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Effect of surface tension and viscosity on bubble growth of single mode Rayleigh-Taylor instability*

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Abstract Based on the Zufiria theoretical model, a new model regarding the asymptotic bubble velocity for the Rayleigh-Taylor (RT) instability is presented by use of the complex velocity potential proposed by Sohn. The proposed model is an extension of the ordinary Zufiria model and can deal with non-ideal fluids. With the control variable method, the effect of the viscosity and surface tension on the bubble growth rate of the RT instability is studied. The result is consistent with Cao's result if we only consider the viscous effect and with Xia's result if we only consider the surface tension effect. The asymptotic bubble velocity predicted by the Zufiria model is smaller than that predicted by the Layzer model, and the result from the Zufiria model is much closer to White's experimental data.

Key words viscosity, surface tension, Rayleigh-Taylor (RT) instability, Zufiria model

Chinese Library Classification O351, O359, O361
2010 Mathematics Subject Classification 76T10, 76E17, 76W05

Nomenclature

A,	Atwood number;	U,	asymptotic bubble velocity;
Bo,	Bond number;	η,	amplitude of bubble;
F,	complex velocity potential;	θ,	stream function;
Fr,	Froude number;	λ,	wave length;
g,	gravity acceleration;	μ,	dynamic viscosity coefficient;
k,	wave number;	ν,	kinetic viscosity coefficient;
R,	curvature radius of bubble;	ρ,	fluid density;
Re,	Reynolds number;	σ,	surface tension;
Q,	source strength;	ϕ,	velocity potential.

1 Introduction

A gravity-driven interfacial instability is known as the Rayleigh-Taylor (RT) instability. The RT instability plays an important role in the astrophysics and inertial confinement fusion. Up to now, a number of theoretical models have been proposed for the nonlinear bubble evolution

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of single mode RT instability[1–2], among which the Layzer model[3] and the Zufiria model[4] are the most common and typical ones.

For the Zufiria model, the bubble is considered as a point source in a uniform free stream, and we can get the detailed development process of the bubble instability based on the complex velocity potential. In fact, the Zufiria model was first proposed for the vacuum bubble. Then, Sohn[5–6] generalized it to the arbitrary Atwood number. Sohn[5] found that the bubble velocity in the RT instability converged to a constant limit, and bubbles attained constant asymptotic curvatures. Sohn[6] performed numerical simulations by using the point vortex method to study the effect of viscosity and surface tension on the bubble growth of the RT instability, and found that both the surface tension and the viscosity decreased the asymptotic bubble velocity.

The Layzer model, which is also based on the potential flow theory, can predict the velocity potential near the bubble tip, and therefore it can describe the bubble development process. However, numerical results are very few for the RT instability with the surface tension for a single mode case. Cao et al.[7] studied the effect of viscosity on the bubble growth rate. Their experimental results showed that the fluid viscosity depressed the bubble velocity, but did not affect the bubble curvature. Xia et al.[8] investigated the Layzer model which was extended to non-ideal fluids, and the effects of the surface tension on the RT instability were investigated. Their results indicated that the surface tension depressed the bubble velocity, but did not affect the bubble curvature. Li and Luo[9] investigated the effect of both viscosity and surface tension on the bubble growth of the RT instability based on Khan's model. They derived the two-dimensional governing equations of bubble movement for non-ideal magnetic fluids, and asymptotic solutions were given for different bubble velocities.

In this paper, we further study the effect of surface tension and viscosity on the single mode RT instability for the Zufiria model. The method is based on complex velocity potentials proposed by Sohn, and this model is generalized into non-ideal fluids. In the following sections, the Zufiria model is analyzed theoretically, and then comparative studies between the Zufiria model, the Layer model of Sohn, and White's experimental data[10] are performed.

2 Bubble velocity in Zufiria model

2.1 Theoretical model

Based on the Zufiria model, we assume that there is a point source under the bubble tip with the strength Q, as shown in Fig. 1. In Fig. 1, g is the gravity acceleration, H is the distance from the point source to the bubble tip, and R is the curvature radius of the bubble. We consider two irrotational fluids in a vertical pipe with the width λ and the infinite length, and the heavy fluid is on the top of the light one. The complex velocity potentials for the heavy and light fluids can be written as

$$F_{\mathrm{h}}(z) = \phi_{\mathrm{h}} + \mathrm{i}\theta_{\mathrm{h}}, \tag{1}$$

$$F_{\mathrm{l}}(z) = \phi_{\mathrm{l}} + \mathrm{i}\theta_{\mathrm{l}}, \tag{2}$$

where ϕ_{h} and ϕ_{l} are the velocity potentials for heavy and light fluids, and θ_{h} and θ_{l} are the stream functions for these two fluids, respectively.

