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Optimal bounded control for maximizing reliability of Duhem hysteretic systems*

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Abstract The optimal bounded control of stochastic-excited systems with Duhem hysteretic components for maximizing system reliability is investigated. The Duhem hysteretic force is transformed to energy-depending damping and stiffness by the energy dissipation balance technique. The controlled system is transformed to the equivalent non-hysteretic system. Stochastic averaging is then implemented to obtain the Itô stochastic equation associated with the total energy of the vibrating system, appropriate for evaluating system responses. Dynamical programming equations for maximizing system reliability are formulated by the dynamical programming principle. The optimal bounded control is derived from the maximization condition in the dynamical programming equation. Finally, the conditional reliability function and mean time of first-passage failure of the optimal Duhem systems are numerically solved from the Kolmogorov equations. The proposed procedure is illustrated with a representative example.

Key words optimal bounded control, reliability, Duhem hysteretic system, stochastic dynamical programming principle

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1 Introduction

Many structural components, such as passive vibration absorber and piezoelectric ceramics, exhibit hysteresis. Many civil and mechanical engineering structures equipped with these components show hysteretic behaviors under serious time-varying loading, such as typhoon, earthquake, and high-intensity noise[1–2]. The hysteretic restoring force is determined by both the instantaneous and past states of the deformation, and a hysteretic loop can be created between the hysteretic restoring force and the displacement under periodic movement. Many kinds of mathematical models have been proposed to characterize the relationship of the hysteretic force and the displacement, including the bi-linear model, the Ramberg-Osgood model,

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Iwan's distributed element model, Ozdemir's model, and so on[3–7]. In all these hysteretic models, the Duhem differential model[8–10] is versatile to cover most existing hysteresis models and can describe the hysteretic behavior more accurately.

Various stochastic optimal control strategies have been proposed[11–13]. However, in mechanical and structural engineering fields, only the linear quadratic Gaussian (LQG) control strategy has been widely applied. In the past decade, an optimal control strategy[14] for nonlinear systems has been developed for the quasi Hamiltonian random system with external and/or parametric excitations by Zhu et al.[15]. The strategy has been applied to semi-active control, bounded control, and robust control for uncertain systems[16]. The LQG control strategy is usually used to reduce the system response or to enhance the system stability, while the optimal control strategy for stochastic systems can also be used to maximize the system reliability, i.e., minimize the first-passage failure[17]. The reliability is described by the probability of the structure in the safety domain during a specified time interval. The bigger the probability is, the better the system reliability is. The design of the optimal control strategy is aimed to enhancing the reliability and making the system safer, i.e., searching the optimal control to enlarge the probability in the safety domain. Almost all the works, however, concentrate on the non-hysteretic systems, and the only work on maximizing the reliability of hysteretic systems is contributed to the Bouc-Wen system[18].

In the present study, the process of deriving the optimal feedback control in order to maximize the reliability of the Duhem system is given. The Duhem hysteretic model is briefly presented. The controlled hysteretic system is transformed to the controlled nonlinear system without hysteresis, and then the controlled system energy's Itô differential equation is deduced according to the theorem of stochastic averaging. The dynamical programming equations with the corresponding solution conditions for the control problem of maximizing system reliability are formulated based on the dynamical programming principle. Finally, the conditional reliability functions and the mean time of the first-passage failure are obtained from the Kolmogorov equation. A tall building, which is simplified as an equivalent single-degree-of freedom system subjected to non-white noise excitations, is discussed to validate the procedures' efficacy.

2 Duhem hysteretic model

The Duhem model for hysteresis satisfies the following equations:

$$\dot{z} = g(x, z, \mathrm{sgn}(\dot{x}))\dot{x} = g_1(x, z)\dot{x}_+ - g_2(x, z)\dot{x}_- = \begin{cases} g_1(x, z)\dot{x}, & \dot{x} > 0, \\ g_2(x, z)\dot{x}, & \dot{x} < 0, \end{cases} \tag{1a}$$

$$\dot{x}_+ = \frac{|\dot{x}| + \dot{x}}{2}, \quad \dot{x}_- = \frac{|\dot{x}| - \dot{x}}{2}, \tag{1b}$$

