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Effects of rotation and gravity on an electro-magneto-thermoelastic medium with diffusion and voids by using the Lord-Shulman and dual-phase-lag models*

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Abstract The effects of rotation and gravity on an electro-magneto-thermoelastic medium with diffusion and voids in a generalized thermoplastic half-space are studied by using the Lord-Shulman (L-S) model and the dual-phase-lag (DPL) model. The analytical solutions for the displacements, stresses, temperature, diffusion concentration, and volume fraction field with different values of the magnetic field, the rotation, the gravity, and the initial stress are obtained and portrayed graphically. The results indicate that the effects of gravity, rotation, voids, diffusion, initial stress, and electromagnetic field are very pronounced on the physical properties of the material.

Key words electromagnetic field, gravity field, rotation, initial stress, voids, diffusion, normal mode analysis, Lord-Shulman (L-S) model, dual-phase-lag (DPL) model

Chinese Library Classification O357.5
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Nomenclature

a,	wave number;
a_{c}, b_{c},	magnitudes of thermoelastic diffusion;
$\boldsymbol{B}$,	magnetic induction vector;
C,	strength of diffusion;
C_{E},	specialized heat per unit mass;
d,	thermoelastic diffusion constant;
e_{ij},	component of the strain tensor;
$\boldsymbol{E}$,	electric intensity vector;
$\boldsymbol{F}_i$,	Lorentz's body force vector;
g,	gravity field;
g^*,	intrinsic equilibrated body force;
$\boldsymbol{h}$,	perturbed magnetic field vector;
$\boldsymbol{H}_0$,	primary constant magnetic field vector;
$\boldsymbol{H}$,	magnetic field vector;
$\boldsymbol{J}$,	electric current density vector;
K,	thermal conductivity;
m,	thermo-void coefficient;
P,	initial stress;
$\boldsymbol{q}_i$,	heat flux vector;
t,	time of wave;
T_0,	reference temperature;
χ,	equilibrated inertia;

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S_i,	component of the equilibrated stress vector;
T,	temperature;
$\alpha, b, \omega_0, \zeta$,	void material parameters;
α_t,	coefficient of linear thermal extension;
α_c,	coefficient of linear diffusion extension;
δ_{ij},	Kronecker delta;
ε_0,	electric permeability;
η,	entropy per unit mass;
λ, μ,	Lame's constants;
μ_r,	magnetic permeability,
ρ,	density;
σ_{ij},	component of the stress tensor;
λ_{ij},	Maxwell's stress tensor;
τ_1,	phase-lag of the heat flux;
τ_Θ,	phase-lag of the temperature gradient;
ω,	complex frequency;
τ_2, τ_η,	diffusion relaxation time;
Φ_v,	change in the volume fraction field;
Ω,	angular velocity.

1 Introduction

In the last five decades, wide spread attention has been paid to the theory of elasticity in the thermal field, where waves propagate only under the thermal effect without considering the mechanical stress at a finite speed. Conventional elastic theories are based on parabolic-type heat equations, and are referred to as generalized theories. For heat wave propagation, generalized thermoelastic theories are more realistic than conventional thermoelastic theories in dealing with practical problems involving very short time intervals and high heat fluxes, e.g., laser units, energy channels, and nuclear reactors. During the 19th century, the interactions between the thermomechanical behaviors and the magnetic behaviors of materials have been widely studied. Biot[1] studied the mechanical properties of porous materials with the consolidation theory of fluid-saturated porous solids. The theory considered the formal arguments of continuum mechanics, in which the concept of distributed body was introduced and a continuum model was represented for granular materials, e.g., sand and powder, and porous materials, e.g., rock and soil. The basic concept underlying this theory is that the bulk density of the material can be written as the product of the density field of the matrix material and the volume fraction field (the ratio of the volume occupied by grains to the bulk volume at a point of the material). The theories of classical and coupled thermoelasticity have been extensively developed due to their applications in structural design.

Recently, there have been many studies on the effect of diffusion. Lord and Shulman[2] obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier's law by a new formula containing the heat flux vector, its time derivative, and a new constant for the relaxation time. Since the heat equation of this theory is of the wave type, it automatically ensures the finite speeds of the propagation of heat and elastic waves. The remaining governing equations for this theory, i.e., the equations of motion and constitutive relations, remain the same as those for coupled and uncoupled theories. Coin and Nunziato[3] studied the effects of linear elastic materials with voids and the change of the void volume, and showed that the void volume was a physical property of the medium independent of the deformation, stress, and temperature. Aouadi[4–5] studied the equations of generalized thermoelastic diffusion based on the Lord-Shulman (L-S) model, proved the uniqueness of the solution of the initial boundary value problem, and derived the dynamic reciprocity theorem for the given model. Aouadi[6] investigated a problem of one-dimensional generalized thermoelastic diffusion in an infinite medium with a spherical cavity under the internal boundary of time-dependent thermal shock. Singh[7–8] studied the reflection of longitudinal and secondary vertical waves on the free surface of an elastic solid with generalized thermodiffusion under the boundary conditions including a system of four non-homogeneous equations for reflection coefficients depending upon the angle of incidence. Nowacki[9–11] depicted the fundamental theorems for the dynamic problem of diffusion in a solid body. Olesiak and Pyryev[12] discussed the cross effects arising from the couplings of the temperature, mass diffusion, and strain in an elastic cylinder. Sherief and Saleh[13] studied the thermoelastic half-space with a permeating substance in contact with the bounding plane with the theory of generalized thermoelastic diffusion, where the bounding surface of the half-space was traction-free and subject to a time-dependent thermal shock. Ram et al.[14] studied the thermomechanical response of the generalized thermo-diffusion elastic theory under one relaxation time due to the time harmonic sources. Bayones[15] studied the viscosity and diffusion effects on the generalized magneto-thermoelastic interactions in an isotropic spherical cavity. Abo-Dahab and Singh[16] studied the interaction between the magnetic field and the elastic field in a solid half-space under thermoelastic diffusion. Xia et al.[17] studied the dynamic response of an infinite elastic body with a cylindrical cavity, and showed the effects of diffusion on the radical displacement, the axial displacement, the radical stress, and the axial stress distribution in an elastic body. Allam et al.[18] studied the interactions among the magnetic field, the electric field, and the initial stress in an elastic solid half-space under thermoelastic diffusion with the Green-Lindsay (GL) model. Abouelregal and Abo-Dahab[19] studied the electro-magneto-thermoelastic problem along an infinite solid cylinder with the dual-phase-lag (DPL) diffusion model considering the modified Ohm's law and the generalized Fourier's law with the consideration of the interactions between the deformation and the magnetic field vector. Abo-Dahab[20] illustrated the effects of the anisotropy, the magnetic field, the gravity field, the non-homogeneity of the medium, the initial stress, the rotation, the incidence direction, and the depth in the phase velocity and the attenuation coefficient of the propagation of S-waves. Kumar and Kumar[21] illustrated the wave propagation and the fundamental solution of thermoelastic diffusion. For more details about the wave propagation considering the interactions among thermal, diffusion, stress, and volume fraction, one can refer to Refs. [22]–[24]. Kumar and Gupta[25–26] represented a generalized form for the classical Fick's diffusion law, and discussed the reflection and transmission of an obliquely incident elastic wave at the interface between an inviscid fluid half-space and a thermoelastic diffusion solid half-space with phase-lag models. Sur and Kanoria[27] studied the interactions among the elastic field, the thermal field, and the diffusion in a homogeneous and isotropic half-space with the three-phase-lag model and the Green-Naghdi models II and III of generalized thermoelasticity. Othman and Abd-Elaziz[28] studied the effects of thermal loading due to the laser pulse in a generalized thermoelastic homogeneous isotropic elastic half-space heated by a non-Gaussian laser beam with voids by using the DPL model. Kumar et al.[29] depicted the effects of thermal and diffusion with two phase lags because of axisymmetric heat supply for a disc by using the DPL model and the DPL diffusion model. The upper and lower surfaces of the disc were traction-free, and were subject to an axisymmetric heat supply. Abouelregal[30] proposed a DPL thermoelastic model for a semi-infinite homogeneous isotropic medium at an exponential heating situation. Kumar and Kansal[31] studied the propagation of longitudinal waves, and obtained the fundamental solution as a function of diffusion and voids. Xiong and Tian[32] studied the transient thermoelastic responses in a fiber-reinforced anisotropic thermoelastic half-space with the consideration of the generalized thermoelasticity without energy dissipation and the effect of a thermal shock. Xiong and Tian[33] showed that the thermal shock and rotation had no effects on the temperature and mass but significant effects on the transient magneto-thermo-elasto-diffusion of rotating porous media without energy dissipation. Xiong et al.[34] studied the reinforcement half-space with diffusion by using the L-S model, the Green-Lord model, and the Green-Naghdi models of types II and III in combination with the DPL model. Abo-Dahab[35] and Abo-Dahab et al.[36] studied the gravity field, the magnetic field, and the rotation of the primary wave.

