Appl. Math. Mech. -Engl. Ed., **32**(3), 301–308 (2011)
DOI 10.1007/s10483-011-1416-8

Applied Mathematics and Mechanics (English Edition)

Stability analysis of radial inflation of incompressible composite rubber tubes*

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Abstract The inflation mechanism is examined for a composite cylindrical tube composed of two incompressible rubber materials, and the inner surface of the tube is subjected to a suddenly applied radial pressure. The mathematical model of the problem is formulated, and the corresponding governing equation is reduced to a second-order ordinary differential equation by means of the incompressible condition of the material, the boundary conditions, and the continuity conditions of the radial displacement and the radial stress of the cylindrical tube. Moreover, the first integral of the equation is obtained. The qualitative analyses of static inflation and dynamic inflation of the tube are presented. Particularly, the effects of material parameters, structure parameters, and the radial pressure on radial inflation and nonlinearly periodic oscillation of the tube are discussed by combining numerical examples.

Key words composite rubber tube, radial inflation, stability, nonlinearly periodic oscillation

Chinese Library Classification O343
2010 Mathematics Subject Classification 74B20

1 Introduction

The mechanical properties of rubber and rubber-like materials are extremely complex. Like nature rubber, synthetic rubber, and synthetic fibre, they all have the typical characteristics of nonlinearity, high elasticity, and large deformation. In real life, there exist many kinds of

* Received Dec. 24, 2010 / Revised Jan. 14, 2011
Project supported by the National Natural Science Foundation of China (Nos. 10872045 and 10721062), the Program for New Century Excellent Talents in University (No. NCET-09-0096), and the Fundamental Research Funds for the Central Universities (No. DC10030104)
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rubber products, such as rubber seal ring, rubber tube, and rubber support. These products are well used in many fields, such as aerospace, precise instrument, national defense and military, machinofacture, medical treatment, and public health. Since the instability of structures composed of rubber materials is viewed as the key mechanism of damage and fracture of materials, the study of the relative problems receives a lot of attention. Rubber and rubber-like materials are also called the Green elastic materials, and their constitutive relations can be completely represented by their strain energy functions. At present, the research findings on the static problems of finite deformation for rubber materials are abundant, such as internal pressure of a cylindrical tube, torsion and bend of a cylinder, inflation of a spherical shell, and cavitation in materials[1–5]. Recently, the dynamic stability problems of radial inflation of incompressible rubber tubes have been presented by some references, such as Refs. [6–8]. In particular, the finite oscillation problem of a cylindrical tube composed of an incompressible Mooney-Rivlin material model was examined by Knowles[6], and the conditions of periodic oscillation of the tube and the formulas of oscillation period and oscillation amplitude were presented. The dynamic response of a hyperelastic cylindrical tube composed of an incompressible neo-Hookean material under periodic loads was studied by Ren[7]. The radial oscillation problem of a cylindrical tube composed of a class of incompressible Ogden materials under periodic step loads was investigated by Yuan et al.[8], and the effects of material parameters, structure parameters, and loading forms on nonlinearly periodic oscillation of the tube were discussed in detail.

Based on the theory of finite deformation for hyperelastic materials, this work first formulates the mathematical model that describes the radial deformation of an incompressible composite cylindrical rubber tube and gets an equation that describes the radial inflation of the tube. Then, we analyze the mechanism of static and dynamic inflation of the tube qualitatively, and discuss the effects of structure and material parameters on finite deformation of the tube by some numerical examples.

2 Mathematical model and solutions

For a composite cylindrical tube, we are concerned with its radial inflation under a prescribed radial pressure p_0 at the inner surface, and the interior and the exterior parts of the tube are composed of different incompressible rubber materials. Assume that the interface of the tube is bonded perfectly during the course of deformation and satisfies the continuity conditions of the radial displacement and the radial stress. Let (R, Θ, Z) and (r, θ, z) be the material and the spatial coordinates corresponding to the undeformed and the deformed configurations. Under the assumption of radially symmetric deformation and of plane strain case, the deformation configurations of the interior and the exterior parts of the tube are, respectively, given by

$$r^{[1]} = r^{[1]}(R,t) > 0, \quad R_1 \leqslant R \leqslant R_2; \quad \theta^{[1]} = \Theta^{[1]}; \quad z^{[1]} = Z^{[1]}, \tag{1}$$

$$r^{[2]} = r^{[2]}(R,t) > 0, \quad R_2 \leqslant R \leqslant R_3; \quad \theta^{[2]} = \Theta^{[2]}; \quad z^{[2]} = Z^{[2]}, \tag{2}$$

where R_1, R_2, and R_3 are the radii of inner, middle, and outer surfaces; $r^{[i]} = r^{[i]}(R,t)$ $(i = 1, 2)$ are undetermined radial motion functions related to time.

