Appl. Math. Mech. -Engl. Ed., **32**(4), 409–418 (2011)
DOI 10.1007/s10483-011-1426-6

Applied Mathematics and Mechanics (English Edition)

MHD stagnation-point flow and heat transfer towards stretching sheet with induced magnetic field*

F. M. ALI[1], R. NAZAR[2], N. M. ARIFIN[1], I. POP[3]

(1. Department of Mathematics, Institute of Mathematical Research, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia;
2. School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia;
3. Faculty of Mathematics, University of Cluj, R-3400 Cluj, CP 253, Romania)

Abstract The problem of the steady magnetohydrodynamic (MHD) stagnation-point flow of an incompressible viscous fluid over a stretching sheet is studied. The effect of an induced magnetic field is taken into account. The nonlinear partial differential equations are transformed into ordinary differential equations via the similarity transformation. The transformed boundary layer equations are solved numerically using the shooting method. Numerical results are obtained for various magnetic parameters and Prandtl numbers. The effects of the induced magnetic field on the skin friction coefficient, the local Nusselt number, the velocity, and the temperature profiles are presented graphically and discussed in detail.

Key words boundary layer, heat transfer, induced magnetic field, numerical solution, magnetohydrodynamic (MHD) flow, stretching sheet

Chinese Library Classification O345
2010 Mathematics Subject Classification 76D10, 76W05

1 Introduction

The study of stretching sheets is important in industries and manufacturing processes. For example, the study of the biomagnetic fluid (biological fluids in the presence of magnetic fields) flow towards a stretching sheet is important in bioengineering and medical applications[1–3]. Further, the study of the magnetohydrodynamic (MHD) flow of an electrically conducting fluid due to the stretching sheet is important in modern metallurgy and metal-working processes. A number of technical processes concerning polymers involve the cooling of continuous strips or filaments by drawing them through a quiescent or moving fluid. The final product depends, to a great extent, on the rate of cooling, which is governed by the structure of the boundary layer near the stretching sheet.

Al-Odat et al.[4] presented numerical solutions for the thermal boundary layer of an exponentially continuous stretching surface in the presence of a magnetic field with variable exponential

* Received Jun. 24, 2010 / Revised Feb. 9, 2011
Project supported by the Fundamental Research Grant Scheme (FRGS) of the Ministry of Higher Education (MOHE) of Malaysia (No. UKM-ST-07-FRGS0036-2009)
Corresponding author R. NAZAR, Associate Professor, Ph. D., E-mail: rmn72my@yahoo.com

temperature at the surface. Li et al.[5] considered another problem of the boundary layer flow in channels with the effect of a transverse magnetic field.

The stagnation-point flow of an incompressible viscous fluid against an infinite flat plate is an important type of flows, which has many engineering applications. Hiemenz[6] was the first to study the two-dimensional (2D) stagnation-point flow against an infinite flat plate. He found an exact solution to the governing Navier-Stokes equations. Eckert[7] extended this problem by including the energy equation, and obtained an exact solution for the thermal field. Many researchers have been working on the stagnation-point flow in various ways. Mahapatra and Gupta[8] considered the MHD case. Mahapatra and Gupta[9–10] studied the heat transfer in the stagnation-point flow over a stretching surface and also the stagnation-point flow in a viscoelastic fluid, respectively. Nazar et al.[11] considered the boundary layer flow near the stagnation-point on a stretching sheet, where time dependence is also taken into account. Ishak et al.[12] studied the boundary layer stagnation-point flow over a stretching sheet for both the cases of assisting and opposing flows, while Wang[13] solved the problem of a stagnation-point flow on a moving plate with a slip condition. In non-Newtonian fluids, Gorla[14] solved the boundary layer flow for power-law fluids at a stagnation-point in the presence of a transverse magnetic field. Recently, Zhu et al.[15–16] presented the analytical solutions of stagnation-point flow over a stretching sheet and also the MHD stagnation-point flow towards a power-law stretching sheet with the effect of slip, respectively, via the homotopy analysis method. These are few examples for the problem where the induced magnetic field is negligible.

