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Effect of inertial particles with different specific heat capacities on heat transfer in particle-laden turbulent flow*

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Abstract The effect of inertial particles with different specific heat on heat transfer in particle-laden turbulent channel flows is studied using the direct numerical simulation (DNS) and the Lagrangian particle tracking method. The simulation uses a two-way coupling model to consider the momentum and thermal interactions between the particles and turbulence. The study shows that the temperature fields display differences between the particle-laden flow with different specific heat particles and the particle-free flow, indicating that the particle specific heat is an important factor that affects the heat transfer process in a particle-laden flow. It is found that the heat transfer capacity of the particle-laden flow gradually increases with the increase of the particle specific heat. This is due to the positive contribution of the particle increase to the heat transfer. In addition, the Nusselt number of a particle-laden flow is compared with that of a particle-free flow. It is found that particles with a large specific heat strengthen heat transfer of turbulent flow, while those with small specific heat weaken heat transfer of turbulent flow.

Key words direct numerical simulation (DNS), Lagrangian tracking approach, heat transfer, particle-laden turbulent flow

Chinese Library Classification O359
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1 Introduction

Particle-laden turbulent flows have received much attention from engineers due to their important applications in industry. Particle-laden turbulent flows are characterized by a continuous fluid phase and a dispersed particle phase, and the momentum and thermal interactions between two phases mainly control the dynamic and thermodynamic characteristics of particle-laden turbulent flows. The extent of these interactions between the turbulence and the particles is affected by various factors, such as the loading ratio, the size and physical properties of the particles, and the flow condition of the fluid[1].

For particle-laden turbulent flows, the modulation of the turbulence has been investigated by experimental and numerical studies[2–4]. Most of the studies mainly focused on the influence

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of the particles on the turbulent velocity field in the past decades. However, the study of heat transfer in particle-laden turbulent flows is relatively few. Hetsroni et al.[5] investigated the thermal interaction between the turbulence and the large diameter particles both by the direction numerical simulation (DNS) and experimentally in the particle-laden turbulent flow. They found a sharp increase of the heat transfer coefficient in the front of the particles. Zonta et al.[6] performed the DNS for non-isothermal particle-laden turbulent channel flows. They observed that the increase or decrease of heat transfer capacity in particle-laden turbulent channel flows depends on the particle diameter size. Kuerten et al.[7] found that the particle mass loading has a significant impact on heat transfer, by means of the DNS of gas-particle turbulent flow in a vertical channel with high specific heat capacity particles. They observed that the enhancement of heat transfer became more obvious for larger particle mass loadings due to the increase of particle thermal feedback contribution. Jaszczur[8] examined the heat transfer processes and thermal interaction between particles and fluid in a non-isothermal fully-developed suspension channel flow. Lessani and Nakhaei[9] analyzed the influence of the particles on the heat transfer rate of non-isothermal fully-developed channel flow using the large-eddy simulation combined with the Lagrangian particle tracking method. They found that the heat transfer rate may increase, decrease, or stay unchanged with the increase of particle mass loading which depends on the particle specific heat.

The literature survey reveals the results of heat transfer modulation in particle-laden turbulent flows induced by different mass loadings or size particles. However, the research on the influence of the particles with different specific heat on heat transfer of turbulence is relatively rare. Therefore, in the present study, we study in particular the effect of the particle specific heat on the properties of temperature fields using the DNS and the particle tracking method. The present paper is organized as follows. The governing equations and the numerical method are described in Section 2. The results of the turbulent thermal statistics (the mean temperature, the temperature fluctuation, the wall-normal heat flux, and the Nusselt number) are discussed in Section 3. Section 4 presents the conclusions of this work.

