ascending values

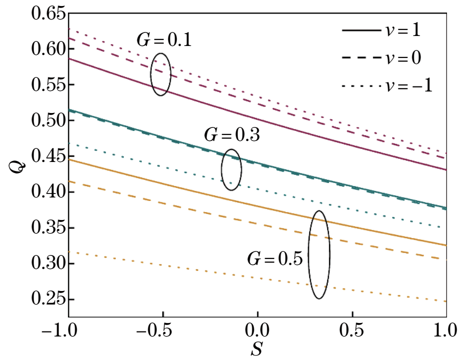

Fig. 9 Base heat transfer rate Q versus S for diverse values of G when $\nu = 1, 0, -1$ (color online)

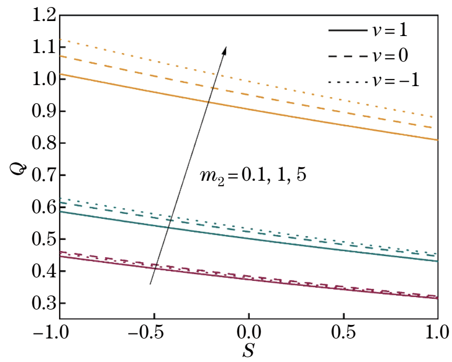

Fig. 10 Base heat transfer rate Q versus S for diverse values of m_2 when $\nu = 1, 0, -1$ (color online)

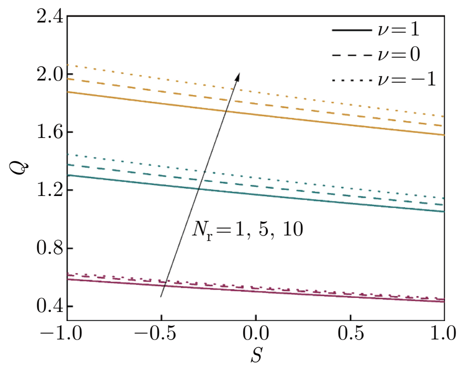

Fig. 11 Base heat transfer rate Q versus S for diverse values of N_{r} when $\nu = 1, 0, -1$ (color online)

of N_{r} uplift the heat transmission rate. Further, the shrinking system assists and the stretching system hinders the fin cooling process. Furthermore, a similar trend is followed by the three exponential fin profiles.

For the sake of future reference, some quantitative data are presented in Table 2. It consists of $\theta'(1)$ for distinct values of the Peclet number for all the shrinking, stagnant, and stretching fin profiles.

Table 2 $\theta'(1)$ values for distinct values of the Peclet number Pe

S	$Pe = 1$			$Pe = 2$		
	$\nu = 1$	$\nu = 0$	$\nu = -1$	$\nu = 1$	$\nu = 0$	$\nu = -1$
-1	0.523 738	0.549 228	0.561 027	0.502 407	0.524 136	0.534 156
-0.5	0.483 645	0.505 739	0.516 099	0.430 598	0.446 745	0.454 231
0	0.447 313	0.466 386	0.475 396	0.371 500	0.383 321	0.388 714
0.5	0.414 389	0.430 791	0.438 553	0.322 797	0.331 325	0.335 068
1	0.384 552	0.398 602	0.405 230	0.282 555	0.288 618	0.291 107

5 Conclusions

The fully wet shrinking, stagnant, and stretching fins of exponential profiles with a varying fin shape parameter subjected to motion and internal heat generation are considered for the thermal analysis by implementing a numerical method. The outcomes are as follows.

(i) The shrinking system has a prominent positive effect on the fin cooling effect, especially when the fin is in motion.

(ii) The radiative environment and the wet nature around the fin accelerate the heat exchange process.

(iii) The generation of heat and the fin movement decelerate the heat output from the fin.

(iv) The lower the values of the power index and the surrounding temperature are, the better the fin performance is.

(v) The increase in the thermal conductivity with temperature assists the rate of heat removal.

(vi) The exponential fin profile with a thin tip outperforms both the rectangular fin profile and the exponential fin profile with a thick tip in decreasing the surface temperature towards the fin tip.

(vii) In general, the exponential fin with a thick tip quickens the heat transmission process more than the other two profiles. But a rectangular fin profile is suited for the applications where the heat loss through convection is less, and an exponential fin profile with a thin tip must be adopted in the scenario of high heat generation within the fin.

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