To date, very little work has been done on the boundary layer flow and heat transfer with the consideration of the induced magnetic field. For example, Kumari et al.[17] investigated the MHD flow and heat transfer over a stretching surface by considering the effect of the induced magnetic field, and Takhar et al.[18] studied the unsteady free convection flow at the stagnation-point under the presence of a magnetic field.

In the present paper, we extend the work by Mahapatra and Gupta[9] to the case of the MHD stagnation-point flow of an electrically conducting fluid in the presence of a transverse magnetic field by taking an induced magnetic field into account. In this study, the magnetic field is applied parallel to the sheet. The partial differential equations are reduced to similarity equations or nonlinear ordinary differential equations, which are solved numerically. The flow depends heavily on the magnetic parameter.

2 Basic equations

Consider the steady 2D stagnation-point flow of an incompressible electrically conducting fluid towards a stretching surface in its own plane with a velocity proportional to the distance from the stagnation-point. The effect of the induced magnetic field is taken into account (see Fig. 1). The basic equations for the flow of a viscous and electrically conducting fluid can be written as follows[19]:

$$\nabla \cdot V = 0, \quad \nabla \cdot H = 0, \tag{1}$$

$$(V \cdot \nabla)V - \frac{\mu}{4\pi\rho}(H \cdot \nabla)H = -\frac{1}{\rho}\nabla P + \nabla^2 V, \tag{2}$$

$$\nabla \times (V \times H) + \mu_{\mathrm{e}}\nabla^2 H = 0, \tag{3}$$

$$(V \cdot \nabla)T = \alpha\nabla^2 T, \tag{4}$$

where V is the fluid velocity vector, H is the induced magnetic field vector, the MHD pressure

$$P = p + \frac{\mu|H|^2}{8\pi},$$

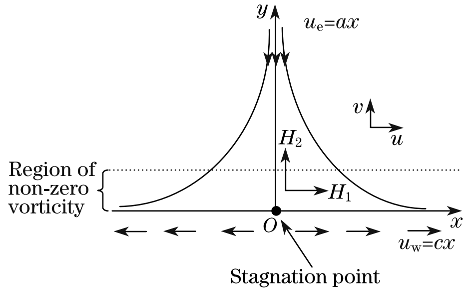

Fig. 1 Physical model and coordinate system

T is the fluid temperature, p is the fluid pressure, μ, ν, σ, ρ, α, and

$$\mu_{\mathrm{e}} = \frac{1}{4\pi\sigma}$$

denote the magnetic permeability, the kinematic viscosity, the electric conductivity, the density, the thermal diffusivity, and the magnetic diffusivity, respectively.

According to the boundary layer approximations, Eqs. (1)–(4) for the problem under consideration can be reduced to[20]

$$\begin{cases} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0, \\ \dfrac{\partial H_1}{\partial x} + \dfrac{\partial H_2}{\partial y} = 0, \end{cases} \tag{5}$$

$$\begin{aligned} & u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - \frac{\mu}{4\pi\rho}\Big(H_1\frac{\partial H_1}{\partial x} + H_2\frac{\partial H_1}{\partial y}\Big) \\ = & \Big(u_{\mathrm{e}}\frac{\mathrm{d}u_{\mathrm{e}}}{\mathrm{d}x} - \frac{\mu H_{\mathrm{e}}}{4\pi\rho}\frac{\mathrm{d}H_{\mathrm{e}}}{\mathrm{d}x}\Big) + \nu\frac{\partial^2 u}{\partial y^2}, \end{aligned} \tag{6}$$

$$u\frac{\partial H_1}{\partial x} + v\frac{\partial H_1}{\partial y} - H_1\frac{\partial u}{\partial x} - H_2\frac{\partial u}{\partial y} = \mu_{\mathrm{e}}\frac{\partial^2 H_1}{\partial y^2}, \tag{7}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{8}$$

where x and y are the Cartesian coordinates along the stretching surface and normal to it, respectively, u and v are the velocity components along x and y, H_1 and H_2 are the magnetic components along x and y, and $u_{\mathrm{e}}(x)$ and $H_{\mathrm{e}}(x)$ are the x-velocity and the x-magnetic field at the edge of the boundary layer. The boundary conditions of Eqs. (5)–(8) are as follows:

