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Throughflow and g-jitter effects on binary fluid saturated porous medium*

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Abstract A non-autonomous complex Ginzburg-Landau equation (CGLE) for the finite amplitude of convection is derived, and a method is presented here to determine the amplitude of this convection with a weakly nonlinear thermal instability for an oscillatory mode under throughflow and gravity modulation. Only infinitesimal disturbances are considered. The disturbances in velocity, temperature, and solutal fields are treated by a perturbation expansion in powers of the amplitude of the applied gravity field. Throughflow can stabilize or destabilize the system for stress free and isothermal boundary conditions. The Nusselt and Sherwood numbers are obtained numerically to present the results of heat and mass transfer. It is found that throughflow and gravity modulation can be used alternately to heat and mass transfer. Further, oscillatory flow, rather than stationary flow, enhances heat and mass transfer.

Key words weakly nonlinear theory, throughflow, complex Ginzburg-Landau equation (CGLE), gravity modulation

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Nomenclature

Latin symbols

A,	amplitude of convection;
δ,	amplitude of gravity modulation;
g,	acceleration due to gravity;
R_0,	critical Rayleigh number;
d,	depth of fluid layer;
q,	fluid velocity;
(x, z),	horizontal and vertical coordinates;
Nu,	Nusselt number;
Pe,	Péclet number, $Pe = \frac{w_0 d}{\kappa_{\mathrm{T}}}$;
p,	reduced pressure;
Sh,	Sherwood number;
ΔS,	solutal difference across porous media;
Rs,	solutal Rayleigh number, $Rs = \frac{\beta_{\mathrm{S}} g \Delta S d K}{\nu \kappa_{\mathrm{S}}}$;
T,	temperature;
ΔT,	temperature difference across porous media;
Ra,	thermal Rayleigh number, $Ra = \frac{\alpha_{\mathrm{T}} g \Delta T d K}{\nu \kappa_{\mathrm{T}}}$;
t,	time;
a,	wavenumber.

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Greek symbols

α_{T}, coefficient of thermal expansion;
Γ, diffusivity ratio, $\Gamma = \frac{\kappa_{\mathrm{S}}}{\kappa_{\mathrm{T}}}$;
ω, dimensionless oscillatory frequency;
μ, dynamic viscosity of fluid;
κ_{T}, effective thermal diffusivity;
ρ, fluid density;
Ω, frequency of modulation;
γ, heat capacity ratio;
ν, kinematic viscosity, $\nu = \frac{\mu}{\rho_0}$;
χ, perturbation parameter;
ε, porosity of porous media;
s, slow time;
β_{S}, solutal expansion coefficient;
ψ, stream function;
$\hat{k}$, vertical unit vector.

Other symbols

∇^2, $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

Subscripts

b, basic state;
c, critical;
0, reference value.

Superscripts

$*$, dimensionless quantity;
$'$, perturbed quantity.

1 Introduction

The problem of thermal instability in porous media was well documented by Vafai[1–2], Pop and Ingham[3], Ingham and Pop[4], Vadász[5], and Nield and Bejan[6]. The concept of regulating convective instabilities is an important topic in thermal and engineering sciences. Such regulations like thermal, gravitational, rotational, and magnetic field modulations can be used in order to control convection. Davis[7] pointed out that the dynamics of stabilization and destabilization may lead to dramatic changes of behavior depending on the proper tuning of the amplitude and frequency of the modulation. If an imposed modulation can destabilize an otherwise stable state, then there is a major enhancement of heat/mass/momentum transport. If an imposed modulation can stabilize an otherwise unstable state, then higher efficiencies can be attained in various processing techniques. The present paper considers the effect of gravity modulation (where the time-periodic gravity modulation in this problem can be realized by vertically oscillating the porous media). Thus, related to gravity modulation, the studies by Gresho and Sani[8] and Clever et al.[9] showed that the gravity modulation acts on the entire volume of fluid and may have a stabilizing or destabilizing effect depending on the amplitude and frequency of the forcing. Similar studies related to gravity modulation were done by Malashetty and Padmavathi[10], Yang[11], Bhadauria et al.[12], Bhadauria[13], Bhadauria et al.[14], Bhadauria and Kiran[15–16], Bhadauria et al.[17], and Kiran[18].

Convection concerns the process of combined heat and mass transfer which are driven by buoyancy forces and are usually referred as double diffusive convection. In this case, the mass friction gradient and the temperature gradient are independent. In some practical problems, such as seawater flow, mantle flow in the Earth's crust, in devicing an effective method (Shivakumara and Khalili[19]) of disposing waste material and extraction of energy and engineering applications, the double diffusive convection plays an important role. The linear and nonlinear stabilities of double diffusive convection in porous media have been studied extensively in the presence of uniform temperature and concentration gradients by Nield and Bejan[6] and Shivakumara and Sumithra[20]. Siddheshwar et al.[21] investigated temperature and gravity modulation effects on double diffusive convection in porous media. They found that both modulations can be used simultaneously to enhance or diminish heat and mass transfer in the system while considering a weakly nonlinear theory for stationary mode. Bhadauria[22] also analyzed the additional effects of internal heating and anisotropy of Siddheshwar et al.[21]. He found that internal heat and anisotropy also can be used to enhance or diminish heat and mass transfer in the system. Malashetty et al.[23] studied the effect of rotation on double diffusive convection while considering the linear theory for onset of convection and the nonlinear theory for finite amplitude convection. The effect of double diffusive magnetoconvection under thermal modulation was investigated by Bhadauria and Kiran[24] for stationary mode of convection. Bhadauria and Kiran[25] investigated double diffusive magnetoconvection under the effects of gravity modulation and chaotic oscillatory mode of convection. They found that gravity modulation can be used to control thermal instability and dynamics of the problem with suitable ranges of modulation parameters.

The throughflow effect on double diffusive convection in porous media is an important concept due to its applications in engineering, geophysics, and seabed hydrodynamics. Throughflow plays an important role in the directional solidification of concentrated alloys, in which the mushy zone exists and it is regarded as a porous layer with double diffusive origin. The basic state temperature profile of throughflow changes from linear to nonlinear with layer height, which in turn affects the stability of the system significantly. The effect of throughflow on the onset of convection in a horizontal porous layer has been studied by Wooding[26], Sutton[27], Homsy and Sherwood[28], and Jones and Persichetti[29]. Nield[30] and Shivakumara[31] showed that a small amount of throughflow can have a destabilizing effect if the boundaries are of different types and a physical explanation has been given. They also found that the effect of throughflow is not invariably stabilizing and depends on the nature of the boundaries. Khalili and Shivakumara[32] investigated the effect of throughflow and internal heat generation on the onset of convection in porous media. They showed that throughflow destabilizes the system, even if the boundaries are of the same type, a result which is not true in the absence of an internal heat source. The non-Darcian effects on convective instability in porous media with throughflow were investigated in order to account for inertia and boundary effects by Shivakumara[33]. The effect of throughflow on the stability of double diffusive convection in a porous layer was investigated by Shivakumara and Khalili[19] for different types of hydrodynamic boundary conditions. They found that throughflow is destabilizing even if the lower and upper boundaries are of the same type and stabilizing as well as destabilizing, irrespective of its direction, when the boundaries are of different types. Khalili and Shivakumara[34] investigated throughflow in the porous layer governed by the Darcy-Forchheimer equation and the Beavers-Joseph condition was applied at the interface of fluid and the porous layer. They found that destabilization arises due to throughflow, and the ratio of fluid layer thickness to porous layer thickness plays an important role in deciding the stability of the system depending on the Prandtl number. Hill[35] investigated linear and nonlinear thermal instabilities of vertical throughflow in a fluid-saturated porous layer, while Hill et al.[36] extended the problem for penetrative convection by considering density to be quadratic in temperature. Brevdo and Ruderman[37–38] analyzed convective instability in porous media with inclined temperature gradient and vertical throughflow.