In this paper, the effects of voids, electromagnetic field, gravity field, rotation, and the initial stress on the diffusion in the generalized thermoplastic half-space are studied by using the L-S and DPL models. Numerical calculations are performed, and the resulting quantities are displayed graphically. The results obtained with different values of the external parameters are compared, and significant effects are observed.

2 Formulation of the problem

Let us consider a homogeneous generalized thermoelastic half-space rotating with an angular velocity vector $\boldsymbol{\Omega} = \Omega\,\boldsymbol{n}$, where $\boldsymbol{n}$ is the unit vector and represents the direction of the axis of rotation. The rectangular Cartesian coordinate system (x, y, z) is adopted, where the y-axis vertically points into the medium (see Fig. 1). The displacement equation of motion with rotation has two additional terms, i.e., the centripetal force with the angular velocity $\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u})$ and Corioli's acceleration $2\boldsymbol{\Omega}\times\dot{\boldsymbol{u}}$, where $\boldsymbol{u} = (u,\, 0,\, w)$ is the dynamic displacement vector and $\boldsymbol{\Omega} = (0,\, \Omega,\, 0)$ is the angular velocity. The normal origin is located at the plane surface of the generalized thermoelastic half-space.

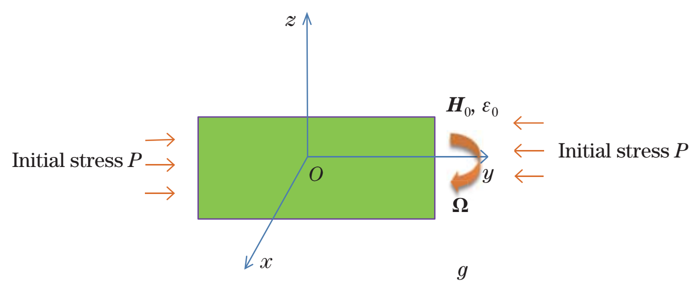

Fig. 1 Formulation of the problem

3 Basic equations

The governing equations for a homogeneous generalized thermoelastic half-space with the diffusion flux, voids, and Lorentz's body forces at the reference temperature T_0 are as follows:

$$\sigma_{ij} = (\lambda e - \gamma T - P + b\Phi_{\rm v} - \beta_1 C)\delta_{ij} + 2\mu e_{ij} - P\omega_{ij}, \tag{1}$$

$$p_{\rm c} = -\beta_1 e + b_{\rm c}C - a_{\rm c}T - b_2^*\Phi_{\rm v}, \tag{2}$$

$$\rho\eta = \gamma e + \alpha T + m\Phi_{\rm v} + a_{\rm c}C, \tag{3}$$

$$g^* = -be - \xi\,\Phi_{\rm v} + mT - \omega_0\dot{\Phi}_{\rm v} + b_2^*C, \tag{4}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{5}$$

$$\omega_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j}), \tag{6}$$

$$S_i = \alpha\Phi_{{\rm v},i}. \tag{7}$$

The Maxwell's equation of electromagnetism is

$$\lambda_{ij} = \mu_r(H_i h_j + H_j h_i - (H_{\Re}h_{\Re})\,\delta_{ij}). \tag{8}$$

The equation of motion is

$$\sigma_{ij,j} + \boldsymbol{F}_i = \rho(\ddot{\boldsymbol{u}}_i + (\boldsymbol{\Omega}\times\boldsymbol{\Omega}\times\boldsymbol{u})_i + (2\boldsymbol{\Omega}\times\dot{\boldsymbol{u}})_i), \tag{9}$$

which tends to

$$\Big(\mu - \frac{P}{2}\Big)u_{i,jj} + \Big(\lambda + \mu + \frac{P}{2}\Big)u_{j,ij} - \gamma T_{,i} + b\Phi_{{\rm v},i} - \beta_1 C_{,i} + F_i + G_i$$
$$= \rho(\ddot{\boldsymbol{u}}_i + (\boldsymbol{\Omega}\times\boldsymbol{\Omega}\times\boldsymbol{u})_i + (2\boldsymbol{\Omega}\times\dot{\boldsymbol{u}})_i), \tag{10}$$

where

$$\boldsymbol{F}_i = (\boldsymbol{J} \times \boldsymbol{B})_i, \quad \boldsymbol{G} = \rho g\Big(\frac{\partial w}{\partial x}, 0, -\frac{\partial u}{\partial x}\Big). \tag{11}$$

The Maxwell's equations of the variance magnetic field and the electric field are

$$\begin{cases} \operatorname{curl} \boldsymbol{h} = \boldsymbol{J} + \varepsilon_0 \dfrac{\partial \boldsymbol{E}}{\partial t}, \quad -\mu_r \dfrac{\partial \boldsymbol{h}}{\partial t} = \operatorname{curl} \boldsymbol{E}, \quad \operatorname{div} \boldsymbol{h} = 0, \quad \operatorname{div} \boldsymbol{E} = 0, \\ \boldsymbol{E} = -\mu_r\Big(\dfrac{\partial \boldsymbol{u}}{\partial t} \times \boldsymbol{H}_0\Big), \quad \boldsymbol{h} = \operatorname{curl}(\boldsymbol{u} \times \boldsymbol{H}_0), \quad \boldsymbol{F}_i = \mu_r(\boldsymbol{J} \times \boldsymbol{H}_0)_i, \end{cases} \tag{12}$$

where

$$\boldsymbol{H} = \boldsymbol{H}_0 + \boldsymbol{h}(x, z, t), \quad \boldsymbol{H}_0 = (0, H_0, 0). \tag{13}$$

With Eq. (12), we have

$$F_x = \mu_r H_0^2\Big(\frac{\partial\, e}{\partial x} - \varepsilon_0 \mu_r \frac{\partial^2 u}{\partial t^2}\Big), \tag{14}$$

$$F_z = \mu_r H_0^2\Big(\frac{\partial\, e}{\partial z} - \varepsilon_0 \mu_r \frac{\partial^2 w}{\partial t^2}\Big), \tag{15}$$

$$F_y = 0. \tag{16}$$

The heat conduction equation of the DPL model is

$$K\Big(1 + \tau_\Theta \frac{\partial}{\partial t}\Big)T_{,ii} = \Big(1 + \tau_1 \frac{\partial}{\partial t}\Big)\Big(\rho C_{\mathrm{E}} \frac{\partial T}{\partial t} + \gamma T_0 \frac{\partial e}{\partial t} + a_{\mathrm{c}} T_0 \frac{\partial C}{\partial t} + m T_0 \frac{\partial \Phi_{\mathrm{v}}}{\partial t}\Big). \tag{17}$$

If $\tau_\Theta = 0$ and $\tau_1 = \tau$ (the first relaxation time), Eq. (17) leads to the L-S model, where $0 \leqslant \tau_\Theta < \tau_1$.

The equation of voids is

$$\alpha \Phi_{\mathrm{v},ii} - b u_{i,\,i} - \zeta \Phi_{\mathrm{v}} - \omega_0 \dot{\Phi}_{\mathrm{v}} + mT + b_2^* C = \rho \chi \ddot{\Phi}_{\mathrm{v}}. \tag{18}$$

The equation of diffusion is

$$\Big(1 + \tau_2 \frac{\partial}{\partial t}\Big)(d\beta_1 e_{,ii} - d b_{\mathrm{c}} C_{,ii} + d a_{\mathrm{c}} T_{,ii} + d b_2^* \Phi_{v,ii}) + \Big(1 + \tau_\eta \frac{\partial}{\partial t}\Big)\dot{C} = 0. \tag{19}$$

When $\tau_2 = 0$, the L-S model holds.

4 Solution of the problem

From Eqs. (10) and (17)–(19), we have

$$\begin{aligned} &\Big(\mu - \frac{P}{2}\Big)\nabla^2 u + \Big(\lambda + \mu + \frac{P}{2}\Big)\frac{\partial e}{\partial x} - \gamma \frac{\partial T}{\partial x} + b \frac{\partial \Phi_{\mathrm{v}}}{\partial x} - \beta_1 \frac{\partial C}{\partial x} + F_x + \rho g \frac{\partial w}{\partial x} \\ &= \rho\Big(\frac{\partial^2 u}{\partial t^2} + 2\Omega \frac{\partial w}{\partial t} - \Omega^2 u\Big), \end{aligned} \tag{20}$$