2 Methodology

In this section, we perform a DNS of fully developed particle-laden turbulent channel flow with heat transfer. The dimensionless equations for the fluid, assumed incompressible and Newtonian, are given as follows[10]:

$$\begin{cases} \dfrac{\partial u_i}{\partial t} = -u_j \dfrac{\partial u_i}{\partial x_j} + \dfrac{1}{Re_\tau} \dfrac{\partial^2 u_i}{\partial x_j^2} - \dfrac{\partial p}{\partial x_i} + \delta_{1i} + F_{\mathrm{p},i}, \\[2ex] \dfrac{\partial T}{\partial t} + u_j \dfrac{\partial T}{\partial x_j} = \dfrac{1}{Re_\tau Pr} \dfrac{\partial^2 T}{\partial x_j^2} + Q_{\mathrm{p}}, \end{cases} \tag{1}$$

where u_i $(i = 1, 2, 3)$ represent the velocity components in the streamwise (x), wall-normal (y), and spanwise (z) directions, respectively, and T is the temperature. The shear Reynolds number $(Re_\tau = u_\tau \delta/\nu)$ is based on the shear velocity u_τ, the half channel height δ, and the fluid kinematic viscosity ν. The Prandtl number $(Pr = C_{p,\mathrm{f}}\mu/k_{\mathrm{f}})$ is based on the specific heat at the constant pressure $C_{p,\mathrm{f}}$, the dynamic viscosity μ, and the thermal conductivity of the fluid k_{f}. p is the kinematic pressure, δ_{1i} is the mean streamwise pressure gradient that drives the flow, and $F_{\mathrm{p},i}$ and Q_{p} are the momentum and thermal feedback terms due to the presence of the particles, respectively.

For particles with the density much larger than that of the fluid and the diameters small compared with the Kolmogorov scale, the most significant force is the Stokes drag, and other forces are not taken into account[11]. The equations of particle motion are

$$\begin{cases} \dfrac{\mathrm{d}x_{\mathrm{p},i}}{\mathrm{d}t} = v_i, \\ \dfrac{\mathrm{d}v_i}{\mathrm{d}t} = \dfrac{\overline{u}_i - v_i}{\tau_{\mathrm{p}}}(1 + 0.15Re_{\mathrm{p}}^{0.687}), \\ \dfrac{\mathrm{d}T_{\mathrm{p}}}{\mathrm{d}t} = \dfrac{T_{\mathrm{f}} - T_{\mathrm{p}}}{\tau_{\mathrm{T}}}(1 + 0.3Re_{\mathrm{p}}^{1/2}Pr^{1/3}), \end{cases} \tag{2}$$

where $x_{\mathrm{p},i}$, v_i, and T_{p} are the particle position, velocity, and temperature, respectively, and $\overline{u}_i$ and T_{f} are the fluid velocity and temperature at the particle position, respectively. The particle velocity relaxation time (τ_{p}) and the particle temperature relaxation time (τ_{T}) are defined by

$$\tau_{\mathrm{p}} = \frac{\rho_{\mathrm{p}} d_{\mathrm{p}}^2}{18\mu}, \quad \tau_{\mathrm{T}} = \frac{C_{p,\mathrm{p}}\rho_{\mathrm{p}} d_{\mathrm{p}}^2}{12k_{\mathrm{f}}},$$

where ρ_{p}, d_{p}, and $C_{p,\mathrm{p}}$ are the particle density, the diameter, and the specific heat capacity, respectively.

$$Re_{\mathrm{p}} = d_{\mathrm{p}}|\boldsymbol{v} - \overline{\boldsymbol{u}}|/\nu$$

is the particle Reynolds number.

In addition, the dimensionless equations of feedback terms in Eq. (1) are[10]

$$F_{\mathrm{p},i} = \frac{1}{VRe_\tau}\sum_{j=1}^{n} 3\pi d_{\mathrm{p}}^j (v_i^j - \overline{u}_i^j)(1 + 0.15Re_{\mathrm{p}}^{0.687}), \tag{3}$$

$$Q_{\mathrm{p}} = \frac{1}{Re_\tau Pr}\frac{1}{V}\sum_{j=1}^{n} \pi d_{\mathrm{p}}^j (T_{\mathrm{p}}^j - T_{\mathrm{f}}^j)(2 + 0.6Re_{\mathrm{p}}^{1/2}Pr^{1/3}), \tag{4}$$

where V is the volume of computational cell which the particles lie in, and n is the number of particles in each computational cell.