In the dynamic coordinate (x, y), as shown in Fig. 1, the interface near the bubble tip can be represented as

$$z(t) = y(t) + \mathrm{i}x(t), \tag{3}$$

and this interface can also be written as[7]

$$\delta(x, y, t) = x^2 + 2R(t)y = 0 \tag{4}$$

with consideration of the shape of the interface.

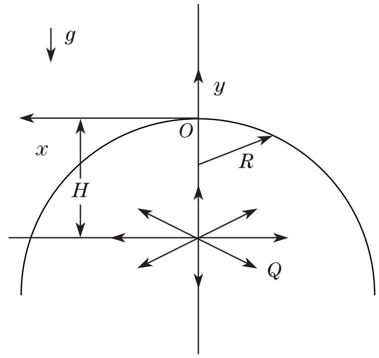

Fig. 1 Illustration of bubble tip for Zufiria model

The governing equations for the interface evolution are

$$\frac{\mathrm{D}\delta}{\mathrm{D}t} = 2x\upsilon_x + 2\frac{\mathrm{D}R}{\mathrm{D}t}y + 2R\upsilon_y = 0, \tag{5}$$

$$\frac{\partial\phi_\mathrm{h}}{\partial t} + \frac{1}{2}(\nabla\phi_\mathrm{h})^2 + \Big(\frac{\mathrm{d}U}{\mathrm{d}t} + g\Big)y + \frac{P_\mathrm{h}}{\rho_\mathrm{h}} = \frac{\rho_\mathrm{l}}{\rho_\mathrm{h}}\Big(\frac{\partial\phi_\mathrm{l}}{\partial t} + \frac{1}{2}(\nabla\phi_\mathrm{l})^2 + \Big(\frac{\mathrm{d}U}{\mathrm{d}t} + g\Big)y + \frac{P_\mathrm{l}}{\rho_\mathrm{l}}\Big), \tag{6}$$

where $\frac{\mathrm{D}}{\mathrm{D}t}$ is the total derivative, υ_x and υ_y are the velocities at the x- and y-directions, respectively, U is the velocity at the bubble tip, P_h and P_l are the normal stresses of heavy and light fluids, respectively, and ρ_h and ρ_l are the densities of heavy and light fluids, respectively. If the surface tension and viscosity of the fluid are considered, the normal stress balance on the interface is given by[6]

$$[\![P]\!] = 2\Big[\!\!\Big[\mu\frac{\partial\upsilon}{\partial y}\Big]\!\!\Big] - \sigma\frac{\eta_{xx}}{(1+\eta_x^2)^{\frac{3}{2}}}, \tag{7}$$

where $[\![D]\!] = D_\mathrm{h} - D_\mathrm{l}$, μ is the coefficient of fluid viscosity, σ is the surface tension of interface, and η is the amplitude of bubble.

The dynamic equation for the interface can be derived from Eqs. (6) and (7),

$$\begin{aligned}&\frac{\partial\phi_\mathrm{h}}{\partial t} + \frac{1}{2}(\nabla\phi_\mathrm{h})^2 + \Big(\frac{\mathrm{d}U}{\mathrm{d}t} + g\Big)y\\ &= \frac{\rho_\mathrm{l}}{\rho_\mathrm{h}}\Big(\frac{\partial\phi_\mathrm{l}}{\partial t} + \frac{1}{2}(\nabla\phi_\mathrm{l})^2 + \Big(\frac{\mathrm{d}U}{\mathrm{d}t} + g\Big)y\Big) - \frac{1}{\rho_\mathrm{h}}\Big(2\Big[\!\!\Big[\mu\frac{\partial\upsilon}{\partial y}\Big]\!\!\Big] - \sigma\frac{\eta_{xx}}{(1+\eta_x^2)^{\frac{3}{2}}}\Big).\end{aligned} \tag{8}$$

Therefore, the evolution of the interface is determined by Eqs. (5) and (8).

We take the complex velocity potentials[5],

$$F_\mathrm{h}(z) = Q_\mathrm{h}\ln(1 - \mathrm{e}^{-k(z+H)}) - Uz, \tag{9}$$

$$F_\mathrm{l}(z) = Q_\mathrm{l}\ln(1 - \mathrm{e}^{-k(z-H)}) + (K-U)z, \tag{10}$$

where Q_h and Q_l are the source strengths of the heavy and light fluid, respectively, $U-K$ represents the uniform flow, K is the source velocity of light fluid, and k is the wave number.