where z and x denote the hysteretic force and the displacement, respectively, and g_1 and g_2 are continuous functions. According to the Duhem model (1a) and (1b), the hysteretic force is determined by g_1 for $\dot{x} > 0$ and g_2 for $\dot{x} < 0$. The corresponding hysteresis loop in the xz-plane consists of two parts, i.e., the ascending line $z_1(x)$ for $\dot{x} > 0$ and the descending line $z_2(x)$ for $\dot{x} < 0$. Both the ascending line and the descending line are independent of the magnitude of velocity $\dot{x}$. The hysteretic force on the ascending line or the descending line is determined by the instantaneous system state, and the local history due to the last changes in the velocity direction is independent of the displacement history before the change. Thus, the Duhem hysteresis model has the characteristics of local memory.

For anti-symmetric Duhem hysteresis models (as shown in Fig. 1), $g_2(x, z) = g_1(-x, -z)$, $z_1(x) = -z_2(-x)$, $a_1 = a_2$, and $x_{10} = x_{20} = x_0$. In this case, the hysteretic force can be replaced by an elastic part and an inelastic part, i.e., $z_1(x) = z^{\mathrm{e}} + z_1^{\mathrm{p}}$, and $z_2(x) = z^{\mathrm{e}} + z_2^{\mathrm{p}}$, in which the superscripts e and p denote elastic part and inelastic part, respectively. The potential energy can be represented as

$$U(x)=\int_0^x z^{\mathrm{e}}(x_1)\mathrm{d}x_1+\int_{-x_0}^{x} z_1^{\mathrm{p}}(x_1)\mathrm{d}x_1,\quad -a\leqslant x\leqslant -x_0, \tag{2a}$$

$$U(x)=\int_0^x z^{\mathrm{e}}(x_1)\mathrm{d}x_1+\int_{x_0}^{(z_2^{\mathrm{p}})^{-1}z_1^{\mathrm{p}}(x)} z_2^{\mathrm{p}}(x_1)\mathrm{d}x_1,\quad -x_0\leqslant x\leqslant a. \tag{2b}$$

The area of the hysteresis loop A_{r} represents the dissipated energy in one cycle by the hysteresis component, which can be written as

$$A_{\mathrm{r}}=\oint z^{\mathrm{p}}(x)\mathrm{d}x=2\int_{-a}^{a} z_1(x)\mathrm{d}x. \tag{3}$$

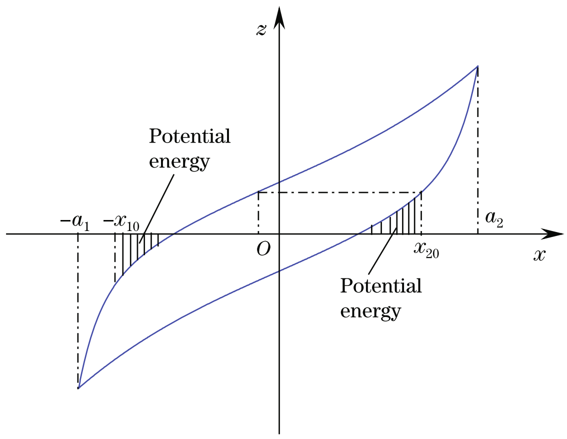

Fig. 1 Representative of Duhem hysteresis loop

In the versatile Duhem model, there is a class of hysteresis called the integrable Duhem hysteresis when the functions g_1 and g_2 make Eqs. (1a) and (1b) be analytically integrable. The integrable Duhem model is quite general, and it includes many existing hysteresis models, such as the Bouc-Wen model and the Yar-Hammond bilinear model[3,6]. Therefore, the integrable Duhem model is used in the present work.

3 Stochastic averaging of controlled Duhem hysteretic system

Discuss a one-degree-of-freedom Duhem hysteretic system subjected to random excitations and control. The equation is

$$\ddot{X}+2\zeta\dot{X}+Z(X,\dot{X})=f_j(X,\dot{X})\xi_j+u,\quad u\in\overline{U}, \tag{4}$$

where X, ζ, and Z denote the system displacement, the damping coefficient, and the Duhem hysteretic restoring force, respectively, $f_j(X,\dot{X})$ represent the amplitudes of external and/or parametric random excitations, which are continuous and differentiable functions of the displacement and the velocity, the zero mean processes $\xi_j(t)$ are stationary excitations with correlation functions $R_{jk}(\tau)=E(\xi_j(t)\xi_k(t+\tau))$, u denotes the external control force in the admissible set of $\overline{U}$, and the subscript "j" is a dummy index indicating a summation with the index running through its domain.