The DNS is performed for turbulence with $Re_\tau = 180$ and $Pr = 0.71$. The computational domain is $2\pi\delta\times 2\delta\times\pi\delta$ and discretized using an Eulerian grid made of 129^3 nodes. The particles with the density $\rho_{\mathrm{p}} = 2\,400$ kg/m^3 and the diameter $d_{\mathrm{p}} = 140$ μm are injected into the flow at a random position throughout the channel. The total number of particles is 200 000, and the average volume fraction of particles in the computational domain is 1.69×10^{-4}. The detailed numerical method can be seen in the previous work[10].

In this study, four cases are simulated to investigate the effect of the particles with different specific heats on heat transfer of turbulence, as shown in Table 1. For the particle-laden flow, Cases 1 ($St = 125$, $R_{cp} = 0.4$), 2 ($St = 125$, $R_{cp} = 1$), and 3 ($St = 125$, $R_{cp} = 2$) consider the influence of the particles on the velocity field and temperature field of turbulence (namely, considering the momentum and thermal interactions between two phases), where $St = \tau_{\mathrm{p}} u_\tau^2/\nu$ represents the particle momentum Stokes number, and $R_{cp} = C_{p,\mathrm{p}}/C_{p,\mathrm{f}}$ is the ratio of particle-to-fluid specific heat, and Case 4 represents the particle-free flow.

Table 1 Definition of some test cases

Case	R_{cp}	$\rho_{\mathrm{p}}/\rho_{\mathrm{f}}$	d_{p}/H	St	St_{T}
1	0.4	1 923	6.0×10^3	125	53
2	1.0	1 923	6.0×10^3	125	133
3	2.0	1 923	6.0×10^3	125	266
4	—	—	—	—	—

3 Results and discussion

3.1 Mean temperature and temperature fluctuation

Figure 1 shows the profile of the fluid mean temperature and the root mean square of temperature fluctuation in Cases 1 ($St = 125$, $R_{cp} = 0.4$), 2 ($St = 125$, $R_{cp} = 1$), and 3 ($St = 125$, $R_{cp} = 2$). In Fig. 1(a), we can observe the differences between the mean temperature profiles in cases with different specific heat particles. With the increase of the particle specific heat, the mean temperature of fluid increases, and the slope of the mean temperature near the wall becomes steeper. In addition, when the particle specific heat is smaller than that of the fluid ($R_{cp} < 1$), the mean temperature of fluid in Case 1 ($St = 125$, $R_{cp} = 0.4$) decreases near the cold wall relative to that in the particle-free flow (Case 4), which causes the gradient of mean temperature along the wall-normal direction to decrease in the near wall region compared with the particle-free flow. However, an opposite trend is observed in Case 3 ($St = 125$, $R_{cp} = 2$) when the particle specific heat is larger than that of fluid ($R_{cp} > 1$). The variation of the mean temperature illustrates that the effect of particle specific heat capacity on the heat transfer process is noticeable. The root mean square of the fluid temperature fluctuations (T_{rms}) for different values of R_{cp} is shown in Fig. 1(b). The particles gradually decrease the intensity of the temperature fluctuations in particle-laden flow with the increase of R_{cp}. Compared with the particle-free flow, the particles with $R_{cp} = 0.4$ in Case 1 increase the fluid temperature fluctuation, and it is opposite for Case 3 with the particles ($R_{cp} = 2$).