$$\Big(\mu - \frac{P}{2}\Big)\nabla^2 w + \Big(\lambda + \mu + \frac{P}{2}\Big)\frac{\partial e}{\partial z} - \gamma\frac{\partial T}{\partial z} + b\frac{\partial \Phi_{\rm v}}{\partial z} - \beta_1\frac{\partial C}{\partial z} + F_z - \rho g\frac{\partial u}{\partial x}$$
$$= \rho\Big(\frac{\partial^2 w}{\partial t^2} - 2\Omega\frac{\partial u}{\partial t} - \Omega^2 w\Big), \tag{21}$$
$$K\Big(1 + \tau_\Theta\frac{\partial}{\partial t}\Big)T_{,ii} = \Big(1 + \tau_1\frac{\partial}{\partial t}\Big)\Big(\rho\, C_{\rm E}\frac{\partial T}{\partial t} + \gamma T_0\frac{\partial e}{\partial t} + a_{\rm c}T_0\frac{\partial C}{\partial t} + mT_0\frac{\partial \Phi_{\rm v}}{\partial t}\Big), \tag{22}$$
$$\alpha\Big(\frac{\partial^2 \Phi_{\rm v}}{\partial x^2} + \frac{\partial^2 \Phi_{\rm v}}{\partial z^2}\Big) - b\,\Big(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\Big) - \zeta\Phi_{\rm v} - \omega_0\frac{\partial \Phi_{\rm v}}{\partial t} + mT + b_2^* C = \rho\chi\frac{\partial^2 \Phi_{\rm v}}{\partial t^2}, \tag{23}$$
$$\Big(1 + \tau_2\frac{\partial}{\partial t}\Big)\Big(d\beta_1\Big(\frac{\partial^2 e}{\partial x^2} + \frac{\partial^2 e}{\partial z^2}\Big) - db_{\rm c}\Big(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial z^2}\Big) + da_{\rm c}\Big(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\Big)$$
$$+ db_2^*\Big(\frac{\partial^2 \Phi_{\rm v}}{\partial x^2} + \frac{\partial^2 \Phi_{\rm v}}{\partial z^2}\Big)\Big) + \Big(1 + \tau_\eta\frac{\partial}{\partial t}\Big)\dot{C} = 0. \tag{24}$$

The constitutive relations are

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda\frac{\partial w}{\partial z} - \gamma T - P + b\Phi_{\rm v} - \beta_1 C, \tag{25}$$
$$\sigma_{yy} = \lambda e - \gamma T - P + b\Phi_{\rm v} - \beta_1 C, \tag{26}$$
$$\sigma_{zz} = (\lambda + 2\mu)\frac{\partial w}{\partial z} + \lambda\frac{\partial u}{\partial x} - \gamma T - P + b\Phi_{\rm v} - \beta_1 C, \tag{27}$$
$$\sigma_{xz} = \mu\Big(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\Big) - \frac{P}{2}\Big(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\Big), \tag{28}$$
$$\sigma_{zx} = \mu\Big(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\Big) - \frac{P}{2}\Big(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\Big), \tag{29}$$
$$\sigma_{xy} = \sigma_{yz} = 0. \tag{30}$$

For assembly, we shall use the following non-dimensional parameters:

$$\begin{cases} x_i' = \dfrac{\omega^*}{c_0}x_i, \quad u_i' = \dfrac{\rho c_0\omega^*}{\gamma T_0}u_i, \quad \Omega' = \dfrac{\Omega}{\omega^*}, \quad \theta = \dfrac{T}{T_0}, \\ \sigma_{ij}' = \dfrac{\sigma_{ij}}{\gamma T_0}, \quad \Phi_{\rm v}' = \dfrac{\chi}{\gamma T_0}\Phi_{\rm v}, \quad C' = \dfrac{\beta_1}{\gamma T_0}C, \quad g' = \dfrac{g}{c_0\omega^*}, \\ (t', \tau', \tau_1', \tau_\Theta', \tau_2', \tau_\eta') = \omega^*(t, \tau, \tau_1, \tau_\Theta, \tau_2, \tau_\eta), \quad b^* = \dfrac{b}{\chi}, \quad \lambda_{ij}' = \dfrac{\lambda_{ij}}{\gamma T_0}. \end{cases} \tag{31}$$

In terms of the non-dimensional quantities defined in Eq. (31), we have

$$\frac{2\mu - P}{2\rho c_0^2}\nabla^2 u + \Big(\frac{2\lambda + 2\mu + P}{2\rho c_0^2} + R_H\Big)\frac{\partial e}{\partial x} - \frac{\partial \theta}{\partial x} + b^*\frac{\partial \Phi_{\rm v}}{\partial x} - \frac{\partial C}{\partial x} + g\frac{\partial w}{\partial x}$$
$$= \Big(\beta^2\frac{\partial^2 u}{\partial t^2} + 2\Omega\frac{\partial w}{\partial t} - \Omega^2 u\Big), \tag{32}$$
$$\Big(\frac{2\mu - P}{2\rho c_0^2}\Big)\nabla^2 w + \Big(\frac{2\lambda + 2\mu + P}{2\rho c_0^2} + R_H\Big)\frac{\partial e}{\partial z} - \frac{\partial \theta}{\partial z} + b^*\frac{\partial \Phi_{\rm v}}{\partial z} - \frac{\partial C}{\partial\, z} - g\frac{\partial u}{\partial x}$$
$$= \Big(\beta^2\frac{\partial^2 w}{\partial t^2} - 2\Omega\frac{\partial u}{\partial t} - \Omega^2 w\Big), \tag{33}$$
$$\Big(1 + \tau_\Theta\frac{\partial}{\partial t}\Big)\nabla^2\theta = \Big(1 + \tau_1\frac{\partial}{\partial t}\Big)(\dot{\theta} + \zeta_1\dot{e} + \zeta_2\dot{\Phi}_{\rm v} + \zeta_3\dot{C}), \tag{34}$$
$$\nabla^2\Phi_{\rm v} - a_1\Big(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\Big) - a_2\Phi_{\rm v} - a_3\frac{\partial \Phi_{\rm v}}{\partial t} + a_4\theta + a_4'' C = a_5\frac{\partial^2 \Phi_{\rm v}}{\partial t^2}, \tag{35}$$
$$\Big(1 + \tau_2\frac{\partial}{\partial t}\Big)(\nabla^2 e + a_6\nabla^2\theta - a_8\nabla^2 C + a_9\nabla^2\Phi_{\rm v}) + a_7(\dot{C} + \tau_\eta\ddot{C}) = 0, \tag{36}$$

where

$$a_1=\frac{b\chi}{\rho\alpha\omega^{*2}},\quad a_2=\frac{\zeta c_0^2}{\alpha\omega^{*2}},\quad a_3=\frac{\omega_0 c_0^2}{\alpha\omega^{*}},\quad a_4=\frac{mc_0^2\chi}{\gamma\alpha\omega^{*2}},\quad a_4''=\frac{b_2^* c_0^2\chi}{\beta_1\alpha\omega^{*2}},$$
$$a_5=\frac{\rho c_0^2\chi}{\alpha},\quad a_6=\frac{a_{\mathrm{c}}\rho c_0^2}{\beta_1\gamma},\quad a_7=\frac{c_0^2 K}{d\beta_1^2 C_{\mathrm{E}}},\quad a_8=\frac{b_{\mathrm{c}}\rho c_0^2}{\beta_1^2},\quad a_9=\frac{b_2^*\rho c_0^2}{\chi\beta_1},$$
$$\beta^2=1+\frac{\varepsilon_0\mu_r^2H_0^2}{\rho},\quad \zeta_1=\frac{\gamma^2T_0}{\rho K\omega^*},\quad \zeta_2=\frac{mT_0\gamma}{\rho C_{\mathrm{E}}\chi},\quad \zeta_3=\frac{a_{\mathrm{c}}T_0\gamma}{\rho C_{\mathrm{E}}\beta_1}.$$

With the Helmholtz representation, we write the displacement vector $\boldsymbol{u}$ as follows:

$$\boldsymbol{u}=\mathrm{grad}\,\boldsymbol{\Phi}+\mathrm{curl}\,\boldsymbol{\Psi},\quad \boldsymbol{\Psi}=(0,\Psi,0),\tag{37}$$

where $\Phi(x,z,t)$ and $\Psi(x,z,t)$ are the displacement potentials. This reduces to

$$(u,w)=\Big(\frac{\partial\Phi}{\partial x}+\frac{\partial\Psi}{\partial z},\frac{\partial\Phi}{\partial z}-\frac{\partial\Psi}{\partial x}\Big),\quad \frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial z^2}=\nabla^2\Phi,\quad \frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}=\nabla^2\Psi.\tag{38}$$

Substituting Eq. (38) into Eqs. (32)–(36) yields

$$\Big(a_{11}\nabla^2-\beta^2\frac{\partial^2}{\partial t^2}+\Omega^2\Big)\Phi-\Big(g\frac{\partial}{\partial x}-2\Omega\frac{\partial}{\partial t}\Big)\Psi-\theta+b^*\Phi_{\mathrm{v}}-C=0,\tag{39}$$

$$\Big(g\frac{\partial}{\partial x}-2\Omega\frac{\partial}{\partial t}\Big)\Phi+\Big(a_{12}\nabla^2-\beta^2\frac{\partial^2}{\partial t^2}+\Omega^2\Big)\Psi=0,\tag{40}$$

$$\Big(1+\tau_\Theta\frac{\partial}{\partial t}\Big)\nabla^2\theta=\Big(1+\tau_1\frac{\partial}{\partial t}\Big)\Big(\dot\theta+\zeta_1\nabla^2\frac{\partial\Phi}{\partial t}+\zeta_2\frac{\partial\Phi_{\mathrm{v}}}{\partial t}+\zeta_3\,\dot C\Big),\tag{41}$$

$$\Big(\nabla^2-a_2-a_3\frac{\partial}{\partial t}-a_5\frac{\partial^2}{\partial t^2}\Big)\Phi_{\mathrm{v}}-a_1\nabla^2\Phi+a_4\theta+a_4''C=0,\tag{42}$$

$$\Big(1+\tau_2\frac{\partial}{\partial t}\Big)(\nabla^2(\nabla^2\Phi)+a_6\nabla^2\theta-a_8\nabla^2C+a_9\nabla^2\Phi_{\mathrm{v}})+\Big(a_7\frac{\partial}{\partial t}+a_7\tau_\eta\frac{\partial^2}{\partial t^2}\Big)C=0.\tag{43}$$

