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Convective transport of nanoparticles in multi-layer fluid flow*

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Abstract Technologically, multi-layer fluid models are important in understanding fluid-fluid or fluid-nanoparticle interactions and their effects on flow and heat transfer characteristics. However, to the best of the authors' knowledge, little attention has been paid to the study of three-layer fluid models with nanofluids. Therefore, a three-layer fluid flow model with nanofluids is formulated in this paper. The governing coupled non-linear differential equations of the problem are non-dimensionalized by using appropriate fundamental quantities. The resulting multi-point boundary value problem is solved numerically by quasi-linearization and Richardson's extrapolation with modified boundary conditions. The effects of the model parameters on the flow and heat transfer are obtained and analyzed. The results show that an increase in the nanoparticle concentration in the base fluid can modify the fluid-velocity at the interface of the two fluids and reduce the shear not only at the surface of the clear fluid but also at the interface between them. That is, nanofluids play a vital role in modifying the flow phenomena. Therefore, one can use nanofluids to obtain the desired qualities for the multi-fluid flow and heat transfer characteristics.

Key words multi-layer fluid model, interaction, nanofluid

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1 Introduction

The study of convective flow and heat transfer in a vertical channel has gained interest in recent years because of its industrial applications occurring in the design of cooling systems for electronic devices, chemical processing equipments, microelectronic cooling, solar energy collection, and nuclear reactors[1–5]. Tao[6] studied the laminar fully developed mixed convection in a vertical channel with a uniform wall temperature. Habchi and Acharya[7] studied the laminar fully developed mixed convection in a vertical channel with asymmetric heating where one plate is heated and the other is adiabatic. Thereafter, numerous investigations have been made on the mixed convection flow with symmetric/asymmetric heating under different physical situations[8–11]. These studies are restricted to a single-fluid model.

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Most of the scientific and technological problems relating petroleum industry, geophysics, plasma physics, etc. involve multi-fluid flow situations. A number of complex interfacing transport phenomena may take place in non-isothermal multi-fluid systems. An important assumption usually met in this model is the interfacial thermal and chemical equilibrium between the fluids. The behavior of two-layer liquids is of major interest in the course of design and operation of the fluid experiments in low-gravity space environment, and the multi-layer of the liquid arrangement provides an improved model for a buoyancy-driven convection process in growing high quality crystals[12–14]. Kimura et al.[15] examined the effects of the ratio of the depth of two layers on the Nusselt number by using a spindle oil-ehtylene glycol combination and a spindle oil-water combination. Malashetty et al.[16–17] studied the magnetoconvection of two immiscible fluids in vertical channels. The physical situation considered by Kimura et al.[15] is one of the possible cases. Another physical phenomenon is the case in which a layer of nanofluids is sandwiched in between another two viscous fluid layers.

Due to the increasing importance of nanofluids, in industry, the convective transport in nanofluids has been studied. Fluids such as oil, water, and ethylene glycol mixture are poor in transmitting heat since their thermal conductivities play an important role in calculating the heat transfer coefficients between the fluid media and the boundary surfaces. Numerous methods have been proposed to improve the thermal conductivities of such fluids by suspending nano/micro sized particles (Cu, Ag, TiO_2, Al_2O_3, etc.) in the liquids. The effective thermal conductivity of nanofluids is expected to enhance the rate of heat transfer compared with the convectional heat transfer liquids. This phenomenon suggests the possibility for the use of nanofluids in advanced nuclear systems.

Nanofluids refer to a liquid containing a suspension of submicronic solid particles. Choi[18] used the term nanofluid to refer to the fluid with suspended nanoparticles. Choi et al.[19] showed that the addition of a small amount (less than 0.01) of nanoparticles to convectional liquids can increase the thermal conductivity of the fluid up to approximately two times (see Refs. [20–21] for details). Buongiorno and Hu[20] argued that the effect on the dispersion of the suspended nanoparticles is too small to the observed enhancement. They also concluded that turbulence is not affected by the presence of nanoparticles, and thus the observed enhancement cannot be explained by this. In another paper, Buongiorno[22] pointed out that the nanoparticle absolute velocity can be viewed as the sum of the base fluid velocity and the relative velocity which he called the slip velocity. He considered in turn seven slip mechanisms: inertia, Brownian motion, thermopherosis, diffusiopherosis, Magnus effect, fluid drainage, and gravity settling.