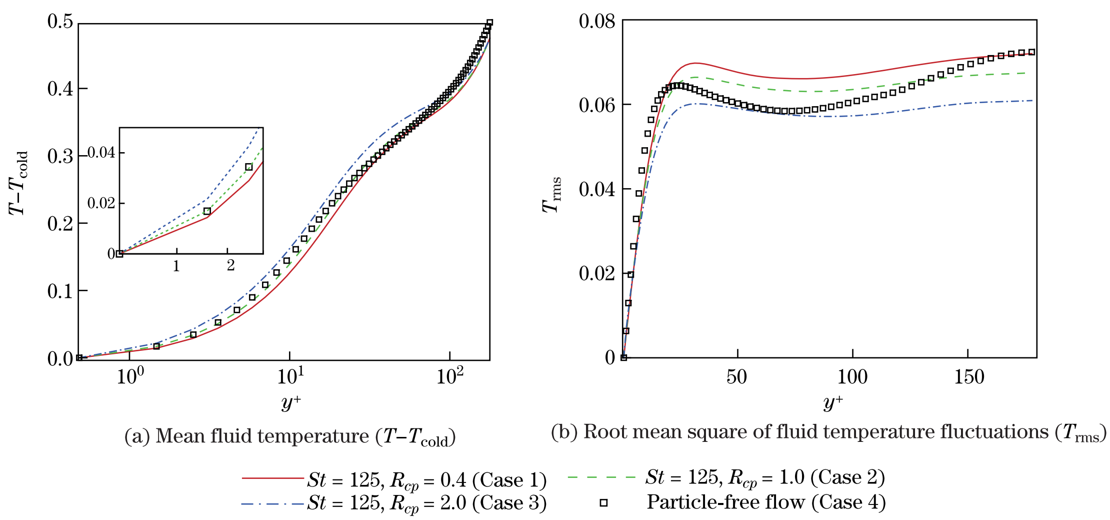


Fig. 1 Mean fluid temperature ($T - T_{\mathrm{cold}}$) and root mean square of fluid temperature fluctuations (T_{rms})

3.2 Effect of particles on wall-normal heat flux

The modification of the wall-normal heat flux induced by particles is investigated on comparison against a particle-free flow (Case 4). The expression for the total wall-normal heat flux in the particle-laden flow can be given as follows:

$$q(y) = \underbrace{\frac{\partial \langle T \rangle}{\partial y}}_{q_v(y)} \underbrace{- Re_\tau Pr \langle T' u_2' \rangle}_{q_{\mathrm{T}}(y)} + \underbrace{Re_\tau Pr \int_{-1}^{y} \langle Q_{\mathrm{p}} \rangle \mathrm{d}y}_{q_{\mathrm{p}}(y)}, \tag{5}$$

where T' and u_2' are the temperature and wall-normal velocity fluctuations of fluid, respectively. The total heat flux ($q(y)$) in Eq. (5) is split into three parts, i.e., the conductive heat flux ($q_\nu(y)$), the turbulent heat flux ($q_{\mathrm{T}}(y)$), and the particle feedback heat flux ($q_{\mathrm{p}}(y)$).

As shown in Fig. 2, in the viscous layer, conductive diffusion makes a dominant contribution to heat transfer. The wall-normal conductive heat flux ($q_\nu(y)$) increases gradually with the increase of particle specific heat in Fig. 2(a). In Case 1 ($St = 125$, $R_{cp} = 0.4$), the gradient of the mean temperature near the wall is decreased by the particles in Fig. 1(a), leading to a decrease in the wall-normal conductive heat flux relative to that of the particle-free flow (Case 4). However, the wall-normal conductive heat flux increases due to the increase of the mean temperature gradient along the wall-normal direction in Case 3 ($St = 125$, $R_{cp} = 2$).