$$\begin{cases} v = 0, \quad u = u_{\mathrm{w}}(x) = cx, \quad \dfrac{\partial H_1}{\partial y} = H_2 = 0, \quad T = T_{\mathrm{w}} \quad \text{at} \quad y = 0, \\ u = u_{\mathrm{e}}(x) = ax, \quad H_1 = H_{\mathrm{e}}(x) = H_0 x, \quad T = T_\infty \quad \text{as} \quad y \to \infty, \end{cases} \tag{9}$$

where a and c are positive constants, and H_0 is the value of the uniform magnetic field at the infinity upstream. Applying the transformations

$$\begin{cases} u = cxf'(\eta), \quad v = -(c\nu)^{\frac{1}{2}} f(\eta), \\ H_1 = H_0 x g'(\eta), \quad H_2 = -(c\nu)^{\frac{1}{2}} g(\eta), \\ \eta = \Big(\dfrac{c}{\nu}\Big)^{\frac{1}{2}} y, \quad \theta(\eta) = \dfrac{T - T_\infty}{T_{\rm w} - T_\infty} \end{cases} \tag{10}$$

to Eqs. (5)–(8), we obtain the following ordinary differential equations:

$$f''' + ff'' - (f')^2 + \frac{a^2}{c^2} + \beta(g'^2 - gg'' - 1) = 0, \tag{11}$$

$$\lambda g''' + fg'' - f''g = 0, \tag{12}$$

$$\theta'' + Prf\theta' = 0. \tag{13}$$

Then, the boundary conditions (9) reduce to

$$\begin{cases} f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = \dfrac{a}{c}, \quad \theta(0) = 1, \\ g(0) = 0, \quad g''(0) = 0, \quad g'(\infty) = 1, \quad \theta(\infty) = 0, \end{cases} \tag{14}$$

where primes denote differentiation with respect to η, λ is the reciprocal magnetic Prandtl number, β is the magnetic parameter, and Pr is the Prandtl number, which are defined as

$$\begin{cases} \lambda = \dfrac{\mu_{\rm e}}{\nu}, \\ Pr = \dfrac{\nu}{\alpha}, \\ \beta = \dfrac{\mu}{4\pi\rho}\Big(\dfrac{H_0}{c}\Big)^2. \end{cases} \tag{15}$$

The magnetic parameter β, which gives the order of the ratio of the magnetic energy and the kinetic energy per unit volume, is related to the Hartmann[21] number Ha and the flow and magnetic Reynolds numbers Re and $Re_{\rm m}$, respectively, in the following way:

$$\begin{cases} \beta = \dfrac{Ha^2}{ReRe_{\rm m}}, \\ Ha = \mu H_0 l \Big(\dfrac{\sigma}{\mu}\Big)^{\frac{1}{2}}, \\ Re = \dfrac{(cl)l}{\nu}, \\ Re_{\rm m} = 4\pi U_\infty l\mu\sigma = \dfrac{(cl)l}{\mu_{\rm e}}, \end{cases} \tag{16}$$

where l is the characteristic length of the stretching surface comparable with the dimensions of the field. It may be noted that for $\beta = 0$ (without a magnetic field), Eq. (11) reduces to that of Mahapatra and Gupta[9]. As $\beta = 0$ implies the absence of the magnetic field, Eq. (12) governing the induced magnetic field is no longer needed.

The physical quantities of interest are the skin friction coefficient $C_{\rm f}$ and the local Nusselt number Nu, which are defined as

$$\begin{cases} C_{\rm f} = \dfrac{\tau_{\rm w}}{\rho u_{\rm w}^2}, \\ Nu = \dfrac{x q_{\rm w}}{k(T_{\rm w} - T_\infty)}, \end{cases} \tag{17}$$

where the wall shear stress $\tau_{\rm w}$ and the wall heat flux $q_{\rm w}$ are given by

$$\begin{cases} \tau_{\rm w} = \mu \left(\dfrac{\partial u}{\partial y}\right)_{y=0}, \\ q_{\rm w} = -k \left(\dfrac{\partial T}{\partial y}\right)_{y=0} \end{cases} \tag{18}$$

with k being the thermal conductivity of the fluid. Using the variables in (10), we obtain

$$\begin{cases} Re_x^{\frac{1}{2}} C_{\rm f} = f''(0), \\ Re_x^{-\frac{1}{2}} Nu = -\theta'(0), \end{cases} \tag{19}$$

where $Re_x = u_{\rm w} x/\nu$ is the local Reynolds number.

