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Thermomagnetic effect with microtemperature in a semiconducting photothermal excitation medium*

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Abstract The main goal of this paper is to focus on the investigation of interaction between a magnetic field and elastic materials with microstructure, whose microelements possess microtemperatures with photothermal excitation. The elastic-photothermal problem in one-dimension is solved by introducing photothermal excitation at the free surface of a semi-infinite semiconducting medium (semiconductor rod). The integral transform technique is used to solve the governing equations of the problem under the effect of the microtemperature field. The analytical expressions for some physical quantities in the physical domain are obtained with the heating boundary surface and free traction. The numerical inversion technique is used to obtain the resulting quantities in the physical domain. The obtained numerical results with some comparisons are discussed and shown graphically.

Key words photothermal theory, carrier density, magnetic field, microtemperature, Laplace transform

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1 Introduction

In 1969, the second law of thermodynamics on the basis of the theory of bodies with inner structure was modified by Grot[1] to include microelements possessing microtemperatures. Grot added the first-moment-of-the-energy equations to the usual balance laws of a continuum with microstructure. Riha[2] investigated heat conduction in thermoelastic materials with inner structure and observed that the experimental data for the silicone rubber containing spherical aluminium particles confirm closely to predicted theoretical thermal conductivity. Iesan and Quintanilla[3] developed a linear theory of thermoelasticity with microtemperatures considering the microstructure of the body and assuming that each microelement possesses a microtemperature. Iesan[4] investigated the theory of micromorphic elastic solids with microtemperature.

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Iesan[5] presented the thermoelasticity theory taking into account the microtemperatures and the inner structure of the materials. Aouadi[6] studied some theorems in microstructure thermoelasticity with microtemperature. Iesan and Quintanilla[7] established a thermomechanical theory of bodies with microstructure and microtemperature. They obtained a uniqueness theorem and instability result. Various investigators studied prominent problems thermoelastic with microtemperature and microstructure with microtemperature[8–9].

Semiconducting polymer nanocomposites (as silicon (Si)) have the characteristic mechanical flexibility and lower cost of fabrication. When a semiconductor surface is exposed to a beam of laser, some electrons will be excited. In this case, the photo-excited free carriers will be generated with non-radiative transitions, and a recombination between electron and hole plasma occurs. Several authors analyzed the deformation of thermoelastic and electronic in semiconductors, using thermal and elastic equations only by neglecting the coupled equations of plasma system. Taking into consideration the plasma effect, the interaction between the plasma fields and thermal fields (photo-thermal-diffusion (PTD)) in isotropic and anisotropic elastic polymer materials is important and has many significant applications in modern physics and soil dynamics.

Gordon et al.[10] and Kreuzer[11] introduced the photothermal method using sensitive photoacoustic spectroscopy. In this process, a beam of laser falls upon a sample of semiconductor. Then, the photothermal process appears. To obtain the values of the temperature and to measure thermal diffusion and electro-chemical properties of nanocomposite materials, the researchers used photothermal methods[12–16]. In this process, the elastic oscillation in the material occurs by the propagation of a thermal wave and the photo excited free carriers produce directly a periodic elastic deformation. Lotfy[17] presented elastic wave motions for a photothermal medium of a dual-phase-lag model with an internal heat source and gravitational field. Hobiny and Abbas[18] studied a photothermal wave in an unbounded semiconductor medium with cylindrical cavity. Abbas[19] investigated photothermal interaction in an unbounded semiconductor medium with cylindrical cavity in the context of dual-phase-lag model. Abo-Dahab and Lotfy[20] and Lotfy[21] studied the problem of two temperature plain strains in a semiconducting medium using photothermal and hydrostatic initial stress.

In this article, the deformation problem is studied with the microtemperature and the inner structure of the materials. Also, the influence of magnetic field to solve the photothermal problem in one-dimensional deformation at the free surface is studied. A semi-infinite semiconducting medium during a photothermal excitation is studied under the effect of thermal shock and free traction. The Laplace transform technique is used to obtain the expressions of non-dimensional normal displacement, normal force stress, carrier density, and temperature distribution. The numerical inversion of the transform integrals is used to obtain numerical results. The obtained results have significant effects in many engineering problems related to microtemperature in isotropic photo-elastic solids under the influence of magnetic field.

2 Basic equations

Consider the medium under investigation to be exposed to a magnetic field such that $H = H_0 + h$, where $H = (0, H_0, 0)$ is the initial magnetic field, and $h(x, y, z)$ is the induced magnetic field acting parallel to the y-axis (see Fig. 1).

We begin our consideration with the linearized electro-dynamic slowly moving medium, neglecting the charge density and the effect of temperature gradient on J. The Maxwell's equations for the electromagnetic field can be written as

$$J = \operatorname{curl} h - \varepsilon_0 \dot{E}, \tag{1}$$

$$\operatorname{curl} E = -\mu_0 \dot{H}, \tag{2}$$

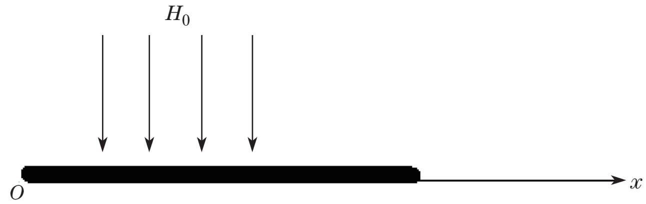

Fig. 1 Schematic of the problem

$$E = -\mu_0(\dot{u} \times H), \tag{3}$$

$$\operatorname{div} H = 0, \tag{4}$$

where μ_0 is the magnetic permeability, ε_0 is the electric permeability, and $\dot{u}$ is the particle velocity of the medium. The dot in the above equations denotes differentiation with respect to time.

Using (1)–(3), the components of the current density vector $J = (J_x, J_y, J_z)$, where J is parallel to the electric field E and can be expressed in terms of the displacement as

$$J_x = 0, \quad J_y = 0, \quad J_z = \frac{\partial h}{\partial x} + \mu_0 \, H_0 \, \varepsilon_0 \, \ddot{u}.$$

The components of the magnetic intensity vector in the medium are

$$H_x = 0, \quad H_z = 0, \quad H_y = H_0 + h(x, y, z). \tag{5}$$

In the following, we will introduce the theoretical analysis of interaction between both the coupled plasma and the thermal and elastic waves at the same time through the transporting process occurring in a semiconductor medium, by taking into consideration the inner structure of the materials, where the microelements possess microtemperatures. The main variables in this discussion are the carrier density $N(r,\, t)$, in which r is the position vector, the temperature distribution $T(r,\, t)$, and the elastic displacement components $u(r,\, t)$.