The effects of quadratic drag and vertical throughflow on double diffusive convection in a horizontal porous layer using the Forchheimer-extended Darcy equation were investigated by Shivakumara and Nanjundappa[39]. The boundaries of the porous layer are considered to be either impermeable or porous, but perfect conductors of heat and solute concentration. They found that irrespective of the nature of boundaries, a small amount of throughflow in either of its direction destabilizes the system, a result which is in contrast to the single component system. Shivakumara and Sureshkumar[40] used the linear study to investigate the convective instability in a horizontal porous medium with viscoelastic fluid of Oldroyd-B type in the presence of vertical throughflow. They found that the effect of throughflow is to suppress the oscillatory convection independent of its direction when the velocity boundary conditions are of the same type. In contrast to this, throughflow in one particular direction augments oscillatory convection if the velocity boundary conditions are not of the same type. Brevdo[41] analyzed the nature of unstable three-dimensional localized disturbances at the onset of convection with inclined temperature gradient and vertical throughflow. He found that the destabilization or convective instability depends on the values of the horizontal Rayleigh number and the Péclet number. It is shown that for marginally supercritical values of the vertical Rayleigh number, the destabilization has the character of absolute instability in all the cases in which the horizontal Rayleigh number is zero or the Péclet number is zero. The vertical throughflow with viscous dissipation in a horizontal porous layer was studied by Barletta et al.[42]. They showed that although generally weak, the effect of viscous dissipation yields an increase in the critical value of the Darcy-Rayleigh number for downward throughflow and a decrease in the case of upward throughflow. The effect of vertical throughflow on the onset of thermal convection in a horizontal layer of an electrically conducting fluid contained between two rigid permeable plates and heated from below in the presence of a uniform vertical magnetic field was studied by Reza and Gupta[43]. They found that the positive throughflow is more stabilizing than the negative throughflow. The effect of vertical heterogeneity of permeability on the onset of convection in a horizontal porous medium uniformly heated from below with vertical throughflow was studied by Nield and Kuznetsov[44] using the linear theory. It is found that to the first-order, a linear variation of the reciprocal of permeability with depth has no effect on the critical Rayleigh number based on the harmonic mean of the permeability, but a quadratic variation increasing in the upwards direction leads to a reduction in the critical Rayleigh number. The effect of vertical throughflow on the onset of convection in a composite porous medium consisting of two horizontal layers was investigated by Nield and Kuznetsov[45]. They found that throughflow has a stabilizing effect whose magnitude may be increased or decreased by the heterogeneity.

In general, throughflow is a subcomponent of interflow, for example, it is the lateral unsaturated flow of water in the soil zone, where a highly permeable geologic unit overlays a less permeable geologic unit, which returns to the surface, as return flow, prior to entering a stream or groundwater. Once water infiltrates into the soil, it is still affected by gravity and either infiltrates to the water table or travels down slope. Throughflow usually occurs during peak hydrologic events, and flow rates are dependent on the hydraulic conductivity of the geologic medium. For this problem, one needs to understand the study of throughflow under gravity modulation. Moreover, the literature shows no study on thermal instability which considers modulation along with vertical throughflow for nonlinear mode of thermal instability. The objective of the present paper is, therefore, to carry out a weakly nonlinear stability analysis of porous media with simultaneous temperature and solute concentration gradients for constant vertical throughflow. Analytic expressions for both the Nusselt and Shearwood numbers are derived from the non-autonomous complex Ginzburg-Landau equation (CGLE)[15–17,46–47] to calculate the finite amplitude. To the best of the author's knowledge till date, no study on oscillatory convection under gravity modulation with vertical throughflow is available in the literature.

2 Mathematical formulation

Consider an infinitely extended horizontal binary fluid saturated in a horizontal porous layer of the thickness d with a constant vertical throughflow of the magnitude w_0 which is either gravity aligned or otherwise in its directions. The porous layer is homogeneous and isotropic. In this problem, only two-dimensional disturbances are considered, i.e., horizontal x and vertical z directions. The graphical representation of the problem is given in Fig. 1. The porous media are heated and salted from below. The lower and upper boundaries are maintained at constant but different temperatures and solutal concentrations, respectively. Using the modified Darcy's model and the Boussinesq approximation for this system, the governing equations of the flow are given by (for the isotropic case of Bhadauria[22])

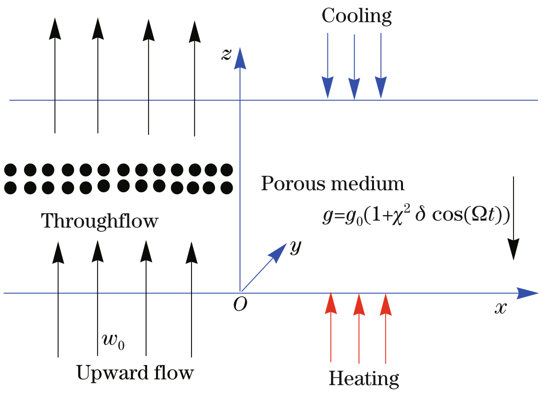


Fig. 1 Sketch of physical problem

$$\nabla \cdot q = 0, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial q}{\partial t} = -\nabla p + \rho g - \frac{\mu}{K} q, \tag{2}$$

$$\gamma \frac{\partial T}{\partial t} + (q \cdot \nabla) T = \kappa_{\mathrm{T}} \nabla^2 T, \tag{3}$$

$$\frac{\partial S}{\partial t} + (q \cdot \nabla) S = \kappa_{\mathrm{S}} \nabla^2 S, \tag{4}$$

$$\rho = \rho_0 (1 - \alpha_{\mathrm{T}}(T - T_0) + \beta_{\mathrm{S}}(S - S_0)), \tag{5}$$

where the physical variables are given in Nomenclature. The flow of fluid through the porous media can be in a steady state because there is a constant flow of fluid in the porous media. Equation (1) states that, in any steady state process, the rate at which mass enters the porous media is equal to the rate at which mass leaves the porous media. When a periodic force is applied to a system, it will typically reach the steady state after going through some transient behavior. This may often be observed in a vibrating system. The heat capacity ratio γ is taken to be 1 for simplicity of the problem. The externally imposed time dependant gravitational field, thermal, and solutal boundary conditions are given by[25]

$$g = g_0 (1 + \chi^2 \delta \cos(\Omega t)) \hat{k}, \tag{6}$$

$$\begin{cases} T = T_0 + \Delta T \quad \text{at} \quad z = 0, \\ T = T_0 \quad \text{at} \quad z = d, \\ S = S_0 + \Delta S \quad \text{at} \quad z = 0, \\ S = S_0 \quad \text{at} \quad z = d, \end{cases} \tag{7}$$

where ΔT and ΔS are the temperature difference and the solute difference across the porous media, respectively, χ is the smallness of amplitude of modulation, and δ and Ω are the amplitude and the frequency of gravity modulation, respectively.

3 Conduction state

The basic state is assumed to be quiescent, and the quantities in this state are given by

$$\begin{cases} q_{\mathrm{b}} = (0, 0, w_0), \quad \rho = \rho_{\mathrm{b}}(z, t), \quad p = p_{\mathrm{b}}(z, t), \\ T = T_{\mathrm{b}}(z, t), \quad S = S_{\mathrm{b}}(z, t). \end{cases} \tag{8}$$

Substituting Eq. (8) into Eqs. (1)–(5) yields the following relation which helps us to define hydrostatic pressure and temperature:

$$\frac{\partial p_{\mathrm{b}}}{\partial z} = \frac{\mu}{K} w_0 - \rho_{\mathrm{b}} g, \tag{9}$$

$$w_0 \frac{\partial T_{\mathrm{b}}}{\partial z} = \kappa_{\mathrm{T}} \frac{\partial^2 T_{\mathrm{b}}}{\partial z^2}, \tag{10}$$

$$w_0 \frac{\partial S_{\mathrm{b}}}{\partial z} = \kappa_{\mathrm{S}} \frac{\partial^2 S_{\mathrm{b}}}{\partial z^2}, \tag{11}$$

$$\rho_{\mathrm{b}} = \rho_0 (1 - \alpha_{\mathrm{T}} (T_{\mathrm{b}} - T_0) + \beta_{\mathrm{S}} (S_{\mathrm{b}} - S_0)). \tag{12}$$