By expanding Eqs. (9) and (10), we have

$$F_{\mathrm{h}}(z) = Q_{\mathrm{h}} \sum_{n=0}^{\infty} \frac{C_{\mathrm{h}n}}{n!} z^n - Uz, \tag{11}$$

$$F_{\mathrm{l}}(z) = Q_{\mathrm{l}} \sum_{n=0}^{\infty} \frac{C_{\mathrm{l}n}}{n!} z^n + (K - U)z, \tag{12}$$

where $C_{\mathrm{h}n}$ and $C_{\mathrm{l}n}$ are the expanding coefficients for heavy and light fluids, respectively. According Ref. [7], we know

$$\frac{\mathrm{d}F}{\mathrm{d}z} = \upsilon_y - \mathrm{i}\upsilon_x. \tag{13}$$

When Eq. (13) is combined with Eqs. (9) and (10), we can get the horizontal and vertical velocities of the heavy and light fluids, i.e., $\upsilon_{\mathrm{h}x}$,$\upsilon_{\mathrm{h}y}$, $\upsilon_{\mathrm{l}x}$, and $\upsilon_{\mathrm{l}y}$. By substituting these velocities into Eq. (5) and expanding these to the first order of x, we get the following equations:

$$C_{\mathrm{h}1} Q_{\mathrm{h}} - U = 0, \tag{14}$$

$$C_{\mathrm{l}1} Q_{\mathrm{l}} + K - U = 0, \tag{15}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} + 3C_{\mathrm{h}2} R Q_{\mathrm{h}} + C_{\mathrm{h}3} R^2 Q_{\mathrm{h}} = 0, \tag{16}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} + 3C_{\mathrm{l}2} R Q_{\mathrm{l}} + C_{\mathrm{l}3} R^2 Q_{\mathrm{l}} = 0. \tag{17}$$

Moreover, by expanding F_{h} and F_{l} to x^5, we have the velocity potentials ϕ_{h} and ϕ_{l} for heavy and light fluids, respectively. By substituting ϕ_{h} and ϕ_{l} into Eq. (8) and expanding these to the first order of x, we get the following equations:

$$\begin{aligned}
&(C_{\mathrm{h}1} + C_{\mathrm{h}2} R)\frac{\mathrm{d}Q_{\mathrm{h}}}{\mathrm{d}t} + Q_{\mathrm{h}}(C_{\mathrm{h}2} + C_{\mathrm{h}3} R)\frac{\mathrm{d}H}{\mathrm{d}t} - C_{\mathrm{h}2}^2 Q_{\mathrm{h}}^2 R + 2\frac{\mu_{\mathrm{h}}}{\rho_{\mathrm{h}}} C_{\mathrm{h}3} Q_{\mathrm{h}} - \frac{3\sigma}{\rho_{\mathrm{h}} R^2} + g \\
= &\frac{\rho_{\mathrm{l}}}{\rho_{\mathrm{h}}}\Big((C_{\mathrm{l}1} + C_{\mathrm{l}2} R)\frac{\mathrm{d}Q_{\mathrm{l}}}{\mathrm{d}t} - Q_{\mathrm{l}}(C_{\mathrm{l}2} + C_{\mathrm{l}3} R)\frac{\mathrm{d}H}{\mathrm{d}t} \\
&+ \frac{\mathrm{d}K}{\mathrm{d}t} - C_{\mathrm{l}2}^2 Q_{\mathrm{l}}^2 R + 2\frac{\mu_{\mathrm{l}}}{\rho_{\mathrm{l}}} C_{\mathrm{l}3} Q_{\mathrm{l}} + g\Big),
\end{aligned} \tag{18}$$