By adopting the energy dissipation balance technique[19], the hysteretic damping effect can be approximated by a damping term provided that the energy dissipations in one period are equal[10]. Then, the Duhem hysteretic system (4) with Eqs. (1a) and (1b) can be substituted by the non-hysteretic nonlinear random system,

$$\ddot{X} + (2\zeta + 2\zeta_1(H))\dot{X} + \frac{\partial U}{\partial X} = f_j(X, \dot{X})\xi_j + u, \tag{5}$$

in which

$$H = \frac{\dot{x}^2}{2} + U(x) \tag{6}$$

represents the system energy, and $2\zeta_1$ is the equivalent quasi-linear damping coefficient, which can be evaluated by the following formula:

$$2\zeta_1(H) = \frac{A_{\mathrm{r}}}{2\int_{-a}^{a}\sqrt{2H - 2U}\mathrm{d}x}. \tag{7}$$

Introduce transformations to the equivalent non-hysteretic nonlinear system[20],

$$\mathrm{sgn}(X)\sqrt{U(X)} = \sqrt{H}\cos\varphi, \quad \dot{X} = -\sqrt{2H}\sin\varphi, \quad 0 \leqslant \varphi < 2\pi. \tag{8}$$

Equation (5) can be substituted by the first-order differential equation for the system energy and phase,

$$\dot{H} = -2H\sin^2\varphi(2\zeta + 2\zeta_1(H)) - u\sqrt{2H}\sin\varphi - \sqrt{2H}\sin\varphi f_j(X, \dot{X})\xi_j, \tag{9a}$$

$$\dot{\varphi} = \frac{1}{\sqrt{2H}}\Big(-\sqrt{2H}\sin\varphi\cos\varphi(2\zeta + 2\zeta_1(H)) + \frac{\frac{\partial U}{\partial X}}{\cos\varphi} - u\cos\varphi\Big) - \frac{\cos\varphi}{\sqrt{2H}}f_j(X, \dot{X})\xi_j. \tag{9b}$$

For the case that the damping and excitations are weak, the system energy can be approximated as a Markov diffusion process[21–22]. Performing time averaging in Eq. (9a) yields the associated Itô equation,

$$\mathrm{d}H = \Big(m(H) + \Big\langle\frac{\partial H}{\partial \dot{X}}u\Big\rangle\Big)\mathrm{d}t + \sigma(H)\mathrm{d}B(t), \tag{10}$$

where $B(t)$ is a unit Wiener process. The drift coefficient $m(H)$ and the diffusion coefficient $\sigma^2(H)$ are as follows:

$$\begin{aligned} m(H) = \Big\langle & -2H\sin^2\varphi(2\zeta + 2\zeta_1(H)) \\ & + \int_{-\infty}^{0}\Big((\sqrt{2H}\sin\varphi f_j(H,\varphi))_{t+\tau}\frac{\partial}{\partial H}(\sqrt{2H}\sin\varphi f_k(H,\varphi))_t \\ & + \Big(\frac{\cos\varphi}{\sqrt{2H}}f_j(H,\varphi)\Big)_{t+\tau}\frac{\partial}{\partial\varphi}(\sqrt{2H}\sin\varphi f_k(H,\varphi))_t\Big)R_{jk}(\tau)\mathrm{d}\tau\Big\rangle_t, \end{aligned} \tag{11}$$

$$\sigma^2(H) = \Big\langle\int_{-\infty}^{\infty}(\sqrt{2H}\sin\varphi f_j(H,\varphi))_{t+\tau}(\sqrt{2H}\sin\varphi f_k(H,\varphi))_t R_{jk}(\tau)\mathrm{d}\tau\Big\rangle_t, \tag{12}$$

in which $\langle\cdot\rangle_t$ represents time averaging.

4 Optimal bounded control strategy for maximizing system reliability

The system energy $H(t)$ slowly varies in the interval $[0, \infty)$. The initial state of the system is set as $H(0) = H_0$ in the prescribed safe interval $[0, H_{\mathrm{c}})$. Once the system energy $H(t)$ is not in $[0, H_c)$, the system fails. The reliability describes the probability of system in $[0, H_c)$ within the specified time interval, which is the crucial concept. In the following part of the paper, the procedures of establishing control strategy to enhance the system reliability of Duhem hysteretic systems are illustrated.

