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Linear stability of triple-diffusive convection in micropolar ferromagnetic fluid saturating porous medium*

S. CHAND

(Department of Mathematics, Shaheed Captain Vikram Batra Government College, Palampur 176061, India)

Abstract The triple-diffusive convection in a micropolar ferromagnetic fluid layer heated and soluted from below is considered in the presence of a transverse uniform magnetic field. An exact solution is obtained for a flat fluid layer contained between two free boundaries. A linear stability analysis and a normal mode analysis method are carried out to study the onset convection. For stationary convection, various parameters such as the medium permeability, the solute gradients, the non-buoyancy magnetization, and the micropolar parameters (i.e., the coupling parameter, the spin diffusion parameter, and the micropolar heat conduction parameter) are analyzed. The critical magnetic thermal Rayleigh number for the onset of instability is determined numerically for a sufficiently large value of the buoyancy magnetization parameter M_1. The principle of exchange of stabilities is found to be true for the micropolar fluid heated from below in the absence of the micropolar viscous effect, the microinertia, and the solute gradients. The micropolar viscous effect, the microinertia, and the solute gradient introduce oscillatory modes, which are non-existent in their absence. Sufficient conditions for the non-existence of overstability are also obtained.

Key words triple-diffusive convection, micropolar ferromagnetic fluid, porous medium, solute gradient, medium permeability, magnetization

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1 Introduction

Micropolar fluids include polymeric fluids and colloidal suspensions that take into account the microscopic effects arising from the local structures and micromotion of the fluid elements. They represent fluids consisting of rigid and randomly oriented or spherical particles suspended in the viscous medium, where the deformation of fluid particles is ignored. Eringen[1] introduced the micropolar fluid theory to describe some physical systems, which do not satisfy the Navier-Stokes equation. The equations governing the micropolar fluid involve the microinertia tensor and a spin vector in addition to the velocity vector. This theory can be used to explain the flow of colloidal fluids, liquid crystals, animal blood, etc. The generalization of the theory including thermal effects has been developed by Kazakia and Ariman[2] and Eringen[3].

* Received Dec. 23, 2011 / Revised Sept. 23, 2012
Corresponding author S. CHAND, Associate Professor, Ph. D., E-mail: suresh-9@hotmail.com

Datta and Sastry[4] initiated the theory of thermomicropolar convection, which was continued by Ahmadi[5], Lebon and Perez-Garcia[6], Bhattachayya and Jena[7], Payne and Straughan[8], Sharma and Kumar[9–10], Sharma and Gupta[11], and Rosenweig[12] for the better understanding of the thermal convection in micropolar fluids.

An interesting possibility in a planer micropolar ferromagnetic fluid flow with an alternation current (AC) magnetic field was studied by Zahn and Greer[13]. They examined a simpler case where the applied magnetic fields along and transverse to the duct axis were spatially uniform and varied sinusoidally with time. In a uniform magnetic field, the magnetization characteristic depends on the particle spin instead of the fluid velocity. Micropolar ferromagnetic fluid stabilities have become an important field of research these days. A particular stability problem is the Rayleigh-Bénard instability in a horizontal thin layer of the fluid heated from below. Chandrasekhar[14] gave a detailed account of thermal convection in a horizontal thin layer of the Newtonian fluid heated from below. A thermo-mechanical interaction for a ferromagnetic fluid was predicted by Finlayson[15] in the presence of a uniform vertical magnetic field. He provided that the magnetization is a function of the temperature and magnetic field and established a temperature gradient across the fluid layer. The thermal convection in the Newtonian ferromagnetic fluid has been studied by many authors[16–25]. The Rayleigh-Bénard convection in a micropolar ferromagnetic fluid layer permeated by a uniform vertical magnetic field with free-free, isothermal, spin-vanishing, and magnetic boundaries was examined by Abraham[26], who observed that the micropolar ferromagnetic fluid layer heated from below is more stable as compared with the classical Newtonian ferromagnetic fluid. Sunil et al.[27] studied the linear stability of double-diffusive convection in a micropolar ferromagnetic fluid saturating a porous medium.

In the standard Bénard problem, the instability is driven by a density difference caused by a temperature difference between the upper and lower planes bounding the fluid. If salt is dissolved in the fluid, there are potentially two destabilizing sources for the density difference, the temperature field, and the salt field. The behavior of the solution in the double-diffusive convection problem is more interesting than that of the standard Bénard problem as the new instability phenomena may occur which has not been presented before. When the temperature and two or more salts or three different salts are presented, the physical and mathematical situations become richer. Pearlstein et al.[28] studied the triple-diffusive convection. They demonstrated that linear instability can occur in discrete sections of the Rayleigh number domain with the fluid being linearly stable in a region between the linear instability ones. This happened because that, for certain parameters, the neutral curve had a finite isolated oscillatory instability curve lying below the usual unbounded stationary convection one. Straughan and Walker[29] derived the equations for the non-Boussinesq convection in a multi-component fluid and investigated the situation analogous to that of Pearlstein et al.[28], but allowing for a density nonlinear in the temperature field. Lopez et al.[30] studied the same problem but with fixed boundary conditions and showed that the effect of the boundary conditions breaks the perfect symmetry. The density of a fluid in reality is never a linear function of temperature. Therefore, the work of Straughan and Walker[29] can be used to the situation where the density is quadratic in temperature. They found that the departure from the linear Boussinesq equation of state changes the perfect symmetry of the heart shaped neutral curve of Pearlstein et al.[28]. O'Sullivan et al.[31] reviewed the numerical techniques and their applications to multi component fluids. Oldenburg and Pruess[32] studied a model for the convection in Darcy's porous medium, where the mechanism involves the temperature and different salts like NaCl, $CaCl_2$, and KCl. A detailed study concerning the convection in a porous medium can be found in the book by Nield and Bejan[33]. Sunil et al.[34–35] studied the thermosolutal convection problem in a ferromagnetic fluid and the thermosolutal convection problem in a ferromagnetic fluid saturating a porous medium. Recently, Chand[36] studied the effect of rotation on the triple-diffusive convection in a magnetized ferrofluid with the internal angular momentum saturating a porous medium.

This paper aims to study the problem of linear stability for the triple-diffusive convection in a micropolar ferromagnetic fluid saturating a porous medium because of the increase in the number of non-isothermal situations wherein the magnetic fluids are used in places of classical fluids.

2 Mathematical formulation of problem

Let us consider an infinite horizontal layer with the thickness d of an electrically non-conducting incompressible thin micropolar ferromagnetic fluid heated and salted from below. The temperature T and the solute concentrations C^1 and C^2 at the bottom and top surfaces $z = \mp\frac{1}{2}d$ are T_0 and T_1, C_0^1 and C_1^1, and C_0^2 and C_1^2, respectively. A uniform temperature gradient $\beta = \left|\frac{\mathrm{d}T}{\mathrm{d}Z}\right|$ and the uniform solute gradients $\beta' = \left|\frac{\mathrm{d}C^1}{\mathrm{d}Z}\right|$ and $\beta'' = \left|\frac{\mathrm{d}C^2}{\mathrm{d}Z}\right|$ are maintained. Due to thermal expansion, the fluid at the bottom is lighter than the fluid at the top. This is in the top heavy arrangement, which is potentially unstable. Moreover, the heavier salt at the lower part of the layer has exactly the opposite effect. These two arrangements thus compete against each other. Here, both the boundaries are taken to be free and perfect conductors of heat. The gravity field $\boldsymbol{g} = (0, 0, -g)$ and the uniform vertical magnetic field intensity $\boldsymbol{H} = (0, 0, H)$ pervade the system.

The mathematical equations governing the motion of micropolar ferromagnetic for the above model are as follows.