$$\begin{aligned}
&\Big(\frac{C_{\mathrm{h}2}}{2} + C_{\mathrm{h}3} R + \frac{C_{\mathrm{h}4} R^2}{6}\Big)\frac{\mathrm{d}Q_{\mathrm{h}}}{\mathrm{d}t} + Q_{\mathrm{h}}\Big(\frac{C_{\mathrm{h}3}}{2} + C_{\mathrm{h}4} R + \frac{C_{\mathrm{h}5} R^2}{6}\Big)\frac{\mathrm{d}H}{\mathrm{d}t} + \frac{1}{2}P_1 \\
= &\frac{\rho_{\mathrm{l}}}{\rho_{\mathrm{h}}}\Big(\Big(\frac{C_{\mathrm{l}2}}{2} + C_{\mathrm{l}3} R + \frac{C_{\mathrm{l}4} R^2}{6}\Big)\frac{\mathrm{d}Q_{\mathrm{l}}}{\mathrm{d}t} \\
&- Q_{\mathrm{l}}\Big(\frac{C_{\mathrm{l}3}}{2} + C_{\mathrm{l}4} R + \frac{C_{\mathrm{l}5} R^2}{6}\Big)\frac{\mathrm{d}H}{\mathrm{d}t} + \frac{1}{2}P_2\Big),
\end{aligned} \tag{19}$$

where

$$P_1 = Q_{\mathrm{h}}^2\Big(C_{\mathrm{h}2}^2 + C_{\mathrm{h}3}^2 R^2 - 2C_{\mathrm{h}2}C_{\mathrm{h}3}R - \frac{4}{3}C_{\mathrm{h}2}C_{\mathrm{h}3}R^2\Big), \tag{20}$$

$$P_2 = Q_{\mathrm{l}}^2\Big(C_{\mathrm{l}2}^2 + C_{\mathrm{l}3}^2 R^2 - 2C_{\mathrm{l}2}C_{\mathrm{l}3}R - \frac{4}{3}C_{\mathrm{l}2}C_{\mathrm{l}3}R^2\Big), \tag{21}$$

and the expanding coefficients $C_{\mathrm{h}n}$ and $C_{\mathrm{l}n}$ $(n = 1, 2, \cdots, 5)$ have the following forms:

$$\begin{cases} C_{\mathrm{h}1} = \dfrac{k}{\mathrm{e}^{kH} - 1}, \\ C_{\mathrm{h}2} = \dfrac{-k^2\mathrm{e}^{kH}}{(\mathrm{e}^{kH} - 1)^2}, \\ C_{\mathrm{h}3} = \dfrac{k^3\mathrm{e}^{kH}(\mathrm{e}^{kH} + 1)}{(\mathrm{e}^{kH} - 1)^3}, \\ C_{\mathrm{h}4} = \dfrac{-k^4\mathrm{e}^{kH}(\mathrm{e}^{2kH} + 4\mathrm{e}^{kH} + 1)}{(\mathrm{e}^{kH} - 1)^4}, \\ C_{\mathrm{h}5} = \dfrac{k^5\mathrm{e}^{kH}(\mathrm{e}^{3kH} + 11\mathrm{e}^{2kH} + 11\mathrm{e}^{kH} + 1)}{(\mathrm{e}^{kH} - 1)^5}, \\ C_{\mathrm{l}1} = -C_{\mathrm{h}1}\mathrm{e}^{kH}, \\ C_{\mathrm{l}2} = -C_{\mathrm{h}2}, \\ C_{\mathrm{l}3} = C_{\mathrm{h}3}, \quad C_{\mathrm{l}4} = -C_{\mathrm{h}4}, \quad C_{\mathrm{l}5} = C_{\mathrm{h}5}. \end{cases} \tag{22}$$

The evolution of the bubble is determined by Eqs. (14)–(19). In the final stage of the RT instability development, the time derivatives of all the variables in Eqs. (14)–(17) are zero. Therefore,

$$3C_{\mathrm{h}2} + C_{\mathrm{h}3}R = 0, \tag{23}$$

$$Q_{\mathrm{l}} = 0. \tag{24}$$

Substituting Eq. (21) into Eqs. (18) and (19) yields

$$C_{\mathrm{h}2}^2 Q_{\mathrm{h}}^2 R - 2\frac{\mu_{\mathrm{h}}}{\rho_{\mathrm{h}}}(C_{\mathrm{h}3} + C_{\mathrm{h}4}R)Q_{\mathrm{h}} = \frac{2A}{A+1}g - \frac{\sigma}{\rho_{\mathrm{h}}}\frac{3}{R^2}, \tag{25}$$

$$C_{\mathrm{h}2}^2 + C_{\mathrm{h}3}^2 R^2 - 2C_{\mathrm{h}2}C_{\mathrm{h}3}R - \frac{4}{3}C_{\mathrm{h}2}C_{\mathrm{h}4}R^2 = 0, \tag{26}$$

where $A = (\rho_{\mathrm{h}} - \rho_{\mathrm{l}})/\rho_{\mathrm{h}} + \rho_{\mathrm{l}})$ is the Atwood number, and μ_{h} is the dynamic viscosity of heavy fluid.