Numerous models and methods have been proposed by many authors to study the convective flows of nanofluids[23–31]. Available literatures on multi-fluid mixed convective flows in a vertical channel show that not much work has been carried out for three-fluid layer models, where nanofluids are sandwiched in between two clear viscous liquids, which have extensive applications in petroleum industry, geophysics, and plasma physics.

Keeping these practical applications in view, in the present paper, we study the mixed convective heat transfer in a nanofluid sandwiched in between viscous fluid layers in a vertical channel. The boundary layer and the Boussinesq approximations are used to simplify the momentum, the energy, and the nanoparticle equations. The physical/rheological properties of the viscous liquid are assumed to be constants. The velocities, stresses, temperatures, and heat fluxes are assumed to be continuous at the interfaces of the nanofluid and the viscous clear fluid. Here, the momentum, energy, and nanoparticle volume fraction equations are coupled and nonlinear. To deal with the coupling and non-linearity, in the multi-point boundary value problem, a numerical technique with quasi-linearization and Richardson's extrapolation is used. It may be noted that the flow and heat transfer characteristics depend on the temperature difference, the Brownian motion, the thermophoresis parameter, the mixed convection parameter, the Prandtl number, and the internal heat source/sink parameter. One of the important findings is that an increase in the nanoparticle concentration (in the base fluid) can modify the fluid velocity at the interface of the two fluids, and can reduce the shear not only at the surface of the clear fluid but also at the interface between them.

2 Formulation and solution

The physical configuration considered in this study is shown in Fig. 1. We consider a convective flow in an infinite vertical channel with a nanofluid layer squeezed between two-viscous fluid layers. The regions

$$-h \leqslant y \leqslant 0, \quad h \leqslant y \leqslant 2h$$

are filled with a clear fluid with the viscosity μ_1 and the thermal diffusivity α_1, and the region $0 \leqslant y \leqslant h$ is occupied by a nanofluid with the viscosity μ_2 and the thermal diffusivity α_2. The boundary walls of the channel are held at different constant temperatures, i.e., T_{w_1} and T_{w_2} ($T_{\mathrm{w}_1} > T_{\mathrm{w}_2}$). The flows in all the regions are assumed to be steady, laminar, and fully developed, and the fluid thermo-physical properties are assumed to be constants except the density variation in the buoyancy force term (in both regions). Furthermore, the fluids in all the three regions are assumed to be driven by a common pressure gradient.

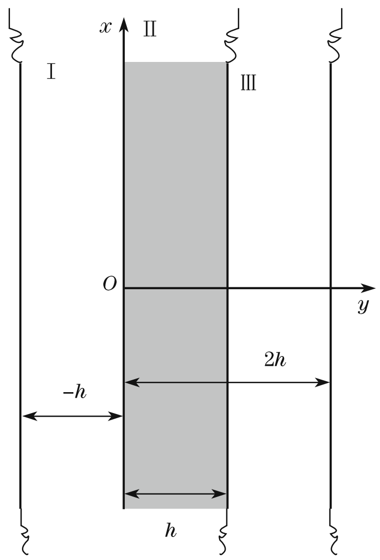

Fig. 1 Geometry of problem where Regions I and III contain clear viscous Newtonian fluids while Region II contains fluid with nanoparticles

With these assumptions, the governing equations of the mass, momentum, energy, and nanoparticle volume fraction reduce to the following equations:

Region I

$$\frac{\mathrm{d}v_1}{\mathrm{d}y} = 0, \tag{1}$$

$$\nu_1 \frac{\mathrm{d}^2 u_1}{\mathrm{d}y^2} - \frac{1}{\rho_1}\frac{\partial p}{\partial x} + g\beta_1 (T_1 - T_{\mathrm{w}_2}) = 0, \tag{2}$$

$$\alpha_1 \frac{\mathrm{d}^2 T_1}{\mathrm{d}y^2} + \frac{Q_1}{\rho_1 c_p}(T_1 - T_{\mathrm{w}_2}) + \frac{\mu_1}{\rho_1 c_p}\Big(\frac{\mathrm{d}u_1}{\mathrm{d}y}\Big)^2 = 0; \tag{3}$$