Select the reliability function as the performance index of the controlled system (10),

$$J(u) = P\left\{H(t,u) \in [0, H_c), 0 \leqslant t \leqslant t_f\right\}. \tag{13}$$

Give the function $V(H,t)$, a value function, as follows:

$$V(H,t) = \sup_{u\in\overline{U}} P\{H(s,u) \in [0,H_c), t < s \leqslant t_f | H(t,u) \in [0,H_c)\}. \tag{14}$$

Based on Bellman' programming principle[11], the dynamical programming equation with respect to $V(H,t)$ derived from the system (10) can be written as

$$\sup_{u\in\overline{U}}\Big(\frac{\partial}{\partial t}+\Big(m(H)+u\frac{\partial H}{\partial \dot{X}}\Big)\frac{\partial}{\partial H}+\frac{1}{2}\sigma^2(H)\frac{\partial^2}{\partial H^2}\Big)V(t,H) = 0, \quad 0 \leqslant t \leqslant T, \quad H \in (0, H_c). \tag{15}$$

The solution conditions of Eq. (15) are

$$V(H_c, t) = 0, \tag{16}$$

$$V(0,t) = \text{finite}, \tag{17}$$

$$V(H,T) = 1, \quad H < H_c. \tag{18}$$

Equations (15) and (18) constitute the definite problem to determine the control strategy for maximizing system reliability.

The control constraint is of the following form:

$$|u| \leqslant b, \quad b > 0. \tag{19}$$

Clearly, the term $u(\frac{\partial H}{\partial \dot{X}})(\frac{\partial V}{\partial H})$ in Eq. (15) is maximal when $|u| = b$, and each term $u(\frac{\partial H}{\partial \dot{X}})(\frac{\partial V}{\partial H})$ is positive. Thus, the optimal control forces can be written as

$$u^* = b\,\mathrm{sgn}\Big(\frac{\partial H}{\partial \dot{X}}\frac{\partial V}{\partial H}\Big), \tag{20}$$

where $\mathrm{sgn}(\cdot)$ is the sign function. It can be seen that the reliability function increases as the system energy H decreases[17], i.e., $\frac{\partial V}{\partial H} < 0$. Thus, Eq. (20) can be rewritten as

$$u^* = -b\,\mathrm{sgn}\Big(\frac{\partial H}{\partial \dot{X}}\Big) = -b\,\mathrm{sgn}(\dot{X}). \tag{21}$$

5 Backward Kolmogorov equation

By inserting Eq. (21) into Eq. (10) for replacing u and averaging the term $u\frac{\partial H}{\partial \dot{X}}$, the following stochastic equation is derived:

$$\mathrm{d}H = (\overline{m}(H))\mathrm{d}t + \sigma(H)\mathrm{d}B(t), \tag{22}$$

where

$$\overline{m}(H)=\Big\langle -2H\sin^2\varphi(2\zeta+2\zeta_1(H))-\sqrt{2H}\,b\,\mathrm{sgn}(\sin\varphi)\sin\varphi$$
$$+\int_{-\infty}^{0}\Big((\sqrt{2H}\sin\varphi f_j(H,\varphi))_{t+\tau}\frac{\partial}{\partial H}(\sqrt{2H}\sin\varphi f_k(H,\varphi))_t$$
$$+\Big(\frac{\cos\varphi}{\sqrt{2H}}f_j(H,\varphi)\Big)_{t+\tau}\frac{\partial}{\partial\varphi}(\sqrt{2H}\sin\varphi f_k(H,\varphi))_t\Big)R_{jk}(\tau)\mathrm{d}\tau\Big\rangle_t, \tag{23a}$$

$$\sigma^2(H)=\Big\langle\int_{-\infty}^{+\infty}(\sqrt{2H}\sin\varphi f_j(H,\varphi))_{t+\tau}(\sqrt{2H}\sin\varphi f_k(H,\varphi))_t R_{jk}(\tau)\mathrm{d}\tau\Big\rangle_t. \tag{23b}$$