We can also get the following equations by combining Eq. (20) with Eq. (23):

$$R = \frac{\sqrt{3}}{k}, \tag{27}$$

$$\mathrm{e}^{kH} = 2 + \sqrt{3}. \tag{28}$$

$C_{\mathrm{h}1}$ to $C_{\mathrm{h}4}$ can be calculated by substituting Eq. (26) into Eq. (22),

$$C_{\mathrm{h}1} = \frac{k}{1+\sqrt{3}}, \quad C_{\mathrm{h}2} = -\frac{k^2}{2}, \quad C_{\mathrm{h}3} = \frac{\sqrt{3}}{2}k^3, \quad C_{\mathrm{h}4} = -2k^4. \tag{29}$$

Finally, by substituting Eq. (29) into Eq. (25) and combining with Eq. (14), we get the asymptotic bubble velocity U in the following form:

$$U = -\frac{6k\nu_{\mathrm{h}}}{1+\sqrt{3}} + \sqrt{\Big(\frac{6k\nu_{\mathrm{h}}}{1+\sqrt{3}}\Big)^2 + \frac{6+4\sqrt{3}}{7+4\sqrt{3}}\Big(\frac{A}{A+1}\frac{2g}{3k} - \frac{\sigma}{\rho_{\mathrm{h}}}\frac{k}{3}\Big)}, \tag{30}$$

where $\nu_{\mathrm{h}} = \mu_{\mathrm{h}}/\rho_{\mathrm{h}}$ is the kinetic viscosity of heavy fluid.

This velocity can also be expressed in the nondimensional form,

$$Fr = -\frac{12\pi}{(1+\sqrt{3})Re} + \sqrt{\Big(\frac{12\pi}{1+\sqrt{3}Re}\Big)^2 + \frac{6+4\sqrt{3}}{7+4\sqrt{3}}\Big(\frac{A}{3\pi(A+1)} - \frac{2\pi}{3Bo}\Big)}. \tag{31}$$

Denote the Froude number as $Fr = U/\sqrt{g\lambda}$, the gravity Reynolds number as $Re = \sqrt{g\lambda^3}/\nu_{\mathrm{h}}$, and the Bond number as $Bo = \rho_{\mathrm{h}} g\lambda^2/\sigma$.

In this paper, both the viscosity and the surface tension effects are considered regarding to the bubble growth. From Eq. (31), we can find that our result is consistent with the result from Ref. [7] if we only consider the viscosity effect, and it is also consistent with the result from Ref. [8] if we neglect the viscosity effect.

2.2 Results

Figure 2 shows the relationship between the bubble Froude number Fr and the Reynolds number Re for the Atwood number $A = 0.5$ in the logarithmic scale. As shown in Fig. 2, the asymptotic bubble velocity is greatly affected by the viscosity when $Re \leqslant 10^3$. However, when $Re > 10^3$, Fr keeps unchanged with the increase of Re, which means that the bubble velocity saturates to a constant value. We can also find that the surface tension can reduce the asymptotic bubble velocity because the value of Fr with $Bo = \infty$ is larger than that with $Bo = 20\pi^2$.

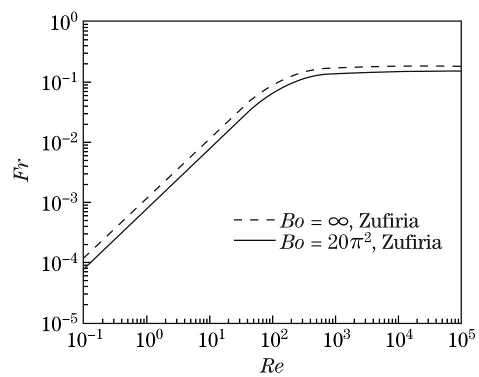

Fig. 2 Relation between Fr and Re for Zufiria model

Figures 3(a) and 3(b) show the relationship between the Froude number Fr and the Atwood number A for the Zufiria model and the Layzer model when $Bo = \infty$ and $Bo = 20\pi^2$, respectively. The solid lines are results from Sohn[6], while the dotted lines are results from Cao et al.[7]. As we can see from Fig. 3(a), the value of bubble velocity computed from the Zufiria model is smaller than that from the Layzer model for both the inviscid and the viscous flows.