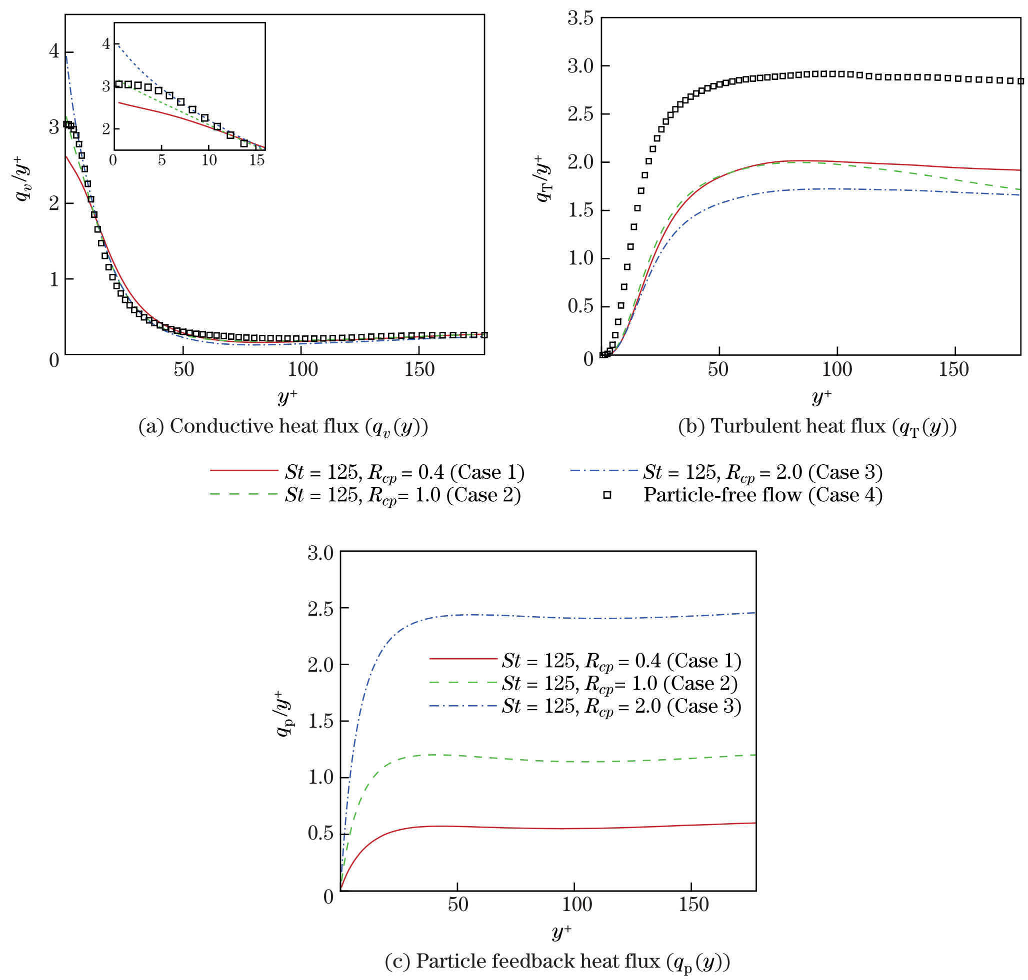


Fig. 2 Wall-normal heat fluxes in both particle-free flow and particle-laden flows

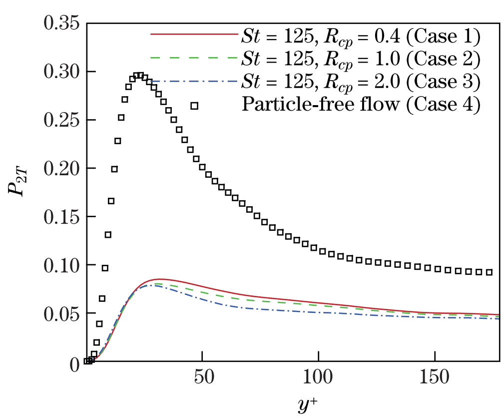
Moreover, the wall-normal turbulent heat flux ($q_{\mathrm{T}}(y)$) decreases with the increase of particle specific heat in Fig. 2(b) at the channel. It can be attributed to the decrease of wall-normal velocity fluctuation and temperature fluctuation induced by the particles. For further explanation of the effect of the particles on the wall-normal turbulent heat flux, we discuss the budget of wall-normal turbulent heat flux. The budget equation of the wall-normal turbulent heat flux is given as follows[12]:

$$\frac{\partial\langle -T'u_2'\rangle}{\partial t} = \underbrace{\langle {u_2'}^2\rangle\frac{\partial\langle T\rangle}{\partial y}}_{P_{2T}} + \underbrace{\frac{1}{Re_\tau}\Big(1+\frac{1}{Pr}\Big)\Big\langle\frac{\partial T'}{\partial x_i}\frac{\partial u_2'}{\partial x_i}\Big\rangle}_{\varepsilon_{2T}}$$