The solutions of Eqs. (10)–(11) subject to the boundary conditions given in Eq. (7) are

$$T_{\mathrm{b}} = T_0 + \Delta T \frac{\mathrm{e}^{Pez} - \mathrm{e}^{Pe}}{1 - \mathrm{e}^{Pe}}, \tag{13}$$

$$S_{\mathrm{b}} = S_0 + \Delta S \frac{\mathrm{e}^{(Pe\Gamma^{-1})z} - \mathrm{e}^{(Pe\Gamma^{-1})}}{1 - \mathrm{e}^{(Pe\Gamma^{-1})}}. \tag{14}$$

4 Dimensionless governing equations

The finite amplitude perturbations on the basic state are superposed in the forms of

$$q = q_{\mathrm{b}} + q', \quad \rho = \rho_{\mathrm{b}} + \rho', \quad p = p_{\mathrm{b}} + p', \quad T = T_{\mathrm{b}} + T', \quad S = S_{\mathrm{b}} + S'. \tag{15}$$

Substitute the above equation (15) and the basic state temperature (Eq. (13)) and solutal (Eq. (14)) equations into Eqs. (1)–(5), and then use the stream function ψ as $u' = \frac{\partial \psi}{\partial z}$, $w' = -\frac{\partial \psi}{\partial x}$ for the two-dimensional flow. The equations are then non-dimensionalized using the following physical variables:

$$(x, y, z) = d(x^*, y^*, z^*), \quad t = \frac{d^2}{\kappa_{\mathrm{T}}} t^*, \quad \psi = \kappa_{\mathrm{T}} \psi^*, \quad T' = \Delta T T^*, \quad S' = \Delta S S^*, \quad \Omega = \frac{\kappa_{\mathrm{T}}}{d^2} \Omega^*.$$

The resulting non-dimensionalized system of equations can be expressed as (dropping the asterisk)

$$\Big(\frac{1}{Pr_{\mathrm{D}}} \frac{\partial}{\partial t} + \nabla^2 \Big) \psi = \Big(Rs \frac{\partial S}{\partial x} - Ra \frac{\partial T}{\partial x} \Big) (1 + \chi^2 \delta \cos(\Omega t)), \tag{16}$$

$$-\frac{\mathrm{d}T_{\mathrm{b}}}{\mathrm{d}z} \frac{\partial \psi}{\partial x} - \Big(\nabla^2 - Pe \frac{\partial}{\partial z} \Big) T = -\frac{\partial T}{\partial t} + \frac{\partial(\psi, T)}{\partial(x, z)}, \tag{17}$$

$$-\frac{\mathrm{d}S_{\mathrm{b}}}{\mathrm{d}z} \frac{\partial \psi}{\partial x} - \Big(\Gamma \nabla^2 - Pe\Gamma^{-1} \frac{\partial}{\partial z} \Big) S = -\frac{\partial S}{\partial t} + \frac{\partial(\psi, S)}{\partial(x, z)}, \tag{18}$$

where $Pr_{\mathrm{D}} = \frac{\varepsilon \nu d^2}{K \kappa_{\mathrm{T}}}$.

The non-dimensionalized parameters in the above equations are given in Nomenclature. It is clear from Eq. (17) and Eq. (18) that througflow and basic state profile of temperature and solutal fields affect the stability problem. The above system can be solved by considering the stress free and isothermal boundary conditions as given below

$$\psi = T = S = 0 \quad \text{at} \quad z = 0, \quad z = 1. \tag{19}$$

5 Derivation of CGLE

Introducing a small perturbation parameter χ that shows deviation from the critical state of onset of convection, the variables for a weak nonlinear state can be expanded as a power series in terms of χ as (Malkus and Veronis [48] and Venezian[49])

$$\begin{cases} Ra = R_0 + \chi^2 R_2 + \chi^4 R_4 + \dots, \\ \psi = \chi\psi_1 + \chi^2\psi_2 + \chi^3\psi_3 + \dots, \\ T = \chi T_1 + \chi^2 T_2 + \chi^3 T_3 + \dots, \\ S = \chi S_1 + \chi^2 S_2 + \chi^3 S_3 + \dots, \end{cases} \tag{20}$$

where R_0 is the critical value of the Darcy-Rayleigh number at which the onset of convection takes place in the absence of gravity modulation. According to Kim et al.[50], Bhadauria and Kiran[15–16,46–47] and Bhadauria et al.[17] introduced the following scale of time as $\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} + \chi^2 \frac{\partial}{\partial s}$, where τ is the fast time scale, and s is the slow time scale.

5.1 Lowest-order system

At the first-order, the nonlinear terms in governing equations vanish. Therefore, the first-order problem reduces to the linear stability problem for the oscillatory mode of thermal instability. The following system is obtained at this order:

$$\begin{bmatrix} \nabla^2 & R_0\dfrac{\partial}{\partial x} & -Rs\dfrac{\partial}{\partial x} \\ -\dfrac{\mathrm{d}T_{\mathrm{b}}}{\mathrm{d}z}\dfrac{\partial}{\partial x} & \Big(\dfrac{\partial}{\partial \tau} - \nabla^2 + Pe\dfrac{\partial}{\partial z}\Big) & 0 \\ -\dfrac{\mathrm{d}S_{\mathrm{b}}}{\mathrm{d}z}\dfrac{\partial}{\partial x} & 0 & \Big(\dfrac{\partial}{\partial \tau} - \Gamma\nabla^2 + Pe\Gamma^{-1}\dfrac{\partial}{\partial z}\Big) \end{bmatrix} \begin{bmatrix} \psi_1 \\ T_1 \\ S_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{21}$$

The solution of the lowest-order system subject to the boundary conditions given in Eq. (19) is assumed to be

$$\psi_1 = \big(A(s)\mathrm{e}^{\mathrm{i}\omega\tau} + \overline{A}(s)\mathrm{e}^{-\mathrm{i}\omega\tau}\big)\sin(ax)\sin(\pi z), \tag{22}$$

$$T_1 = \big(B(s)\mathrm{e}^{\mathrm{i}\omega\tau} + \overline{B}(s)\mathrm{e}^{-\mathrm{i}\omega\tau}\big)\cos(ax)\sin(\pi z), \tag{23}$$

$$S_1 = \big(C(s)\mathrm{e}^{\mathrm{i}\omega\tau} + \overline{C}(s)\mathrm{e}^{-\mathrm{i}\omega\tau}\big)\cos(ax)\sin(\pi z). \tag{24}$$

The undetermined amplitudes are the functions of the slow time scale and are related by the following relations:

$$B(s) = -\frac{4\pi^2 a}{(4\pi^2 + Pe^2)(c + \mathrm{i}\omega)}A(s), \tag{25}$$

$$C(s) = -\frac{4\pi^2 a}{(4\pi^2 + (Pe\Gamma^{-1})^2)(\Gamma c + \mathrm{i}\omega)}A(s), \tag{26}$$

where $c = a^2 + \pi^2$. The critical Rayleigh number R_0 for the oscillatory mode of convection is given by

$$R_0 = \frac{(4\pi^2 + Pe^2)(c - \omega^2 Pr_{\mathrm{D}})}{4a^2\pi^2} + \frac{Rs(4\pi^2 + Pe^2)(\Gamma c^2 + \omega^2)}{(4\pi^2 + (Pe\Gamma^{-1})^2)(\Gamma^2 c^2 + \omega^2)}. \tag{27}$$

The corresponding critical wavenumber will be calculated while minimizing the critical Rayleigh number with respect to the wavenumber. The growth rate of the disturbance ω^2 can be defined as follows:

$$\omega^2 = \frac{4\pi^2 a^2 Rs(1-\Gamma)}{(4\pi^2 + Pe^2)(1 + cPr_{\mathrm{D}}^{-1})} - c^2\Gamma^2. \tag{28}$$

It is to be noted that for existing an oscillatory mode of convection ($\omega^2 > 0$), the values of Pe, Pr_{D}, and the diffusivity ratio Γ must consider to satisfy Eq. (28).