3 Results and discussion

Equations (11)–(13) subjected to the boundary conditions (14) have been solved numerically by the shooting method, as discussed in a paper by Meade et al.[22]. This well-known technique is an iterative algorithm. It attempts to identify appropriate initial conditions for a related initial value problem (IVP) that provides the solution to the original boundary value problem (BVP). The shooting method used in this study is based on Maple's dsolve command with the program Shoot 9, which is built in Maple and developed by Meade et al.[22]. The edge of the boundary layer chosen in this study is between 5 to 10.

To check the validity and accuracy of the numerical results obtained, the values of the skin friction coefficient $f''(0)$ when the magnetic field is absent, i.e., $\beta = 0$, are compared with the previously published results shown in Table 1. They show good agreement.

Table 1 Skin friction coefficient $f''(0)$ for different values of $a/c = 3$ when $\beta = 0$

a/c	Mahapatra and Gupta[9]	Nazar et al.[11]	Ishak et al.[12]	Present
0.1	−0.969 4	−0.969 4	−0.969 4	−0.969 4
0.2	−0.918 1	−0.918 1	−0.918 1	−0.918 1
0.5	−0.667 3	−0.667 3	−0.667 3	−0.667 3
2.0	2.017 5	2.017 6	2.017 5	2.017 5
3.0	4.729 3	4.729 6	4.729 4	4.729 3

Table 2 shows that for fixed $\beta = 1$ and $a/c = 3$, the local Nusselt number $Re_x^{-\frac{1}{2}} Nu$ increases as the Prandtl number Pr increases.

Table 2 Variation of $Re_x^{\frac{1}{2}} C_{\rm f}$ and $Re_x^{-\frac{1}{2}} Nu$ with $a/c = 3$ and $\beta = 1$

Pr	0.07	0.5	2.0	6.8	10.0
$Re_x^{-\frac{1}{2}} Nu$	0.338 14	0.827 48	1.521 47	2.597 80	3.079 02

Table 3 shows the variations of $Re_x^{\frac{1}{2}}C_{\mathrm{f}}$ and $Re_x^{-\frac{1}{2}}Nu$ for various a/c, β, and Pr. As shown in Table 3, when the effect of the magnetic parameter $a/c > 1$ ($a/c = 3$ and $Pr = 0.72$), the skin friction coefficient decreases when β increases, while it shows the opposite phenomenon for $a/c < 1$ ($a/c = 0.5$ and $Pr = 0.72$). It is observed that for $a/c > 1$, we can take large values of β but not for the case of $a/c < 1$.

Table 3 Variation of $Re_x^{\frac{1}{2}}C_{\mathrm{f}}$ and $Re_x^{-\frac{1}{2}}Nu$ with $Pr = 0.72$ and different a/c

$a/c = 3$			$a/c = 0.5$		
β	$Re_x^{\frac{1}{2}}C_{\mathrm{f}}$	$Re_x^{-\frac{1}{2}}Nu$	β	$Re_x^{\frac{1}{2}}C_{\mathrm{f}}$	$Re_x^{-\frac{1}{2}}Nu$
0.1	4.709 28	0.979 02	0.10	−0.575 95	0.591 71
0.5	4.627 64	0.976 17	0.15	−0.509 38	0.602 07
1.0	4.521 58	0.972 40	0.20	−0.407 17	0.618 11
2.0	4.294 31	0.964 05			
5.0	3.433 52	0.928 63			
8.0	1.877 34	0.844 94			

Figures 2–4 show the effect of the magnetic parameter β on the velocity f', g', and temperature θ profiles with $a/c = 3$ and $Pr = 0.72$. It shows that the velocity and g' profiles decrease as β increases. However, it shows the reverse effect for the temperature profiles.