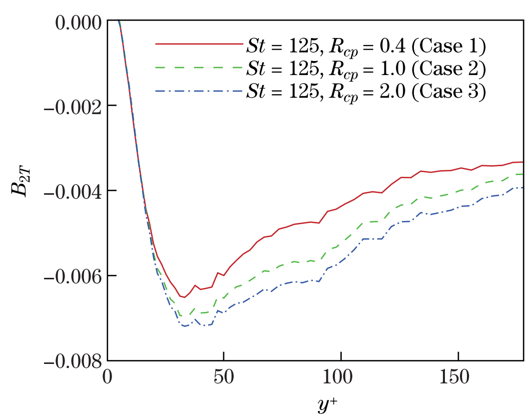
$$+ \underbrace{\Big\langle T'\frac{\partial p'}{\partial y}\Big\rangle}_{\Pi_{2T}} + \underbrace{\frac{\partial\langle {u_2'}^2 T'\rangle}{\partial y}}_{T_{2T}} \underbrace{-\frac{1}{Re_\tau}\frac{\partial}{\partial y}\Big\langle T'\frac{\partial u_2'}{\partial y} + \frac{1}{Pr}u_2'\frac{\partial T'}{\partial y}\Big\rangle}_{D_{2T}}$$

$$\underbrace{-\langle F_{\rm p}'^2 T' + Q_{\rm p}' u_2'\rangle}_{B_{2T}}. \tag{6}$$

The terms on the right-hand side of Eq. (6) denote the production (P_{2T}), dissipation (ε_{2T}), pressure-gradient correlation (Π_{2T}), turbulent diffusion (T_{2T}), viscous diffusion (D_{2T}), and particle feedback diffusion (B_{2T}). The production term is dominant for the budget of wall-normal turbulent heat flux, and it reflects the intensity of wall-normal turbulent heat flux. The particle feedback diffusion reflects the contribution of the particles to the wall-normal turbulent heat flux. It can be found in Eq. (6) that the positive values of the particle feedback diffusion indicate that the particles have positive contribution to the wall-normal turbulent heat flux, and its negative values represent the particles that have negative contribution to wall-normal turbulent heat flux. Therefore, we mainly focus on the production term (P_{2T}) and the particle feedback diffusion (B_{2T}). Figure 3(a) shows that the presence of the particles severely restrains the production term, and the reduction of the production term becomes more obvious with the increase of particle specific heat in Fig. 3(a). Therefore, the reduction of the wall-normal turbulent heat flux in Case 3 is larger than that in Cases 1 and 2. In addition, Fig. 3(b) shows that the particle feedback diffusions have negative values, indicating that the particles have negative contribution to the budget equation of the wall-normal turbulent heat flux. This is consistent with the decrease of the wall-normal turbulent heat flux ($q_{\rm T}(y)$).



(a) Production term P_{2T}



(b) Particle feedback term B_{2T}

Fig. 3 Budget terms of wall-normal turbulent heat flux

The thermal transport of turbulent convection is closely related to the sweep and ejection motions of turbulence. Therefore, we use quadrant analysis to obtain the quantitative information of bursting events (sweeps and ejections). Each quadrant is defined as follows[13]. Q_2 event ($u_1' < 0, u_2' > 0$) and Q_4 event ($u_1' > 0, u_2' < 0$) correspond to ejections and sweeps, respectively. Obviously, Q_2 and Q_4 events contribute to positive productions of the Reynolds stress and the turbulent kinetic energy (TKE). Q_1 event ($u_1' > 0, u_2' > 0$) and Q_3 event ($u_1' < 0, u_2' < 0$) are the negative contribution of TKE. Figure 4 shows that Q_2 and Q_4 events in the particle-laden flow decrease relative to that of the particle-free flow, indicating that the intensity of sweep and ejection motions near the wall is suppressed by the particles. Therefore, the heat transfer capacity of turbulent convection from the near-wall region towards the logarithmic region reduces.