Region II

$$\frac{\mathrm{d}v_2}{\mathrm{d}y}=0, \tag{4}$$

$$\nu_2\frac{\mathrm{d}^2u_2}{\mathrm{d}y^2}-\frac{1}{\rho_2}\frac{\partial p}{\partial x}+g\beta_2(T_2-T_{\mathrm{w}_2})=0, \tag{5}$$

$$\begin{aligned}&\alpha_2\frac{\mathrm{d}^2T_2}{\mathrm{d}y^2}+\tau\Big(D_{\mathrm{B}}\frac{\mathrm{d}C}{\mathrm{d}y}\frac{\mathrm{d}T_2}{\mathrm{d}y}+\Big(\frac{D_{\mathrm{T}}}{T_{\mathrm{w}_2}}\Big)\Big(\frac{\mathrm{d}T_2}{\mathrm{d}y}\Big)^2\\&+Q_2(T_2-T_{\mathrm{w}_2})+\mu_2\Big(\frac{\mathrm{d}u_2}{\mathrm{d}y}\Big)^2\Big)=0,\end{aligned} \tag{6}$$

$$D_{\mathrm{B}}\frac{\mathrm{d}^2C}{\mathrm{d}y^2}+\Big(\frac{D_{\mathrm{T}}}{T_{\mathrm{w}_2}}\Big)\frac{\mathrm{d}^2T_2}{\mathrm{d}y^2}=0; \tag{7}$$

Region III

$$\frac{\mathrm{d}v_3}{\mathrm{d}y}=0, \tag{8}$$

$$\nu_1\frac{\mathrm{d}^2u_3}{\mathrm{d}y^2}-\frac{1}{\rho_1}\frac{\partial p}{\partial x}+g\beta_1(T_3-T_{\mathrm{w}_2})=0, \tag{9}$$

$$\alpha_1\frac{\mathrm{d}^2T_3}{\mathrm{d}y^2}+\frac{Q_3}{\rho_1c_p}(T_3-T_{\mathrm{w}_2})+\frac{\mu_1}{\rho_1c_p}\Big(\frac{\mathrm{d}u_3}{\mathrm{d}y}\Big)^2=0, \tag{10}$$

where u_i, T_i, and Q_i $(i=1,2,3)$ are, respectively, the velocities (in the x-direction), the temperatures, and the internal heat generation/absorption of Regions I, II, and III. ρ_i, ν_i, α_i, and β_i $(i=1,2,3)$ are, respectively, the densities, the viscosities, the thermal diffusivities, and the coefficients of thermal expansion of Regions I, II, and III. Here, g is the acceleration due to gravity, c_p is the specific heat at constant pressure, C is the nanoparticle volume fraction, D_{B} is the Brownian diffusion coefficient, and D_{T} is the thermophoretic diffusion coefficient. τ is the heat capacity ratio expressed by

$$\tau=\frac{(\rho c_p)_{\mathrm{p}}}{(\rho c_p)_{\mathrm{f}}},$$

where the subscripts p and f denote the nanoparticles and the base fluid, respectively. The appropriate boundary and the interface conditions for the problem are

$$\begin{cases}u_1(y)=0,\quad T_1(y)=T_{\mathrm{w}_2}\quad\text{at}\quad y=-h;\\ u_2(y)=u_1(y),\quad T_2(y)=T_1(y),\quad C(y)=C_{\mathrm{w}},\\ \qquad\mu_1\dfrac{\mathrm{d}u_1}{\mathrm{d}y}=\mu_2\dfrac{\mathrm{d}u_2}{\mathrm{d}y},\quad\alpha_1\dfrac{\mathrm{d}T_1}{\mathrm{d}y}=\alpha_2\dfrac{\mathrm{d}T_2}{\mathrm{d}y}\quad\text{at}\quad y=0;\\ u_2(y)=u_3(y),\quad T_2(y)=T_3(y),\quad C(y)=0,\\ \qquad\mu_2\dfrac{\mathrm{d}u_2}{\mathrm{d}y}=\mu_1\dfrac{\mathrm{d}u_3}{\mathrm{d}y},\quad\alpha_2\dfrac{\mathrm{d}T_2}{\mathrm{d}y}=\alpha_1\dfrac{\mathrm{d}T_3}{\mathrm{d}y}\quad\text{at}\quad y=h;\\ u_3(y)=0,\quad T_3(y)=T_{\mathrm{w}_1}\quad\text{at}\quad y=2h.\end{cases} \tag{11}$$