The constitutive relations are

$$\sigma_{xx}=b_0u_{,x}+b_1w_{,z}-\theta-\frac{P}{\gamma T_0}+b^*\Phi_{\mathrm{v}}-C,\tag{44}$$

$$\sigma_{yy}=b_1\nabla^2\Phi-\theta-\frac{P}{\gamma T_0}+b^*\Phi_{\mathrm{v}}-C,\tag{45}$$

$$\sigma_{zz}=b_0w_{,z}+b_1u_{,x}-\theta-\frac{P}{\gamma T_0}+b^*\Phi_{\mathrm{v}}-C,\tag{46}$$

$$\sigma_{xz}=b_2(u_{,z}+w_{,x})-b_3(w_{,x}-u_{,z}),\tag{47}$$

$$\sigma_{zx}=b_2(u_{,z}+w_{,x})-b_3(u_{,z}-w_{,x}),\tag{48}$$

$$\sigma_{xy}=\sigma_{yz}=0,\tag{49}$$

where

$$(b_0,b_1,b_2,b_3)=\frac{1}{\rho c_0^2}\Big(\lambda+2\mu,\lambda,\mu,\frac{P}{2}\Big),\quad a_{11}=\frac{\lambda+2\mu}{\rho\,c_0^2}+R_H,\quad a_{12}=\frac{2\mu-P}{2\rho c_0^2}.$$

5 Normal mode analysis

The solution of the considered physical variable decomposed in terms of the normal modes has the following form:

$$\begin{aligned}&(u,\,w,\,e,\,\theta,\,\Phi,\,\Psi,\,h,\,E,\,\sigma_{ij},\,\Phi_{\mathrm{v}},\,C)(x,\,z,\,t)\\&=(u^*,\,w^*,\,e^*,\,\theta^*,\,\Phi^*,\,\Psi^*,\,h^*,\,E^*,\,\sigma_{ij}^*,\,\Phi_{\mathrm{v}}^*,\,C^*)(z)\mathrm{e}^{\omega t+\mathrm{i}ax},\end{aligned}\tag{50}$$

where ω and a are in the x-direction, and $u^*(z)$, $w^*(z)$, $e^*(z)$, $\theta^*(z)$, $\Phi^*(z)$, $\Psi^*(z)$, $h^*(z)$, $E^*(z)$, $\sigma^*_{ij}(z)$, $\Phi^*_{\mathrm{v}}(z)$, and $C^*(z)$ are the maximum field quantities.

Substituting Eq. (50) into Eqs. (39)–(49) yields

$$(a_{11}D^2-\Lambda_1)\Phi^*-\Lambda_2\Psi^*-\theta^*+b^*\Phi^*_{\mathrm{v}}-C^*=0, \tag{51}$$

$$\Lambda_2\Phi^*+(a_{12}D^2-\Lambda_3)\Psi^*=0, \tag{52}$$

$$-\Lambda_5(D^2-a^2)\Phi^*+(D^2-\Lambda_4)\theta^*+\Lambda_6\Phi^*_{\mathrm{v}}+\Lambda_8C^*=0, \tag{53}$$

$$(D^2-\Lambda_7)\Phi^*_{\mathrm{v}}+(-a_1D^2+a_1a^2)\Phi^*+a_4\theta^*+a_4''C^*=0, \tag{54}$$

$$(D^4-2a^2D^2+a^4)\Phi^*+a_6(D^2-a^2)\theta^*+(\Lambda_9-a_8D^2)C^*+a_9(D^2-a^2)\Phi^*_{\mathrm{v}}=0, \tag{55}$$

$$\sigma^*_{xx}=\mathrm{i}ab_0u^*+b_1Dw^*-\theta^*-\frac{P}{\gamma T_0}+b^*\Phi^*_{\mathrm{v}}-C^*, \tag{56}$$

$$\sigma^*_{yy}=b_1(D^2-a^2)\Phi^*-\theta^*-\frac{P}{\gamma T_0}+b^*\Phi^*_{\mathrm{v}}-C^*, \tag{57}$$

$$\sigma^*_{zz}=b_0Dw^*+\mathrm{i}ab_1u^*-\theta^*-\frac{P}{\gamma T_0}+b^*\Phi^*_{\mathrm{v}}-C^*, \tag{58}$$

$$\sigma^*_{xz}=(b_2+b_3)Du^*+(b_2-b_3)\mathrm{i}aw^*, \tag{59}$$

$$\sigma^*_{zx}=(b_2-b_3)Du^*+(b_2+b_3)\mathrm{i}aw^*, \tag{60}$$

$$\sigma^*_{xy}=\sigma^*_{yz}=0, \tag{61}$$

where

$$\Lambda_1=a_{11}a^2+\beta^2\omega^2-\Omega^2,\quad \Lambda_2=\mathrm{i}ag-2\Omega\omega,\quad \Lambda_3=a_{12}a^2+\beta^2\omega^2-\Omega^2,$$
$$\Lambda_4=a^2+\frac{\omega\omega_2}{\omega_1},\quad \Lambda_5=\frac{\zeta_1\omega\omega_2}{\omega_1},\quad \Lambda_6=-\frac{\zeta_2\omega\omega_2}{\omega_1},\quad \Lambda_7=a^2+a_2+a_3\omega+a_5\omega^2,$$
$$\Lambda_8=-\frac{\zeta_3\omega\omega_2}{\omega_1},\quad \Lambda_9=a_7\frac{\omega\omega_2^*}{\omega_1^*}+a_8a^2,\quad \omega_1=1+\tau_\Theta\omega,$$
$$\omega_2=1+\tau_1\omega,\quad \omega_1^*=1+\tau_2\omega,\quad \omega_2^*=1+\tau_\eta\omega.$$

Eliminating $\Psi^*(z)$, $\Phi^*_{\mathrm{v}}(z)$, $C^*(z)$, and $\theta^*(z)$ in Eqs. (51)–(55), we get the differential equation for $\Phi^*(z)$ as follows:

$$(D^{10}-AD^8+BD^6-CD^4+ED^2-L)(\Phi^*(z))=0. \tag{62}$$

Similarly, we have

$$(D^{10}-AD^8+BD^6-CD^4+ED^2-L)(\Psi^*(z),\theta^*(z),\Phi^*_{\mathrm{v}}(z),C^*(z))=0, \tag{63}$$

where

$$A=-\frac{1}{F}(a_{12}(r_4-r_1r_2-\Lambda_8r_3+\Lambda_5r_6+r_5')-\Lambda_3r_1),$$
$$B=\frac{1}{F}(a_{12}(L_1+L_2)+a_{12}L_{12}+L_{13}-r_7+\Lambda_3(r_1r_2+\Lambda_8r_3-r_4+\Lambda_5r_6)),$$
$$C=-\frac{1}{F}(a^2(a^2r_{10}+L_{10})+L_{11}+r_{12}r_2+\Lambda_3L_{19}+r_8r_{11}+\Lambda_3\Lambda_7(\Lambda_5r_6-\Lambda_8r_3-\Lambda_4r_1)$$
$$+r_5'(1+2a^2a_{12}\Lambda_4)+a_4''b^*L_7+L_{14}),$$
$$E=\frac{1}{F}(\Lambda_3L_{20}+L_7(\Lambda_7r_9-L_{17})+\Lambda_1\Lambda_9L_9-r_8(L_{15}+a^2a_9a_4''-L_3)$$
$$+a^2(L_{15}(a_{12}(\Lambda_1-a_{12}L_9)+a_{11}\Lambda_3+L_4-a^2L_{18}))),$$
$$L=-\frac{1}{F}(a^4\Lambda_3(L_9+r_{13}+L_{16})+a^2(\Lambda_3r_{14}-a_6\Lambda_2^2\Lambda_7\Lambda_8-r_8L_{15})-\Lambda_9r_8L_9).$$