The continuity equation for an incompressible fluid is

$$\nabla \cdot \boldsymbol{q} = 0. \tag{1}$$

The momentum and internal angular momentum equations are

$$\frac{\rho_0}{\epsilon}\frac{\partial \boldsymbol{q}}{\partial t} = -\nabla p + \rho \boldsymbol{g} + \nabla \cdot (\boldsymbol{H}\boldsymbol{B}) - \frac{1}{k_1}(\zeta + \eta)\boldsymbol{q} + 2\zeta(\nabla \times \omega), \tag{2}$$

$$\rho_0\Big(\frac{\partial}{\partial t} + \frac{1}{\epsilon}(\boldsymbol{q} \cdot \nabla)\Big)\omega = 2\zeta\Big(\frac{1}{\epsilon}\nabla \times q - 2\omega\Big) + \mu_0(\boldsymbol{M} \times \boldsymbol{H}) + (\lambda' + \eta')\nabla(\nabla \cdot \omega) + \eta'\nabla^2\omega. \tag{3}$$

The equations of temperature and solute concentrations for an incompressible micropolar ferromagnetic fluid are

$$\begin{aligned}&\Big(\rho_0 C_{V,H} - \mu_0 \boldsymbol{H}\Big(\frac{\partial \boldsymbol{M}}{\partial t}\Big)_{V,H}\Big)\frac{\mathrm{D}T}{\mathrm{D}t} + (1-\epsilon)\rho_{\mathrm{S}}C_{\mathrm{S}}\frac{\partial T}{\partial t} + \mu_0 T\Big(\frac{\partial \boldsymbol{M}}{\partial t}\Big)_{V,H}\frac{\mathrm{D}H}{\mathrm{D}t}\\ &= K_1\nabla^2 T + \delta(\nabla \times \omega)\cdot\nabla T,\end{aligned} \tag{4}$$

$$\begin{aligned}&\Big(\rho_0 C_{V,H} - \mu_0 \boldsymbol{H}\Big(\frac{\partial \boldsymbol{M}}{\partial C^1}\Big)_{V,H}\Big)\frac{\mathrm{D}C^1}{\mathrm{D}t} + (1-\epsilon)\rho_{\mathrm{S}}C_{\mathrm{S}}\frac{\partial C^1}{\partial t} + \mu_0 C^1\Big(\frac{\partial \boldsymbol{M}}{\partial C^1}\Big)_{V,H}\frac{\mathrm{D}H}{\mathrm{D}t}\\ &= K_1'\nabla^2 C^1,\end{aligned} \tag{5}$$

$$\begin{aligned}&\Big(\rho_0 C_{V,H} - \mu_0 \boldsymbol{H}\Big(\frac{\partial \boldsymbol{M}}{\partial C^2}\Big)_{V,H}\Big)\frac{\mathrm{D}C^2}{\mathrm{D}t} + (1-\epsilon)\rho_{\mathrm{S}}C_{\mathrm{S}}\frac{\partial C^2}{\partial t} + \mu_0 C^2\Big(\frac{\partial \boldsymbol{M}}{\partial C^2}\Big)_{V,H}\frac{\mathrm{D}H}{\mathrm{D}t}\\ &= K_1''\nabla^2 C^2,\end{aligned} \tag{6}$$

where the subscript S denotes the solid material.

In terms of the temperature T and the concentrations C^1 and C^2, we suppose that the density of the mixture is given by (known as the density equation of state)

$$\rho = \rho_0(1 - \alpha(T - T_{\mathrm{a}}) + \alpha'(C^1 - C_{\mathrm{a}}^1) + \alpha''(C^2 - C_{\mathrm{a}}^2)), \tag{7}$$

where $\rho, \rho_0, \boldsymbol{q}, \omega, t, p, \eta, \zeta, \lambda', \eta', \delta, I, \mu_0, \boldsymbol{B}, C_{V,H}, M, K_1, K_1', K_1'', \alpha, \alpha'$, and α'' are the fluid density, the reference density, the velocity, the microrotation, the time, the pressure, the shear kinematic viscosity coefficient, the coupling viscosity coefficient or vortex viscosity, the bulk spin viscosity coefficient, the shear spin viscosity coefficient, the micropolar heat conduction coefficient, the moment of inertia (microinertia constant), the magnetic permeability, the magnetic induction, the heat capacity at a constant volume and magnetic field, the magnetization, the thermal conductivity, the solute conductivity, the thermal expansion coefficient, and the concentration expansion coefficient analogous to the thermal expansion coefficient, respectively. T_{a} is the average temperature given by $T_{\mathrm{a}} = \frac{T_0+T_1}{2}$, where T_0 and T_1 are the constant average temperatures of the lower and upper surfaces of the layer, and C_{a}^1 and C_{a}^2 are the average concentrations given by $C_{\mathrm{a}}^1 = \frac{C_0^1+C_1^1}{2}$ and $C_{\mathrm{a}}^2 = \frac{C_0^2+C_1^2}{2}$, where C_0^1, C_1^1 and C_0^2, C_1^2 are the constant average concentrations of the lower and upper surfaces of the layer. The additional term in Eq. (2) pertinent to a ferromagnetic fluid is the magnetic stress, which has been derived by different authors[36–38]. There are many situations in which the basic equations can be simplified considerably. These situations occur when the variability in the density and in the other coefficients is due to moderate amounts of variations in the temperature. There is an exception that the variability of ρ in the gravitational body force term in the equation of motion occurs. Therefore, we may treat ρ as a constant in all terms in the equation of motion except the one in the external force. In Eq. (2), we use the Boussinesq approximation by allowing the density to change only in the gravitational body force term. The inertial force becomes relatively insignificant as compared with the viscous drag when the permeability of the porous material is low. The term $\frac{1}{\epsilon^2}(\boldsymbol{q} \cdot \nabla)\boldsymbol{q}$ is generally small. Thus, it is better to drop it in the numerical calculation.

Darcy's model is used for a porous medium of very low permeability. For a medium of very large stable particle suspension, the permeability tends to be small and hence justifies the use of Darcy's model. This is because of the viscous drag force which is negligibly small in comparison with Darcy's resistance due to the large particle suspension. Darcy's law governs the flow of the ferromagnetic fluid through an isotropic and homogeneous porous medium. Brinkman[39] proposed the introduction of the term $\frac{\eta}{\epsilon}\nabla^2\boldsymbol{q}$ (now known as the Brinkman term) in addition to the Darcian term $-\frac{\eta}{K_1}\boldsymbol{q}$ to be mathematically compatible and physically consistent with the Navier-Stokes equations. However, the Brinkman term contributes to very little effect for the flow through a porous medium, whereas the main effect is due to the Darcian term. Therefore, Darcy's law is proposed heuristically to govern the flow of this micropolar ferromagnetic fluid saturating a porous medium.

Maxwell's equation simplified for a non-conducting fluid with no displacement current is

$$\nabla \cdot B = 0, \tag{8a}$$

$$\nabla \cdot H = 0. \tag{8b}$$

Here, the magnetic induction is given by

$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}).$$

The presence of the ferromagnetic fluid in general can distort an external field if the magnetic interaction (dipole-dipole) takes place. However, it is negligible for the small particle concentration as is assumed here. The magnetization is aligned with the magnetic field, but allows a dependence on the magnitude of the magnetic field, temperature, and salinity so that

$$\boldsymbol{M} = \frac{\boldsymbol{H}}{H} M(H, T, C^1, C^2). \tag{9}$$

The magnetic equation of state is linearized about the magnetic field H_0, the average temperature $T_{\rm a}$, and the average concentrations $C_{\rm a}^1$ and $C_{\rm a}^2$ to

$$M = M_0 + \chi(H - H_0) - K_2(T - T_{\rm a}) + K_3(C^1 - C_{\rm a}^1) + K_4(C^2 - C_{\rm a}^2), \tag{10}$$

where the magnetic susceptibility, the pyromagnetic coefficient, and the salinity magnetic coefficients are defined by

$$\begin{cases} \chi \equiv \Big(\dfrac{\partial M}{\partial H}\Big)_{H_0,T_{\rm a}}, \quad K_2 \equiv -\Big(\dfrac{\partial M}{\partial T}\Big)_{H_0,T_{\rm a}}, \\ K_3 \equiv \Big(\dfrac{\partial M}{\partial C^1}\Big)_{H_0,C_{\rm a}^1}, \quad K_4 \equiv \Big(\dfrac{\partial M}{\partial C^2}\Big)_{H_0,C_{\rm a}^2}. \end{cases} \tag{11}$$

Here, H_0 is the uniform magnetic field of the fluid layer when placed in an external magnetic field $H = H_0^{\rm ext}\widehat{\boldsymbol{k}}$, where $\widehat{\boldsymbol{k}}$ is a unit vector in the z-direction, $H = |\boldsymbol{H}|$, $M = |\boldsymbol{M}|$, and $M_0 = M(H_0, T_{\rm a}, C_{\rm a}^1, C_{\rm a}^2)$. The basic state is assumed to be quiescent and is given by

$$\begin{cases} \boldsymbol{q} = \boldsymbol{q}_{\rm b} = (0,0,0), \quad \boldsymbol{\omega} = \boldsymbol{\omega}_{\rm b} = (0,0,0), \quad \rho = \rho_{\rm b}(z), \\ p = p_{\rm b}(z), \quad T = T_{\rm b}(z) = -\beta z + T_{\rm a}, \\ C^1 = C_{\rm b}^1(z) = -\beta z + C_{\rm a}^1, \quad C^2 = C_{\rm b}^2(z) = -\beta z + C_{\rm a}^2, \\ \beta = \dfrac{T_0 - T_1}{d}, \quad \beta' = \dfrac{C_1^1 - C_0^1}{d}, \\ \beta'' = \dfrac{C_1^2 - C_0^2}{d}, \quad \boldsymbol{H}_{\rm b} = \Big(H_0 - \dfrac{K_2\beta z}{1+\chi} + \dfrac{K_3\beta' z}{1+\chi} + \dfrac{K_4\beta'' z}{1+\chi}\Big)\widehat{\boldsymbol{k}}, \\ \boldsymbol{M}_{\rm b} = \Big(M_0 + \dfrac{K_2\beta z}{1+\chi} - \dfrac{K_3\beta' z}{1+\chi} - \dfrac{K_4\beta'' z}{1+\chi}\Big)\widehat{\boldsymbol{k}}, \quad H_0 + M_0 = H_0^{\rm ext}, \end{cases} \tag{12}$$

where the subscript b denotes the basic state.