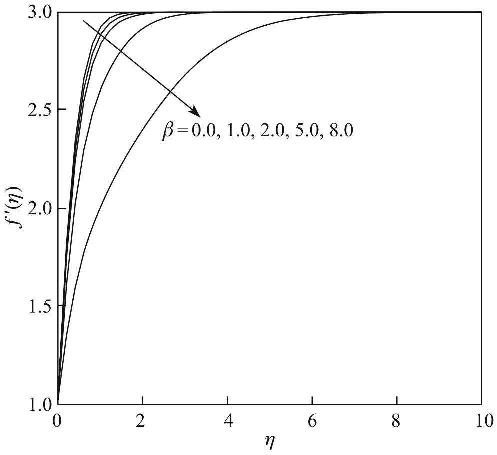

Fig. 2 Velocity profiles for different values of β when $a/c = 3$ and $Pr = 0.72$

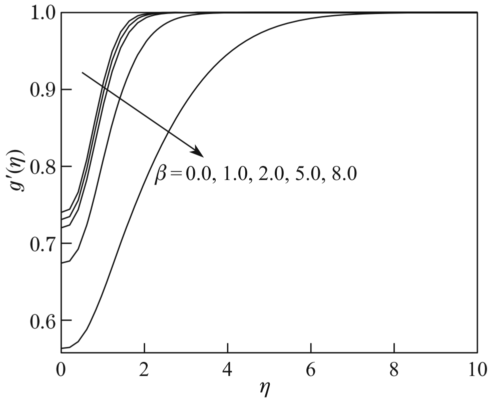

Fig. 3 g' profiles for different values of β when $a/c = 3$ and $Pr = 0.72$

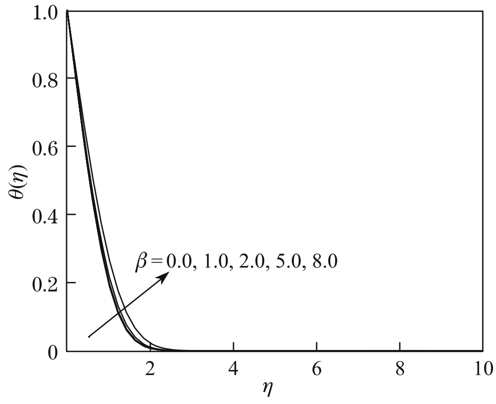

Fig. 4 Temperature profiles for different values of β when $a/c = 3$ and $Pr = 0.72$

Figures 5–7 show the effect of the magnetic parameter β on the velocity f', g', and temperature θ profiles, respectively, with $a/c = 0.5$ and $Pr = 0.72$. It can be seen clearly that the velocity f' and g' profiles increase as β increases, while the temperature θ profiles decrease as β increases.

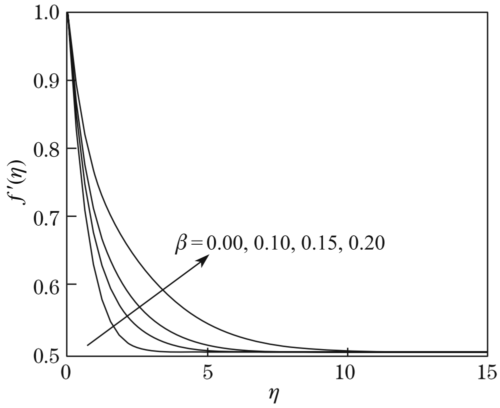

Fig. 5 Velocity profiles for different values of β when $a/c = 0.5$ and $Pr = 0.72$