Since

$$F = a_{12}r_1, \quad r_1 = 1 - a_8a_{11}, \quad r_2 = \Lambda_4 + \Lambda_7, \quad r_3 = 1 + a_6a_{11}, \quad r_6 = a_6 + a_8,$$
$$r'_5 = a_1a_9 + a''_4b^* - a_9a_{11}a''_4, \quad r_7 = a_8\Lambda_2^2, \quad r_4 = a_{11}\Lambda_9 - a_1a_8b^* - 2a^2 + a_8\Lambda_1,$$
$$r_5 = a_1(\Lambda_9 + a_8\Lambda_4 - a_6\Lambda_8 + a^2a_8) - a_4\Lambda_8 + a_4a_8\Lambda_5, \quad L_9 = a_4\Lambda_6 - \Lambda_4\Lambda_7,$$
$$L_1 = -r_1L_9 + b^*r_5 + \Lambda_7\Lambda_8r_3 + r_2(2a^2 - a_8\Lambda_1 - a_{11}\Lambda_9), \quad r_8 = \Lambda_1\Lambda_3 + \Lambda_2^2,$$
$$L_2 = a^4 + a^2\Lambda_8(1 + r_3) + a_6\Lambda_1\Lambda_8 - r_6(\Lambda_5\Lambda_7 + a^2\Lambda_5 + a_1\Lambda_6) - a^2a_6\Lambda_5 - \Lambda_9(\Lambda_1 + \Lambda_5),$$
$$r_9 = \Lambda_4 + \Lambda_8, \quad L_7 = a^4a_{12} + 2a^2\Lambda_3, \quad L_3 = a_8L_9 - \Lambda_4\Lambda_9 + a_6\Lambda_7\Lambda_8 + a^2a_6\Lambda_8,$$
$$r_{10} = \Lambda_3 + a_{12}(r_2 + \Lambda_8 - a_6\Lambda_5),$$
$$L_5 = a_4a_{11}\Lambda_6 + a_1\Lambda_4b^* + a_4\Lambda_5b^* - \Lambda_1\Lambda_7 - a_1\Lambda_6 - a_{11}\Lambda_4\Lambda_7 - a^2\Lambda_5,$$
$$r_{14} = a_{12}\Lambda_5\Lambda_7 + \Lambda_3\Lambda_5 + a_1a_{12}\Lambda_6,$$
$$L_8 = a_4\Lambda_6 + a_4\Lambda_8b^* + a_6(\Lambda_5\Lambda_7 + a_1\Lambda_6 + a_1\Lambda_8b^*),$$
$$r_{11} = \Lambda_9 + a_8r_2 - a_6\Lambda_8, \quad r_{12} = \Lambda_9(a_{11}\Lambda_3 + a_{12}\Lambda_1),$$
$$L_4 = a_6a_{11}\Lambda_4\Lambda_7\Lambda_8 - a_{12}(\Lambda_9(\Lambda_5\Lambda_7 - a_1\Lambda_4b^* - a_4\Lambda_5b^* + \Lambda_6) - a_6\Lambda_1\Lambda_7\Lambda_8)$$
$$\quad + b^*\Lambda_3(a_1(\Lambda_9 + a_8\Lambda_4 - 2a_6\Lambda_8) + a_4a_8\Lambda_5),$$
$$L_6 = a^4\Lambda_8 - a^2(2a_4\Lambda_6 - a^2a_6\Lambda_5 - 2a_1a_6\Lambda_6 - a_1a_8\Lambda_6 - a_8\Lambda_5\Lambda_7 - 2a_6\Lambda_5\Lambda_7 - 2a_4\Lambda_8b^*),$$
$$L_{10} = a_{12}r_{13} + (a_6 + r_6)r_{14} - (1 + r_3)(\Lambda_3\Lambda_8 + a_{12}\Lambda_7\Lambda_8) - 2\Lambda_3\Lambda_7,$$
$$L_{11} = a_{12}(\Lambda_9(a_{11}\Lambda_4\Lambda_7 - a_4\Lambda_3b^* - a_1\Lambda_4b^* - a_4a_{11}\Lambda_6 + \Lambda_5\Lambda_7 + a_1\Lambda_6)$$
$$\quad - \Lambda_1(a_8L_9 + a_6\Lambda_7\Lambda_8)) + a_1\Lambda_3(a_8\Lambda_7 + a_6\Lambda_6),$$
$$r_{13} = 2L_9 + b^*(2a_4\Lambda_8 - a_1a_8\Lambda_4 - a_4a_8\Lambda_5 + 2a_1a_4\Lambda_8 - a_1\Lambda_9) + \Lambda_5\Lambda_9 - a_6\Lambda_1\Lambda_8,$$
$$L_{12} = -a_9(a_1\Lambda_8 + a_4\Lambda_5 + 2a^2a_1) + a''_4(a_9(\Lambda_1 + a\Lambda_5 + a_{11}\Lambda_8 + a^2a_{11}) + a_6\Lambda_5b^* + r_3\Lambda_6),$$
$$L_{13} = -2a^2a_{11}a''_4b^* - r'_5(\Lambda_3 + a_{11}\Lambda_4),$$
$$L_{14} = a^2(a_9(a''_4a_{11}\Lambda_4 - a_{12}(2a''_4\Lambda_5 + a''_4\Lambda_1 + a_4a_{11}\Lambda_8 - 2a_1\Lambda_8 - 2a_4\Lambda_5 - a^2a_1)$$
$$\quad - a_6a''_4a_{12}(a_{11}\Lambda_6 + 2\Lambda_5b^*) - 2a''_4a_{12}\Lambda_6)$$
$$\quad - a_9(a''_4\Lambda_2^2 + a_4a_{12}\Lambda_1\Lambda_8 + a''_4a_{12}\Lambda_1\Lambda_5) - a_6a''_4a_{12}\Lambda_1\Lambda_6,$$
$$L_{15} = a_4a_9\Lambda_8 + a_6a''_4\Lambda_6 + a_9a''_4\Lambda_4,$$
$$L_{16} = a_1a_9r_9\Lambda_3 + a_4a_9\Lambda_3\Lambda_5 - a''_4\Lambda_3\Lambda_6 + a''_4\Lambda_3\Lambda_4b^* - a_9a''_4\Lambda_3\Lambda_5 - a_6a''_4\Lambda_3\Lambda_5b^*,$$
$$L_{17} = a_1a_9\Lambda_8 - a''_4\Lambda_6 + a_4a_9\Lambda_5 - a_9a''_4\Lambda_5 - L_{15},$$
$$L_{18} = a_1a_9\Lambda_3 + a''_4\Lambda_3b^* + a_1a_9a_{12}\Lambda_4 + a_{12}a''_4\Lambda_4b^*, \quad L_{19} = a_4r_1\Lambda_6 - r_5b^* - L_{12},$$
$$L_{20} = a^4r_2 + \Lambda_9L_5 + L_6 - 2\Lambda_4r'_5,$$

Eq. (62) can be factorized as follows:

$$(D^2 - \Upsilon_1^2)(D^2 - \Upsilon_2^2)(D^2 - \Upsilon_3^2)(D^2 - \Upsilon_4^2)(D^2 - \Upsilon_5^2)\Phi^*(z) = 0, \tag{64}$$

where Υ_j^2 $(j = 1, 2, 3, 4, 5)$ are the roots of the characteristic equation (64).

The solution of Eq. (64) when $z \to \infty$ is

$$\Phi^*(z) = \sum_{j=1}^{5} R_j \mathrm{e}^{-\Upsilon_j z}, \tag{65}$$

$$\Psi^*(z) = \sum_{j=1}^{5} R_j H_{1j} \mathrm{e}^{-\Upsilon_j z}, \tag{66}$$

$$\Phi_{\mathrm{v}}^*(z) = \sum_{j=1}^{5} R_j H_{2j} \mathrm{e}^{-\Upsilon_j z}, \tag{67}$$

$$C^*(z) = \sum_{j=1}^{5} R_j H_{3j} \mathrm{e}^{-\Upsilon_j z}, \tag{68}$$

$$\theta^*(z) = \sum_{j=1}^{5} R_j H_{4j} \mathrm{e}^{-\Upsilon_j z}, \tag{69}$$

$$u^*(z) = \sum_{j=1}^{5} M_{1j} R_j \mathrm{e}^{-\Upsilon_j z}, \tag{70}$$

$$w^*(z) = \sum_{j=1}^{5} M_{2j} R_j \mathrm{e}^{-\Upsilon_j z}, \tag{71}$$

$$\sigma_{xx}^* = \sum_{j=1}^{5} M_{3j} R_j \mathrm{e}^{-\Upsilon_j z}, \tag{72}$$

$$\sigma_{yy}^* = \sum_{j=1}^{5} M_{4j} R_j \mathrm{e}^{-\Upsilon_j z}, \tag{73}$$

$$\sigma_{zz}^* = \sum_{j=1}^{5} M_{5j} R_j \mathrm{e}^{-\Upsilon_j z}, \tag{74}$$

$$\sigma_{xz}^* = -\sum_{j=1}^{5} M_{6j} R_j \mathrm{e}^{-\Upsilon_j z}, \tag{75}$$

where

$$H_{1j} = \frac{\Lambda_2}{\Lambda_3 - a_{12} k_j^2},$$

$$H_{2j} = \frac{(\Gamma_2\Gamma_4 - \Lambda_5\Gamma_1)(a_4'' a_6 \Gamma_1 - a_4\Gamma_6) + (a_1 a_6 \Gamma_1^2 + a_4\Gamma_7)(\Lambda_8 - \Gamma_2) - \Lambda_2\Gamma_2(a_4'' a_6\Gamma_1 - a_4\Gamma_6)H_{1j}}{(\Lambda_8 - \Gamma_2)(a_6\Gamma_1\Gamma_3 - a_4 a_9\Gamma_1) - (a_4'' a_6\Gamma_1 - a_4\Gamma_6)(b^*\Gamma_2 + \Lambda_6)},$$