The conditional reliability function for the controlled system (22) $R(t|H_0)=P\{H(\tau,u^*)\in[0,H_\mathrm{c}),\ \tau\in(0,t]|H_0\in[0,H_\mathrm{c})\}$ satisfies the following Kolmogorov deferential equation[17]:

$$\frac{\partial R(t|H_0)}{\partial t}=\overline{m}(H_0)\frac{\partial R(t|H_0)}{\partial H_0}+\frac{\sigma^2(H_0)}{2}\frac{\partial^2 R(t|H_0)}{\partial H_0^2}, \tag{24}$$

where $\overline{m}(H_0)$ and $\sigma^2(H_0)$ are obtained from Eqs. (23a) and (23b) with H replaced by H_0. The initial condition of Eq. (24) is

$$R(0|H_0)=1,\quad H_0\in(0,H_\mathrm{c}), \tag{25}$$

and the associated boundary conditions are

$$R(t|H_\mathrm{c})=0, \tag{26a}$$

$$R(t|0)=\text{finite}. \tag{26b}$$

The one-dimensional initial-boundary value problem, Eqs. (24)–(26), can be calculated numerically.

The conditional probability density of the first-passage time is the gradient of the conditional reliability function,

$$p(T|H_0)=-\frac{\partial R}{\partial t}\Big|_{t=T}. \tag{27}$$

Then, the mean of first-passage time (t_MFP) can be obtained from $p(T|H_0)$ through Eq. (27),

$$t_\mathrm{MFP}(T|H_0)=\int_0^T tp(t|H_0)\mathrm{d}t. \tag{28}$$

The relative increase of t_MFP, which reflects the capacity and effects of the optimal bounded controller to enlarge t_MFP, is used to evaluate the control effectiveness and defined as

$$K=\frac{|(t_\mathrm{MFP}(T|H_0))_\mathrm{u}-(t_\mathrm{MFP}(T|H_0))_\mathrm{c}|}{(t_\mathrm{MFP}(T|H_0))_\mathrm{u}}, \tag{29}$$

where the subscripts “u” and “c” are the uncontrolled and controlled situation, respectively, and $|\cdot|$ represents the absolute value of “$\cdot$”. t_MFP of the controlled system is obtained from Eq. (28), and that of the uncontrolled system is obtained from Eqs. (22)–(28) by omitting the control term, i.e., replacing $\overline{m}(H_0)$ in Eq. (23a) by $m(H_0)$ in Eq. (11). Obviously, a higher K indicates a better control strategy.

6 Example

Consider a tall building under random excitations. The hysteretic behavior plays an important role in the structural response and cannot be ignored. It is reasonable that the building is idealized as a hysteresis column supporting[23] (as shown in Fig. 2). The transverse motion of the column is the following partial differential equation: $EI\frac{\partial^4 W}{\partial y^4} + F_1(t)\frac{\partial^2 W}{\partial y^2} + m\ddot{W} + c\dot{W} = F_2(t)$, where m denotes the mass of column per unit length, and c is the damping coefficient. Assume that W is dominated by the first vibration mode, $W(y,t) = X(t)\sin(\frac{\pi y}{l})$. Then, the non-dimensional equation of the controlled column-mass model is

$$\ddot{X} + 2\zeta\dot{X} + Z(X,\dot{X}) = \xi_1(t) + X\xi_2(t) + u, \tag{30}$$

where $\xi_i(t)\,(i=1,2)$ are the independent non-white noise excitations, and the intensities are $2D_i$. The ascending function of the anti-symmetric integrable Duhem hysteresis model with nonlinear elasticity is taken as[10]

$$g_1(x,z) = k_1 + 3k_3x^2 + \frac{\gamma}{\beta}(1 - \beta(z - k_1x - k_3x^3)), \tag{31}$$

where k_1 and k_3 are linear and nonlinear stiffnesses, and α, β, and γ are hysteresis constants. The hysteretic force z is obtained from Eqs. (1a) and (1b),

$$z_1(x) = k_1x + k_3x^3 + \frac{1}{\beta}(1 - \mathrm{e}^{-\gamma(x+x_0)}), \quad \dot{x} > 0, \tag{32a}$$

$$z_2(x) = k_1x + k_3x^3 - \frac{1}{\beta}(1 - \mathrm{e}^{\gamma(x-x_0)}), \quad \dot{x} < 0. \tag{32b}$$