3 Perturbation equations and normal mode analysis method

We now examine the stability of the basic state and assume that the perturbation quantities are small. We can write

$$\begin{cases} \boldsymbol{q} = \boldsymbol{q}_{\rm b} + \boldsymbol{q}', \quad \boldsymbol{\omega} = \boldsymbol{\omega}_{\rm b} + \boldsymbol{\omega}', \quad \rho = \rho_{\rm b} + \rho', \\ p = p_{\rm b}(z) + p', \quad T = T_{\rm b}(z) + \theta, \quad C^1 = C_{\rm b}^1(z) + \gamma, \\ C^2 = C_{\rm b}^2(z) + \gamma', \quad \boldsymbol{H} = \boldsymbol{H}_{\rm b}(z) + \boldsymbol{H}', \quad \boldsymbol{M} = \boldsymbol{M}_{\rm b}(z) + \boldsymbol{M}', \end{cases} \tag{13}$$

where $\boldsymbol{q}' = (u, v, w)$, $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)$, and ρ', θ, γ, γ', $\boldsymbol{H}'$, and $\boldsymbol{M}'$ are the perturbations in the velocity q, the spin ω, the pressure p, the temperature T, the concentrations C^1 and C^2, the magnetic field intensity H, and the magnetization M, respectively. The change in the density ρ' caused mainly by the perturbation θ, γ, and γ' in the temperature and concentrations, respectively, is given by

$$\rho' = -\rho_0(\alpha\theta - \alpha'\gamma - \alpha''\gamma'). \tag{14}$$

Then, the linearized perturbation equations (by neglecting the second-order small quantities) of the micropolar ferromagnetic fluid become

$$\frac{\rho_0}{\epsilon}\frac{\partial u}{\partial t} = -\frac{\partial p'}{\partial x} + \mu_0(M_0 + H_0)\frac{\partial H_1'}{\partial z} - \frac{1}{k_1}(\zeta + \eta)u + 2\zeta\Omega_1', \tag{15}$$

$$\frac{\rho_0}{\epsilon}\frac{\partial v}{\partial t} = -\frac{\partial p'}{\partial y} + \mu_0(M_0 + H_0)\frac{\partial H_2'}{\partial z} - \frac{1}{k_1}(\zeta + \eta)v + 2\zeta\Omega_2', \tag{16}$$

$$\frac{\rho_0}{\epsilon}\frac{\partial w}{\partial t} = -\frac{\partial p'}{\partial y} + \mu_0(M_0+H_0)\frac{\partial H_3'}{\partial z} - \frac{1}{k_1}(\zeta+\eta)w + 2\zeta\Omega_3$$
$$- \mu_0\frac{K_2\beta}{1+\chi}(H_3'(1+\chi) - K_2\beta) + \mu_0\frac{K_3\beta'}{1+\chi}(H_3'(1+\chi)+K_3\gamma)$$
$$+ \mu_0\frac{K_4\beta''}{1+\chi}(H_3'(1+\chi)+K_4\gamma') - \mu_0\frac{K_2K_3}{1+\chi}(\beta'\theta+\beta\gamma)$$
$$+ \mu_0\frac{K_3K_4}{1+\chi}(\beta''\gamma+\beta\gamma') - \mu_0\frac{K_2K_4}{1+\chi}(\beta''\theta+\beta\gamma)$$
$$+ \rho_0 g(\alpha\theta - \alpha'\gamma - \alpha''\gamma'), \tag{17}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{18}$$

$$\rho C_1\frac{\partial \theta}{\partial t} - \mu_0 T_0 K_2\epsilon\frac{\partial}{\partial t}\Big(\frac{\partial \phi_1'}{\partial z}\Big) = K_1\nabla^2\theta + \Big(\frac{\rho C_2\beta - \mu_0 T_0 K_2^2\beta}{1+\chi}\Big)\omega - \delta\beta\Omega_3', \tag{19}$$

$$\rho C_1'\frac{\partial \gamma}{\partial t} - \mu_0 C_0^1 K_3\epsilon\frac{\partial}{\partial t}\Big(\frac{\partial \phi_2'}{\partial z}\Big) = K_1'\nabla^2\gamma + \Big(\frac{\rho C_2'\beta' - \mu_0 C_0^1 K_3^2\beta'}{1+\chi}\Big)\omega, \tag{20}$$

$$\rho C_1''\frac{\partial \gamma'}{\partial t} - \mu_0 C_0^2 K_2\epsilon\frac{\partial}{\partial t}\Big(\frac{\partial \phi_3'}{\partial z}\Big) = K_1''\nabla^2\gamma' + \Big(\frac{\rho C_2''\beta'' - \mu_0 C_0^2 K_2^2\beta''}{1+\chi}\Big)\omega, \tag{21}$$

where

$$\rho C_1 = \epsilon\rho_0 C_{V,H} + (1-\epsilon)\rho_{\rm S} C_{\rm S} + \epsilon\mu_0 K_2 H_0, \quad \rho C_2 = \rho_0 C_{V,H} + \mu_0 K_2 H_0,$$
$$\rho C_1' = \epsilon\rho_0 C_{V,H} + (1-\epsilon)\rho_{\rm S} C_{\rm S} - \mu_0 K_3 H_0, \quad \rho C_2' = \rho_0 C_{V,H} - \mu_0 K_3 H_0 \rho,$$
$$C_1'' = \epsilon\rho_0 C_{V,H} + (1-\epsilon)\rho_{\rm S} C_{\rm S} - \mu_0 K_4 H_0, \quad C_2'' = \rho_0 C_{V,H} - \mu_0 K_4 H_0.$$

Equations (9) and (10) yield

$$H_3' + M_3' = (1+\chi)H_3' - K_2\theta, \tag{22}$$
$$H_3' + M_3' = (1+\chi)H_3' + K_3\gamma, \tag{23}$$
$$H_3' + M_3' = (1+\chi)H_3' + K_4\gamma', \tag{24}$$
$$H_i' + M_i' = \Big(1 + \frac{M_0}{H_0}\Big)H_i', \quad i = 1,2,3. \tag{25}$$

Here, we assume

$$K_2(T_{\rm b} - T_{\rm a}) \ll (1+\zeta)H_0, \quad K_3\beta' d \ll (1+\zeta)H_0,$$
$$K_4\beta'' d \ll (1+\zeta)H_0, \quad \Omega = \Omega_1'\Omega_2'\Omega_3' = \nabla\times\omega'.$$

Thus, the analysis is restricted to the physical situation in which the magnetization induced by the temperature and concentration variations is small compared with that induced by the external magnetic field. Equation (8b) means that we can write $H' = \nabla(\phi_1' - \phi_2' - \phi_3')$, where ϕ_1' is the perturbed magnetic potential, and ϕ_2' and ϕ_3' are the perturbed magnetic potentials analogous to solute.

Eliminating u, v, and p' between Eqs. (15)–(17), using Eq. (18), taking curl once on Eq. (3), and considering only the kth component, we obtain

$$\begin{aligned}
&\Big(\frac{\rho_0}{\epsilon}\frac{\partial}{\partial t}+\frac{1}{k_1}(\zeta+\eta)\Big)\nabla^2 w\\
&=-\mu_0\frac{K_2\beta}{1+\chi}\nabla_1^2\Big((1+\chi)\frac{\partial}{\partial z}(\phi_1'-\phi_2'-\phi_3')-K_2\theta\Big)\\
&\quad+\mu_0\frac{K_3\beta'}{1+\chi}\nabla_1^2\Big((1+\chi)\frac{\partial}{\partial z}(\phi_1'-\phi_2'-\phi_3')-K_3\gamma\Big)\\
&\quad+\mu_0\frac{K_4\beta''}{1+\chi}\nabla_1^2\Big((1+\chi)\frac{\partial}{\partial z}(\phi_1'-\phi_2'-\phi_3')-K_4\gamma'\Big)\\
&\quad-\mu_0\frac{K_2K_3}{1+\chi}\nabla_1^2(\beta'\theta+\beta\gamma)+\mu_0\frac{K_4K_3}{1+\chi}\nabla_1^2(\beta''\gamma+\beta'\gamma')\\
&\quad-\mu_0\frac{K_2K_4}{1+\chi}\nabla_1^2(\beta''\theta+\beta'\gamma)+\rho_0 g\nabla_1^2(\alpha\theta-\alpha'\gamma-\alpha''\gamma')+2\zeta\nabla^2\Omega_3',
\end{aligned}\tag{26}$$

$$\rho_0\frac{\partial\Omega_3'}{\partial t}=-2\zeta\Big(\frac{1}{\epsilon}\nabla^2\omega+2\Omega_3'\Big)+\eta'\nabla^2\Omega_3'.\tag{27}$$