$$H_{3j} = \frac{(a_6\Gamma_1\Gamma_3 - a_4 a_9\Gamma_1)(\Lambda_2\Gamma_2 H_{1j} + (\Gamma_2\Gamma_4 - \Lambda_5\Gamma_1)) + (a_1 a_6\Gamma_1^2 + a_4\Gamma_7)(b^*\Gamma_2 + \Lambda_6)}{(a_4'' a_6\Gamma_1 - a_4\Gamma_6)(b^*\Gamma_2 + \Lambda_6) - (\Lambda_8 - \Gamma_2)(a_6\Gamma_1\Gamma_3 - a_4 a_9\Gamma_1)},$$

$$H_{4j} = -\frac{\Gamma_7 + \Gamma_6 H_{3j} + a_9\Gamma_1 H_{2j}}{a_6\Gamma_1}, \quad M_{1j} = \mathrm{i}a - \Upsilon_j H_{1j}, \quad M_{2j} = \Upsilon_j + \mathrm{i}a H_{1j},$$

$$M_{3j} = \mathrm{i}a b_0 M_{1j} + b_1\Upsilon_j M_{2j} - H_{4j} - \frac{P}{\gamma T_0} + b^* H_{2j} - H_{3j},$$

$$M_{4j} = b_1\Gamma_1 - H_{4j} - \frac{P}{\gamma T_0} + b^* H_{2j} - H_{3j},$$

$$M_{5j} = \mathrm{i}a b_1 M_{1j} + b_0\Upsilon_j M_{2j} - H_{4j} - \frac{P}{\gamma T_0} + b^* H_{2j} - H_{3j},$$

$$M_{6j} = (b_2 + b_3)\Upsilon_j M_{1j} + \mathrm{i}a M_{2j}(b_2 - b_3).$$

In the above equations,

$$\Gamma_1 = \Upsilon_j^2 - a^2, \quad \Gamma_2 = \Upsilon_j^2 - \Lambda_4, \quad \Gamma_3 = \Upsilon_j^2 - \Lambda_7, \quad \Gamma_4 = a_{11}\Upsilon_j^2 - \Lambda_1,$$
$$\Gamma_5 = (\Lambda_6 + \Lambda_8 b^*), \quad \Gamma_6 = (\Lambda_9 - \Upsilon_j^2 a_8), \quad \Gamma_7 = (\Upsilon_j^4 - 2a^2\Upsilon_j^2 + a^4).$$

6 Applications

We take the boundary conditions at $z=0$ as follows:

$$\begin{cases} \theta(x,0,t)=f(x,0,t)=f^{*}\mathrm{e}^{\omega t+\mathrm{i}ax}, \quad (\sigma_{xx}+\lambda_{xx})(x,0,t)=-\dfrac{P}{\gamma T_0}, \\ (\sigma_{xz}+\lambda_{xz})(x,0,t)=0, \quad \dfrac{\partial C}{\partial z}=0, \quad \dfrac{\partial \Phi_{\mathrm{v}}}{\partial z}=0, \end{cases} \tag{76}$$

where $f(x,t)$ is an arbitrary function, f^{*} is a constant, $\lambda_{xx}=\mu_{\mathrm{e}}H_0^2 e$, $\lambda_{xz}=0$, and $e=\nabla^2\Phi$.

With the help of Eqs. (65)–(67), considering the boundary conditions in Eq. (76), we can obtain the following equations:

$$\sum_{j=1}^{5} H_{4j}R_j=f^{*}, \tag{77}$$

$$\sum_{j=1}^{5} (M_{3j}+R_H\Gamma_1)\,R_j=0, \tag{78}$$

$$\sum_{j=1}^{5} M_{6j}R_j=0, \tag{79}$$

$$\sum_{j=1}^{5} \Upsilon_j H_{2j}R_j=0, \tag{80}$$

$$\sum_{j=1}^{5} \Upsilon_j H_{3j}R_j=0, \tag{81}$$

where the parameters $R_1, R_2, \cdots, R_5$ can be determined by Eqs. (77)–(81). Therefore, we obtain the expressions of the displacement components, the force stress, the temperature, the volume fraction field, and the concentration of diffusion.

7 Numerical results and discussion

We take the values of the physical parameters for copper as follows[13]:

$$\begin{aligned} &\lambda=7.76\times10^{10}\,\mathrm{N\cdot m^{-2}}, \quad \mu=3.86\times10^{10}\,\mathrm{kg\cdot m^{-1}\cdot s^{-2}}, \quad C_{\mathrm{E}}=383.1\,\mathrm{J\cdot kg^{-1}\cdot K^{-1}}, \\ &K=386\,\mathrm{W\cdot m^{-1}\cdot K^{-1}}, \quad \alpha_{\mathrm{t}}=1.78\times10^{-5}\,\mathrm{K^{-1}}, \quad \rho=8\,954\,\mathrm{kg\cdot m^{-3}}, \\ &T_0=293\,\mathrm{K}, \quad f^{*}=1, \quad \omega=\omega_0+\mathrm{i}\xi, \quad \omega_0=4, \quad \xi=-4.1, \quad a=0.77, \\ &\tau_1=0.95, \quad \tau_\Theta=0.05, \quad t=0.001, \quad x=1.5, \quad 0\leqslant z\leqslant 5. \end{aligned}$$

The void parameters are

$$\begin{aligned} &b=1.138\,49\times10^{10}, \quad \omega_0=0.078\times10^{-3}, \quad \chi=1.756\times10^{-15}, \\ &\alpha=3.688\times10^{-5}, \quad m=2\times10^{6}, \quad \zeta=1.475\times10^{10}. \end{aligned}$$

The diffusion parameters are

$$\begin{aligned} &b_{\mathrm{c}}=0.9\times10^{6}, \quad a_{\mathrm{c}}=1.2\times10^{4}, \quad \tau_2=0.5, \quad \tau_\eta=0.9, \\ &\alpha_{\mathrm{c}}=1.98\times10^{-4}, \quad d=0.85\times10^{-8}, \quad b_2^{*}=2.9\times10^{12}. \end{aligned}$$

A MATLAB package is used for the calculations.

Figures 2–8 show the calculation results predicted by two different generalized thermoelasticity models, i.e., the L-S model and the DPL model. In these figures, the solid curves represent the results of the DPL model, the dashed curves represent the results of the L-S model, and all variables are non-dimensional. Due to the boundary conditions, the stress components σ_{xx} and σ_{xz} based on both the L-S model and the DPL model start from zero. The values of u, w, θ, σ_{xx}, σ_{xz}, $\Phi_{\rm v}$, and C converge to zero. When $\beta^2 = 1$, i.e., $\varepsilon_0 = 1$, $\mu_r = 1$, and $H_0 = 0$, the effects are without the electromagnetic field. When $\beta^2 = 1.3$, i.e., $\varepsilon_0 = 1$, $\mu_r = 1$, and $H_0 = 50$, and $\beta^2 = 1.7$, i.e., $\varepsilon_0 = 1$, $\mu_r = 1$, and $H_0 = 80$, the effects are with the electromagnetic field.

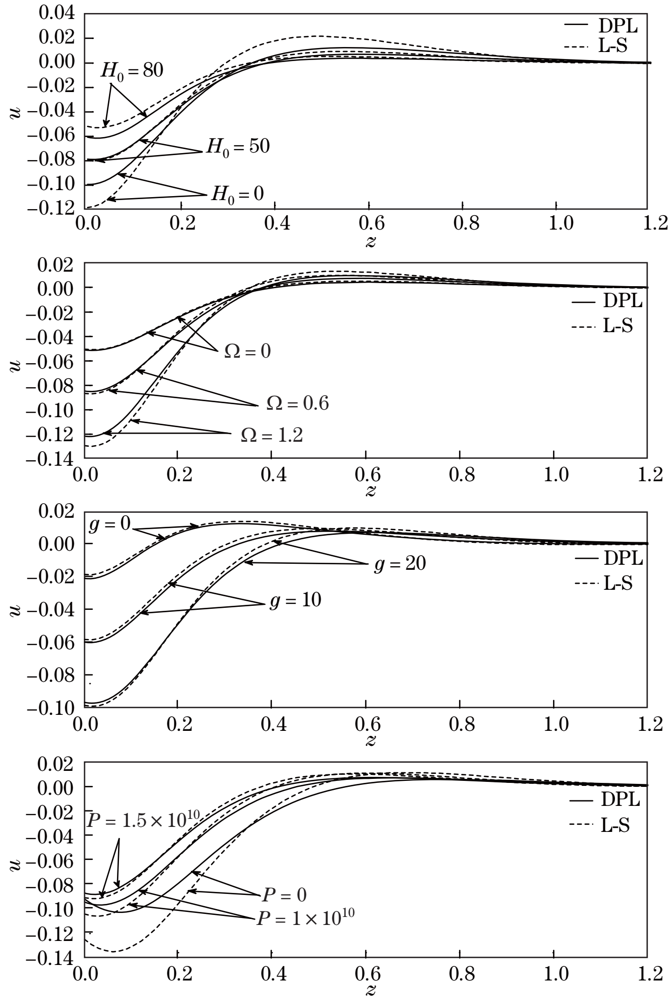

Fig. 2 Variations of the horizontal displacement u with different values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P

Figure 2 shows the variations of the horizontal displacement u along the z-axis with various values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P by using the L-S and DPL models. It can be seen that the horizontal displacement u increases when H_0 and P increase, but decreases when Ω and g increase. Moreover, for the magnetic field H_0, the rotation Ω, and the initial stress P, the values of u obtained with the L-S model are larger than those obtained with the DPL model, while for the gravity g, the values of u obtained with the DPL model are larger than those with the L-S model. When z tends to infinity, the values of

u approach zero.