To account for the nanoparticle concentration, the form of the coupled equations (6)–(7) accounting for the nanoparticle concentration are taken to follow from the theory outlined in Buongiorno[22]. To non-dimensionalize the governing equations, the following fundamental quantities are used:

$$\begin{cases} u_1^* = \dfrac{u_1}{\hat{u}_1}, \quad u_2^* = \dfrac{u_2}{\hat{u}_1}, \quad u_3^* = \dfrac{u_3}{\hat{u}_1}, \\ y^* = \dfrac{y}{h}, \quad \phi = \dfrac{C}{C_{\mathrm{w}}}, \quad P = -\dfrac{h_1^2}{\mu_1 \hat{u}_1}\dfrac{\partial p}{\partial x}, \\ Re = \dfrac{\hat{u}_1 h}{\nu_1}, \quad \theta_1 = \dfrac{T_1 - T_{\mathrm{w}_2}}{T_{\mathrm{w}_1} - T_{\mathrm{w}_2}}, \\ \theta_2 = \dfrac{T_2 - T_{\mathrm{w}_2}}{T_{\mathrm{w}_1} - T_{\mathrm{w}_2}}, \quad \theta_3 = \dfrac{T_2 - T_{\mathrm{w}_2}}{T_{\mathrm{w}_1} - T_{\mathrm{w}_2}}, \\ Gr = \dfrac{g\beta_1 (T_{\mathrm{w}_1} - T_{\mathrm{w}_2}) h_1^3}{\nu_1^2}. \end{cases} \tag{12}$$

Substituting the above quantities into Eqs. (1)–(10), after dropping asterisks, we get the following non-dimensionalized equations:

Region I

$$\frac{\mathrm{d}^2 u_1}{\mathrm{d}y^2} + P + \lambda\theta_1 = 0, \tag{13}$$

$$\frac{\mathrm{d}^2 \theta_1}{\mathrm{d}y^2} + Pr\Big(\delta_1\theta_1 + Ec\Big(\frac{\mathrm{d}u_1}{\mathrm{d}y}\Big)^2\Big) = 0; \tag{14}$$

Region II

$$\frac{\mathrm{d}^2 u_2}{\mathrm{d}y^2} + m_1 P + b_1 a_1 \lambda\theta_2 = 0, \tag{15}$$

$$\frac{\mathrm{d}^2 \theta_2}{\mathrm{d}y^2} + \frac{Pr}{a_1 d_1}\Big(N_{\mathrm{b}}\frac{d\theta_2}{dy}\frac{\mathrm{d}\phi}{\mathrm{d}y} + N_{\mathrm{t}}\Big(\frac{\mathrm{d}\theta_2}{\mathrm{d}y}\Big)^2 + \delta_2\theta_2 + Ec\Big(\frac{\mathrm{d}u_2}{\mathrm{d}y}\Big)^2\Big) = 0, \tag{16}$$

$$\frac{\mathrm{d}^2 \phi}{\mathrm{d}y^2} + \frac{N_{\mathrm{t}}}{N_{\mathrm{b}}}\frac{\mathrm{d}^2\theta_2}{\mathrm{d}y^2} = 0; \tag{17}$$

Region III

$$\frac{\mathrm{d}^2 u_3}{\mathrm{d}y^2} + P + \lambda\theta_3 = 0, \tag{18}$$

$$\frac{\mathrm{d}^2 \theta_3}{\mathrm{d}y^2} + Pr\Big(\delta_3\theta_3 + Ec\Big(\frac{\mathrm{d}u_3}{\mathrm{d}y}\Big)^2\Big) = 0. \tag{19}$$

In the above equations, λ, Pr, Ec, N_{b}, and N_{t} are the mixed convection parameter, the Prandtl number, the Eckert number, the Brownian motion parameter, and the thermophoresis parameter, and δ_1, δ_2, and δ_3 are the heat source/sink parameters in Regions I, II, and III, respectively. They are defined by

$$\begin{cases} \lambda = \dfrac{Gr}{Re}, \quad N_{\rm b} = \dfrac{\tau D_{\rm B} C_{\rm w}}{\nu_1}, \\ N_{\rm t} = \dfrac{\tau D_{\rm T}(T_{{\rm w}_1} - T_{{\rm w}_2})}{\nu_1 T_{{\rm w}_2}}, \\ Ec = \dfrac{\hat{u}_1^2}{c_p(T_{{\rm w}_1} - T_{{\rm w}_2})}, \\ Pr = \dfrac{\nu_1}{\alpha_1}, \quad \delta_1 = \dfrac{Q_1 h^2}{\rho_1 c_p \nu_1}, \\ \delta_2 = \dfrac{Q_2 \tau h^2}{\rho_1 c_p \nu_1}, \quad \delta_3 = \dfrac{Q_3 h^2}{\rho_1 c_p \nu_1}. \end{cases} \tag{20}$$