5.2 Second-order system

At the second-order system, the nonlinear effects are obtained in terms of Jacobian. In this case, the following relations are obtained for the temperature and solutal fields:

$$\psi_2 = 0, \tag{29}$$

$$\Big(\frac{\partial}{\partial\tau} - \nabla^2\Big)T_2 = \frac{\partial(\psi_1, T_1)}{\partial(x,z)}, \tag{30}$$

$$\Big(\frac{\partial}{\partial\tau} - \Gamma\nabla^2\Big)S_2 = \frac{\partial(\psi_1, S_1)}{\partial(x,z)}. \tag{31}$$

Using the first-order solutions, the second-order solutions are obtained. From the above relations, according to Kim et al.[50], Bhadauria and Kiran[15–16,46–47], and Kiran[51], one can deduce that the velocity, temperature, and solutal fields have the terms of the frequency 2ω and are independent of the fast time scale. Thus, introduce the temperature and solutal concentration terms at the second-order system as

$$T_2 = (T_{20} + T_{22}\mathrm{e}^{2\mathrm{i}\omega\tau} + \overline{T}_{22}\mathrm{e}^{-2\mathrm{i}\omega\tau})\sin(2\pi z), \tag{32}$$

$$S_2 = (S_{20} + S_{22}\mathrm{e}^{2\mathrm{i}\omega\tau} + \overline{S}_{22}\mathrm{e}^{-2\mathrm{i}\omega\tau})\sin(2\pi z), \tag{33}$$

where (T_{20}, T_{22}) and (S_{20}, S_{22}) are the temperature and solutal fields of the terms, which have the frequency 2ω and are independent of the fast time scale, respectively. The second-order solutions are defined using the expressions of T_2 and S_2 in Eqs. (30)–(31). For measuring temperature and concentration transports, define the horizontally averaged Nusselt and Sherwood numbers for the oscillatory mode of convection as follows:

$$Nu = 1 + \frac{\Big(\frac{a_{\mathrm{c}}}{2\pi}\int_0^{\frac{2\pi}{a_{\mathrm{c}}}} \big(\frac{\partial T_2}{\partial z}\big)\,\mathrm{d}x\Big)_{z=0}}{\Big(\frac{a_{\mathrm{c}}}{2\pi}\int_0^{\frac{2\pi}{a_{\mathrm{c}}}} \big(\frac{\mathrm{d}T_{\mathrm{b}}}{\mathrm{d}z}\big)\,\mathrm{d}x\Big)_{z=0}}, \tag{34}$$

$$Sh = 1 + \frac{\Big(\frac{a_{\mathrm{c}}}{2\pi}\int_0^{\frac{2\pi}{a_{\mathrm{c}}}} \big(\frac{\partial S_2}{\partial z}\big)\,\mathrm{d}x\Big)_{z=0}}{\Big(\frac{a_{\mathrm{c}}}{2\pi}\int_0^{\frac{2\pi}{a_{\mathrm{c}}}} \big(\frac{\mathrm{d}S_{\mathrm{b}}}{\mathrm{d}z}\big)\,\mathrm{d}x\Big)_{z=0}}. \tag{35}$$

5.3 Third-order system

For the third-order system, the following relations are obtained:

$$\begin{bmatrix} \nabla^2 & R_0\dfrac{\partial}{\partial x} & -Rs\dfrac{\partial}{\partial x} \\ -\dfrac{\mathrm{d}T_\mathrm{b}}{\mathrm{d}z}\dfrac{\partial}{\partial x} & \Big(\dfrac{\partial}{\partial \tau}-\nabla^2+Pe\dfrac{\partial}{\partial z}\Big) & 0 \\ -\dfrac{\mathrm{d}S_\mathrm{b}}{\mathrm{d}z}\dfrac{\partial}{\partial x} & 0 & \Big(\dfrac{\partial}{\partial \tau}-\Gamma\nabla^2+Pe\Gamma^{-1}\dfrac{\partial}{\partial z}\Big) \end{bmatrix} \begin{bmatrix} \psi_3 \\ T_3 \\ S_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \end{bmatrix}, \tag{36}$$

where the expressions for R_{31}, R_{32}, and R_{33} are given in Appendix A. Now, under the solvability condition[15–18,46–47], for the existence of third-order solution, derive the following non-autonomous CGLE that describes the temporal variation of the amplitude $A(s)$ of the convection cell[15–18,46–47,52],

$$\frac{\mathrm{d}A(s)}{\mathrm{d}s} - \gamma_1^{-1}F(s)A(s) + \gamma_1^{-1}k_1|A(s)|^2A(s) = 0, \tag{37}$$

where the coefficients $\gamma_1, F(s)$, and k_1 are given in Appendix A. Write $A(s)$ in the phase-amplitude of the following form:

$$A(s) = |A(s)|\mathrm{e}^{\mathrm{i}\theta}. \tag{38}$$

Now substituting the above expression of $A(s)$ into Eq. (37), we obtain the following equations for the amplitude $|A(s)|$ of convection:

$$\frac{\mathrm{d}|A(s)|^2}{\mathrm{d}s} - 2p_\mathrm{r}|A(s)|^2 + 2l_\mathrm{r}|A(s)|^4 = 0, \tag{39}$$

$$\frac{\mathrm{d}(ph(A(s)))}{\mathrm{d}s} = p_\mathrm{i} - l_\mathrm{i}|A(s)|^2, \tag{40}$$

where

$$\gamma_1^{-1}F(s) = p_\mathrm{r} + \mathrm{i}p_\mathrm{i}, \quad \gamma_1^{-1}k_1 = l_\mathrm{r} + \mathrm{i}l_\mathrm{i},$$

in which the subscripts i and r represent imaginary and real parts of the quality, respectively, and $ph(\cdot)$ represents the phase. As Eq. (37) is non-autonomous, it is evaluated numerically using NDSolve of MATHEMATICA 8.0 with the suitable initial condition $A(s) = a_0$. Without loss of generality, $R_2 = R_0$ is assumed in the calculations, and this is done to keep the parameters to the minimum. From Eq. (20), for $R_2 = R_0$, one gets $\frac{Ra}{1+\chi^2} = R_0$. This means that the actual Rayleigh number is diminished as a result of this assumption. In the event of R_2 becoming negative, it means that the actual Rayleigh number is enhanced.

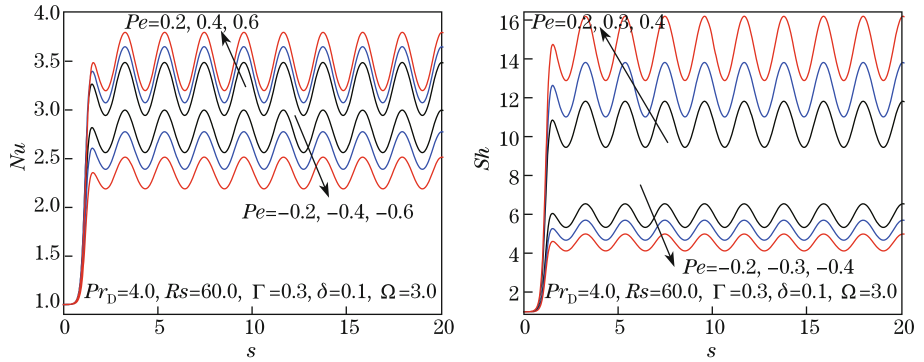
6 Results and discussion

When a horizontal porous medium is heated uniformly from below and cooled from above, a cellular regime of steady convection is set up at the value of the Rayleigh number exceeding a critical value. To determine this amplitude of convection, one has to develop a nonlinear theory to analyze the nonlinear interactions of fluid motion with temperature and concentration. A method is presented here to determine the amplitude of this convection and analyze heat and mass transfer. The combined effect of gravity modulation and vertical throughflow on thermal convection in an infinite horizontal fluid saturated porous medium is investigated. The presence of throughflow is to alter the basic temperature and solutal gradients from linear to nonlinear with respect to the porous layer height. Using the CGLE, a weakly nonlinear stability analysis is performed to investigate the effect of gravity modulation and vertical throughflow on heat and mass transports. Since the porous medium is assumed to be closely packed, the Darcy-model is considered. It is observed that, for existing the oscillatory mode of convection, the oscillatory frequency (ω) must be positive. Hence, the values of $Pe, Pr_{\rm D}$, and the diffusivity ratio Γ must consider to satisfy Eq. (28). Also, the values of δ and Ω are considered to be small. For small values of amplitude and frequencies, the heat and mass transfer can reach the maximum. A small amount of throughflow is in a particular direction either to destabilize or stabilize the system. Hence, the values of Pe are taken around 0.1. The numerical results for Nu and Sh obtained from the expressions given in Eqs. (34) and (35) by solving Eq. (39) are presented in Figs. 2–12. The effect of each parameter on heat and mass transports is shown in Figs. 2–12, wherein the plots of Nu and Sh versus the slow time s are presented. It is found from the figures that, the values of Nu and Sh start with one and remain constant for quite some time, showing the conduction state initially. Then, the values of Nu and Sh increase with the time, thus showing the convection state, and finally become constant on further increasing. Thus, the steady state is obtained.