From Eq. (20), we have

$$(1+\chi)\frac{\partial^2\phi_1'}{\partial z^2}+\Big(1+\frac{M_0}{H_0}\Big)\nabla^2\phi_1'-K_2\frac{\partial\theta}{\partial z}=0,\tag{28}$$

$$(1+\chi)\frac{\partial^2\phi_2'}{\partial z^2}+\Big(1+\frac{M_0}{H_0}\Big)\nabla^2\phi_2'-K_3\frac{\partial\gamma}{\partial z}=0,\tag{29}$$

$$(1+\chi)\frac{\partial^2\phi_3'}{\partial z^2}+\Big(1+\frac{M_0}{H_0}\Big)\nabla^2\phi_3'-K_4\frac{\partial\gamma'}{\partial z}=0.\tag{30}$$

Analyzing the disturbances into normal modes, we assume

$$f(x,y,z,t)=f(z,t)\mathrm{e}^{\mathrm{i}(k_x x+k_y y)},\tag{31}$$

where $f(z,t)$ represents $W(z,t)$, $\Theta(z,t)$, $\Gamma(z,t)$, $\Psi(z,t)$, $\phi_1(z,t)$, $\phi_2(z,t)$, $\phi_3(z,t)^2$, or $\Omega_3(z,t)$, k_x and k_y are the wave numbers along the x- and y-directions, respectively, and $k=\sqrt{(k_x^2+k_y^2)}$ is the resultant wave number.

With Eq. (31), Eqs. (19)–(21) and (28)–(30) become

$$\begin{aligned}
&\Big(\frac{\zeta+\eta}{k_1}(\zeta+\eta)\nabla^2\frac{\partial}{\partial t}-\frac{\zeta+\eta}{k_1}\Big(\frac{\partial^2}{\partial z^2}-k^2\Big)\Big)\Big(\frac{1}{\epsilon}\Big(\frac{\partial^2}{\partial z^2}-k^2\Big)\Big)W\\
&=\mu_0\frac{k_2\beta}{1+\chi}\Big((1+\chi)\frac{\partial}{\partial z}(\phi_1-\phi_2-\phi_3)-K_2\Theta\Big)k^2\\
&\quad+\mu_0\frac{k_3\beta'}{1+\chi}\Big((1+\chi)\frac{\partial}{\partial z}(\phi_1-\phi_2-\phi_3)-K_3\Gamma\Big)k^2\\
&\quad+\mu_0\frac{k_4\beta''}{1+\chi}\Big((1+\chi)\frac{\partial}{\partial z}(\phi_1-\phi_2-\phi_3)-K_4\Psi\Big)k^2\\
&\quad+\mu_0\frac{k_3k_2}{1+\chi}(\beta'\Theta+\beta\Gamma)k^2+\mu_0\frac{k_3k_4}{1+\chi}(\beta''\Gamma+\beta'\Psi)k^2+\mu_0\frac{k_2k_4}{1+\chi}(\beta''\Theta+\beta\Psi)k^2\\
&\quad-\rho_0 gk^2(\alpha\Theta-\alpha'\Gamma-\alpha''\Psi)+2\zeta\Big(\frac{\partial}{\partial z^2}-k^2\Big)\Omega_3,
\end{aligned}\tag{32}$$

$$\rho_0\frac{\partial\Omega_3}{\partial t}=-2\zeta\Big(\frac{1}{\epsilon}\Big(\frac{\partial^2}{\partial z^2}-k^2\Big)\omega+2\Omega_3\Big)+\eta'\Big(\frac{\partial^2}{\partial z^2}-k^2\Big)\Omega_3, \tag{33}$$

$$\rho C_1\frac{\partial\Theta}{\partial t}-\mu_0T_0K_2\epsilon\frac{\partial}{\partial z}\phi_1=K_1\Big(\frac{\partial^2}{\partial z^2}-k^2\Big)\Theta+\Big(\rho C_1\beta-\mu_0\frac{T_0K_2^2\beta}{1+\chi}\Big)W-\delta\beta\Omega_3, \tag{34}$$

$$\rho C_1'\frac{\partial\Gamma}{\partial t}-\mu_0C_0^1K_3\epsilon\frac{\partial\phi_2}{\partial z}=K_1'\Big(\frac{\partial^2}{\partial z^2}-k^2\Big)\Gamma+\Big(\rho C_1'\beta'-\mu_0\frac{C_0K_3^2\beta'}{1+\chi}\Big)W, \tag{35}$$

$$\rho C_1''\frac{\partial\Psi}{\partial t}-\mu_0C_0^2K_4\epsilon\frac{\partial\phi_3}{\partial z}=K_1''\Big(\frac{\partial^2}{\partial z^2}-k^2\Big)\Psi+\Big(\rho C_1''\beta''-\mu_0\frac{T_0K_4^2\beta''}{1+\chi}\Big)W, \tag{36}$$

$$(1+\chi)\frac{\partial^2\phi_1}{\partial z^2}-\Big(1+\frac{M_0}{H_0}\Big)k^2\phi_1-K_2\frac{\partial\Theta}{\partial z}=0, \tag{37}$$

$$(1+\chi)\frac{\partial^2\phi_2}{\partial z^2}-\Big(1+\frac{M_0}{H_0}\Big)k^2\phi_2-K_3\frac{\partial\Gamma}{\partial z}=0, \tag{38}$$

$$(1+\chi)\frac{\partial^2\phi_3}{\partial z^2}-\Big(1+\frac{M_0}{H_0}\Big)k^2\phi_3-K_4\frac{\partial\Psi}{\partial z}=0. \tag{39}$$

Equations (32)–(39) give the following dimensionless equations:

$$\begin{aligned}&\Big(\frac{1}{\epsilon}\frac{\partial}{\partial t^*}+\frac{1+N_1}{k_1^*}(D^2-a^2)\Big)(D^2-a^2)W^*\\ =\ & a\sqrt{R}((M_1-M_4)D\phi_1^*-(1+M_1-M_4)T^*)\\ &+a\sqrt{S_1}((M_1'-M_4')D\phi_2^*+(1-M_1'+M_4')C^{1*})+a\sqrt{S_2}((M_1''-M_4'')D\phi_3^*\\ &+(1-M_1''+M_4'')C^{2*})+2N_1(D^2-a^2)\Omega_3^*,\end{aligned} \tag{40}$$

$$\frac{\partial\Omega_3^*}{\partial t^*}=-2N_1\Big(\frac{1}{\epsilon}(D^2-a^2)W^*+2\Omega_3^*\Big)+N_3(D^2-a^2)\Omega_3^*, \tag{41}$$

$$P_{\rm r}'\frac{\partial T^*}{\partial t^*}-\epsilon P_{\rm r}M_2\frac{\partial(D\phi_1^*)}{\partial t^*}=(D^2-a^2)T^*+a\sqrt{R}(1-M_2)W^*-a\sqrt{R}N_5\Omega_3^*, \tag{42}$$

$$P_{S_1}'\frac{\partial C^{1^*}}{\partial t^*}-\epsilon P_{S_1}M_2'\frac{\partial(D\phi_2^*)}{\partial t^*}=(D^2-a^2)C^{1^*}+a\sqrt{S_1}(1-M_2')W^*, \tag{43}$$

$$P_{S_2}'\frac{\partial C^{2^*}}{\partial t^*}-\epsilon P_{S_2}M_2''\frac{\partial(D\phi_3^*)}{\partial t^*}=(D^2-a^2)C^{2^*}+a\sqrt{S_2}(1-M_2'')W^*, \tag{44}$$

$$D^2\phi_1^*-a^2M_3\phi_1^*-DT^*=0, \tag{45}$$

$$D^2\phi_2^*-a^2M_3\phi_2^*-DC^{1^*}=0, \tag{46}$$

$$D^2\phi_3^*-a^2M_3\phi_3^*-DC^{2^*}=0, \tag{47}$$

where the following non-dimensional quantities and non-dimensional parameters are introduced:

$$t'=\frac{vt}{d^2},\quad W^*=\frac{Wd}{v},\quad \phi_1^*=\frac{(1+\chi)K_1a\sqrt{R}}{K_2\rho C_1\beta vd^2}\phi_1,\quad \phi_2^*=\frac{(1+\chi)K_1'a\sqrt{S_1}}{K_3\rho C_1'\beta'vd^2}\phi_2,$$