The deformation gradient tensors $F^{[i]}(i = 1, 2)$ corresponding to the radial symmetric deformation (1) and (2) are as follows:

$$F^{[i]} = \mathrm{diag}\,(\lambda_1^{[i]}, \lambda_2^{[i]}, \lambda_3^{[i]}) = \mathrm{diag}\,\Big(\frac{\partial r^{[i]}(R,t)}{\partial R}, \frac{r^{[i]}(R,t)}{R}, 1\Big), \tag{3}$$

where $\lambda_1^{[i]}$, $\lambda_2^{[i]}$, and $\lambda_3^{[i]}$ are known as the principal stretches of the deformation gradient tensors.

The incompressibility condition requires that the determinants of the deformation gradient tensors are equal to 1, i.e., $\lambda_1^{[i]}\lambda_2^{[i]}\lambda_3^{[i]} = 1$, with Eq. (3), we have

$$r^{[i]} = r^{[i]}(R, t) = (R^2 + c^2(t) - R_1^2)^{\frac{1}{2}}, \quad t \geqslant 0, \tag{4}$$

where $c(t) \geqslant 0$ is an undetermined function. As $i = 1$, $R_1 \leqslant R \leqslant R_2$, while as $i = 2$, $R_2 \leqslant R \leqslant R_3$.

2.1 Constitutive models of composite rubbers

Since the interior and the exterior parts of the tube are, respectively, composed of two different materials, and the deformation field is assumed to be continuous at the interface, the associated strain energy functions are expressed as follows:

$$W = W(\lambda_1, \lambda_2, \lambda_3) = \begin{cases} W^{[1]}(\lambda_1, \lambda_2, \lambda_3), & R_1 < R < R_2, \\ W^{[2]}(\lambda_1, \lambda_2, \lambda_3), & R_2 < R < R_3. \end{cases} \tag{5}$$

The principal components of the Cauchy stress tensors of the composite cylindrical tube are, respectively, given by

$$\begin{cases} \tau_{rr}^{[i]}(r^{[i]}, t) = \lambda_1^{[i]} \dfrac{\partial W^{[i]}}{\partial \lambda_1^{[i]}} - p^{[i]}(r^{[i]}, t), \\ \tau_{\theta\theta}^{[i]}(r^{[i]}, t) = \lambda_2^{[i]} \dfrac{\partial W^{[i]}}{\partial \lambda_2^{[i]}} - p^{[i]}(r^{[i]}, t), \end{cases} \tag{6}$$

where $p^{[i]}(r^{[i]}, t)$ $(i = 1, 2)$ are the hydrostatic pressures associated with the incompressible materials. Moreover, the continuity condition of the radial stress at the material interface requires that

$$\tau_{rr}^{[1]}(r^{[1]}(R_2, t), t) = \tau_{rr}^{[2]}(r^{[2]}(R_2, t), t). \tag{7}$$

2.2 Governing equation and initial-boundary conditions

In the absence of body force, the differential equation that governs the radial motion of the composite cylindrical tube in each individual phase is given by

$$\frac{\partial \tau_{rr}^{[i]}(r^{[i]}, t)}{\partial r^{[i]}} + \frac{1}{r^{[i]}}\left(\tau_{rr}^{[i]}(r^{[i]}, t) - \tau_{\theta\theta}^{[i]}(r^{[i]}, t)\right) = \rho^{[i]} \frac{\partial^2 r^{[i]}}{\partial t^2}, \tag{8}$$

where $\rho^{[i]}$ $(i = 1, 2)$ is the constant mass density of the materials.