The oscillatory Rayleigh number (Eq. (27)) increases with Pe, and it is independent of the throughflow direction. This may be due to the fact that, throughflow is to confine a significant thermal gradient to a thermal boundary layer at the boundary towards which the throughflow is directed. The effective length scale is thus smaller than the thickness of the porous layer. Hence, the Rayleigh number will be much smaller than the actual value of the Rayleigh number. Therefore, large values of the Rayleigh number are needed for the onset of convection when the throughflow strength increases, which are the results obtained by Khalili and Shivakumara[34] for free-free boundaries. The opposite results were obtained by Nield[30] in the case of a fluid layer for small amount of throughflow. Reza and Gupta[43] also stated that, with increasing the throughflow velocity, a temperature boundary layer forms at one of the boundaries. This reduces the effective thickness of the stratified layer, while the characteristic temperature difference across the layer remains constant. Thus, one would expect that the critical Rayleigh number increases with the increase in Pe. Shivakumara and Sureshkumar[40] pointed out that the reason for the opposite effect may be due to the distortion of steady-state basic temperature distribution from linear to nonlinear because of throughflow. A measure of throughflow is given by the basic state temperature, and this can be interpreted as a rate of energy transfer into the disturbance by interaction of the perturbation convective motion with the basic temperature gradient. The maximum temperature occurs at a place where the perturbed vertical velocity is high, and this leads to an increase in the energy supply for destabilization. A similar case for the solutal concentration is presented, where the conduction state profile of concentration given in Eq. (14) is nonlinear.

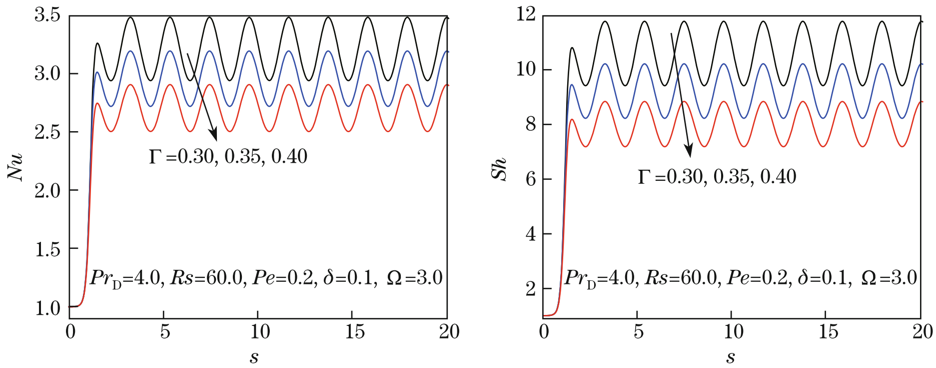
The basic state thermal and solutal concentration distributions are obtained for representative values of Pe and Γ and are presented graphically in Figs. 2 and 3 in order to understand their influence on the stability of the system. For throughflow, the basic state distributions become nonlinear and deviate from each other with an increase in the Péclet number Pe. In fact, the nonlinearity in the base-state solute concentration stratification becomes more dominant as compared to the temperature stratification with a decrease in Γ. It is found that Nu and Sh start with one, showing the conduction state, increase with the time s, and then become oscillatory, showing the convection state. The effect of upward throughflow ($Pe > 0$) is to enhance, and the downward throughflow ($Pe < 0$) is to diminish heat and mass transfer in the system as given in Figs. 2(a) and 2(b). The effect of the diffusivity ratio Γ is to diminish heat and mass transfer in the system given in Figs. 3(a) and 3(b). These results conform the results of Bhadauria[22] and Bhadauria and Kiran[24]. The corresponding results of $Pr_{\rm D}$ are presented in Figs. 4(a) and 4(b), and it is observed that Nu and Sh increase upon increasing $Pr_{\rm D}$. These results conform the results of Bhadauria[22] and Bhadauria and Kiran[53]. The effect of the solutal

Rayleigh number Rs in Figs. 5(a) and 5(b) is to increase heat and mass transfer. Although the stabilizing gradient of the solute concentration delays the onset of convection, the strong finite amplitude flows (Bhadauria and Kiran[24]), for large values of Rayleigh number, tend to mix the solute and redistribute it so that the interior layers of the fluid are more neutrally stratified. As a result, the enhancing effect of the solute concentration is greatly decreased. Hence, the fluid will convect more due to the increase in heat and mass transfer as Rs increases. This is possible only for nonlinear theories.



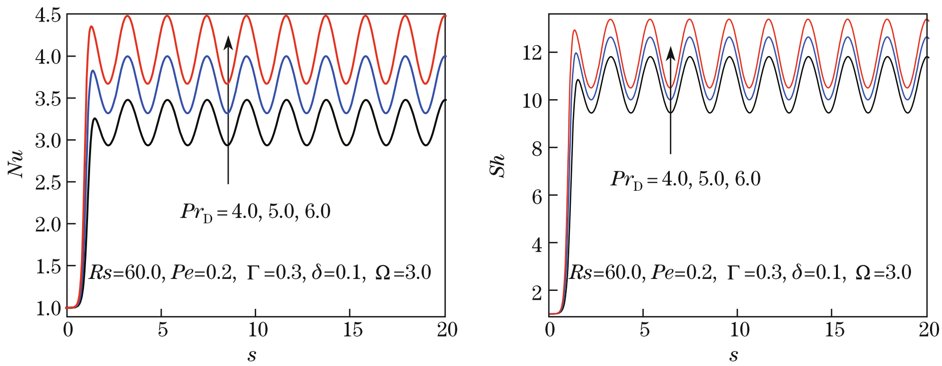
(a) Nu (b) Sh

Fig. 2 Effect of Péclet number on heat and mass transfer



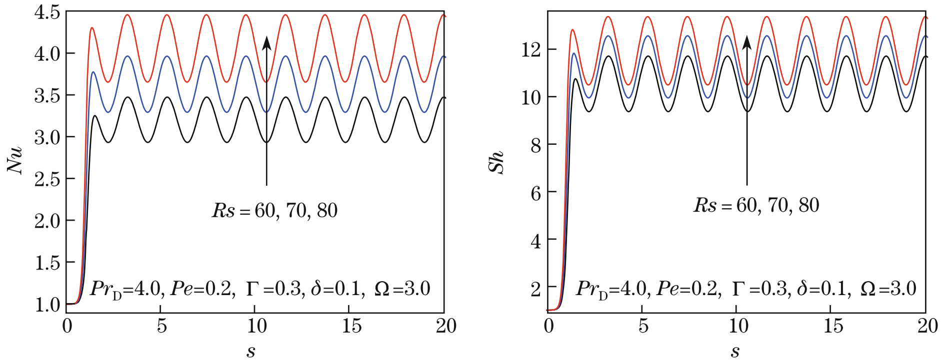
(a) Nu (b) Sh

Fig. 3 Effect of diffusivity ratio on heat and mass transfer



(a) Nu (b) Sh

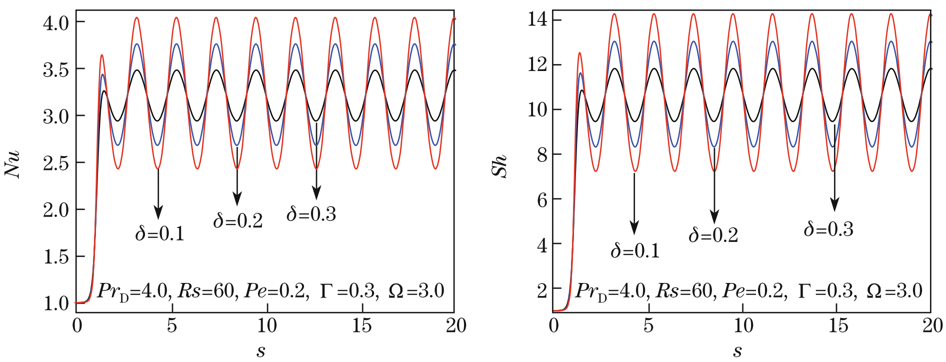
Fig. 4 Effect of Vadasz number Pr_D on heat and mass transfer