$$\phi_3^*=\frac{(1+\chi)K_1''a\sqrt{S_2}}{K_4\rho C_1''\beta''vd^2}\phi_3,\quad R=\frac{g\alpha\beta d^4\rho C_1}{vK_1},\quad S_1=\frac{g\alpha'\beta'd^4\rho C_1'}{vK_1'},\quad S_2=\frac{g\alpha''\beta''d^4\rho C_1''}{vK_1''},$$

$$T^*=\frac{K_1a\sqrt{R}}{\rho C_1\beta vd}\Theta,\quad C^{1^*}=\frac{K_1'a\sqrt{S_1}}{\rho C_1'\beta'vd}\Gamma,\quad C^{2^*}=\frac{K_1''a\sqrt{S_2}}{\rho C_1''\beta''vd}\Psi,\quad a=kd,\quad z'=\frac{z}{d},\quad D=\frac{\partial}{\partial z^*},$$

$$P_{\mathrm{r}}=\frac{v}{K_1}\rho C_1,\quad P_{S_1}=\frac{v}{K_1'}\rho C_1',\quad P_{S_2}=\frac{v}{K_1''}\rho C_1'',\quad M_1=\frac{\mu_0K_2^2\beta}{(1+\chi)\alpha\rho_0 g},$$

$$M_1'=\frac{\mu_0K_3^2\beta'}{(1+\chi)\alpha'\rho_0 g},\quad M_1''=\frac{\mu_0K_4^2\beta''}{(1+\chi)\alpha''\rho_0 g},\quad M_2=\frac{\mu_0T_0K_2^2}{(1+\chi)\rho C_1},\quad M_2'=\frac{\mu_0C_0^1K_3^2}{(1+\chi)\rho C_1},$$

$$M_2''=\frac{\mu_0C_0^2K_4^2}{(1+\chi)\rho C_1''},\quad M_3=\frac{1+\frac{M_0}{H_0}}{1+\chi},\quad M_4=\frac{\mu_0K_2K_3\beta'}{(1+\chi)\alpha\rho_0 g},\quad M_4'=\frac{\mu_0K_3K_4\beta''}{(1+\chi)\alpha'\rho_0 g},$$

$$M_4''=\frac{\mu_0K_2K_4\beta}{(1+\chi)\alpha''\rho_0 g},\quad M_5=\frac{M_4}{M_1}=\frac{M_1'}{M_4'}=\frac{K_3\beta'}{K_2\beta}=\frac{M_1''}{M_4''}=\frac{K_4\beta''}{K_2\beta},\quad N_1=\frac{\zeta}{\eta},$$

$$N_3=\frac{\eta'}{\eta d^2},\quad N_5=\frac{\delta}{\rho C_2 d^2},\quad I'=\frac{I}{d^2},\quad \Omega_3^*=\frac{\Omega_3 d^3}{v}.$$

4 Exact solution for free boundaries

The simplest boundary conditions are chosen here, namely, free-free, no-spin, and isothermal with the infinite magnetic susceptibility χ in the perturbed field to keep the problem analytically tractable, and serve the purpose of providing a qualitative insight in to the problem. The case of two free boundaries is of little physical interest. However, it is mathematically important because one can derive an exact solution, whose properties guide our analysis. The exact solutions to Eqs. (40)–(47) subjected to the boundary conditions are

$$W^*=D^2W=T^*=C^{1^*}=C^{2^*}=\Omega_3^*=D\phi_1^*=D\phi_2^*=D\phi_3^*=0\quad\text{at}\quad z=\pm\frac{1}{2},\tag{48}$$

which are written in the forms of

$$\begin{aligned}&W^*=A_1\mathrm{e}^{\sigma t^*}\cos(\pi z^*),\quad T^*=B_1\mathrm{e}^{\sigma t^*}\cos(\pi z^*),\quad D\phi_1^*=C_1\mathrm{e}^{\sigma t^*}\cos(\pi z^*),\\&\Omega_3^*=D_1\mathrm{e}^{\sigma t^*}\cos(\pi z^*),\quad D\phi_2^*=E_1\mathrm{e}^{\sigma t^*}\cos(\pi z^*),\quad \phi_1^*=\frac{C_1}{\pi}\mathrm{e}^{\sigma t^*}\sin(\pi z^*),\\&\phi_2^*=\frac{E_1}{\pi}\mathrm{e}^{\sigma t^*}\sin(\pi z^*),\quad C^{1^*}=F_1\mathrm{e}^{\sigma t^*}\cos(\pi z^*),\quad C^{2^*}=G_1\mathrm{e}^{\sigma t^*}\cos(\pi z^*),\\&D\phi_3^*=H_1\mathrm{e}^{\sigma t^*}\cos(\pi z^*),\quad \phi_3^*=\frac{H_1}{\pi}\mathrm{e}^{\sigma t^*}\sin(\pi z^*).\end{aligned}\tag{49}$$

Here, A_1, B_1, C_1, D_1, E_1, F_1, G_1, and H_1 are constants, and σ is the growth rate which, in general, is a complex constant. Substituting these values in the linearized perturbation dimensionless equations and dropping asterisks for convenience, we get the equation involving the coefficients A_1, B_1, C_1, D_1, E_1, F_1, G_1, and H_1. For the existence of non-trivial solutions, the determinant of the coefficients A_1, B_1, C_1, D_1, E_1, F_1, G_1, and H_1 must vanish. This determinant on simplification yields

$$\mathrm{i}T_5\sigma_i^5+T_4\sigma_i^4-\mathrm{i}T_3\sigma_i^3-T_2\sigma_i^2+\mathrm{i}T_1\sigma_i+T_0=0.\tag{50}$$

Here,

$$T_5=\frac{b}{\epsilon}L_4'L_3'L_2'L_1',\tag{51a}$$

$$T_4=b\Big(\frac{b}{\epsilon}(L_4'L_1(L_3'+L_2')+L_3'L_2')I_1+\Big((1+N_1)\frac{I_1}{P_1}+\Big(\frac{4N_1+N_3'b}{\epsilon}\Big)\Big)L_4'L_3'L_2'\Big),\tag{51b}$$

$$T_3=b^3\Big((L_4'L_1(L_3'+L_2')+L_3'L_2')(1+N_1)\frac{I_1}{P_1}+L_1(L_4'L_1+L_2'+L_3')I_1\Big)$$

$$+ b^2\Big((L_4'L_1(L_3'+L_2')+L_3'L_2')\frac{4N_1+N_3'b}{\epsilon}$$

$$+ L_4'L_3'L_2'\Big(\frac{1+N_1}{P_1}(4N_1+N_3'b)-\frac{4N_1^2}{\epsilon}\Big)\Big)$$

$$+ (-L_4'L_3'L_2'(1-M_2)(xR_1)+L_2'(1-M_2')(xS_1)$$

$$+ (L_5L_3'+L_1^2L_1'-L_1'L_1)L_4'(xS_1)(xS_2))I_1, \tag{51c}$$

$$T_2 = b^4(L_4'L_1^2+L_2'L_1)(1+N_1)\frac{I_1}{P_1}+b^3\Big((L_4'L_1+L_2')L_1\frac{4N_1+N_3'b}{\epsilon}$$

$$+ L_4'L_1(L_2'+L_3')\Big(\frac{1+N_1}{P_1}(4N_1+N_3'b)-\frac{4N_1^2}{\epsilon}\Big)+L_3'L_1(1+N_1)\frac{I_1}{P_1}\Big)$$

$$+ b^2\Big(L_3'L_1\frac{4N_1+N_3'b}{\epsilon}+L_3'L_2'\Big(\frac{1+N_1}{P_1}(4N_1+N_3'b)-\frac{4N_1^2}{\epsilon}\Big)\Big)$$

$$+ b\Big((L_2'L_3(1-M_2')(xS_1)-(1-M_2)(L_4'L_1+L_3')L_2(xR_1))I_1$$

$$+ (L_5L_4L_1^2-L_4L_1'(1-M_2'')(L_1-1))(xS_1)(xS_2)I_1+L_4'L_3'L_2\frac{2N_1N_5'}{\epsilon}(xR_1)\Big)$$

$$+ (L_1L_4L_3'+(L_5L_4L_1'(L_1-1))(4N_1+N_3'b)(xS_1)(xS_2)$$

$$- L_4'L_3'L_2(1-M_2)(4N_1+N_3'b)(xR_1)$$

$$+ (1-M_2')L_4'L_3(L_2'(4N_1+N_3'b)+I_1)(xS_1)), \tag{51d}$$

$$T_1 = b^4\Big(L_1^2\frac{4N_1+N_3'b}{\epsilon}+(L_4'L_1^2+L_2'L_1+L_3'L_1)\Big(\frac{1+N_1}{P_1}(4N_1+N_3'b)-\frac{4N_1^2}{\epsilon}\Big)\Big)$$