Assume that the composite cylindrical tube is in an undeformed state, and at time $t = 0$, we have

$$r^{[i]}(R, 0) = R, \quad \dot{r}^{[i]}(R, 0) = 0. \tag{9}$$

Using Eq. (4) yields

$$c(0) = R_1, \quad \dot{c}(0) = 0. \tag{10}$$

Since the inner surface of the tube is subjected to a suddenly applied constant pressure p_0, and the outer surface is traction free, the corresponding boundary conditions are given by

$$\tau_{rr}^{[1]}(r^{[1]}(R_1, t), t) = -p_0, \quad \tau_{rr}^{[2]}(r^{[2]}(R_3, t), t) = 0, \quad t \geqslant 0. \tag{11}$$

2.3 Solutions

With Eq. (4), we have

$$\frac{\partial^2 r^{[i]}(R,t)}{\partial t^2}=\frac{\partial\Big(c\ddot{c}\ln r^{[i]}+\Big(\ln r^{[i]}+(r^{[i]})^{-2}\dfrac{c^2}{2}\Big)\dot{c}^2\Big)}{\partial r^{[i]}},\quad i=1,2. \tag{12}$$

Integrating Eq. (8) with respect to $r^{[i]}$ from $r^{[i]}(R_i,t)$ to $r^{[i]}(R_{i+1},t)$ $(i=1,2)$, and then adding the two obtained expressions, finally, using the continuity Eq. (7) of radial stress and the boundary condition Eq. (11) yields

$$\begin{aligned}
&p_0+\int_{r_1^{[1]}}^{r_2^{[1]}}\Big(\lambda_1^{[1]}\frac{\partial W^{[1]}}{\partial\lambda_1^{[1]}}-\lambda_2^{[1]}\frac{\partial W^{[1]}}{\partial\lambda_2^{[1]}}\Big)\frac{\mathrm{d}r^{[1]}}{r^{[1]}}+\int_{r_2^{[2]}}^{r_3^{[2]}}\Big(\lambda_1^{[2]}\frac{\partial W^{[2]}}{\partial\lambda_1^{[2]}}-\lambda_2^{[2]}\frac{\partial W^{[2]}}{\partial\lambda_2^{[2]}}\Big)\frac{\mathrm{d}r^{[2]}}{r^{[2]}}\\
=&\rho^{[1]}\Big(c\ddot{c}\ln\Big(\frac{r_2^{[1]}}{c}\Big)+\Big(\ln\Big(\frac{r_2^{[1]}}{c}\Big)+\frac{1}{2}\big((r_2^{[1]})^{-2}c^2-1\big)\Big)\dot{c}^2\Big)\\
&+\rho^{[2]}\Big(c\ddot{c}\ln\Big(\frac{r_3^{[2]}}{r_2^{[2]}}\Big)+\Big(\ln\Big(\frac{r_3^{[2]}}{r_2^{[2]}}\Big)+\frac{1}{2}\big((r_3^{[2]})^{-2}-(r_2^{[2]})^{-2}\big)\Big)c^2\dot{c}^2\Big),
\end{aligned} \tag{13}$$

where

$$\begin{cases}
r_1^{[1]}=r_1^{[1]}(R_1,t)=c(t),\\
r_2^{[1]}=r_2^{[1]}(R_2,t)=r_2^{[2]}=r_2^{[2]}(R_2,t)=(R_2^2-R_1^2+c^2(t))^{\frac{1}{2}},\\
r_3^{[2]}=r_3^{[2]}(R_3,t)=(R_3^2-R_1^2+c^2(t))^{\frac{1}{2}}.
\end{cases} \tag{14}$$

Obviously, Eq. (13) in fact is a second-order nonlinear ordinary differential equation for $c(t)$, which describes the exact relation between the radial pressure p_0 acting on the inner surface of the composite tube and the inner radius $c(t)$. Using Eqs. (4) and (5) can yield the whole information of the radial motion of the tube.

3 Qualitative analysis of inflation of tube

For studying convenience, we introduce the following dimensionless notions:

$$\begin{cases}
x(t)=\dfrac{c(t)}{R_1},\\
\delta_1=\dfrac{R_2^2}{R_1^2}-1,\\
\delta_2=\dfrac{R_3^2}{R_2^2}-1.
\end{cases} \tag{15}$$

It is easy to show that the following expressions are valid, i.e.,

$$\frac{r_3^2}{R_3^2}=\frac{x^2+\delta_1+\delta_2+\delta_1\delta_2}{(1+\delta_1)(1+\delta_2)},\quad \frac{(r_2^{[i]})^2}{R_2^2}=\frac{x^2+\delta_1}{1+\delta_1},\quad i=1,2. \tag{16}$$

Further, let

$$\omega=\omega(R,c)=\begin{cases}
\dfrac{r^{[1]}(R)}{R},\quad R_1\leqslant R\leqslant R_2,\\
\dfrac{r^{[2]}(R)}{R},\quad R_2\leqslant R\leqslant R_3.
\end{cases}$$