Figure 3 shows the variations of the vertical displacement w with different values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P along the axial z by using the L-S and DPL models. It is noticed that the vertical displacement w increases when g increases, while decreases when H_0, Ω, and P increase. Moreover, for the magnetic field H_0, the rotation Ω, and the initial stress P, the values of w obtained with the L-S model are larger than those obtained with the DPL model, while for the gravity g, the values of w obtained with the DPL model are larger than those with the L-S model. When z tends to infinity, the values of w approach zero. Besides, the effects of the magnetic field and the initial stress are greater than those of the rotation and the gravity on the vertical displacement w.

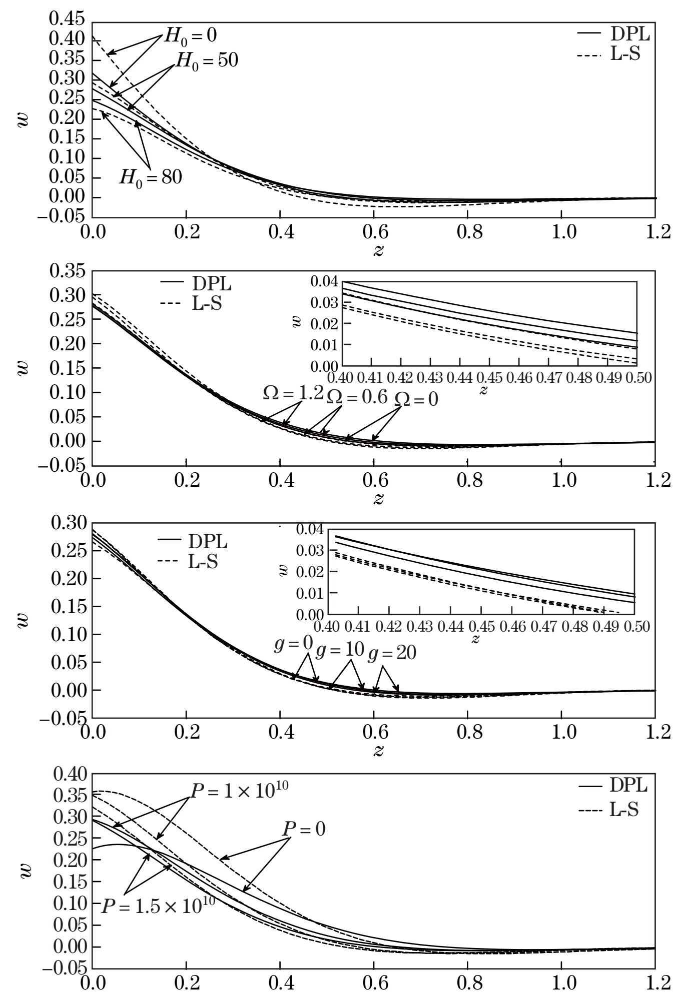

Fig. 3 Variations of the vertical displacement w with different values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P

Figure 4 shows the variations of the temperature distribution θ with different values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P along the axial z by using the L-S and DPL models. It is noticed that θ increases when g increases, while decreases when H_0, Ω, and P increase. Moreover, for the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P, the values of θ obtained with the L-S model are larger than those obtained with the DPL model. When z tends to infinity, θ approaches zero.

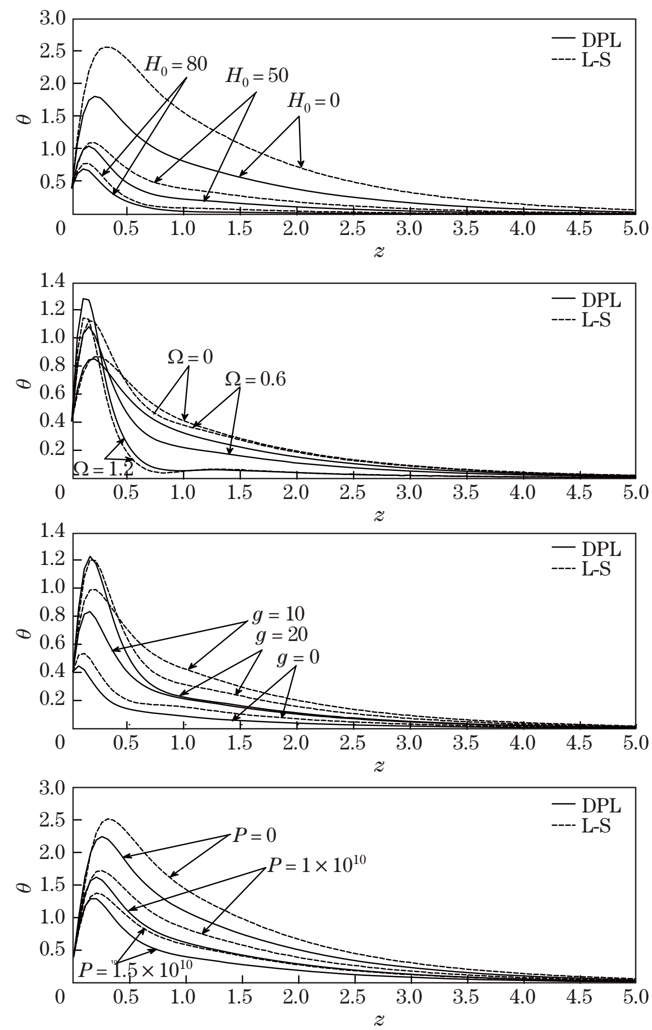

Fig. 4 Variations of the temperature θ with different values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P

Figure 5 shows the variations of the fraction field Φ_{v} with different values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P by using the L-S and DPL models. It is noticed that Φ_{v} increases when H_0, P, and Ω increase, while decreases when g increases. Moreover, for the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P, the values of Φ_{v} obtained with the DPL model are larger than those obtained with the L-S model.

Figure 6 shows the variations of the stress component σ_{xx} with different values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P by using the L-S and DPL models. It is noticed that σ_{xx} increases when H_0 and P increase, while decreases when g increases. When Ω increases, σ_{xx} first decreases, and then increases. Moreover, for the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P, the values of σ_{xx} obtained with the DPL model are larger than those obtained with the L-S model.

Figure 7 displays the variations of the stress component σ_{xz} with different values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P by using the L-S and

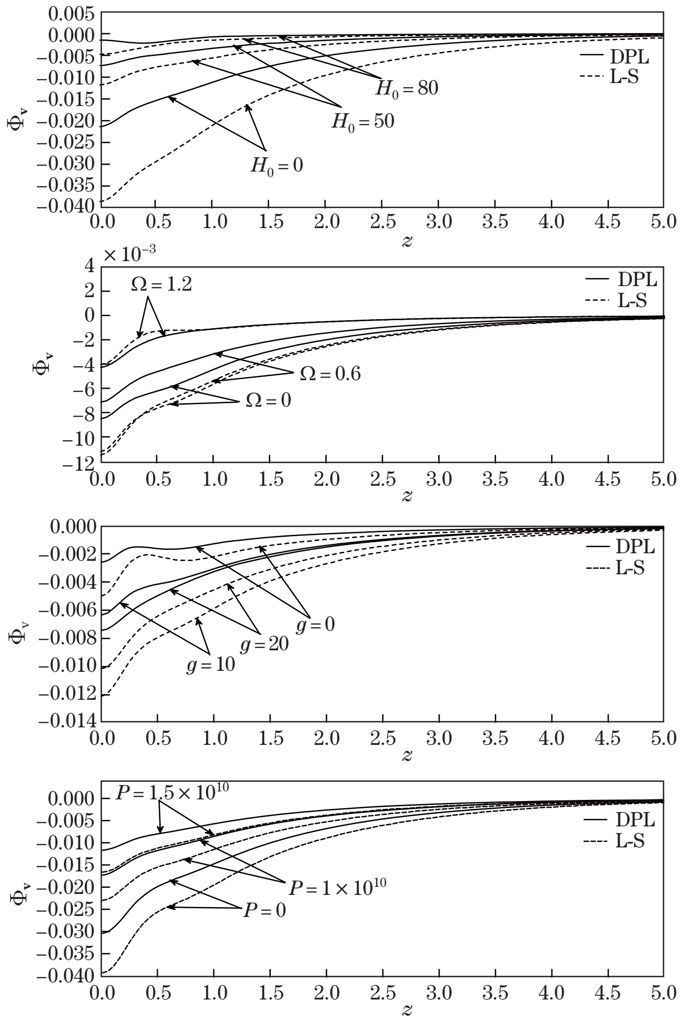

Fig. 5 Variations of the fraction field Φ_{v} with different values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P

DPL models. It is noticed that σ_{xz} decreases when H_0 and P increase, while increases when Ω and g increase. Moreover, for the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P, the values of σ_{xz} obtained with the L-S model are larger than those obtained with the DPL model. When z tends to infinity, σ_{xz} approaches zero.