$$+ b^2\Big(((1-M_2')I_1L_1L_3-(1-M_2'')I_1L_4L_1'(L_1-1))(xS_1)$$

$$+ \Big(\frac{2N_1N_5'}{\epsilon}(L_4'L_1+L_1'L_3)-L_1L_2(1-M_2)I_1\Big)xR_1\Big)$$

$$+ b((L_5L_4L_1^2-(1-M_2'')L_4L_1'(L_1-1))(4N_1+N_3'b)(xS_1)(xS_2)$$

$$+ (L_2'L_3+L_4'L_1L_3)((1-M_2')(4N_1+N_3'b)(xS_1))$$

$$- (1-M_2)L_2L_1(L_4'+L_3')(4N_1+N_3'b)(xR_1)), \tag{51e}$$

$$T_0 = b^5\Big(L_1^2\Big(\frac{1+N_1}{P_1}(4N_1+N_3'b)-\frac{4N_1^2}{\epsilon}\Big)\Big)+b^3\Big(L_1\frac{2N_1N_5'}{\epsilon}(xR_1)\Big)$$

$$+ b^2(((1-M_2')L_1L_3xS_1-(1-M_2'')L_1L_4(L_1-1)(xS_1)(xS_2))(4N_1+N_3'b)$$

$$- (1-M_2)L_1L_2(4N_1+N_3'b)xR_1), \tag{51f}$$

where

$$R_1=\frac{R}{\pi^4},\quad S_1=\frac{S_1}{\pi^4},\quad S_2=\frac{S_2}{\pi^4},\quad x=\frac{a^2}{\pi^2},\quad I_1=\mathrm{i}\pi^2 I,\quad \sigma_i=\frac{\sigma}{\pi^2},\quad N_3'=\pi^2N_3,$$

$$N_5'=\pi^2N_5,\quad b=1+x,\quad P_1=\pi^2k_1,\quad L_1=1+xM_3,\quad L_1'=PS_1(1+xM_3),$$

$$L_2=1+xM_3+xM_3M_1(1-M_5),\quad L_2'=(P_\mathrm{r}'-\epsilon P_\mathrm{r}M_2)+xP_\mathrm{r}'M_3,$$

$$L_3 = 1 + xM_3' + xM_3M_1'\Big(\frac{1}{M_5'} - 1\Big), \quad L_3' = (P_{S_1}' - \epsilon P_{S_1}M_2') + xP_{S_1}'M_3,$$

$$L_4 = 1 + xM_3 + xM_3M_1''\Big(\frac{1}{M_5'} - 1\Big), \quad L_4' = (P_{S_2}' - \epsilon P_{S_2}M_2'') + xP_{S_2}'M_3,$$

$$L_5 = P_{\mathrm{r}}(1 - M_2'').$$

5 Results and discussion

5.1 Case of stationary convection

When the instability sets in as the stationary convection in the case of $M_2 \cong 0$ and $M_2' \cong 0$, the marginal state will be characterized by $\sigma_i = 0$[14]. Then, the Rayleigh number R_1 is given by

$$\begin{aligned}R_1 = &\Big(b^3(1 + xM_3)\Big(\frac{1+N_1}{P_l}(4N_1 + N_3'b) - \frac{4N_1^2}{\epsilon}\Big) + \Big(\Big(1 + xM_3 + xM_1'M_3\Big(\frac{1}{M_5} - 1\Big)\Big)xS_1 \\ &+ \Big(1 + xM_3 + xM_1''M_3\Big(\frac{1}{M_5} - 1\Big)\Big)xS_2\Big)(4N_1 + N_3'b)\Big) \\ &\Big/\Big(x(1 + xM_3 + xM_1M_3(1 - M_5))\Big(4N_1 + b\Big(N_3' - \frac{2N_1N_5'}{\epsilon}\Big)\Big)\Big),\end{aligned} \tag{52}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x, the buoyancy magnetization parameter M_1, the non-buoyancy magnetization parameter M_3, the medium permeability P_l, the solute gradient parameters S_1 and S_2, the ratio of the salinity effect on the magnetic field to the pyromagnetic coefficient M_5, the coupling parameter N_1 (coupling between vorticity and spin effects), the spin diffusion parameter N_3', and the micropolar heat conduction parameter N_5' (coupling between spin and heat fluxes). The parameters N_1 and N_3' measure the micropolar viscous effect and the micropolar diffusion effect, respectively.

The classical results with respect to Newtonian fluids can be obtained as the limiting case of the present study. Setting $N_1 = 0$ and $S_1 = 0$ and keeping N_3' arbitrary in Eq. (52), we get

$$R_1 = \frac{(1+x)^3(1+xM_3)}{xP_l(1 + xM_3(1 + M_1))}, \tag{53}$$

which is the expression for the Rayleigh number of ferromagnetic fluids (see Finlayson[15]).

Setting $M_3 = 0$ in Eq. (53), we get

$$R_1 = \frac{(1+x)^3}{xP_l}, \tag{54}$$

which is the classical Rayleigh Bénard result[14] for the Newtonian fluid case.

Before we investigate the effects of various parameters, we first make some comments on the parameters N_1, N_3', and N_5' arising due to suspended particles. Assuming the Clausius-Duhem inequality, Eringen[3] presented certain thermodynamic restrictions which lead to non-negativeness of N_1, N_3', and N_5'. It is obvious that the couple stress comes into play at small values of N_3'. This supports the condition that $0 \leqslant N_1 \leqslant 1$ and that N_3' is a small positive real number. The parameter N_5' has to be finite because the increase in the concentration has to be practically stopped somewhere and hence it has to be a positive finite real number. The range of the values for the other parameters is the same as those in classical ferroconvection problems involving Newtonian ferromagnetic fluids[30–31]. M_1' and M_1'' are the effects of magnetization due to salinity. They are allowed to vary from 0.1 to 0.5 and take the values less than the magnetization parameter M_3. M_5 represents the ratio of the salinity effects on the magnetic field to the pyromagnetic coefficient, which varies from 0.1 to 0.5. The salinity Rayleigh numbers S_1 and S_2 vary from 0 to 500.

To investigate the effect of the solute gradients, the non-buoyancy magnetization coefficient, the coupling parameter, the spin parameter, and the micropolar heat conduction parameter, we examine the behavior of $\frac{\mathrm{d}R_1}{\mathrm{d}P_1}$, $\frac{\mathrm{d}R_1}{\mathrm{d}S_1}$, $\frac{\mathrm{d}R_1}{\mathrm{d}S_2}$, $\frac{\mathrm{d}R_1}{\mathrm{d}M_3}$, $\frac{\mathrm{d}R_1}{\mathrm{d}N_1}$, $\frac{\mathrm{d}R_1}{\mathrm{d}N_3'}$, and $\frac{\mathrm{d}R_1}{\mathrm{d}N_5'}$ analytically. Equation (52) yields

$$\frac{\mathrm{d}R_1}{\mathrm{d}P_1} = \frac{b^2 L_1(1+N_1)(4N_1+N_3'b)}{xP_1^2 L_2\big(4N_1+b\big(N_3'-\frac{2N_1N_5'}{\epsilon}\big)\big)},$$

$$\frac{\mathrm{d}R_1}{\mathrm{d}S_1} = \frac{\big(1+xM_3+xM_1'M_3\big(\frac{1}{M_5}-1\big)\big)(4N_1+N_3'b)}{L_2\big(4N_1+b\big(N_3'-\frac{2N_1N_5'}{\epsilon}\big)\big)}, \tag{55}$$

$$\frac{\mathrm{d}R_1}{\mathrm{d}S_2} = \frac{\big(1+xM_3+xM_1''M_3\big(\frac{1}{M_5}-1\big)\big)(4N_1+N_3'b)}{L_2\big(4N_1+b\big(N_3'-\frac{2N_1N_5'}{\epsilon}\big)\big)}. \tag{56}$$

This shows that, for the stationary convection, the stable solute gradients have a stabilizing effect if

$$N_3' > 2N_1N_5'.$$

In the absence of the micropolar viscous effect coupling parameter N_1, the stable solute gradients always have a stabilizing effect on the system. Equation (52) also yields

$$\begin{aligned}\frac{\mathrm{d}R_1}{\mathrm{d}M_3} = &\Big((1-M_5)\Big(b^3M_1\Big(\frac{1}{P_1}((1+N_1)N_3'b+4N_1)+4N_1^2\Big(\frac{1}{P_1}-\frac{b}{\epsilon}\Big)\Big)\Big)\\ &+\Big(xS_1\Big(M_1-\frac{M_1'(1+xM_3)}{M_5}\Big)+xS_2\Big(M_1-\frac{M_1''(1+xM_3)}{M_5}\Big)\Big)(4N_1+N_3'b)\Big)\\ &\Big/\Big((1+xM_3+xM_1M_3(1-M_5))^2\Big(4N_1+b\Big(N_3'-\frac{2N_1N_5'}{\epsilon}\Big)\Big)\Big),\end{aligned} \tag{57}$$

which is negative if

$$N_3' > \frac{2N_1N_5'}{\epsilon}, \quad \frac{1}{P_1} > \frac{b}{\epsilon}, \quad M_4 > M_1'(1+xM_3), \quad M_4 > M_1''(1+xM_3). \tag{58}$$

When conditions (58) are satisfied, the non-buoyancy magnetization has a destabilizing effect. In the absence of magnetization due to salinity ($M_1' = 0$ and $M_1'' = 0$) and micropolar viscous effect ($N_1' = 0$), the non-buoyancy magnetization always has a destabilizing effect on the system.