The ratios of different physical quantities that appear in the governing equations are, respectively, the thermal diffusivity, the viscosities, the kinematic viscosity, and the coefficient of thermal expansion, which are defined as follows:

$$\begin{cases} b_1 = \dfrac{\beta_2}{\beta_1}, \quad C_1 = \dfrac{\rho_2}{\rho_1}, \\ d_1 = \dfrac{\alpha_2}{\alpha_1}, \quad m_1 = \dfrac{\mu_2}{\mu_1}, \quad a_1 = m_1 C_1. \end{cases} \tag{21}$$

The boundary conditions in the non-dimensional form are

$$\begin{cases} \theta_1(y) = 0, \quad u_1(y) = 0 \quad \text{at} \quad y = -1; \\ u_1(y) = u_2(y), \quad \theta_1(y) = \theta_2(y), \quad \phi(y) = 1, \\ \qquad \dfrac{{\rm d}u_1}{{\rm d}y} = \dfrac{1}{m_1}\dfrac{{\rm d}u_2}{{\rm d}y}, \quad \dfrac{{\rm d}\theta_1}{{\rm d}y} = \dfrac{1}{d_1}\dfrac{{\rm d}\theta_2}{{\rm d}y} \quad \text{at} \quad y = 0; \\ u_2(y) = u_3(y), \quad \theta_2(y) = \theta_3(y), \quad \phi(y) = 0, \\ \qquad \dfrac{{\rm d}u_2}{{\rm d}y} = m_1\dfrac{{\rm d}u_3}{{\rm d}y}, \quad \dfrac{{\rm d}\theta_2}{{\rm d}y} = d_1\dfrac{{\rm d}\theta_3}{{\rm d}y} \quad \text{at} \quad y = 1; \\ \theta_3(y) = 1, \quad u_3(y) = 0 \quad \text{at} \quad y = 2. \end{cases} \tag{22}$$

3 Numerical procedure

The system of non-linear ordinary differential equations (13)–(19) is solved by the quasi-linearization method[32]. Since the equations (14), (16), and (19) are non-linear, we can obtain a system of second-order linear ordinary differential equations which can be solved by a simple iterative method after linearizing them. To derive the recurrence system, let us denote the ith iteration of u_1 as $u_1^{[i]}$. For the $(i+1)$th iteration, we get

$$\frac{\mathrm{d}^2u_1^{[i+1]}}{\mathrm{d}y^2}+P+\lambda\theta_1^{[i+1]}=0, \tag{23}$$

$$\frac{\mathrm{d}^2\theta_1^{[i+1]}}{\mathrm{d}y^2}+Pr\Big(\delta_1\theta_1^{[i+1]}+2Ec\frac{\mathrm{d}u_1^{[i]}}{\mathrm{d}y}\frac{\mathrm{d}u_1^{[i+1]}}{\mathrm{d}y}-Ec\Big(\frac{\mathrm{d}u_1^{[i]}}{\mathrm{d}y}\Big)^2\Big)=0, \tag{24}$$

$$\frac{\mathrm{d}^2u_2^{[i+1]}}{\mathrm{d}y^2}+m_1P+b_1a_1\lambda\theta_2^{[i+1]}=0, \tag{25}$$

$$\begin{aligned}&\frac{\mathrm{d}^2\theta_2^{[i+1]}}{\mathrm{d}y^2}+\frac{Pr}{a_1d_1}\Big(\delta_2\theta_2^{[i+1]}+N_{\mathrm{b}}\Big(\frac{\mathrm{d}\phi^{[i]}}{\mathrm{d}y}\frac{\mathrm{d}\theta_2^{[i+1]}}{\mathrm{d}y}+\frac{\mathrm{d}\phi^{[i+1]}}{\mathrm{d}y}\frac{\mathrm{d}\theta_2^{[i]}}{\mathrm{d}y}\Big)\\&+2N_{\mathrm{t}}\frac{\mathrm{d}\theta_2^{[i]}}{\mathrm{d}y}\frac{\mathrm{d}\theta_2^{[i+1]}}{\mathrm{d}y}+2Ec\frac{\mathrm{d}u_2^{[i]}}{\mathrm{d}y}\frac{\mathrm{d}u_2^{[i+1]}}{\mathrm{d}y}\Big)\\&-\frac{Pr}{a_1d_1}\Big(N_{\mathrm{b}}\frac{\mathrm{d}\theta_2^{[i]}}{\mathrm{d}y}\frac{\mathrm{d}\phi^{[i]}}{\mathrm{d}y}+N_{\mathrm{t}}\Big(\frac{\mathrm{d}\theta_2^{[i]}}{\mathrm{d}y}\Big)^2+Ec\Big(\frac{\mathrm{d}u_2^{[i]}}{\mathrm{d}y}\Big)^2\Big)=0,\end{aligned} \tag{26}$$