It leads to

$$\Big(\lambda_1^{[i]}\frac{\partial W^{[i]}}{\partial\lambda_1^{[i]}}-\lambda_2^{[i]}\frac{\partial W^{[i]}}{\partial\lambda_2^{[i]}}\Big)\frac{\mathrm{d}r^{[i]}}{r^{[i]}}=\frac{W_1^{[i]}\mathrm{d}\omega}{\omega^2-1}, \tag{17}$$

where $W_1^{[i]}=\dfrac{\mathrm{d}W^{[i]}(\omega)}{\mathrm{d}\omega}$. Using the notions in Eqs. (15)–(17), we can rewrite Eq. (13) as

$$\begin{aligned}p_0+G(x,\delta_1,\delta_2)=&A^{[1]}(x,\delta_1,\delta_2)\ddot{x}+B^{[1]}(x,\delta_1,\delta_2)\dot{x}^2\\&+A^{[2]}(x,\delta_1,\delta_2)\ddot{x}+B^{[2]}(x,\delta_1,\delta_2)\dot{x}^2,\end{aligned} \tag{18}$$

where

$$G(x,\delta_1,\delta_2)=\int_x^{\big(\frac{x^2+\delta_1}{1+\delta_1}\big)^{\frac{1}{2}}}\frac{W_1^{[1]}\mathrm{d}\omega}{\omega^2-1}+\int_{\big(\frac{x^2+\delta_1}{1+\delta_1}\big)^{\frac{1}{2}}}^{\big(\frac{x^2+\delta_1+\delta_2+\delta_1\delta_2}{(1+\delta_1)(1+\delta_2)}\big)^{\frac{1}{2}}}\frac{W_1^{[2]}\mathrm{d}\omega}{\omega^2-1}, \tag{19}$$

$$A^{[1]}(x,\delta_1,\delta_2)=\frac{1}{2}\rho^{[1]}R_1^2x\ln\Big(1+\frac{\delta_1}{x^2}\Big), \tag{20}$$

$$B^{[1]}(x,\delta_1,\delta_2)=\frac{1}{2}\rho^{[1]}R_1^2\Big(\ln\Big(1+\frac{\delta_1}{x^2}\Big)-\frac{\delta_1}{x^2+\delta_1}\Big), \tag{21}$$

$$A^{[2]}(x,\delta_1,\delta_2)=\frac{1}{2}\rho^{[2]}R_1^2x\ln\Big(1+\delta_2\frac{1+\delta_1}{x^2+\delta_1}\Big), \tag{22}$$

$$B^{[2]}(x,\delta_1,\delta_2)=\frac{1}{2}\rho^{[2]}R_1^2\Big(\ln\Big(1+\delta_2\frac{1+\delta_1}{x^2+\delta_1}\Big)-\frac{\delta_2+\delta_1\delta_2}{x^2+\delta_1+\delta_2+\delta_1\delta_2}\frac{x^2}{x^2+\delta_1}\Big). \tag{23}$$

The initial conditions in Eq. (10) become

$$x(0)=1,\quad \dot{x}(0)=0. \tag{24}$$

To better understand the conclusions obtained in this work, here we consider the interior and the exterior parts of the tube are, respectively, composed of two classes of transversely isotropic power-law material models, and the strain energy functions are as follows:

$$W(\lambda_1,\lambda_2,\lambda_3)=\begin{cases}\mu_1\Big(\dfrac{(\lambda_1^{[1]})^{2n_1}+(\lambda_2^{[1]})^{2n_1}-2}{2n_1}+\beta_1((\lambda_1^{[1]})^2-1)^2\Big), & R_1<R<R_2,\\[2ex] \mu_2\Big(\dfrac{(\lambda_1^{[2]})^{2n_2}+(\lambda_2^{[2]})^{2n_2}-2}{2n_2}+\beta_2((\lambda_1^{[2]})^2-1)^2\Big), & R_2<R<R_3,\end{cases} \tag{25}$$

where μ_1 and μ_2 are shear modulus for infinitesimal deformations of the two materials, respectively; n_1 and n_2 are given material constants, respectively; β_1 and β_2 are material parameters describing the anisotropic degree about radial direction, respectively. In particular, Eq. (25) is the power-law material model proposed by Chou-Wang and Horgan[9] if $\beta_1=\beta_2=0$, and is the classic neo-Hookean material model[5] if $n_1=n_2=1$.