Taking into consideration the medium properties (the medium is linear, homogeneous, and has isotropic properties), the new model of the governing equations of the photothermal excitation with microtemperature theory and heat conduction is as follows[6,15–17,22–23].

(i) The balance equation of the first moment of energy is

$$\begin{cases} k_6 \, \nabla^2 w + (k_4 \, + k_5 \,)\nabla(\nabla \cdot w) - k_3 \nabla T - k_2 w - b\dot{w} = 0, \\ q = k\nabla T + k_1 w, \\ Q = (k_1 - k_2)w + (k - k_3)\nabla T. \end{cases} \tag{6}$$

(ii) For semiconductor materials, the photo-excited free carriers produce a periodic elastic deformation. The plasma equation and heat source excitations are

$$\frac{\partial N(r,t)}{\partial t} = D_{\mathrm{E}} \nabla^2 N(r,t) - \frac{N(r,t)}{\tau} + \kappa \, T(r,t). \tag{7}$$

(iii) The strain generates a cyclic variation in temperature, also taking into consideration that microelements possess microtemperatures. The body couple, heat conduction, carrier density, and microtemperature equations are

$$\rho \, C_{\mathrm{e}} \frac{\partial T(r,t)}{\partial t} = k\nabla^2 T(r,t) + \frac{E_{\mathrm{g}}}{\tau} N(r,t) - \gamma T_0 \nabla \cdot \frac{\partial u(r,t)}{\partial t} \, + k_1 \cdot \nabla w. \tag{8}$$

(iv) Taking into account the Lorentz force, the equation of motion in the case of plasma wave with an external magnetic field is[24–25]

$$\rho\,\ddot{u}(r,t) = \mu\nabla^2 u(r,t) + (\mu+\lambda)\,\nabla\,(\nabla\cdot\, u(r,t)) - \gamma\nabla T(r,t) - \delta_n \nabla N(r,t) + F. \tag{9}$$

In (6)–(9), $\kappa = \frac{\partial n^*}{\partial T}\frac{T}{\tau}$ is the coupling parameter of the thermal activation in the case of harmonic modulation laser that could be neglected when the temperature is relatively low. n^* is the equilibrium carrier concentration at the temperature T. D_{E} is the carrier diffusion coefficient. τ is the photo-generated carrier lifetime. E_{g} is the gap energy of the semiconductor medium. μ and λ are the well-known Lamé elastic constants. ρ is the density, k is the thermal conductivity of the sample, and T_0 is the absolute temperature. $\gamma = (3\lambda + 2\mu)\,\alpha_{\mathrm{T}}$ is the volume thermal expansion, where α_{T} is the coefficient of linear thermal expansion. C_{e} is the specific heat at the constant strain of the solid plate, and δ_n is the difference of the deformation potential of conduction and valence band. $k_n\,(n = 1, 2, \cdots, 6)$ are the constitutive coefficients, and w is the microtemperature vector. q and Q are the heat flux moment and mean heat flux vectors, respectively.

Restricting our study in one dimension, the x-direction, the displacement component $u(x,z,t)$ and the strain e are defined as $u = (u, 0, 0)$ and $e = u_x$. Also, we define the Lorentz force as $F = \mu_0(J \times H)$ which may be written in one dimension as follows:

$$F = \mu_0(J\times H) \equiv \Big(-\mu_0 H_0 \frac{\partial h}{\partial x} - \varepsilon_0 \mu_0^2 H_0^2 \frac{\partial^2 u}{\partial t^2},\, 0,\, 0\Big) \equiv F_x e_i. \tag{10}$$

The one-dimensional form of the heat flux moment q_x and the mean heat flux Q_x is

$$q_x = q = k\frac{\partial T}{\partial x} + k_1 w, \quad Q_x = Q = (k_1 - k_2)w_1 + (k - k_3)\frac{\partial T}{\partial x}. \tag{11}$$

Using (10), (6)–(9) can be expressed in one dimension as

$$\frac{\partial N}{\partial t} = D_{\mathrm{E}}\frac{\partial^2 N}{\partial x^2} - \frac{N}{\tau} + \kappa T, \tag{12}$$

$$\rho\, C_{\mathrm{e}}\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + \frac{E_{\mathrm{g}}}{\tau}N - \gamma T_0 \frac{\partial e}{\partial t} + k_1\frac{\partial w}{\partial x}, \tag{13}$$

$$k^*\frac{\partial^2 w}{\partial x^2} - k_3\frac{\partial T}{\partial x} - k_2 w - b\frac{\partial w}{\partial t} = 0, \tag{14}$$

$$\rho\,\frac{\partial^2 u}{\partial t^2} = (2\mu+\lambda)\frac{\partial^2 u}{\partial x^2} - \gamma\frac{\partial T}{\partial x} - \delta_n\frac{\partial N}{\partial x} - \mu_0 H_0\frac{\partial h}{\partial x} - \varepsilon_0\mu_0^2 H_0^2\frac{\partial^2 u}{\partial t^2}. \tag{15}$$

The one-dimensional constitutive equation takes the form of

$$\sigma = \sigma_{xx} = (2\mu+\lambda)e - (3\lambda+2\mu)(\alpha_{\mathrm{T}}\,T + d_n N). \tag{16}$$

Differentiating (15) with respect to x, we obtain

$$\rho\,\frac{\partial^2 e}{\partial t^2} = (2\mu+\lambda)\frac{\partial^2 e}{\partial x^2} - \gamma\frac{\partial^2 T}{\partial x^2} - \delta_n\frac{\partial^2 N}{\partial x^2} + \mu_0 H_0^2\frac{\partial^2 e}{\partial x^2} - \varepsilon_0\mu_0^2 H_0^2\frac{\partial^2 e}{\partial t^2}, \tag{17}$$

where $k^* = k_4 + k_5 + k_6$, and $h = -H_0\, e$, in which e is the dilatation.