(a) *Nu* (b) *Sh*

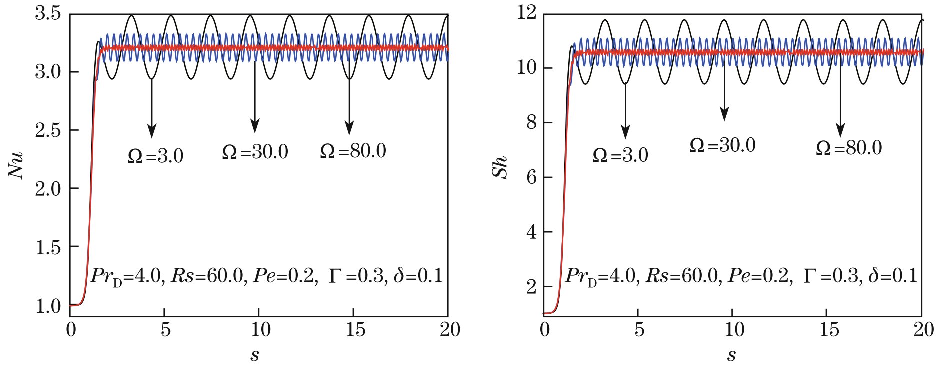
Fig. 5 Effect of solutal Rayleigh number on heat and mass transfer

Further, in Fig. 6, the effect of amplitude of gravity modulation is to increase the magnitudes of Nu and Sh, thus increase the heat and mass transports. Also, from Fig. 7, it is observed that, with the increase in the value of Ω, the magnitudes of Nu and Sh decrease, and the effect of frequency of modulation is to decrease the heat and mass transport. At high frequency, the effect of gravity modulation on thermal instability disappears altogether. These results agree with the



(a) *Nu* (b) *Sh*

Fig. 6 Effect of amplitude of modulation on heat and mass transfer



(a) *Nu* (b) *Sh*

Fig. 7 Effect of frequency of modulation on heat and mass transfer linear theory results of Malashetty and Padmavathi[10] and Venezian[49], where the correction in the critical value of Rayleigh number due to gravity and thermal modulations becomes almost zero at high frequencies. Figure 8 shows the comparison between the stationary ($\omega = 0$) and oscillatory modes of convection. It is found that, the oscillatory mode of convection increases heat and mass transfer rather than the stationary mode of convection. The reason behind this is that for the oscillatory mode of convection, an additional quantity oscillatory frequency ω^2 (the growth rate of disturbances) plays a critical role in the Rayleigh number and in the amplitude of convection. These are the results obtained by Bhadauria and Kiran[15–16,46–47], Bhadauria et al.[17], and Kiran[18].

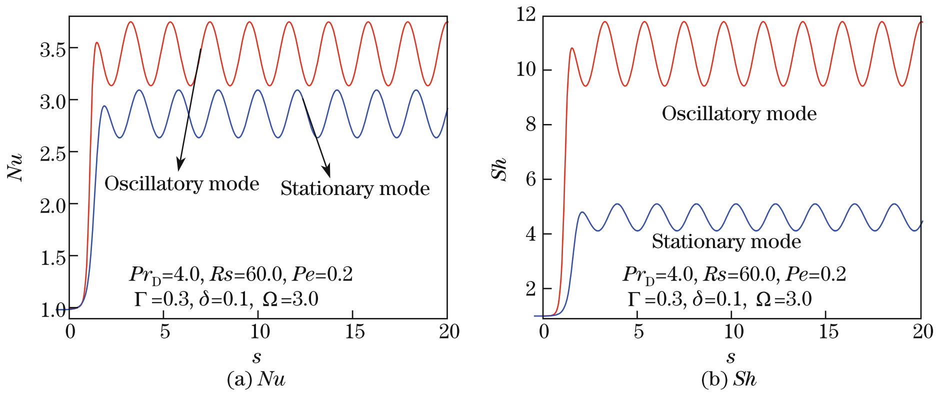


Fig. 8 Comparison between stationary and oscillatory modes of convection

Figure 9 presents a comparison between the analytical solution of the unmodulated case and the numerical solution of the modulated system. It is observed that, the values of the Nusselt and Sherwood numbers for the unmodulated case are larger than those in the modulated case. These are the results qualitatively similar to Bhadauria[22] for the unmodulated case and Srivastava et al.[54] for the modulated case. Davis[7] stated that, for nonlinear flows, the modulated (Gresho and Sani[8]) flows transport less heat than their corresponding unmodulated flows. The present results conform the results of Davis[7].

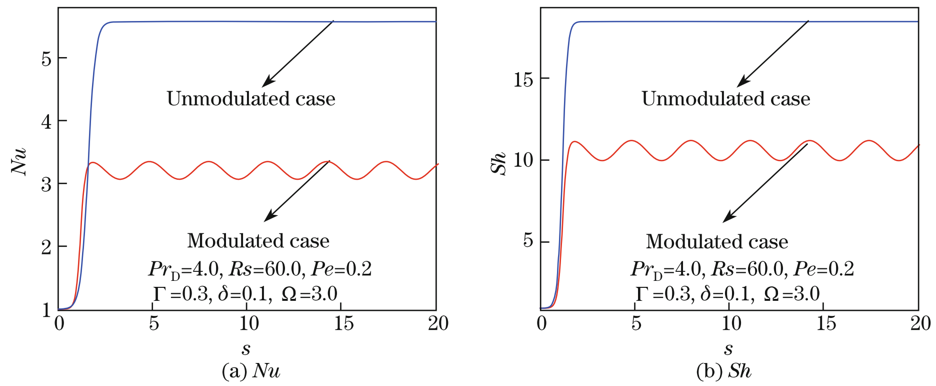


Fig. 9 Comparison between modulated and unmodulated systems

In Figs. 10–12, the streamlines and the corresponding isotherms and isohalines are depicted for gravity modulation, respectively, at various stages of the slow time $s = 0.0, 0.9, 1.2, 1.5, 2.5,$ and $s = 3.5$ for $Pe = 0.2$, $Rs = 60, Pr_D = 4.0, \Gamma = 0.3, \delta = 0.1$, $\Omega = 3.0$, and $\chi = 0.3$.

One can easily observe from the figures that, initially the magnitude of streamlines is small for small values of the time s as given in Figs. 10(a)–10(b) and isotherms in Figs. 10(a)–10(b) and isohalines in Figs. 11(a)–11(b) are flat showing the system in the conduction state. However, as time passes, the magnitude of streamlines increases given in Figs. 10(b)–10(d), and the isotherms in Figs. 11(b)–11(d) and the isohalines in Figs. 12(b)–12(d) lose their evenness, thus showing that the convection is in progress. Convection becomes faster on further increasing the value of time s. At later point of time, the system achieves the steady state beyond $s = 2.5$ as there is no change in the streamlines (Figs. 10(e)–10(f)), the isotherms (Figs. 11(e)–11(f)), and the isohalines (Figs. 12(e)–12(f)).