3.1 Static radial inflation

This subsection studies the quasi-static inflation of the composite tube under the radial pressure p_0. In this case, Eq. (18) reduces to

$$p_0+G(x,\delta_1,\delta_2)=0. \tag{26}$$

If $x\in(1,+\infty)$, it is easy to show that $G(1,\delta_1,\delta_2)=0$, and the following inequalities

$$\Big(\frac{x^2+\delta_1}{1+\delta_1}\Big)^{\frac{1}{2}}<x,\quad \Big(\frac{x^2+\delta_1+\delta_2+\delta_1\delta_2}{(1+\delta_1)(1+\delta_2)}\Big)^{\frac{1}{2}}<\Big(\frac{x^2+\delta_1}{1+\delta_1}\Big)^{\frac{1}{2}}$$

are valid for any given values of δ_1 and δ_2. This means that $G(x, \delta_1, \delta_2) < 0$; in other words, Eq. (26) has real roots.

On the other hand, it can be shown that the following conclusions are valid for any given values of δ_1 and δ_2 by using the properties of improper integrals.

(I) If the highest powers of the strain energy functions associated with the two materials do not exceed 2, which means that $n_1, n_2 \leqslant 1$, then Eq. (26) has a horizontal asymptote, particularly,

(i) if the highest powers of the strain energy functions are all less than 2, namely, n_1, $n_2 < 1$, then $p_0 \to 0$ as $x \to +\infty$;

(ii) if one of the highest powers of the strain energy functions is less than 2, and the other one is greater than 2, then $p_0 \to 0.5\mu_1 \ln(1+\delta_1)$ $(n_1 = 1, n_2 < 1)$ or $p_0 \to 0.5\mu_2 \ln(1+\delta_2)$ $(n_1 < 1, n_2 = 1)$ as $x \to +\infty$;

(iii) if the highest powers of the strain energy functions are all equal to 2, namely, $n_1 = n_2 = 1$, then $p_0 \to 0.5\mu_1 \ln(1+\delta_1) + 0.5\mu_2 \ln(1+\delta_2)$ as $x \to +\infty$.

(II) If at least one of the highest powers of the strain energy functions is greater than 2, namely, $n_1 > 1$ or $n_2 > 1$, then Eq. (26) has an oblique asymptote, i.e., $p_0 \to +\infty$ as $x \to +\infty$.

If the material parameters and one of the structure parameters are given, for example, $n_2 = 1$, $\beta_2 = 0$, and $\delta_1 = 1$, Figs. 1 and 2, respectively, show the relation curves of P versus x satisfying Eq. (26) for different values of $\mu = \frac{\mu_2}{\mu_1}$ and n_1, where $P = \frac{p_0}{\mu_1}$.

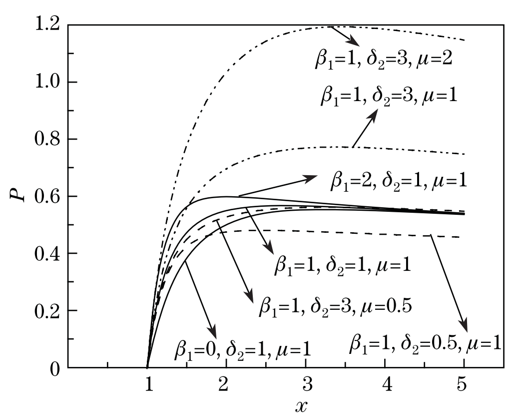

Fig. 1 Relation curves of P versus x for material parameters satisfying $n_1, n_2 \leqslant 1$, where $n_1 = 0.75$

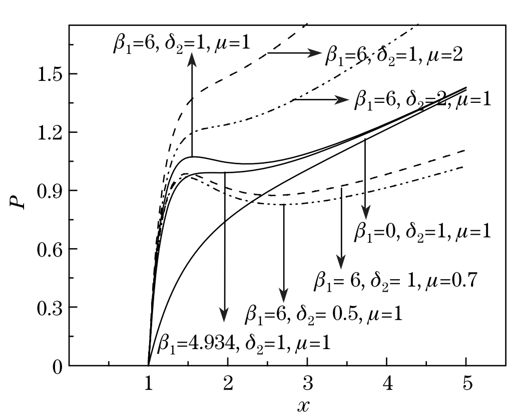

Fig. 2 Relation curves of P versus x for material parameters satisfying one of conditions $n_1 > 1$ or $n_2 > 1$ or n_1, $n_2 > 1$, where $n_1 = 1.5$