For convenience, we introduce the following dimensionless forms:

$$\begin{cases} (x', u') = \dfrac{(x,u)}{C_{\mathrm{T}} t^*}, \quad t' = \dfrac{t}{t^*}, \quad T' = \dfrac{\gamma T}{2\mu+\lambda}, \\ N' = \dfrac{\delta_n N}{2\mu+\lambda}, \quad \sigma' = \dfrac{\sigma}{\mu}, \quad h' = \dfrac{h}{\rho C_{\mathrm{T}}^2}, \quad w' = \dfrac{1}{t^* C_{\mathrm{T}}}. \end{cases} \tag{18}$$

Using (18) and dropping the dashed, the non-dimensional expressions of (12)–(14) and (16)–(17) are

$$\Big(\nabla^2 - q_1 - q_2\frac{\partial}{\partial t}\Big)N + \varepsilon_3 T = 0, \tag{19}$$

$$\Big(\nabla^2 - \frac{\partial}{\partial t}\Big)T + \varepsilon_2 N - \varepsilon_1\frac{\partial e}{\partial t} + \varepsilon_4\frac{\partial w}{\partial x} = 0, \tag{20}$$

$$\Big(\alpha\nabla^2 - R_H\,\frac{\partial^2}{\partial t^2}\Big)e - \nabla^2 T - \nabla^2 N = 0, \tag{21}$$

$$\Big(\nabla^2 - \varepsilon_7\frac{\partial}{\partial t} - \varepsilon_6\Big)w - \varepsilon_5\frac{\partial T}{\partial x} = 0, \tag{22}$$

$$\sigma = \frac{(2\mu+\lambda)}{\mu}e + -\frac{(2\mu+\lambda)}{\mu}(T+N), \tag{23}$$

where

$$\begin{aligned}
&q_1 = \frac{kt^*}{D_{\mathrm{E}}\rho\tau C_{\mathrm{e}}}, \quad q_2 = \frac{k}{D_{\mathrm{E}}\rho C_{\mathrm{e}}}, \quad \varepsilon_1 = \frac{\gamma^2 T_0 t^*}{k\rho}, \quad \varepsilon_2 = \frac{\alpha_{\mathrm{T}} E_{\mathrm{g}} t^*}{d_n\rho\tau C_{\mathrm{e}}}, \quad \varepsilon_3 = \frac{d_n k\kappa t^*}{\alpha_{\mathrm{T}}\rho C_{\mathrm{e}} D_{\mathrm{E}}},\\
&C_{\mathrm{T}}^2 = \frac{2\mu+\lambda}{\rho}, \quad C_{\mathrm{L}}^2 = \frac{\mu}{\rho}, \quad \beta^2 = \frac{C_{\mathrm{T}}^2}{C_{\mathrm{L}}^2}, \quad \delta_n = (2\mu+3\lambda)d_n, \quad t^* = \frac{k}{\rho C_{\mathrm{e}} C_{\mathrm{T}}^2},\\
&\alpha = 1 + \frac{\mu_0 H_0^2}{2\mu+\lambda}, \quad \varepsilon_4 = \frac{k_1\gamma {t^*}^2}{k\rho}, \quad \varepsilon_5 = \frac{k_3\,(2\mu+\lambda)}{k^*\gamma}, \quad \varepsilon_6 = \frac{k_2 C_{\mathrm{T}}^2\,{t^*}^2}{k^*},\\
&\varepsilon_7 = \frac{bC_{\mathrm{T}}^2\,t^*}{k^*}, \quad R_H = 1 + \varepsilon_0\mu_0^2 H_0^2/\rho.
\end{aligned}$$

Here, ε_1 and ε_3 are the thermal activation coupling parameters (the thermoelastic coupling parameter and the thermoelectric coupling parameter, respectively). We call the new parameters ε_i ($i = 4,\ 5,\ 6,\ 7$) the constitutive coupling parameters.

Since the considered medium is homogeneous, the initial conditions take the forms of

$$\begin{cases}
u(x,\,t)|_{t=0} = \left.\dfrac{\partial u(x,\,t)}{\partial t}\right|_{t=0} = 0, \quad T(x,t)|_{t=0} = \left.\dfrac{\partial T(x,\,t)}{\partial t}\right|_{t=0} = 0,\\
\sigma(x,t)|_{t=0} = \left.\dfrac{\partial \sigma(x,\,t)}{\partial t}\right|_{t=0} = 0, \quad N(x,t)|_{t=0} = \left.\dfrac{\partial N(x,\,t)}{\partial t}\right|_{t=0} = 0,\\
w(x,t)|_{t=0} = \left.\dfrac{\partial w(x,\,t)}{\partial t}\right|_{t=0} = 0.
\end{cases} \tag{24}$$

3 Solution to the problem

To solve the dimensionless governing equations, we use the Laplace transform defined for any function $\zeta(x,t)$ as

$$L(\zeta(x,t)) = \overline{\zeta}(x,s) = \int_0^\infty \zeta(x,t)\exp(-st)\,\mathrm{d}t, \tag{25}$$

where s is the Laplace parameter.

Therefore, (19)–(23) take the form

$$(\Delta^2 - q_1 - q_2 s)\overline{N} + \varepsilon_3 \overline{T} = 0, \tag{26}$$

$$(\Delta^2 - s)\overline{T} + \varepsilon_2 \overline{N} - \varepsilon_1 s\overline{e} + \varepsilon_4 \Delta\overline{w} = 0, \tag{27}$$

$$(\alpha\Delta^2 - R_H s^2)\overline{e} - \Delta^2\overline{T} - \Delta^2 \overline{N} = 0, \tag{28}$$

$$(\Delta^2 - \varepsilon_7 s - \varepsilon_6)\overline{w} - \varepsilon_5\Delta\overline{T} = 0, \tag{29}$$

$$\overline{\sigma} = \alpha_5(\overline{e} - (\overline{T} + \overline{N})), \tag{30}$$

where $\Delta = \frac{\mathrm{d}}{\mathrm{d}x}$, and $\Delta^2 = \frac{\mathrm{d}^2}{\mathrm{d}x^2}$.