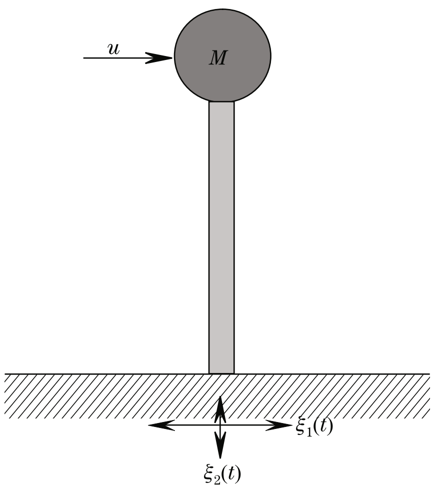

Fig. 2 Tall building subjected to random excitations and control

The potential energy and the dissipated energy of hysteresis component in one cycle are, respectively,

$$U(x) = \frac{1}{2}k_1x^2 + \frac{1}{4}k_3x^4 + \frac{1}{\beta}(x+x_0) + \frac{1}{\beta\gamma}(\mathrm{e}^{-\gamma(x+x_0)} - 1), \quad -a \leqslant x < -x_0, \tag{33a}$$

$$U(x) = \frac{1}{2}k_1x^2 + \frac{1}{4}k_3x^4 - \frac{1}{\beta\gamma}\ln(2 - \mathrm{e}^{-\gamma(x+x_0)}) + \frac{1}{\beta\gamma}(1 - \mathrm{e}^{-\gamma(x+x_0)}), \quad -x_0 \leqslant x \leqslant a, \tag{33b}$$

$$A_{\mathrm{r}} = \frac{4}{\beta\gamma}(1 + a\gamma) - \frac{4}{\beta\gamma}\mathrm{e}^{\gamma(a-x_0)}, \tag{34}$$

in which the residual hysteresis displacement x_0 and the displacement amplitude a for certain H are determined by

$$x_0 = -a + \frac{1}{\gamma}\ln\frac{1+\mathrm{e}^{2a\gamma}}{2}, \tag{35a}$$

$$H = \frac{1}{2}k_1a^2 + \frac{1}{4}k_3a^4 - \frac{1}{\beta}(a - x_0) + \frac{1}{\beta\gamma}(\mathrm{e}^{\gamma(a-x_0)} - 1). \tag{35b}$$

The system energy H is the averaged Itô equation for the controlled system in the form of Eq. (22), and the associated drift and associated diffusion coefficients of the system (30) are, respectively[24],

$$\overline{m}(H) = \frac{1}{T(H)}\Big(-4\zeta\int_{-a}^{a}\sqrt{2H-2U(x)}\mathrm{d}x - A_{\mathrm{r}} + 2D_2\int_{-a}^{a}\frac{x^2}{\sqrt{2H-2U(x)}}\mathrm{d}x\Big) + D_1 - \frac{ab}{T(H)}, \tag{36a}$$

$$\sigma^2(H) = \frac{2}{T(H)}\int_{-a}^{a}(2D_1 + 2D_2x^2)\sqrt{2H-2U(x)}\mathrm{d}x. \tag{36b}$$

The conditional reliability function $R(t|H_0)$ for the system (30) is determined by Eq. (24). The corresponding initial and boundary conditions are given in Eqs. (25) and (26), in which $\overline{m}(H_0)$ and $\sigma^2(H_0)$ coincide with Eqs. (36a) and (36b) except that H is replaced by H_0. The relations of the function $p(T|H_0)$ and the function $R(t|H_0)$ have been shown in Eq. (27). The function $R(t|H_0)$, the function $p(T|H_0)$, the mean time of the first-passage $t_{\mathrm{MFP}}(T|H_0)$, and the control effectiveness K can be evaluated by the numerical technique.