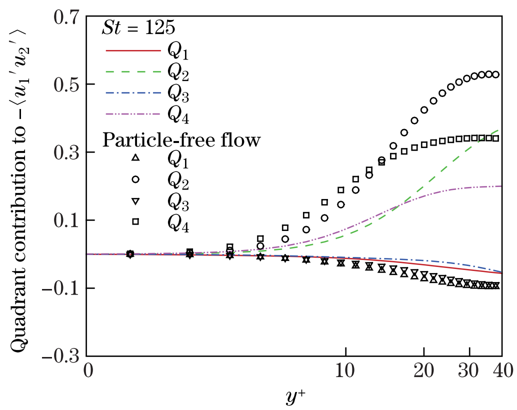


Fig. 4 Quadrant contributions to Reynolds stress where lines represent particle-laden flow and symbols represent particle-free flow

However, in Cases 2 ($St = 125$, $R_{cp} = 1$) and 3 ($St = 125$, $R_{cp} = 2$), the wall-normal turbulent heat flux ($q_{\mathrm{T}}(y)$) decreases relative to that in the particle-free flow (Case 4) in Fig. 2(b), while the total wall-normal heat flux ($q(y)$) increases ($q(y) \approx 3.4$ for Case 2, $q(y) \approx 4.3$ for Case 3, and $q(y) \approx 3.1$ for Case 4). The wall-normal particle feedback heat flux ($q_{\mathrm{p}}(y)$) (see Fig. 2(c)) is larger than the reduction of the wall-normal turbulent heat flux. The essence of thermal feedback is heat exchange between the particles and the surrounding turbulence. As the particle specific heat increases, the particle thermal response time becomes longer. Thus, the difference of temperature between the particles and the surrounding fluid increases in Fig. 5(a). That further makes the heat exchange (Q_{p}) between two phases become more active, as shown in Fig. 5(b). Therefore, the wall-normal particle feedback heat flux ($q_{\mathrm{p}}(y)$) increases as R_{cp} increases in Fig. 2(c).

3.3 Nusselt number

The Nusselt number is an important parameter to characterize the capacity of heat transfer. It measures here the heat transfer capacity from the hot wall to the cold wall in the channel flow. For the particle-laden turbulent flow, the Nusselt number (Nu) can be decomposed into three parts[7],

$$Nu = \underbrace{1}_{Nu_\nu} + \underbrace{Re_\tau Pr \int_{-1}^{1} -\langle T' u_2' \rangle \,\mathrm{d}y}_{Nu_{\mathrm{T}}} \\ + \underbrace{Re_\tau Pr \int_{-1}^{1} \int_{-1}^{y} \langle Q_{\mathrm{p}} \rangle \mathrm{d}y\mathrm{d}y,}_{Nu_{\mathrm{p}}} \tag{7}$$

where the three components of contribution are expressed as Nu_{ν}, Nu_{T}, and Nu_{p}, due to conductive contribution, turbulent contribution, and particle feedback contribution, respectively. Table 2 shows different behaviors of Nusselt number for the particle-laden turbulent flow (Cases 1, 2, 3, and 4). It is found that the contribution from turbulent convective motion on heat transfer (Nu_{T}) becomes smaller when the particle specific heat increases. However, the total Nusselt number (Nu) and the particle feedback Nusselt number (Nu_{p}) monotonically increase with the increase of the particle specific heat. As presented in Table 2, the increase of the total Nusselt number (Nu) can be attributed to the increase of the feedback Nusselt number (Nu_{p}).