Then, (26)–(29) can be reduced as

$$(\Delta^2 - \alpha_1)\overline{N} + \varepsilon_3 \overline{T} = 0, \tag{31}$$

$$(\Delta^2 - s)\overline{T} + \varepsilon_2 \overline{N} - \alpha_2\overline{e} + \varepsilon_4 \Delta\overline{w} = 0, \tag{32}$$

$$(\alpha\Delta^2 - \alpha_3)\overline{e} - \Delta^2\overline{T} - \Delta^2 \overline{N} = 0, \tag{33}$$

$$(\Delta^2 - \alpha_4)\overline{w} - \varepsilon_5\Delta\overline{T} = 0, \tag{34}$$

where $\alpha_1 = q_1 + sq_2$, $\alpha_2 = s\varepsilon_1$, $\alpha_3 = R_H s^2$, $\alpha_4 = \alpha_1 - \varepsilon_3$, and $\alpha_5 = \frac{2\mu+\lambda}{\mu}$.

The physical quantities $\overline{e}(x, s)$, $\overline{N}(x, s)$, $\overline{w}(x, s)$, and $\overline{T}(x, s)$ of the coupled differential equations (31)–(34) can be written as

$$(\Delta^8 - \Gamma_1\Delta^6 + \Gamma_2\Delta^4 - \Gamma_3\Delta^2 + \Gamma_4)\{\overline{e}, \overline{T}, \overline{N}, \overline{w}\}(x, s) = 0, \tag{35}$$

where

$$\Gamma_1 = (\alpha(\alpha_1 + \alpha_4 + s - \varepsilon_4\varepsilon_5) + 2\alpha_2)\alpha^{-1}, \tag{36}$$

$$\Gamma_2 = (\alpha_2(\alpha_1 + 2\alpha_4 + \alpha_5 + s - \varepsilon_4\varepsilon_5) + \alpha(\alpha_4(\alpha_1 + s) + \alpha_1(s + \varepsilon_4\varepsilon_5) - \varepsilon_2\varepsilon_3))\alpha^{-1}, \tag{37}$$

$$\Gamma_3 = (\alpha_2(\alpha_1(s - \varepsilon_4\varepsilon_5) + \varepsilon_2\varepsilon_3) + \alpha_4(\alpha_2(\alpha_1 + \alpha_5 + s) + \alpha(\alpha_1 s - \varepsilon_2\varepsilon_3)))\alpha^{-1}, \tag{38}$$

$$\Gamma_4 = (\alpha_2\alpha_4(\alpha_1 s - \varepsilon_2\varepsilon_3))\alpha^{-1}. \tag{39}$$

Factorize (35) as

$$(\Delta^2 - m_1^2)(\Delta^2 - m_2^2)(\Delta^2 - m_3^2)(\Delta^2 - m_4^2)\{\overline{e}, \overline{T}, \overline{N}, \overline{w}\}(x, s) = 0, \tag{40}$$

where $m_i^2\,(i = 1, 2, 3, 4)$ are the roots of the characteristic equation, and the only positive real part will be considered in our discussion.

The characteristic equation of (40) is

$$m^8 - \Gamma_1 m^6 + \Gamma_2 m^4 - \Gamma_3 m^2 + \Gamma_4 = 0. \tag{41}$$

Since (35) is linear, the solutions can be written as

$$\{\overline{e}, \overline{T}, \overline{N}, \overline{w}\} = \sum_{i=1}^{4} \{\overline{e}_i, \overline{T}_i, \overline{N}_i, \overline{w}_i\}. \tag{42}$$

(42) is bounded as $x \to \infty$, and the solution to this equation can be written as

$$\overline{T}(x) = \sum_{i=1}^{4} D_i(s)\, \exp(-m_i x). \tag{43}$$

In a similar way,

$$\overline{N}(x) = \sum_{i=1}^{4} D_i'(s)\, \exp(-m_i x), \tag{44}$$

$$\overline{w}(x) = \sum_{i=1}^{4} D_i''(s)\, \exp(-m_i x), \tag{45}$$

$$\overline{e}(x) = \sum_{i=1}^{4} D_i'''(s)\, \exp(-m_i x), \tag{46}$$

where $D_i(s)$, $D_i'(s)$, $D_i''(s)$, and $D_i'''(s)$ are parameters to be determined from the boundary conditions and depending on s.

Also, the elastic displacement components can be obtained from the relation

$$\overline{e}(x) = \frac{\mathrm{d}\overline{u}}{\mathrm{d}x} = \sum_{i=1}^{4} D_i'''(s)\, \exp(-m_i x). \tag{47}$$

Integrating both sides of (47), we get

$$\overline{u} = -\sum_{i=1}^{4} \frac{D_i'''(s)}{m_i}\, \exp(-m_i x). \tag{48}$$

The relations between the parameters $D_i(s)$, $D_i'(s)$, $D_i''(s)$, and $D_i'''(s)$ can be obtained by using (43)–(47) and (31)–(34) as follows:

$$D_i'(s) = H_{1i} D_i(s), \tag{49}$$

$$D_i''(s) = H_{2i} D_i(s), \tag{50}$$

$$D_i'''(s) = H_{3i} D_i(s), \tag{51}$$

where

$$H_{1i} = -\frac{\varepsilon_3}{(m_i^2 - \alpha_1)}, \tag{52}$$

$$H_{2i} = \frac{\varepsilon_5 m_i}{(m_i^2 - \alpha_4)}, \tag{53}$$

$$H_{3i} = \frac{m_i^2 (m_i^2 - \alpha_5)}{(m_i^2 - \alpha_1)(\alpha m_i^2 - \alpha_3)}, \tag{54}$$

in which $i = 1, 2, 3, 4$. The field variables can be written as

$$\overline{N}(x, s) = \sum_{i=1}^{4} H_{1i}\, D_i(s)\, \exp(-m_i x), \tag{55}$$

$$\overline{w}(x, s) = \sum_{i=1}^{4} H_{2i}\, D_i(s)\, \exp(-m_i x), \tag{56}$$

$$\overline{e}(x, s) = \sum_{i=1}^{4} H_{3i}\, D_i(s)\, \exp(-m_i x), \tag{57}$$

$$\overline{u}(x, s) = -\sum_{i=1}^{4} \frac{H_{3i} D_i(s)}{k_i}\, \exp(-m_i x), \tag{58}$$

$$\overline{\sigma} = \sum_{i=1}^{4} H_{4i}\, D_i(s)\, \exp(-m_i x), \tag{59}$$

where

$$H_{4i} = \alpha_5(H_{3i} - H_{1i} - 1), \quad i = 1, 2, 3, 4. \tag{60}$$

4 Application

The medium is initially at rest, homogeneous, and load free. To obtain the values of the unknown parameters D_i, we apply the Laplace transform to both sides to obtain the following boundaries.