Suppose that the boundary of system first-passage failure is $H_{\mathrm{c}} = 0.5$, and the other parameter values are $\gamma = 2.0$, $\beta = 0.3$, $D_1 = 0.2$, $D_2 = 0.05$, $k_1 = 2.0$, $k_3 = 0.05$, and $H_0 = 0$. Equation (24) combined with its initial-boundary conditions in Eqs. (25) and (26) can be solved. The numerical results for the $R(t|H_0)$ function, the $p(T|H_0)$ function, and the function $t_{\mathrm{MFP}}(T|H_0)$ are shown in Figs. 3–5, respectively. It can be seen from Figs. 3–5 that the control strategy greatly enhances the system reliability and t_{MFP} of the Duhem system. When the magnitude b of the control force decreases, the reliability decreases more quickly, and t_{MFP} decreases. The effectiveness of the optimal bounded control strategy is shown in Fig. 6. The control efficacy decreases as the control magnitude b increases.

7 Conclusions

In this study, the optimal bounded control of Duhem hysteretic systems for maximizing the reliability is developed. The controlled hysteretic system is substituted by a controlled nonlinear system without hysteresis, and the controlled system energy's Itô equation is deduced. The optimal bounded control for enlarging system reliability is obtained by the dynamical programming principle. Then, the conditional reliability function governed by the Kolmogorov equation is obtained. The results associated with the system reliability of uncontrolled and controlled systems obtained from the present procedure and the results from the Monte-Carlo simulation are in good agreement. Also, it can be concluded that the reliability of Duhem hysteretic systems will be really enhanced by the optimal bounded control. Finally, we note that the proposed technique cannot be adopted to derive the optimal unbounded control strategy, since the unbounded control strategy makes the dynamical programming equation unsolvable.

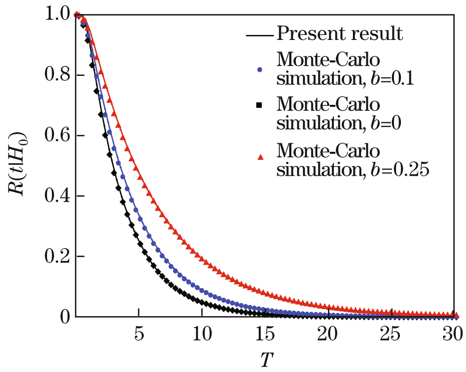

Fig. 3 Conditional reliability functions $R(t|H_0)$ of uncontrolled and optimally-controlled systems (30)

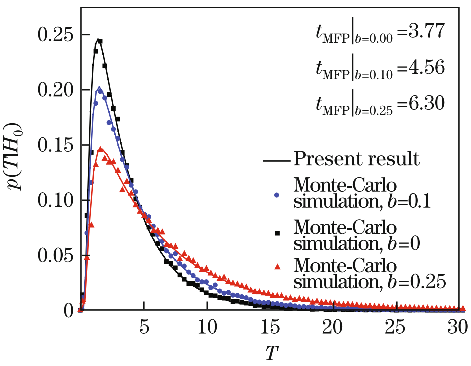

Fig. 4 Conditional probability densities $p(T|H_0)$ of first-passage time of uncontrolled and optimally-controlled systems (30)

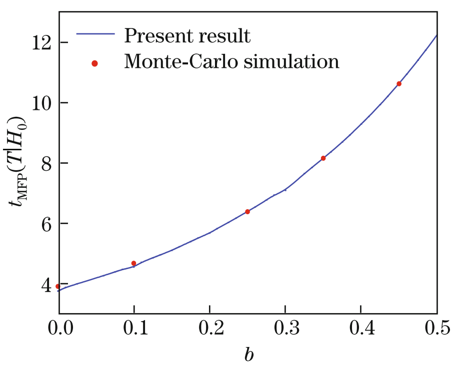

Fig. 5 Mean of first-passage time $t_{\mathrm{MFP}}(T|H_0)$ of optimally-controlled system (30) with different values of control magnitude b

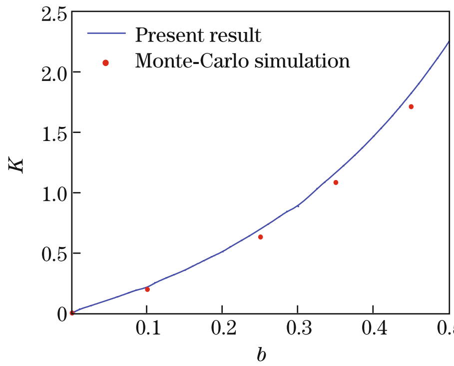

Fig. 6 Control effectiveness K versus control magnitude b

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