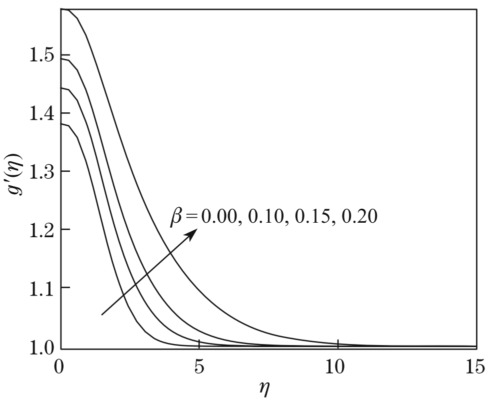

Fig. 6 g' profiles for different values of β when $a/c = 0.5$ and $Pr = 0.72$

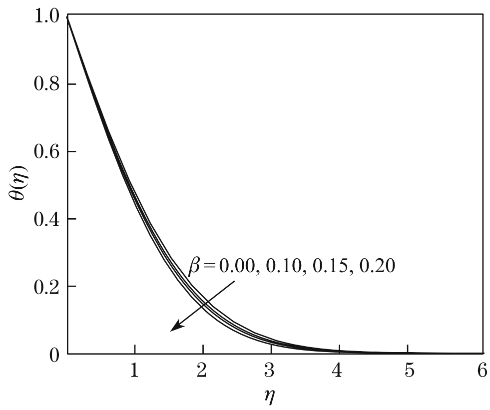

Fig. 7 Temperature profiles for different values of β when $a/c = 0.5$ and $Pr = 0.72$

In Figs. 8–11, we consider the fixed values of Pr and β for various a/c. Figure 8 shows the flow with a boundary layer structure. It can be seen from Fig. 8 that the boundary layer thickness decreases as a/c increases for the case of $a/c > 1$. From [9], physically, this phenomenon can be explained as follows: for fixed value of c corresponding to the stretching of the surface, the increase in a in relation to c implies the increase in the straining motion near the stagnation region that can increase the acceleration of the external stream. Therefore, the increase in a/c has the effect of thinning the boundary layer. However, for $a/c < 1$, the flow has an inverted boundary layer structure, where the stretching velocity cx of the surface exceeds the velocity ax of the external stream. In Fig. 9, it can be seen that all the g' profiles decrease with the increase in a/c. However, for the case of $a/c > 1$, the opposite trend occurs after a certain point. In Fig. 10, it can be seen that the plotted f profiles decrease when a/c decreases. Figure 11 shows that the thermal boundary layer thickness decreases when a/c increases.

From Figs. 12 and 13, again both $a/c = 3$ and $a/c = 0.5$ are considered. It is found that the thermal boundary layer thickness also decreases as Pr increases. If Pr increases, the thermal diffusivity will decrease. This leads to the decrease of the energy transfer ability, and results in the reducing thermal boundary layer.

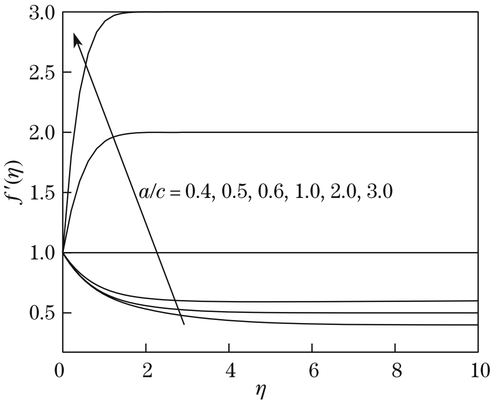

Fig. 8 Velocity profiles for different values of a/c when $Pr = 0.72$ and $\beta = 0.1$

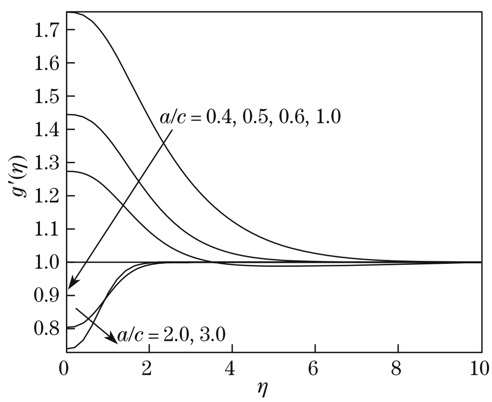

Fig. 9 g' profiles for different values of a/c when $Pr = 0.72$ and $\beta = 0.1$

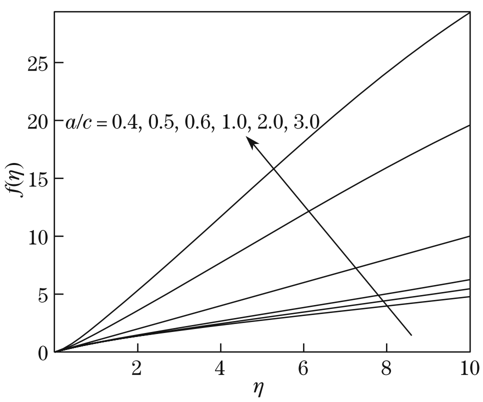

Fig. 10 Variation of f for different values of a/c when $Pr = 0.72$ and $\beta = 0.1$