(i) Accordingly, the generation of thermal waves in the structure, the thermal boundary at the surface $x = 0$ subject to a thermal shock is

$$\overline{T}(s) = T_0\overline{h}(s). \tag{61}$$

Therefore,

$$D_1 + D_2 + D_3 + D_4 = \frac{T_0}{s}. \tag{62}$$

(ii) The normal stress component on the surface is traction free,

$$\overline{\sigma}(s) = 0. \tag{63}$$

Therefore,

$$H_{41}\, D_1 + H_{42}\, D_2 + H_{43}\, D_3 + H_{44}\, D_4 = 0. \tag{64}$$

(iii) At the boundary, the photogenerated carrier takes the form

$$\overline{N}(a,\, s) = \frac{\lambda\!\!\!^{-}}{D_{\mathrm{E}}}\overline{R}(s), \tag{65}$$

$$H_{11}\, D_1 + H_{12}\, D_2 + H_{13}\, D_3 + H_{14}\, D_4 = \frac{\lambda\!\!\!^{-}}{s\varepsilon_3 D_{\mathrm{E}}}. \tag{66}$$

(iv) At the plane surface, the heat flux moment q is free,

$$q_x = q = 0, \tag{67}$$

$$(-km_1+k_1H_{21})D_1+(-km_2+k_1H_{22})D_2+(-km_3+k_1H_{23})D_3+(-km_4+k_1H_{24})D_4=0, \tag{68}$$

where $h(t)$ and $R(s)$ are the Heaviside unit step functions, and $\lambda\!\!\!^{-}$ is a chosen constant. Solving (62), (64), (66), and (68) yields the values of the four constants D_i $(i = 1, 2, 3, 4)$. By using the inverse Laplace transform, we obtain the physical quantity expressions.

5 Numerical inversion of the Laplace transform

To obtain the non-dimensional physical quantity expressions in the rod, such as the temperature, the displacement, the normal stress, the heat flux moment, and the carrier intensity, we use the inverse Laplace method (see Honig and Hirdes[26]).

In the Laplace domain, any function $f(x, t')$ can be inverted as

$$f(x,\, t') = L^{-1}\{\overline{f}(x, s)\} = \frac{1}{2\pi\mathrm{i}}\int_{n-\mathrm{i}\infty}^{n+\mathrm{i}\infty} \exp(st')\overline{f}(x, s)\mathrm{d}s. \tag{69}$$

Presume $s = n + \mathrm{i}M$ (n, $M \in \mathbb{R}$). Then, (39) takes the form

$$f(x,\, t') = \frac{\exp(nt')}{2\pi}\int_{-\infty}^{\infty}\exp(\mathrm{i}\beta t)\overline{f}(x, n + \mathrm{i}\beta)\mathrm{d}\beta. \tag{70}$$

Using the Fourier series to expand the function $f(x,\, t')$ in the interval $[0,\, 2t']$, we can write

$$f(x,\, t') = \frac{\mathrm{e}^{nt'}}{t'}\Big(\frac{1}{2}\overline{f}(x,\, n) + \mathrm{Re}\sum_{k=1}^{N}\overline{f}\Big(x,\, n + \frac{\mathrm{i}k\pi}{t'}\Big)(-1)^{n}\Big), \tag{71}$$

where Re represents the real part, $\mathrm{i} = \sqrt{-1}$, and n is a sufficiently large integer. Numerous numerical experiments have shown that the value of n should satisfy the relation $nt' \approx 4.7$.

6 Numerical results and discussion

In the following discussion, we choose the semiconductor material as silicon crystal (Si) for numerical simulation. The physical constants (parameters in SI unit) of the problem are listed as follows[24–25]:

$\lambda = 3.64 \times 10^{10}$ N/m^2, $\mu = 5.46 \times 10^{10}$ N/m^2, $\rho = 2\ 330$ kg/m^3, $\tau = 5 \times 10^{-5}$ s,

$T_0 = 800$ K, $d_n = -9 \times 10^{-31}$ m^3, $D_{\mathrm{E}} = 2.5 \times 10^{-3}$ m^2/s, $E_{\mathrm{g}} = 1.11$ eV,

$C_{\mathrm{e}} = 695$ J/(kg·K), $\alpha_{\mathrm{T}} = 4.14 \times 10^{-6}$ K^{-1}, $s = 2$ m/s, $t = 0.008$ s,

$\mu_0 = 4\pi \times 10^{-7}$ H/m, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\lambdabar = 2$ m/s, $n^* = 10^{20}$ m^{-3},

$k_1 = 3.5 \times 10^{-3}$ N/s, $k_2 = 4.5 \times 10^{-3}$ N/s, $k_3 = 5.5 \times 10^{-3}$ N/(s·K),

$k_4 = 6.5 \times 10^{-3}$ N/(s·m^2), $k_5 = 7.6 \times 10^{-3}$ N/(s·m^2), $k_6 = 9.6 \times 10^{-3}$ N/(s·m^2).

Figure 2 shows the variations of the temperature T, the displacement component u, the stress component σ, the microtemperature component w, the heat flux moment q, and the carrier density N with respect to x for different values of the photogenerated carrier lifetime. The values of the temperature T and the microtemperature component w increase when $0 \leqslant x < 2$ and decrease in the remaining range as the carrier lifetime increases near the application of the source. The values of u and σ decrease at the beginning and then increase after the oscillation in the assumed region. The carrier density N decreases for $0 \leqslant x \leqslant 1$ and oscillates outside this interval for all values of carrier lifetime.