As $A \to 0$, bubble velocities predicted from these two models are almost the same. However, the discrepancy increases with increasing A. This conclusion is consistent with the results from Sohn[6] and Cao et al.[7].

When $Bo = 20\pi^2$, the value of bubble velocity predicted by the Zufiria model is also smaller than that predicted by the Layzer model for both $Re = 1\,000$ and $Re = \infty$, as shown in Fig. 3(b). Moreover, for $Re = 1\,000$, the bubble velocity remains zero at the beginning. As $A \to 0.05$, the bubble velocity begins to increase for the Layzer model. For the Zufiria model, the bubble velocity begins to increase when $A > 0.1$.

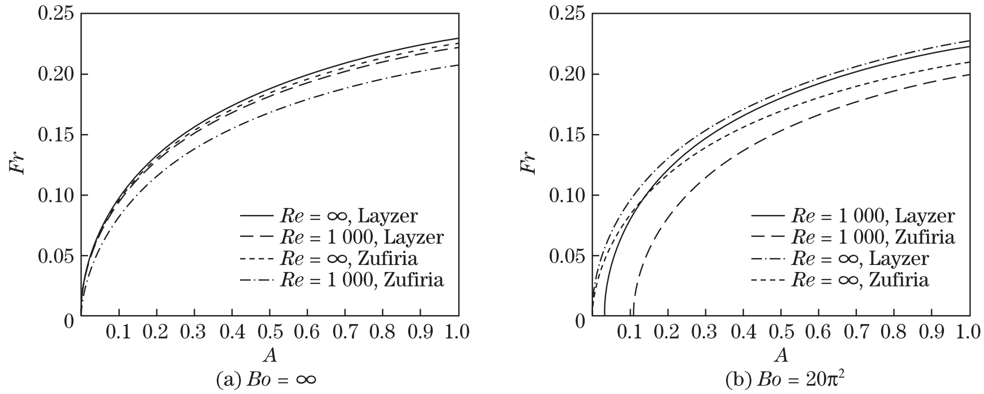

Fig. 3 Changes of Fr as function of Atwood number

In order to compare our results with the existing experimental data, the following physical properties[10] are used in our model to compute the asymptotic velocity: the viscosity coefficient of heavy fluid $\mu_{\mathrm{h}} = 5 \times 10^{-3}$ Pa·s, the density of heavy fluid $\rho_{\mathrm{h}} = 2\,735\,\mathrm{kg/m^3}$, the wave length $\lambda = 2.12\,\mathrm{cm}$, the gravity acceleration $g = 980\,\mathrm{cm/s^2}$, and the Atwood number $A = 0.99$. Figure 4 shows the asymptotic bubble velocity as a function of time. The solid line in Fig. 4 is computed from the Zufiria model, the dash line is from the Layzer model, and the solid circle is the experimental result. As we can see from Fig. 4, the values of velocity predicted by both models are larger than the experimental values, and the result of the Zufiria model is closer to the experimental result. This probably is because of the introduction of the complex velocity potential for the Zufiria model, which may cause the artificial velocity diffusion.

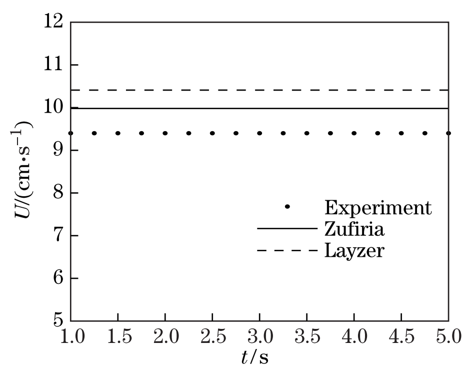

Fig. 4 Velocity predicted by Zufiria model, Layzer model, and experimental method

3 Conclusions

In this paper, we present an analytical model for the asymptotic velocity and curvature of the bubble of single mode RT instability. The new model is based on the Zufiria model and considers the effects of viscosity and surface tension on the bubble growth rate of RT instability. The differences between the Zufiria model and the Layzer model are analyzed. Our results indicate that the asymptotic bubble velocity is decreased with the increase of viscosity and the surface tension. The value of asymptotic velocity predicted by the Zufiria model is always smaller than that of the Layzer model, and the result of the Zufiria model is closer to White's experimental result.

Future work will focus on the following two aspects:

(i) Study the effect of fluid vorticity on the RT instability based on the Zufiria model.

(ii) Study the combination effect of all the factors that affect the interface evolution process by use of both the Zufiria and Layzer models.

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