It follows from Eq. (52) that

$$\begin{aligned}\frac{\mathrm{d}R_1}{\mathrm{d}N_1} = &\Big(b^3L_1\Big(\frac{b^2}{P_1}N_1'\Big(\frac{2N_5'}{\epsilon}+N_3'\Big)\Big)+8N_1\Big(\frac{1}{P_1}-\frac{b}{\epsilon}\Big)\Big(2N_1+b\Big(N_3'-\frac{N_1N_5'}{\epsilon}\Big)\Big)\\ &+\Big(b^2\frac{2N_1N_5'}{\epsilon}\Big)\Big(xS_1\Big(1+xM_3+xM_1'M_3\Big(\frac{1}{M_5}-1\Big)\Big)\\ &+xS_2\Big(1+xM_3+xM_1''M_3\Big(\frac{1}{M_5}-1\Big)\Big)\Big)\Big)\\ &\Big/\Big(xL_2\Big(4N_1+b\Big(N_3'-\frac{2N_1N_5'}{\epsilon}\Big)\Big)^2\Big),\end{aligned} \tag{59}$$

which is always positive if

$$N_3' > \frac{2N_1N_5'}{\epsilon}, \quad \frac{1}{P_1} > \frac{b}{\epsilon}. \tag{60}$$

It shows that the coupling parameter has a stabilizing effect, when conditions (60) hold. In a non-porous medium, Eq. (59) yields that $\frac{\mathrm{d}R_1}{\mathrm{d}N_1}$ is always positive, implying the stabilizing effect of the coupling parameter. Thus, the medium permeability and porosity play a significant role in the developing condition for the stabilizing behavior of the coupling parameter.

It follows from Eq. (52) that

$$\frac{\mathrm{d}R_1}{\mathrm{d}N_3'} = -\frac{2b^5L_1N_1(N_5' + N_1(N_5' - 2P_1))}{\epsilon x L_2\big(4N_1 + b\big(N_3' - \frac{2N_1N_5'}{\epsilon}\big)\big)^2}, \tag{61}$$

which is negative if $N_5' > 2P_1$.

This shows that the spin diffusion has a stabilizing effect when the condition (61) holds.

Equation (52) also gives

$$\begin{aligned}\frac{\mathrm{d}R_1}{\mathrm{d}N_5'} = &\Big(2b^4L_1M_1\Big(b^3M_1\Big(\frac{1}{P_1}((1+N_1)N_3'b + 4N_1) + 4N_1^2\Big(\frac{1}{P_1} - \frac{b}{\epsilon}\Big)\Big)\Big) \\ &+ 2b(4N_1 + bN_3')\Big(xS_1\Big(1 + xM_3 + xM_1'M_3\Big(\frac{1}{M_5} - 1\Big)\Big) \\ &+ xS_2\Big(1 + xM_3 + xM_1''M_3\Big(\frac{1}{M_5} - 1\Big)\Big)\Big)\Big) \\ &\Big/\Big(\epsilon x L_2\Big(4N_1 + b\Big(N_3' - \frac{2N_1N_5'}{\epsilon}\Big)\Big)^2\Big),\end{aligned} \tag{62}$$

which is always positive if

$$\frac{1}{P_1} > \frac{b}{\epsilon}. \tag{63}$$

Equation (63) shows that the micropoalr heat conduction always has a stabilizing effect. It is observed that in a non-porous medium, $\frac{\mathrm{d}R_1}{\mathrm{d}N_5'}$ is always positive, implying the stabilizing effect of the micropolar heat conduction.

When M_1 is sufficiently large, we have

$$\begin{aligned}R_{\mathrm{m}} &= R_1M_1 \\ &= \Big(b^3\Big((1 + xM_3)\Big(\frac{1+N_1}{P_1}(4N_1 + N_3'b) - \frac{4N_1^2}{\epsilon}\Big)\Big) \\ &\quad + (4N_1 + bN_3')\Big(xS_1\Big(1 + xM_3 + xM_1'M_3\Big(\frac{1}{M_5} - 1\Big)\Big) \\ &\quad + xS_2\Big(1 + xM_3 + xM_1''M_3\Big(\frac{1}{M_5} - 1\Big)\Big)\Big)\Big) \\ &\quad \Big/\Big(x^2M_3(1 - M_5)\Big(4N_1 + b\Big(N_3' - \frac{2N_1N_5'}{\epsilon}\Big)\Big)\Big),\end{aligned} \tag{64}$$

where R_{m} is the magnetic thermal Rayleigh number.

As a function of x, $R_{\rm m}$ given by Eq. (64) attains its maximum when

$$P_6x^6 + P_5x^5 + P_4x^4 + P_3x^3 + P_2x^2 + P_1x + P_0 = 0. \tag{65}$$

The coefficients P_0, P_1, P_2, P_3, P_4, P_5, and P_6 are quite lengthy. Thus, they are not written here and are evaluated by the numerical calculation.

The values of the critical wave number for the onset of instability are determined numerically by using the Newtonian Raphson method with the condition $\frac{{\rm d}R_{\rm m}}{{\rm d}x} = 0$. With x_1 determined as a solution to Eq. (65), Eq. (64) gives the required critical magnetic thermal Rayleigh number $N_{\rm c}$ which depends upon M_3, P_l S_1, S_2, and the micropolar parameters N_1, N_3', and N_5'.

5.2 Principle of exchange of stabilities

We shall examine the possibility of oscillatory modes, if any, on stability problems due to the presence of micropolar parameters and solute gradients. Equating the imaginary parts of Eq. (50), we obtain

$$\begin{aligned}&\sigma_i\Big(\frac{b}{\epsilon}L_4'L_3'L_2'I_1\sigma_i^4 - \Big(b^3\Big(((L_4'L_1(L_3'+L_2') + L_3'L_2')(1+N_1)\frac{I_1}{P_l} + L_1(L_4'L_1 + L_2' + L_3')I_1\Big)\\ &+ b^2\Big((L_4'L_1(L_3'+L_2') + L_3'L_2')\Big(\frac{4N_1+N_3'b}{\epsilon}\Big) + L_4'L_3'L_2'\Big(\Big(\frac{1+N_1}{P_l}\Big)(4N_1+N_3'b) - \frac{4N_1^2}{\epsilon}\Big)\Big)\\ &+ L_2'(1-M_2')xS_1 + (L_5L_3' + L_1^2L_1' - L_1'L_1)L_4'(xS_1)(xS_2))I_1\Big)\sigma_i^2 + b^4\Big(L_1^2\Big(\frac{4N_1+N_3'b}{\epsilon}\Big)\\ &+ (L_4'L_1^2 + L_2'L_1 + L_3'L_1)\Big(\Big(\frac{1+N_1}{P_l}\Big)(4N_1+N_3'b) - \frac{4N_1^2}{\epsilon}\Big)\Big) + b^2\Big(((1-M_2')I_1L_1L_3\\ &- (1-M_2'')I_1L_1L_4(L_1-1))xS_1 + \Big(\frac{2N_1N_5'}{\epsilon}(L_4'L_1 + L_1'L_3) - L_1L_2(1-M_2)I_1\Big)xR_1\\ &+ b((L_5L_4L_1^2 - (1-M_2'')L_4L_1'(L_1-1))(4N_1+N_3'b)(xS_1)(xS_2) + (L_2'L_3 + L_4'L_1L_3)\\ &\cdot((1-M_2')(4N_1+N_3'b)xS_1) - (1-M_2)L_2L_1(L_4'+L_3')(4N_1+N_3'b)xR_1\Big) = 0.\end{aligned} \tag{66}$$

It is evident from Eq. (66) that σ_i may be either zero or non-zero, meaning that the modes may be either oscillatory or non-oscillatory. In the absence of the micropolar viscous effect ($N_1 = 0$), the microinertia ($I_1 = 0$), and the solute gradients ($S_1 = 0$, $S_2 = 0$, $L_3' = 0$, and $L_4' = 0$), we obtain the result as

$$\sigma_i\Big(\frac{b}{\epsilon}L_1 + \frac{L_2'}{\epsilon}\Big) = 0. \tag{67}$$

Here, the quantity inside the bracket is positive definite because the typical values of M_2 are $+10^{-6}$[15]. Hence,

$$\sigma_i = 0, \tag{68}$$

which implies that the oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied for the micropolar ferromagnetic fluid heated from below in the absence of the micropolar viscous effect, the microinertia, and the solute gradients. Thus, from Eq. (67), we conclude that the oscillatory modes are introduced due to the presence of the micropolar viscous effect, the microinertia, and the solute gradient, which are non-existent in their absence.