Figure 3 represents the variation of T, u, σ, w, q, and N with respect to x with (WMF) and without (WNMF) the magnetic field. Near the source of application, as a result of the magnetic effect, the values of the temperature T and the displacement component u are higher in comparison with no magnetic field effect. The behavior and variation of the carrier density u and the carrier density N are oscillatory in nature for both cases of WNMF and WMF. The behaviors of the stress component σ and the heat flux moment q are opposite oscillatory and different in their magnitude values.

Figure 4 depicts the variation of the temperature T, the displacement component u, the stress component σ, the microtemperature component w, the heat flux moment q, and the carrier density N with respect to x for different values of the thermoelectric coupling parameter under the influence of magnetic field when $\varepsilon_5 = 0.04$. The values of the temperature T and the microtemperature component w initially increase and then decrease in the remaining region as the thermoelectric coupling increases. The behaviors of the displacement component u, the stress component σ, the heat flux moment q, and the carrier density N are similar in nature. The values of the stress component σ and the carrier density N decrease initially for the range $0 \leqslant x < 2$ and then oscillate in the considered range for all cases of thermoelectric coupling.

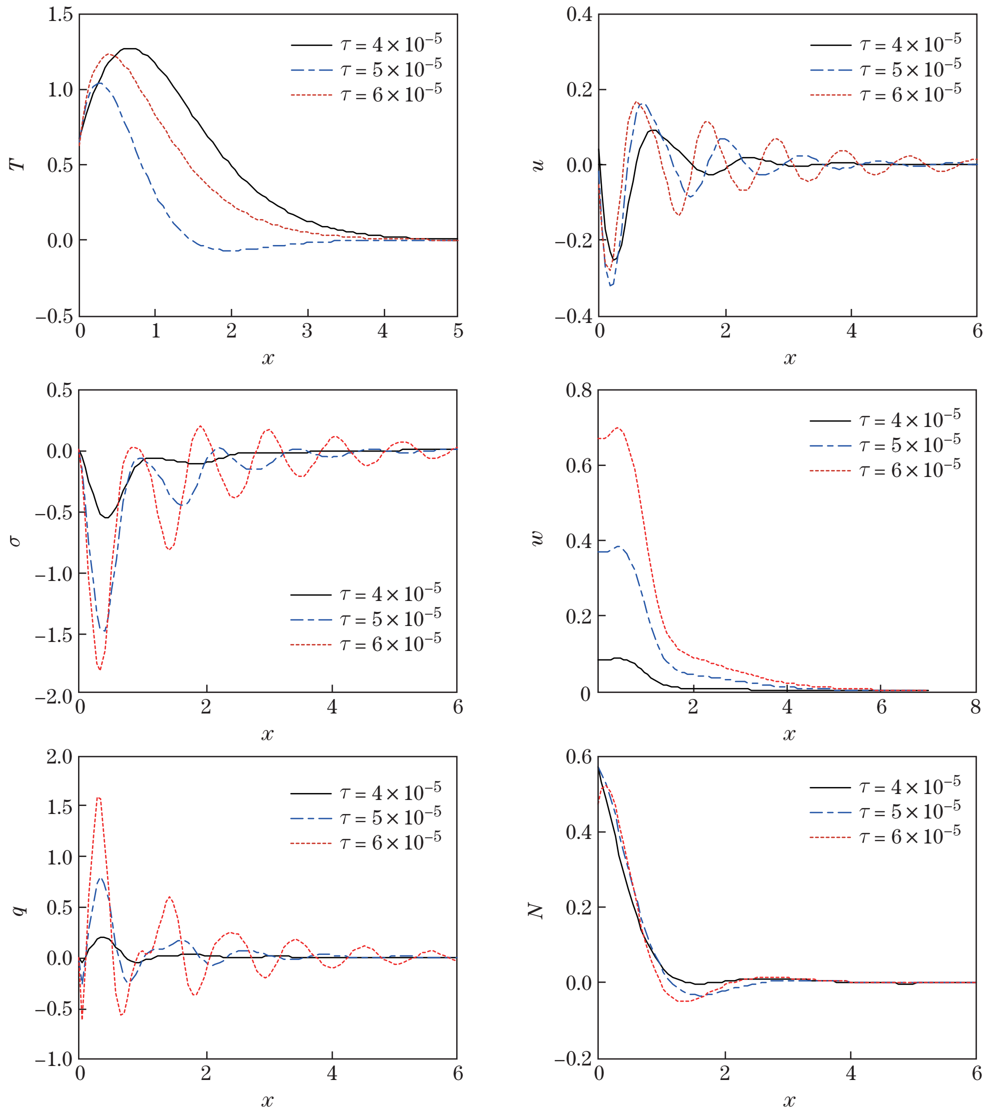

Fig. 2 Some physical quantity distributions with variation of the photogenerated carrier lifetime in second when $\varepsilon_3 = -0.04$ and $\varepsilon_5 = 0.04$ under the effect of magnetic field

Reversed behaviors are observed for both the displacement component u and the heat flux moment q. The variations of the physical field quantities are similar in nature for different values of the thermoelectric coupling parameter and propagate in a wave form. The thermochemical coupling parameter observed in this group has a significant effect.

Figure 5 shows the variations of T, u, σ, w, q, and N with respect to x for different values of the constitutive coupling parameter ε_5 under the influence of magnetic field when $\varepsilon_3 = 0.04$. The values of the temperature T and the microtemperature component w increase initially and then decrease smoothly for the whole region. As the constitutive coupling increases, the values of the temperature T decrease for $0 \leqslant x < 2$, while T takes higher values in the remaining range, and the values of the microtemperature component w are lower for the whole region. The behaviors and variations of the displacement component u, the stress component σ, and the heat flux moment q are oscillatory in nature for all cases of constitutive coupling. The carrier density N initially decreases for the range $0 \leqslant x \leqslant 1$ and then increases in the remaining region.