$$\frac{\mathrm{d}^2\phi^{[i+1]}}{\mathrm{d}y^2}+\frac{N_{\mathrm{t}}}{N_{\mathrm{b}}}\frac{\mathrm{d}^2\theta_2^{[i+1]}}{\mathrm{d}y^2}=0, \tag{27}$$

$$\frac{\mathrm{d}^2u_3^{[i+1]}}{\mathrm{d}y^2}+P+\lambda\theta_3^{[i+1]}=0, \tag{28}$$

$$\frac{\mathrm{d}^2\theta_3^{[i+1]}}{\mathrm{d}y^2}+Pr\Big(\delta_3\theta_3^{[i+1]}+2Ec\frac{\mathrm{d}u_3^{[i]}}{\mathrm{d}y}\frac{\mathrm{d}u_3^{[i+1]}}{\mathrm{d}y}-Ec\Big(\frac{\mathrm{d}u_3^{[i]}}{\mathrm{d}y}\Big)^2\Big)=0 \tag{29}$$

with the boundary conditions (22). The system (23)–(29) is solved numerically (with the accuracy 10^{-6}) by Richardson's extrapolation method (see Bulirsch and Stoer[33]) with

$$\begin{cases}u_1^{[0]}(y)=u_2^{[0]}(y)=u_3^{[0]}(y)=0,\\ \phi^{[0]}(y)=1-y,\\ \theta_1^{[0]}(y)=\dfrac{d_1}{1+2d_1}(1+y),\\ \theta_2^{[0]}(y)=\dfrac{1}{1+2d_1}(d_1+y),\\ \theta_3^{[0]}(y)=1+\dfrac{d_1}{1+2d_1}(y-2).\end{cases}$$

4 Results and discussion

The fully developed convective nanofluid flow and heat transfer in a vertical channel squeezed between two-viscous fluid layers is studied numerically. The velocity, the temperature, the shear stress, and the heat fluxes across the interface are assumed to be continuous. Numerical computation has been carried out to analyze the multi-fluid model by using the method described in the above section for various values of the pertinent parameters, namely, the Brownian motion parameter $N_{\rm b}$, the thermophoresis parameter $N_{\rm t}$, the mixed convection parameter λ, the heat source/sink parameters δ_i, and the ratios of physical quantities (i.e., viscosities, thermal diffusivities, and coefficients of thermal expansion). The Prandtl number Pr, the Eckert number Ec, and the non-dimensional pressure gradient P in all the regions are chosen as 0.7, 0.1, and 5.0, respectively, throughout the computation. The numerical results are presented in Figs. 2–7 to elucidate the interesting features of the convective nanofluid flow and heat transfer. Our results are in good agreement with the results of Umavathi et al.[34] without nanoparticles.

Figures 2–4 are the graphical representations of the stream-wise velocity profiles for different values of the thermo-physical properties (ratios of viscosities, coefficients of thermal expansion, and kinematic viscosities) and the parameters (i.e., the Brownian motion parameter $N_{\rm b}$, the thermophoresis parameter $N_{\rm t}$, and the mixed convection parameter λ). The general trend is that the parabolic nature of the stream-wise velocity is more pronounced with the parameters associated with the thermo-physical properties. The velocity profiles increase initially at the left (the clear fluid region) wall, then become a parabolic in nature in the mid channel (the nanofluid region), and finally go to zero at the right channel (the clear fluid region) wall.

In Fig. 2, the stream-wise velocity profiles are presented for different values of the Brownian motion parameter $N_{\rm b}$. From this figure, we can see that the velocity profiles are considerably reduced with an increase in $N_{\rm b}$. The presence of nanoparticles in the fluid opposes the transport phenomena through the parameter $N_{\rm b}$. The effect of the increasing values of $N_{\rm b}$ is to decrease the velocity, and hence reduces the velocity thickness of the nanofluid region. These results are fully consistent with the physical situation. The stream-wise velocity profiles for different values of the thermophoresis parameter $N_{\rm t}$ for different values of the viscosity ratios are shown graphically in Fig. 3. The effect of increasing the values of $N_{\rm t}$ is to increase the velocity component u. This is due to the presence of nanoparticles and the temperature difference in the nanofluid regions. The results presented in this paper make it clear that the effect of the Brownian motion parameter $N_{\rm b}$ is to decrease the velocity in the nanofluid regions, but the effect of the thermophoresis parameter $N_{\rm t}$ is to increase the velocity in the nanofluid regions.