5.3 Case of overstability

The present section is devoted to find the possibility that the observed instability may really be overstable. Since we wish to determine the Rayleigh number for the onset of the instability through the state of pure oscillations, it is sufficient to find the conditions for which Eq. (50) will admit of the solutions with σ_i being real.

Equating the real and imaginary parts of Eq. (50) and eliminating R_1 between them, we obtain

$$A_3 c_1^3 + A_2 c_1^2 + A_1 c_1 + A_0 = 0, \tag{69}$$

where $c_1 = \sigma_i^2$. Since σ_i is real for overstability, the three values of $c_1 = \sigma_i^2$ are positive. The product of roots of Eq. (69) is $-\frac{A_0}{A_3}$, where

$$A_3 = -bL_4'^2 L_3'^2 L_2' I_1((L_1 + (1+N_1)L_2')(1-M_2) + (2N_1N_3')L_2'), \tag{70}$$

$$\begin{aligned}
A_0 = {} & \Big(b^3 L_1 \frac{2N_1N_5'}{\epsilon} - b^2(1-M_2)L_1L_2(4N_1+N_3'b)\Big)\Big(b^4\Big(L_1^2\frac{4N_1+N_3'b}{\epsilon} \\
& + (L_4'L_1^2 + L_2'L_1 + L_3'L_1)\Big(\frac{1+N_1}{P_1}(4N_1+N_3'b) - \frac{4N_1^2}{\epsilon}\Big)\Big) + b^2(((1-M_2')I_1L_1L_3 \\
& - (1-M_2'')I_1L_1L_4(L_1-1))(xS_1)) + b((L_5L_4L_1^2 - (1-M_2'')L_4L_1'(L_1-1)) \\
& \cdot (4N_1+N_3'b)(xS_1)(xS_2) + (L_2'L_3 + L_4'L_1L_3)(1-M_2')(4N_1+N_3'b)(xS_1)\Big) \\
& - \Big(b^5L_1^2\Big(\frac{1+N_1}{P_1}(4N_1+N_3'b) - \frac{4N_1^2}{\epsilon}\Big) + b^2(((1-M_2')L_1L_3(xS_1) \\
& - (1-M_2'')L_1L_4(L_1-1))(xS_1)(xS_2))(4N_1+N_3'b)\Big)\Big(b^2\frac{2N_1N_5'}{\epsilon}((L_4'L_1 + L_1'L_3) \\
& - L_1L_2(1-M_2)I_1) - b(1-M_2)L_2L_1(L_4'+L_3')(4N_1+N_3'b)\Big).
\end{aligned} \tag{71}$$

The coefficients A_2 and A_1 are quite lengthy and are not needed in the discussion of overstability. Thus, they are not written here.

Since σ_i is real for overstability, the three values of $c_1 = \sigma_i^2$ are positive. The product of roots of Eq. (69) is $-\frac{A_0}{A_3}$. If it is to be negative, then A_3 and A_0 are of the same sign. Now, the product is negative if

$$N_3'(1-M_2) > \frac{4N_1N_5'}{\epsilon}, \quad N_3'L_2' > I_1L_1, \quad L_1 > N_1L_2',$$
$$L_2' > L_3'\Big(1 + \frac{N_3'L_2'}{I_1L_1}\Big), \quad L_2' > L_4'\Big(1 + \frac{N_3'L_2'}{I_1L_1}\Big),$$

if

$$N_3' > \frac{4N_1N_5'}{\epsilon(1-M_2)}, \quad N_3' > \frac{I_1(1+xM_3)}{P_r' - \epsilon P_r' M_2}, \quad \frac{1}{P_r'} > N_1,$$
$$P_r' > P_{S_1}'\Big(1 + \frac{\eta' P_r'}{\eta I}\Big), \quad P_r' > P_{S_1}'\Big(1 + \frac{\eta' P_r'}{\eta I}\Big) + \epsilon P_r M_2, \quad P_r' > P_{S_2}'\Big(1 + \frac{\eta' P_r'}{\eta I}\Big)$$
$$P_r' > P_{S_2}'\Big(1 + \frac{\eta' P_r'}{\eta I}\Big) + \epsilon P_r M_2,$$

which implies that

$$N_3' > \max\Big(\frac{4N_1N_5'}{\epsilon(1-M_2)}, \frac{I_1(1+xM_3)}{P_r'-\epsilon P_r' M_2}\Big), \quad \frac{1}{P_r} > N_1,$$

$$K_1 < K_1'\Big(\frac{\rho C_1 - \frac{\epsilon\mu_0 T_0 K_2^2}{1+\chi}}{\rho C_1'\big(1+\frac{\eta' P_r'}{\eta I}\big)}\Big), \quad K_1 < K_1''\Big(\frac{\rho C_1 - \frac{\epsilon\mu_0 T_0 K_2^2}{1+\chi}}{\rho C_1''\big(1+\frac{\eta' P_r'}{\eta I}\big)}\Big).$$

However, $P_r' > P_{S_1}'\big(1+\frac{\eta' P_r'}{\eta I}\big)$ and $P_r' > P_{S_2}'\big(1+\frac{\eta' P_r'}{\eta I}\big)$ are already satisfied in the above condition because the typical values of M_2 are $+10^{-6}$[15].

Thus, for

$$N_3' > \max\Big(\frac{4N_1N_5'}{\epsilon(1-M_2)}, \frac{I_1(1+xM_3)}{P_r'-\epsilon P_r M_2}\Big), \quad \frac{1}{P_r'} > N_1,$$

$$K_1 < K_1'\Big(\frac{\rho C_1 - \frac{\epsilon\mu_0 T_0 K_2^2}{1+\chi}}{\rho C_1'\big(1+\frac{\eta' P_r'}{\eta I}\big)}\Big), \quad K_1 < K_1''\Big(\frac{\rho C_1 - \frac{\epsilon\mu_0 T_0 K_2^2}{1+\chi}}{\rho C_1''\big(1+\frac{\eta' P_r'}{\eta I}\big)}\Big),$$

overstability cannot occur, and the principle of the exchange of stabilities is valid. The above conditions are the sufficient conditions for the non-existence of overstability, the violation of which does not necessarily imply the occurrence of overstability. In the absence of the magnetic parameters and the micropolar parameters in the non-porous medium, the above conditions reduce to $K_1 < K_1'$ and $K_1 < K_1''$, i.e., the thermal conductivity is less than the solute conductivity.

6 Conclusions

In this paper, the linear stability of the triple-diffusive convection in a micropolar ferromagnetic fluid saturating a porous medium is investigated. Here, the simplest boundary conditions, namely, free-free, no-spin, and isothermal with the infinite magnetic susceptibility χ in the perturbed field, keep the problem analytically tractable and serve the purpose of providing a qualitative insight into the problem. The case of two free boundaries is of little physical interest. However, it is mathematically important because one can derive an exact solution whose properties guide our analysis. In conclusion, we see that convection can be encouraged in a micropolar ferromagnetic fluid by means of the spatial variation in magnetization, which can be induced when the magnetization of the fluid depends on the temperature and solute concentrations, and a uniform temperature gradient and a uniform solute gradient are established across the layer. This problem represents the thermal-salinity-micro rotational-mechanical interaction in the porous medium arising through the stress tensor, salinity, and micro rotation. The behaviors of various parameters such as the medium permeability, the solute gradients, the non-buoyancy magnetization, the coupling parameter, the spin diffusive parameter, and the micropolar heat conduction on the onset of convection are analyzed analytically and numerically. The destabilizing behavior of the medium permeability and the stabilizing behavior of the solute gradient are virtually unaffected by the magnetic parameters, while are significantly affected by the micropolar parameters. The presence of coupling between the vorticity and spin effects (micropolar viscous effect), the microinrtia, and the solute gradient may bring overstability in the system. The principle of exchange of stabilities is found to be true for the micropolar ferromagnetic fluid heated from below in the absence of the micropolar viscous effect, the microinertia, and the solute gradients. Thus, oscillatory modes are introduced due to the presence of the micropolar viscous effect, the microinertia, and the solute gradients, which are non-exitent in their absence. We conclude that the micropolar parameters and the solute gradient have a profound influence on the onset of convection in the porous medium.

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