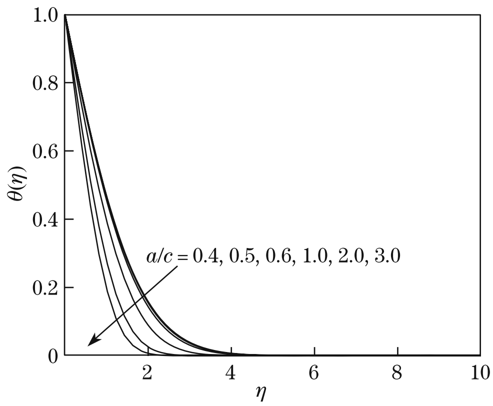

Fig. 11 Temperature profiles for different values of a/c when $Pr = 0.72$ and $\beta = 0.1$

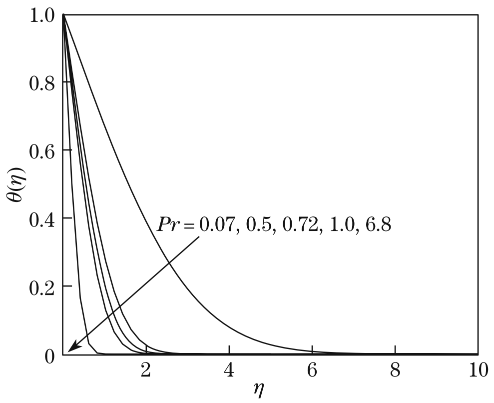

Fig. 12 Temperature profiles for several values of Pr when $a/c = 3.0$ and $\beta = 1.0$

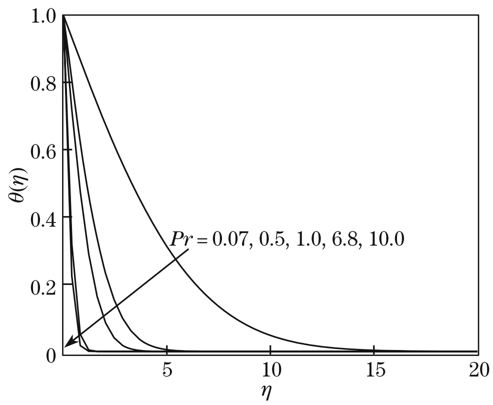

Fig. 13 Temperature profiles for several values of Pr when $a/c = 0.5$ and $\beta = 0.1$

4 Conclusions

A numerical study is performed for the problem of the MHD stagnation-point flow of an incompressible viscous fluid over a stretching sheet in the presence of an induced magnetic field. Our results with absent magnetic parameters are compared with previously well-known results. The agreement is found to be very good. The effects of the magnetic parameter β on the velocity, g', and temperature profiles for both $a/c > 1$ and $a/c < 1$ are investigated. It can be seen that all the skin friction coefficients decrease as β increases for $a/c > 1$; however, it shows the opposite phenomenon for $a/c < 1$. In this study, we find that there are significant effects of the magnetic parameter on the velocity, g', and temperature profiles. It is also found that the thermal boundary layer thickness reduces when Pr increases for both the cases of $a/c < 1$ and $a/c < 1$.

Acknowledgements The authors gratefully acknowledge the beneficial comments and suggestions given by the anonymous reviewers.

References

[1] Tzirtzilakis, E. E. and Kafoussias, N. G. Biomagnetic fluid flow over a stretching sheet with nonlinear temperature dependent magnetization. *Journal of Applied Mathematics and Mechanics* (*ZAMP*), **54**, 551–565 (2003)

[2] Tzirtzilakis, E. E. and Tanoudis, G. B. Numerical study of biomagnetic fluid flow over a stretching sheet with heat transfer. *International Journal for Numerical Methods in Heat and Fluid Flow*, **13**(7), 830–848 (2003)

[3] Tzirtzilakis, E. E. and Kafoussias, N. G. Three-dimensional magnetic fluid boundary layer flow over a linearly stretching sheet. *ASME Journal of Heat Transfer*, **132**, 011702-1–011702-8 (2010)