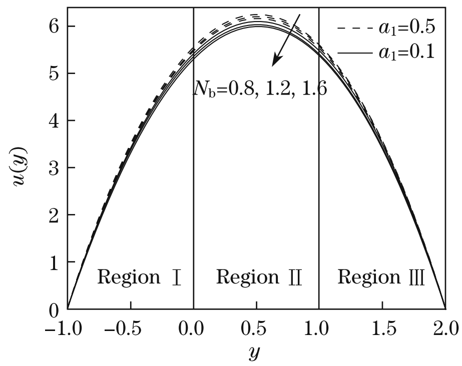

Fig. 2 Velocity profiles for different values of $N_{\rm b}$ and a_1 with $N_{\rm t} = 0.1$ and $\delta_2 = 0.1$

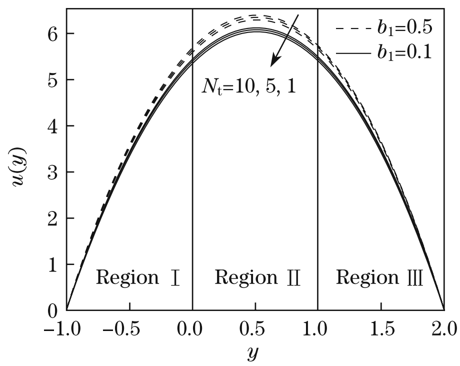

Fig. 3 Velocity profiles for different values of $N_{\rm t}$ and b_1 with $N_{\rm b} = 0.1$ and $\delta_2 = 0.1$

Figure 4 shows the effect of the mixed convection parameter λ on the stream-wise velocity. It is observed that an increase in λ increases the velocity profiles in the nanofluid region of the channel. Physically, $\lambda > 0$ means heating the fluid, $\lambda < 0$ means cooling the fluid, and $\lambda = 0$ corresponds to the absence of mixed convection currents. An increase in λ means an increase in the temperature difference $(T_{\rm w_1} - T_{\rm w_2})$. This leads to the enhancement of the stream-wise velocity due to the enhanced convection, and thus increases the parabolic nature of the velocity profiles in the nanofluid region.

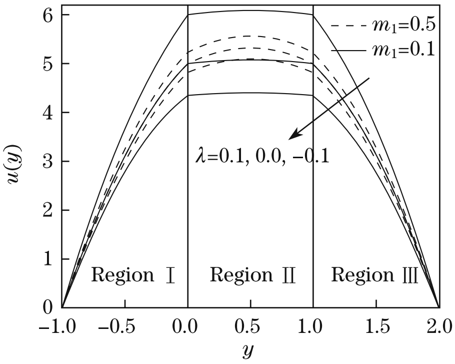

Fig. 4 Velocity profiles for different values of λ and m_1 with $N_b = N_t = 0.1$

The numerical results for the temperature profiles are depicted in Figs. 5–7 for different values of the thermo-physical parameters and the governing parameters. It is observed from these figures that the temperature distribution is zero at the left channel wall and attains the value of unity at the right channel wall.

Figures 5 and 6 depict the changes in the temperature $\theta(y)$ for different values of the thermo-physical and other governing parameters. All physical parameters have similar qualitative effects on the temperature profiles as those seen in the velocity profiles. It is noticed by the comparative study of these two figures with those of Figs. 2 and 3 that the velocity profiles are significantly pronounced as seen the temperature profiles in the nanofluid regions.