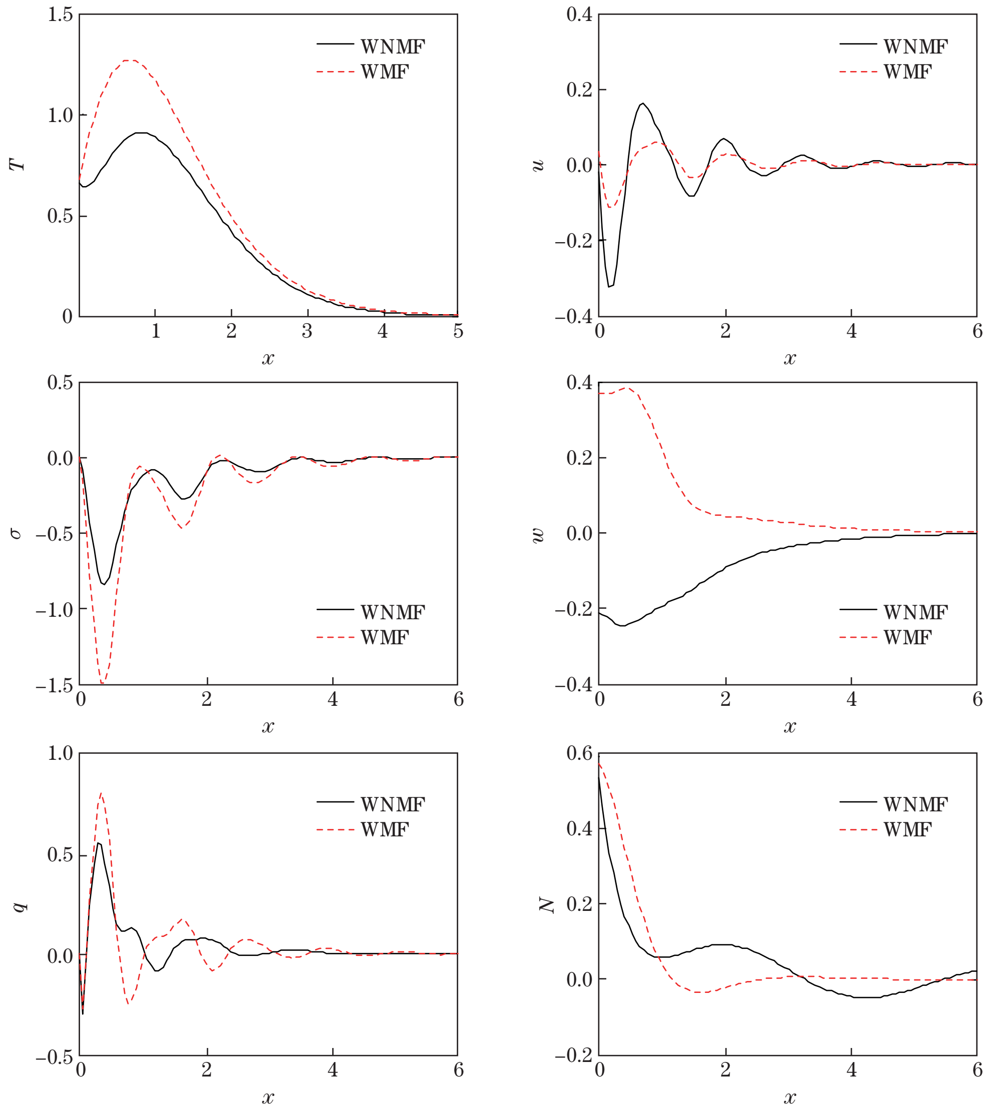

Fig. 3 Some physical quantity distributions in comparison of WNMF and WMF when $\varepsilon_3 = -0.04$, $\varepsilon_5 = 0.04$, and $\Omega = 0.2$

From these figures illustrated, any small difference in constitutive and thermoelectric coupling parameters is important and influential on the thermal wave propagations in semiconductor composites. Also, these parameters have great influence on the physical quantity distributions.

7 Conclusions

A thermoelastic model with microtemperature and photothermal excitation is governed by a system of linear partial differential equations. This system is coupled in a consistent way with thermoelasticity to additional microstructure vector field with additional balance law and balance first moment of energy. The problem is formulated in one dimension and solved using the integral technique. The photothermal excitation at the free surface of a semi-infinite semiconducting medium is considered. The expressions of some physical quantities are obtained with heating boundary surface and free traction. It is noticed that the resulting quantities are