[4] Al-Odat, M. Q., Damseh, R. A., and Al-Azab, T. A. Thermal boundary layer on an exponentially stretching continuous surface in the presence of magnetic field effect. *International Journal of Applied Mechanics and Engineering*, **11**, 289–299 (2006)

[5] Li, B. T., Zheng, L. C., and Zhang, X. X. Multiple solutions of laminar flow in channels with a transverse magnetic field. *Chinese Physics Letters*, **26**, 094101-1–094101-3 (2009)

[6] Hiemenz, K. Die grenzschicht an einem in den gleichförmigen flüssigkeitsstrom eingetauchten geraden kreiszylinder. *Polytechnic Journal*, **326**, 321–324 (1911)

[7] Eckert, E. R. G. Die berechnung des warmeubergangs in der laminaren grenzschicht umstromter korpe. *VDI Forschungsheft*, **416**, 1–23 (1942)

[8] Mahapatra, T. R. and Gupta, A. S. Magnetohydrodynamic stagnation point flow towards a stretching sheet. *Acta Mechanica*, **152**, 191–196 (2001)

[9] Mahapatra, T. R. and Gupta, A. S. Heat tansfer in stagnation-point flow towards a stretching sheet. *Heat and Mass Transfer*, **38**, 517–521 (2002)

[10] Mahapatra, T. R. and Gupta, A. S. Stagnation-point flow of a viscoelastic fluid towards a stretching surface. *International Journal of Non-Linear Mechanics*, **39**, 811–820 (2004)

[11] Nazar, R., Amin, N., Filip, D., and Pop, I. Unsteady boundary layer flow in the region of the stagnation point on a stretching sheet. *International Journal of Engineering Science*, **42**, 1241–1253 (2004)

[12] Ishak, A., Nazar, R., and Pop, I. Mixed convection boundary layers in the stagnation-point flow towards a stretching vertical sheet. *Meccanica*, **41**, 509–518 (2006)

[13] Wang, C. Y. Stagnation slip flow and heat transfer on a moving plate. *Chemical Engineering Science*, **61**(23), 7668–7672 (2006)

[14] Gorla, R. S. R. Non-Newtonian fluid at a stagnation point in the presence of a transverse magnetic field. *Mechanics Research Communications*, **3**, 1–6 (1976)

[15] Zhu, J., Zheng, L. C., and Zhang, X. X. Analytical solution to stagnation-point flow and heat transfer over a stretching sheet based on homotopy analysis. *Applied Mathematics and Mechanics* (*English Edition*), **30**(4), 463–474 (2009) DOI 10.1007/s10483-009-0407-2

[16] Zhu, J., Zheng, L. C., and Zhang, Z. G. Effects of slip condition on MHD stagnation-point flow over a power-law stretching sheet. *Applied Mathematics and Mechanics* (*English Edition*), **31**(4), 439–448 (2010) DOI 10.1007/s10483-010-0404-z

[17] Kumari, M., Takhar, H. S., and Nath, G. MHD flow and heat transfer over a stretching surface with prescribed wall temperature or heat flux. *Wärme und Stoffübertragung*, **25**(6), 331–336 (1990)

[18] Takhar, H. S., Kumari, M., and Nath, G. Unsteady free convection flow under the influence of a magnetic field. *Archive of Applied Mechanics*, **63**, 313–321 (1993)

[19] Cowling, T. G. *Magnetohydrodynamics*, Interscience Publication, New York (1957)

[20] Davies, T. V. The magneto-hydrodynamic boundary layer in the two-dimensional steady flow past a semi-infinite plat plate I, uniform conditions at infinity. *Proceedings of the Royal Society A*, **273**, 496–508 (1962)

[21] Hartmann, J. Hg-dynamics I, theory of the laminar flow of an electrically conducting liquid in a homogenous magnetic field. *Matematisk-Fysiske Meddelelser*, **15**(6), 1–28 (1937)

[22] Meade, D. B., Haran, B. S., and White, R. E. The shooting technique for the solution of two-point boundary value problems. *Maple Technology*, **3**, 85–93 (1996)