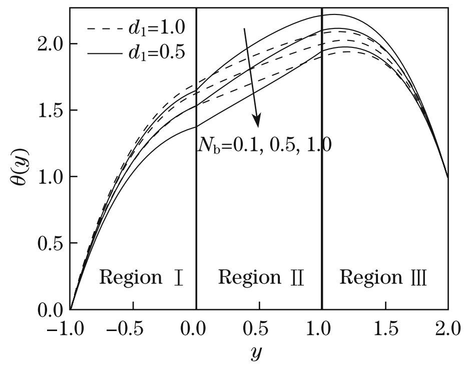

Fig. 5 Temperature profiles for different values of N_b and d_1 with $N_t = 0.5$ and $\delta_2 = 0.1$

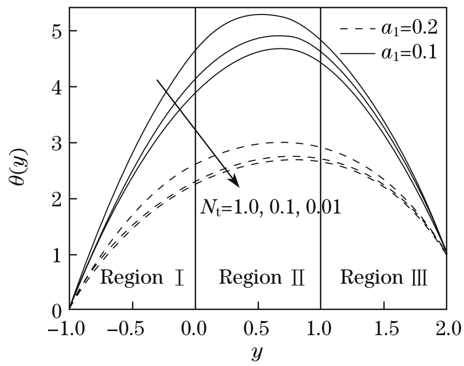

Fig. 6 Temperature profiles for different values of N_t and a_1 with $N_b = 0.5$ and $\delta_2 = 0.1$

Figure 7 exhibits the changes in the temperature with y for different values of the heat source/sink parameter δ_2 and the mixed convection parameter λ. From this figure, we observe that the temperature profiles are lower throughout the channel for negative values of δ_2 (heat sink) and higher for positive values of δ_2 (heat source). The effect of increasing the values of the heat source/sink parameter δ_2 is to increase the temperature profiles in the nanofluid region. From this figure, we also notice that an increase in the values of λ results in a decrease in the thermal layer thickness in the nanofluid region, and this results in an increase in the magnitude of the wall temperature gradient and produces an increase in the heat transfer rate.

Figure 8 shows the effect of the Brownian motion parameter and the thermophoresis parameter on the concentration profiles. From this figure, we can see that decreases in both the parameters enhance the thickness of the nanoparticle volume fraction and hence the concentration profiles.

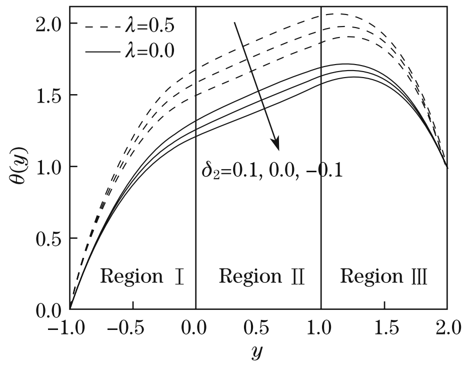

Fig. 7 Temperature profiles for different values of λ and δ_2 with $N_t = N_b = 0.1$

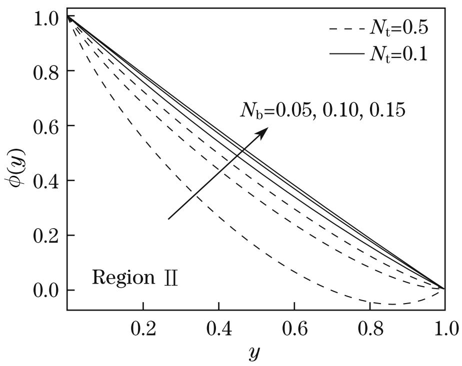

Fig. 8 Concentration profiles for different values of N_b and N_t with $\delta_2 = 0.1$

5 Conclusions

The convective flow and heat transfer of a nanofluid packed between two viscous Newtonian fluid layers in a vertical channel are mathematically analyzed. The governing equations are non-dimensionalized by using the proper non-dimensional quantities. The governing coupled nonlinear differential equations of the problem are non-dimensionalized by using the appropriate fundamental quantities. The resulting multi-point boundary value problem is solved numerically by quasi-linearization and Richardson's extrapolation method with modified boundary conditions. The numerical results are presented through graphs for various values of pertinent parameters, namely, the Brownian motion parameter, the thermophoresis parameter, the mixed convection parameter, and the ratios of different physical quantities. Some of the interesting findings are as follows:

(i) The effects of the Brownian motion parameter are to decelerate the flow and to reduce the heat transfer in the nanofluid region.

(ii) The effects of the thermophoresis parameter and the mixed convection parameter are to enhance the flow and heat transfer in the nanofluid region.

(iii) The effect of the Brownian motion parameter is to enhance the nanoparticle concentration even in the presence of thermophoresis parameter.

(iii) For multi-fluid layer models, adding nanoparticles to one of the fluid regions may be a possible way to control the velocity of the fluid at the interface.

(iv) Interestingly, an increase in the thermophoresis parameter reduces the magnitude of the velocity at the interface. Hence, a thin layer of fluids with nanoparticles can be used to modify the flow properties of clear fluid layers.

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