Figure 8 shows the variations of the strength of diffusion C with different values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P by using the L-S and the DPL models. It is noticed that C decreases when H_0, Ω, g, and P increase. Moreover, for the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P, the values of C obtained with the DPL model are larger than those obtained with the L-S model. When z tends to infinity, C approaches zero.

8 Conclusions

In this paper, the effects of the electromagnetic field, the gravity field, the rotation, and the initial stress on an electro-magneto-thermoelastic medium with diffusion and voids in a

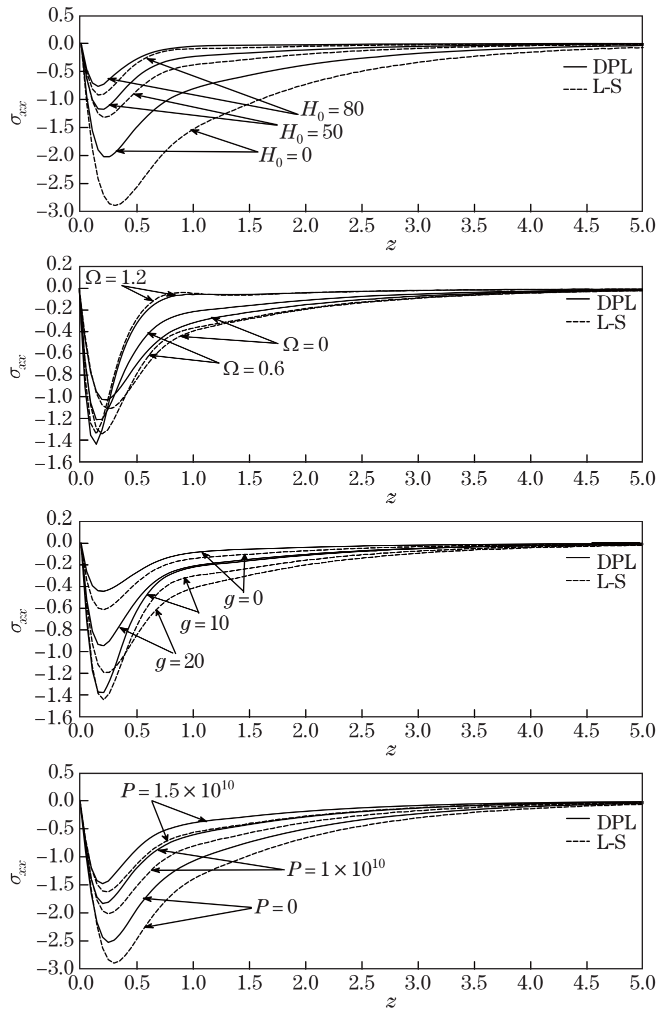

Fig. 6 Variations of the stress component σ_{xx} with different values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P

generalized thermoelastic half-space are studied by using the L-S and DPL models. The analysis indicates some conclusions as follows:

(i) The effects of diffusion and voids on the physical quantities are significant. The horizontal displacement u increases when H_0 and g increase, while decreases when Ω and g increase. The vertical displacement w increases when g increases, while decreases when H_0, Ω, and P increase. Φ_{v} increases when H_0, P, and Ω increase, while decreases when g increases. θ decreases when H_0, P, and Ω increase, while increases when g increases. σ_{xz} decreases when H_0 and P increase, while increases when Ω and g increase. σ_{xx} increases when H_0 and P increase, while decreases when Ω and g increase. When H_0, Ω, g, and P increase, C decreases.

(ii) The solutions obtained with the L-S model and the DPL model have similar tendencies along the z-direction. When z tends to infinity, the values of the studied physical quantities approach zero.

(iii) The results are useful in the design of new materials, and the techniques applied in the present article are applicable to a wide range of problems in thermodynamics and thermoelasticity.

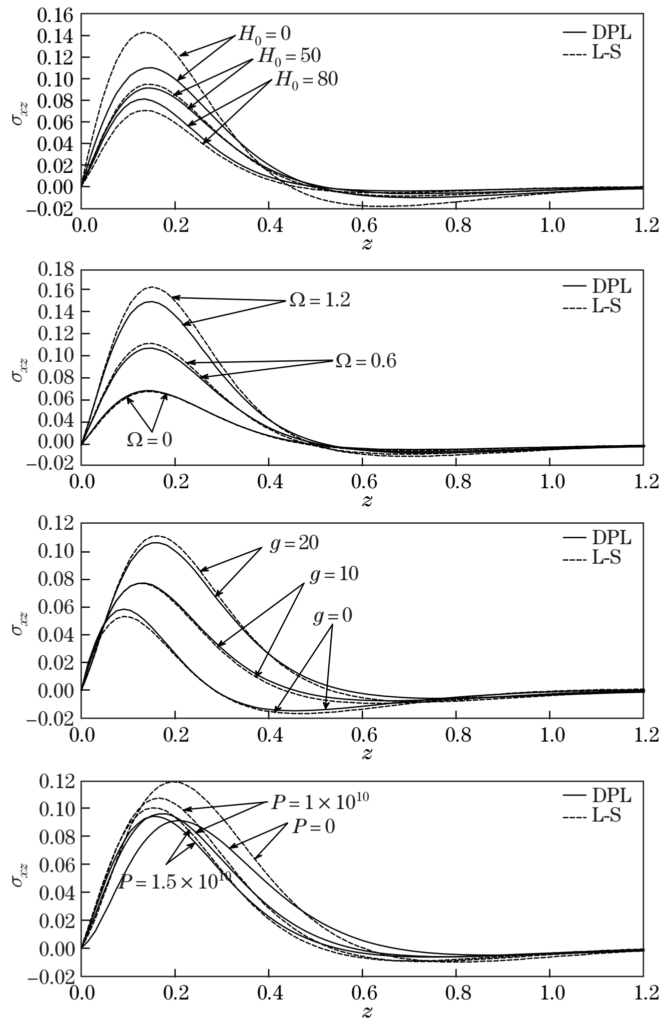

Fig. 7 Variations of the stress component σ_{xz} with different values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P

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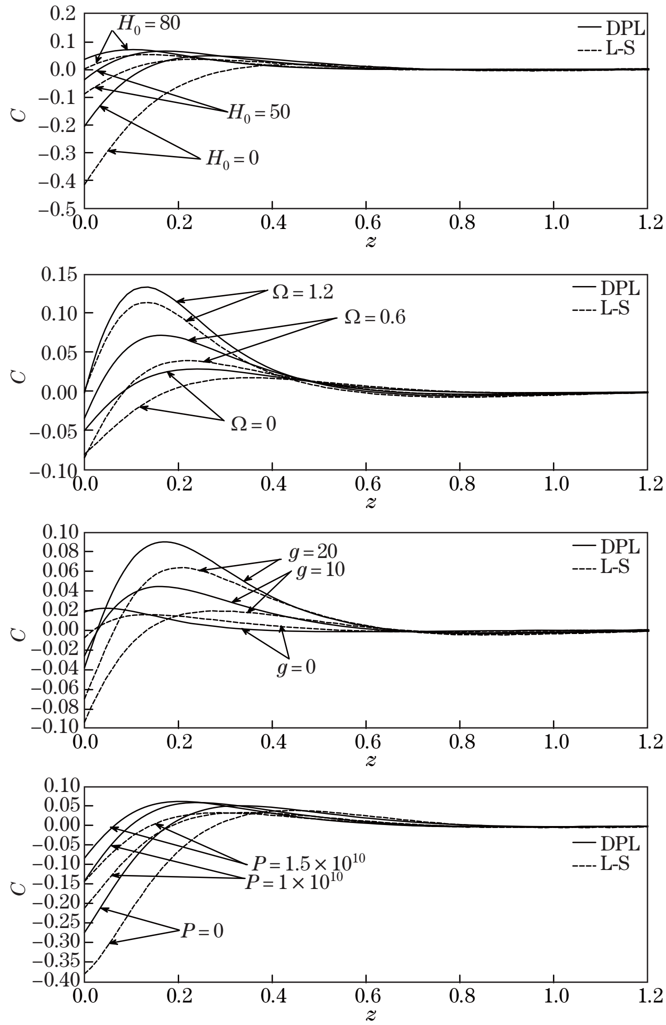

Fig. 8 Concentration of C with different values of the magnetic field H_0, the rotation Ω, the gravity g, and the initial stress P

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