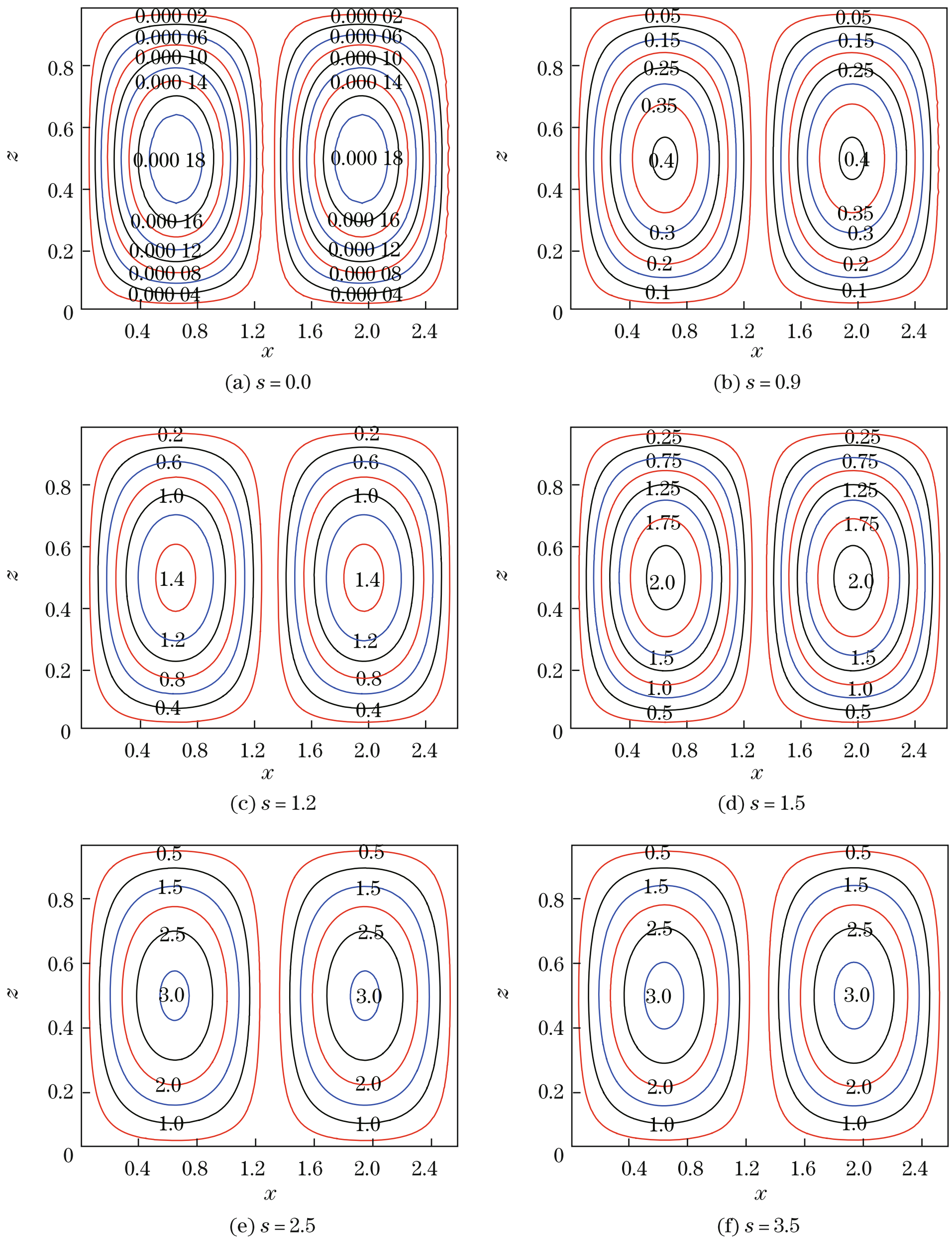


Fig. 10 Streamlines at various values of time s for $Pr_{\mathrm{D}} = 4.0$, $Rs = 60$, $Pe = 0.2$, $\Gamma = 0.3$, $\delta = 0.1$, $\Omega = 3.0$, and $\chi = 0.3$

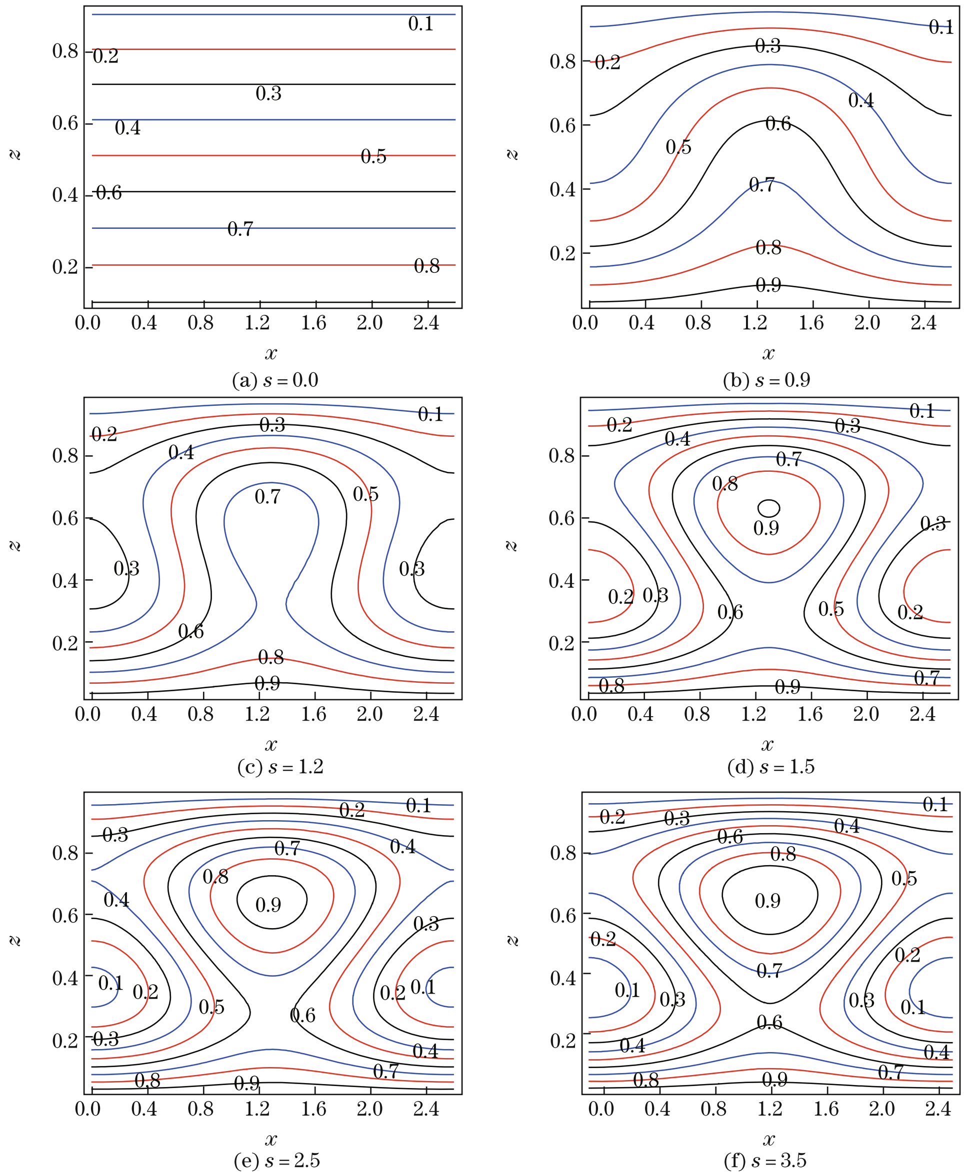


Fig. 11 Isotherms at various values of time s for $Pr_{\mathrm{D}} = 4.0$, $Rs = 60$, $Pe = 0.2$, $\Gamma = 0.3$, $\delta = 0.1$, $\Omega = 3.0$, and $\chi = 0.3$

7 Conclusions

The combined effects of throughflow and gravity modulation are investigated for the oscillatory mode of convection in the porous media while performing a weakly nonlinear stability analysis resulting in the complex Ginzburg-Landau amplitude equation. The following conclusions are made:

(i) The effect of the upward throughflow ($Pe > 0$) enhances heat and mass transfer, and the downward throughflow ($Pe < 0$) diminishes heat and mass transfer. Thus, throughflow has a dual behavior on heat and mass transfer.