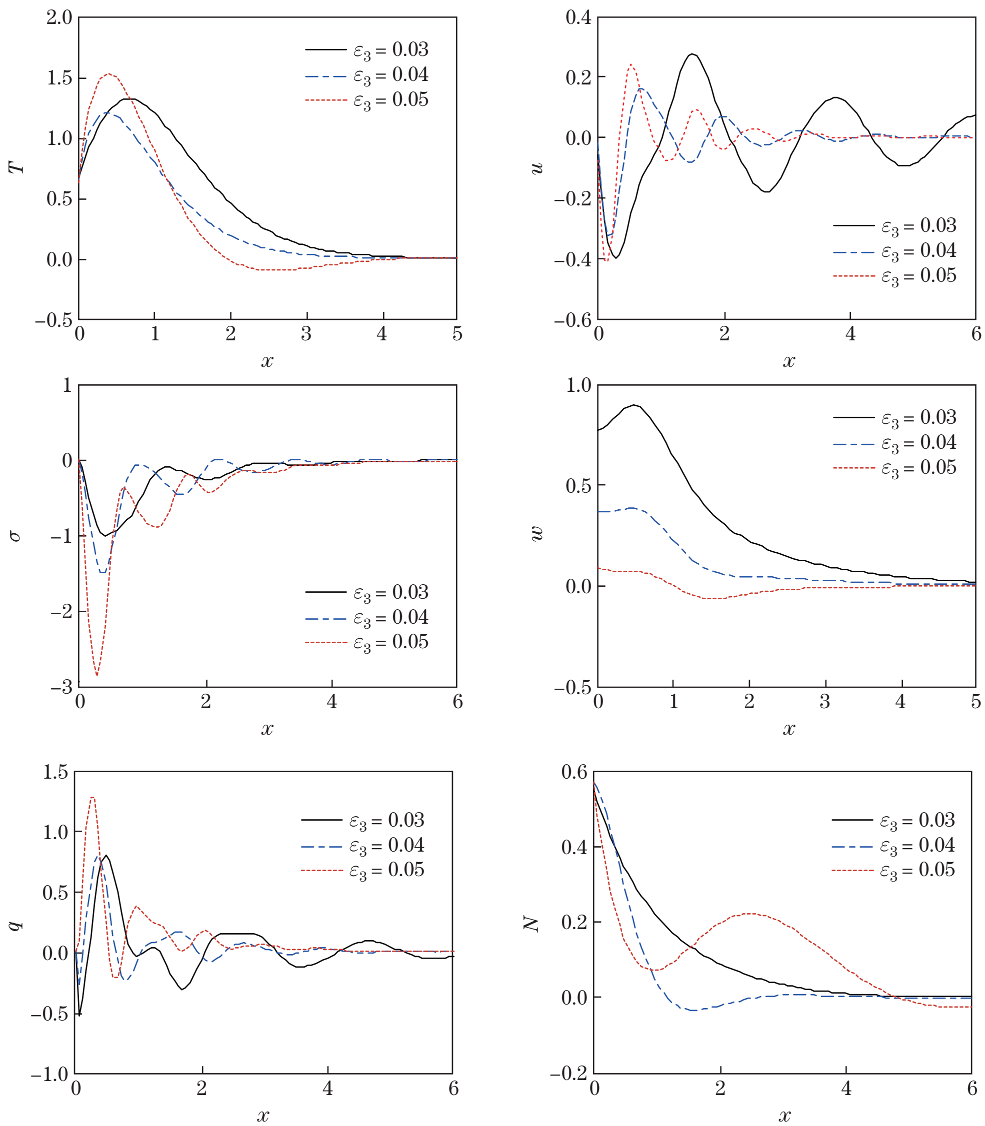

Fig. 4 Some physical quantity distributions with variation of the thermoelectric coupling parameter under influence of the magnetic field when $\varepsilon_5 = 0.04$ and $\Omega = 0.2$

influenced by the magnetic field and carrier density function significantly. The temperature T, the displacement component u, the stress component σ, the microtemperature component w, the heat flux moment q, and the carrier density N are computed numerically and depicted graphically. It is noticed that the resulting physical quantities are significantly influenced by the microtemperature and carrier density function. From the figures, it is observed that some physical quantities are oscillatory in nature with the increase in the photogenerated carrier lifetime and thermoelastic coupling and constitutive coupling parameters. As the thermoelastic coupling and constitutive coupling parameters increase, the values of temperature and microtemperature components decrease in the whole region. The values of temperature and microtemperature components increase as a result of applying the magnetic field, while in the absence of the magnetic field, the values decrease.

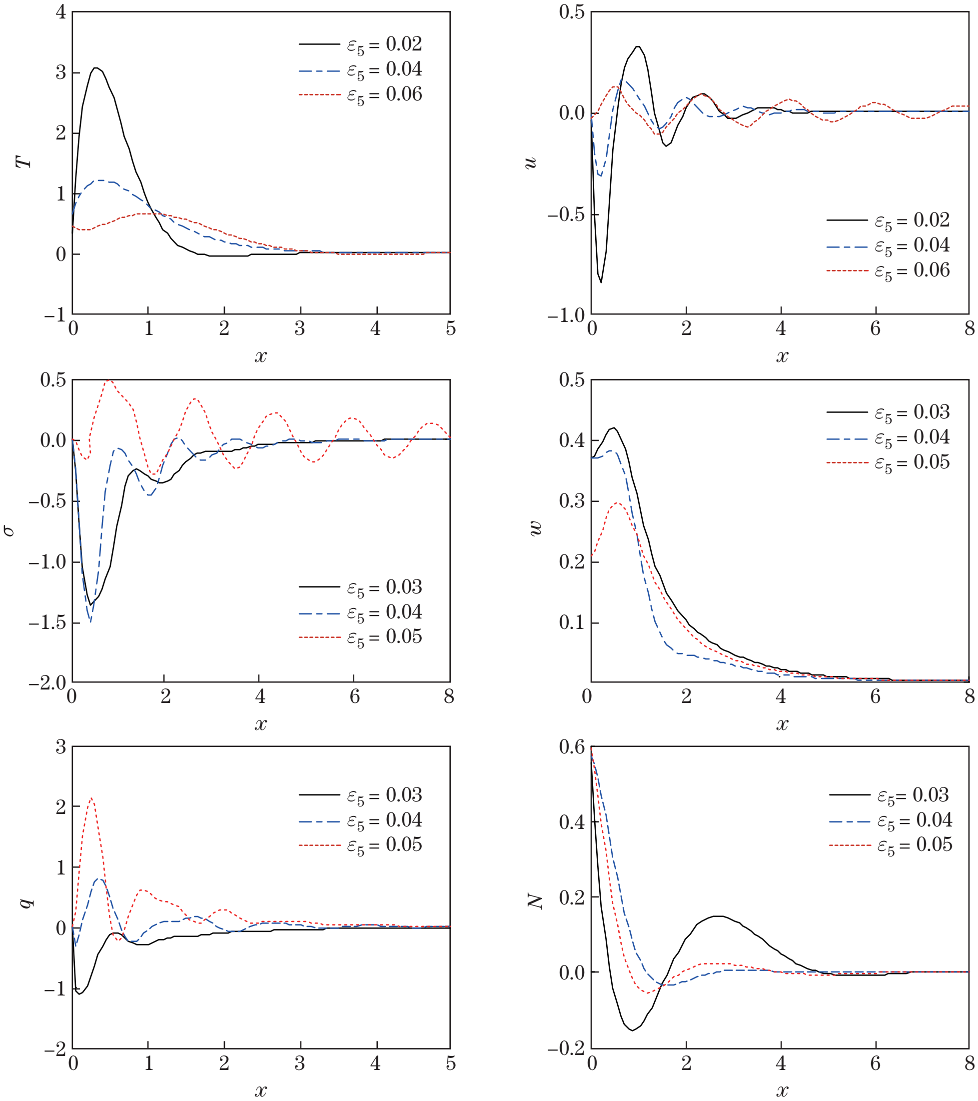

Fig. 5 Some physical quantity distributions with variation of the constitutive coupling parameter ε_5 under influence of the magnetic field when $\varepsilon_3 = 0.04$ and $\Omega = 0.2$

The studied problem has important applications for semiconductor nanocomposites in modern physics through a photothermal process in many industrial fields. More applications of the considered problem are in modern technology such as electronic industry, integrated circuits, and semiconductor materials. The microtemperatures are very important in geophysics science and earthquake engineering.

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