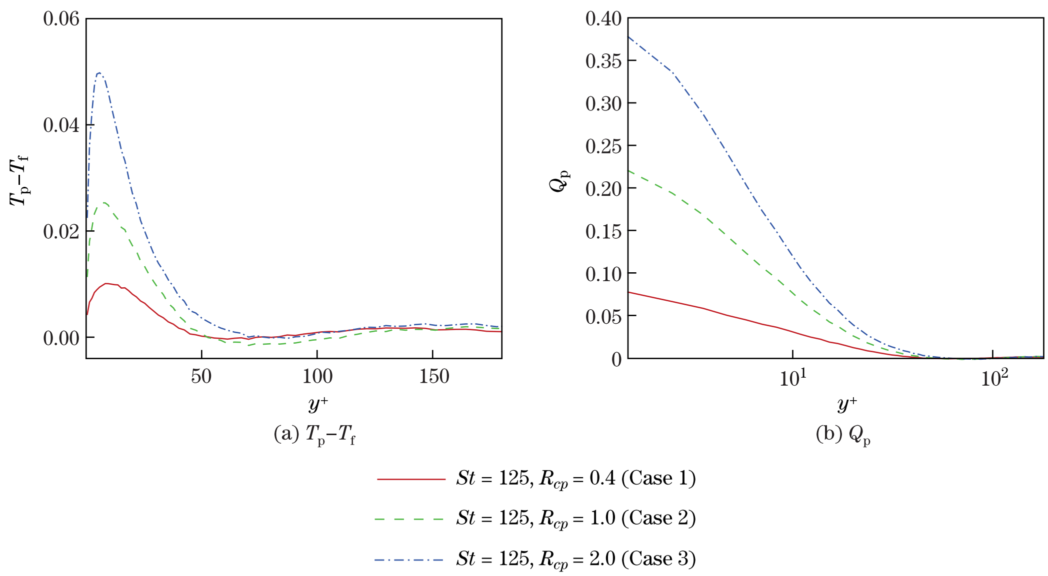


Fig. 5 Relative temperature difference between particles and surrounding fluid ($T_{\mathrm{p}}-T_{\mathrm{f}}$) and thermal feedback coupling terms by particles (Q_{p})

Table 2 Different contributions to Nusselt number for four cases

Case	Nu	Nu_{ν}	Nu_{T}	Nu_{p}
1	5.46	1.00	3.39	1.07
2	6.56	1.00	3.26	2.30
3	8.54	1.00	2.93	4.61
4	6.17	1.00	5.17	—

We compare the Nusselt number in the particle-laden flow with that in the particle-free flow (Case 4). It is seen that the total Nusselt number (Nu) in Case 1 ($St = 125$, $R_{cp} = 0.4$) decreases by about 12% relative to that in the particle-free flow, while it increases by about 40% in Case 3 ($St = 125$, $R_{cp} = 2$). It indicates that the efficiency of heat transfer in the particle-laden flow relative to that in the particle-free flow can be enhanced when the particles with high heat capacity are added to flows. In addition, it is found in Case 3 that the particle feedback Nusselt number ($Nu_{\mathrm{p}} = 4.61$) is larger than the turbulent Nusselt number ($Nu_{\mathrm{T}} = 2.93$). It indicates that the contribution to heat transfer from the particle feedback is dominant and more important than the turbulent contribution in the particle-laden flow with large specific heat particles.

4 Conclusions

In this paper, heat transfer modification of the particle-laden turbulent channel flow is investigated using the DNS and the Lagrangian particle tracking based on a point-force approximation. The study focuses on the effect of the particle specific heat on the properties of temperature fields. The results show that the particle-laden flow with different specific heat particles exhibits different characteristics of thermodynamics. By comparison with the results of particle-free flow, the normal-wall turbulent heat flux ($q_{\mathrm{T}}(y)$) decreases gradually in the particle-laden flow with the increase of particle specific heat due to the decrease of temperature fluctuation. Moreover, the wall-normal turbulent heat flux is associated with the bursting motion of turbulence. We find that the particles reduce the intensity of bursting motion of turbulence in the particle-laden flow relative to the particle-free flow, which results in the suppression of heat transfer of turbulent convection in the particle-laden flow. In addition, the total normal-wall heat flux ($q(y)$) increases in the particle-laden flow with large specific heat particles relative to that in the particle-free flow. The reason is that the wall-normal particle feedback heat flux ($q_{\mathrm{p}}(y)$) is larger than the reduction of the wall-normal turbulent heat flux ($q_{\mathrm{T}}(y)$).

We further analyze the Nusselt number to investigate the efficiency of the heat transfer between the hot and cold walls. The results indicate that the total Nusselt number (Nu) and its particle feedback part (Nu_{p}) increase with the increase of the particle specific heat, while a decreasing trend is observed for the turbulent part (Nu_{T}). In addition, we find that the efficiency of heat transfer in the particle-laden flow with $R_{cp} = 2$ particles increases by about 40% relative to that in the particle-free flow, while it decreases by about 12% in the particle-laden flow with $R_{cp} = 0.4$ particles.

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