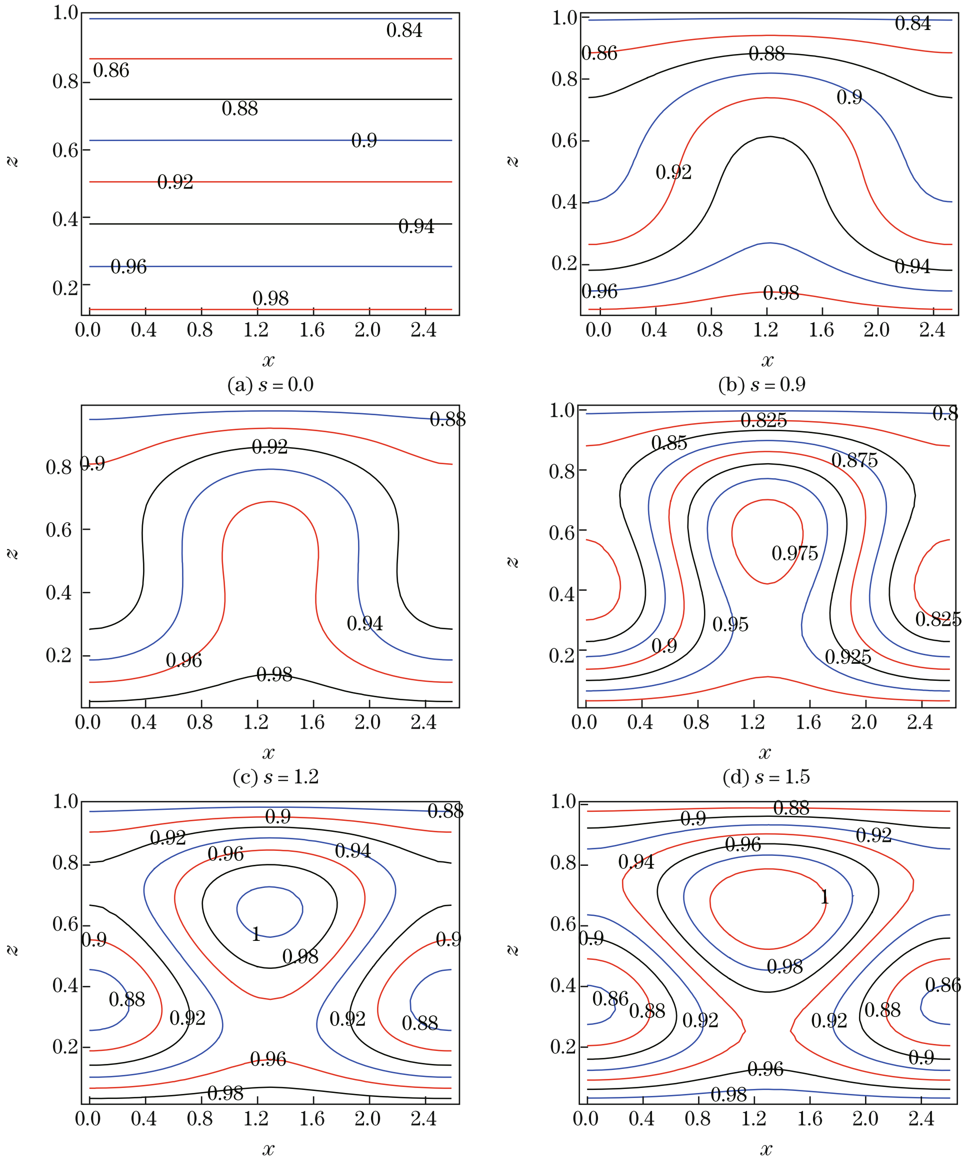


Fig. 12 Isotherms at various values of time s for $Pr_{\mathrm{D}} = 4.0$, $Rs = 60$, $Pe = 0.2$, $\Gamma = 0.3$, $\delta = 0.1$, $\Omega = 3.0$, and $\chi = 0.3$

(ii) An increment in the amplitude δ of modulation is to enhance heat and mass transfer.

(iii) The frequency Ω of modulation decreases heat and mass transfer as its value increases.

(iv) The oscillatory mode of convection is more effective than the stationary mode of convection.

(v) Throughflow and gravity modulation can be used to regulate heat and mass transfer in the system effectively.

(vi) For nonlinear fluid flows, gravity modulated system transports less heat and mass transport than their corresponding unmodulated flows.

(vii) Gravity modulated flows are similar to lower boundary temperature modulation[51] flows.

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Appendix A

The expressions given in Eqs. (34)–(35) are simplified as

$$\begin{aligned}
Nu(s) &= 1 + \frac{\mathrm{e}^{Pe} - 1}{Pe(4\pi^2 + Pe^2)} \\
&\quad \cdot \Big(\frac{2\pi^2 a^2 c}{(c^2 + \omega^2)} + \frac{2\pi^4 a^2}{\sqrt{4\pi^4 + \omega^2}\sqrt{c^2 + \omega^2}} \Big) |A(s)|^2, \\
Sh(s) &= 1 + \frac{\mathrm{e}^{(Pe\Gamma^{-1})} - 1}{Pe\Gamma^{-1}(4\pi^2 + (Pe\Gamma^{-1})^2)} \\
&\quad \cdot \Big(\frac{2\pi^2 a^2 c}{(\Gamma^2 c^2 + \omega^2)} + \frac{2\pi^4 a^2}{\sqrt{4\pi^4\Gamma^2 + \omega^2}\sqrt{\Gamma^2 c^2 + \omega^2}} \Big) |A(s)|^2.
\end{aligned}$$

The expressions given in Eq. (36) are

$$\begin{aligned}
R_{31} &= -\frac{1}{Pr_{\mathrm{D}}} \frac{\partial}{\partial s}(\nabla^2 \psi_1) + Rs\delta\cos(\Omega s)\frac{\partial S_1}{\partial x} \\
&\quad - (R_2 + R_0\delta\cos(\Omega s))\frac{\partial T_1}{\partial x}, \\
R_{32} &= \frac{\partial \psi_1}{\partial x}\frac{\partial T_2}{\partial z} - \frac{\partial T_1}{\partial s}, \\
R_{33} &= \frac{\partial \psi_1}{\partial x}\frac{\partial S_2}{\partial z} - \frac{\partial S_1}{\partial s}.
\end{aligned}$$

The coefficients given in Eq. (37) are

$$\gamma_1 = \frac{c}{Pr_{\mathrm{D}}} + \frac{4\pi^2 R_0 a^2}{(4\pi^2 + Pe^2)(c + \mathrm{i}\omega)^2} - \frac{4\pi^2 Rs a^2}{(4\pi^2 + (Pe\Gamma^{-1})^2)(\Gamma c + \mathrm{i}\omega)^2},$$

$$F(s) = \frac{4\pi^2 R_0 a^2}{(4\pi^2 + Pe^2)(c + \mathrm{i}\omega)}(1 + \delta\cos(\Omega s)) - \frac{4\pi^2 Rs a^2}{(4\pi^2 + (Pe\Gamma^{-1})^2)(\Gamma c + \mathrm{i}\omega)}\delta\cos(\Omega s),$$

$$k_1 = \frac{a^4\pi^2 c R_0}{(4\pi^2 + Pe^2)(c^2 + \omega^2)(c + \mathrm{i}\omega)} + \frac{a^4\pi^4 R_0}{(4\pi^2 + Pe^2)(2\pi^2 + \mathrm{i}\omega)(c + \mathrm{i}\omega)^2} - \frac{a^4 c\pi^2 Rs}{(4\pi^2 + (Pe\Gamma^{-1})^2)(\Gamma^2 c^2 + \omega^2)(\Gamma c + \mathrm{i}\omega)} - \frac{a^4\pi^4 Rs}{(4\pi^2 + (Pe\Gamma^{-1})^2)(2\pi^2\Gamma + \mathrm{i}\omega)(\Gamma c + \mathrm{i}\omega)^2}.$$