3.2 Dynamical radial inflation

Let $y = \dot{x}$. Then, Eq. (18) is equivalent to the following system of the first-order differential equations, i.e.,

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} y \\ C(x, y) \end{pmatrix}, \tag{27}$$

where

$$C(x, y) = \frac{-(B^{[1]}(x, \delta_1, \delta_2) + B^{[2]}(x, \delta_1, \delta_2))y^2 + p_0 + G(x, \delta_1, \delta_2)}{A^{[1]}(x, \delta_1, \delta_2) + A^{[2]}(x, \delta_1, \delta_2)}. \tag{28}$$

Obviously, the equilibrium point of the system is given by $(x, y) = (\overline{x}, 0)$, where $\overline{x}$ is a root of Eq. (26). Furthermore, the eigenvalues of the Jacobian matrix of Eq. (27) are as follows:

$$\lambda_{1,2} = \pm\Big(\frac{G_x(\overline{x}, \delta_1, \delta_2)}{A^{[1]}(\overline{x}, \delta_1, \delta_2) + A^{[2]}(\overline{x}, \delta_1, \delta_2)}\Big)^{\frac{1}{2}}. \tag{29}$$

From Fig. 1, we know that there exists a maximum value on each relation curve of P versus x, written as P_{m}. As $P < P_{\mathrm{m}}$, if the relation curve increases monotonically, then $G_x(\overline{x}, \delta_1, \delta_2) < 0$. In this case, λ_1 and λ_2 are two pure imaginary roots with opposite sign. Thus, the equilibrium point $(\overline{x}, 0)$ is a center of the linearized system of Eq. (27). While if the relation curve decreases monotonically, then $G_x(\overline{x}, \delta_1, \delta_2) > 0$, this implies that λ_1 and λ_2 are two real eigenvalues with opposite sign. Thus, the equilibrium point $(\overline{x}, 0)$ is a saddle of the linearized system of Eq. (27) and so it is a saddle of Eq. (27). However, as $P > P_{\mathrm{m}}$, the equilibrium point does not exist anymore, which means that the solutions to Eq. (27) will increase infinitely with the increasing time. In other words, the tube will be destroyed ultimately.

The analysis of Fig. 2 is similar to that of Fig. 1. Relative discussion can also be found in Ref. [8].

From the qualitative theory of differential equations, we know that a center of the linearized equation of a nonlinear differential equation may be a center of the nonlinear differential equation, but it may also be a focus of the nonlinear differential equation. Therefore, it requires to further discuss the case of the equilibrium point being a center of the linearized equation.

Multiplying Eq. (18) by $x\dot{x}$ yields the first integral of Eq. (18) satisfying the initial conditions in Eq. (24), i.e.,

$$\int_1^x zG(z, \delta_1, \delta_2)\mathrm{d}z + \frac{1}{2}p_0(x^2 - 1) = \frac{1}{2}(A^{[1]}(x, \delta_1, \delta_2) + A^{[2]}(x, \delta_1, \delta_2))x\dot{x}^2. \tag{30}$$

It is easy to see that Eq. (30) is symmetric with respect to $\dot{x}$. From the symmetric principle of nonlinear differential equations, we know that if the equilibrium point is a center of the linearized equation of Eq. (27), then it is also a center of the nonlinear Eq. (27). In this case, the solution to Eq. (18) satisfying the initial conditions in Eq. (24) is periodic, that is to say, the motion of the composite tube under the inner surface pressure is a nonlinearly periodic oscillation.

4 Conclusions

The mechanism of static and dynamic inflation of the incompressible composite rubber tube is examined through analyzing the equations describing the radial inflation of the tube qualitatively and the numerical examples. This work proposes the following conclusions:

(i) If the highest powers of the strain energy functions associated with the two materials do not exceed 2, then there exists a critical pressure. That is to say, the radial inflation mode of the tube with time is a nonlinearly periodic oscillation as the radial pressure is less than the critical pressure, while the tube will inflate infinitely with the increasing time and will be destroyed ultimately as the radial pressure is larger than the critical pressure.

(ii) If at least one of the highest powers of the strain energy functions is greater than 2, then for any given pressures, the radial inflation mode of the tube with time is a nonlinearly periodic oscillation. Moreover, the amplitude of oscillation will increase discontinuously under certain conditions.

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