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Vibration characteristics of FGM circular cylindrical shells filled with fluid using wave propagation approach *

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Abstract The vibration characteristics of a functionally graded material circular cylindrical shell filled with fluid are examined with a wave propagation approach. The shell is filled with an incompressible non-viscous fluid. Axial modal dependence is approximated by exponential functions. A theoretical study of shell vibration frequencies is analyzed for simply supported-simply supported, clamped-simply supported, and clamped-clamped boundary conditions with the fluid effect. The validity and the accuracy of the present method are confirmed by comparing the present results with those available in the literature. Good agreement is observed between the two sets of results.

Key words functionally graded material, Love's shell theory, cylindrical shell, volume fraction law, natural frequency, wave propagation

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Introduction

The study of the vibration characteristics of functionally graded cylindrical shells filled with fluid is of great importance and well-established as a field of research in structural dynamics. The shells filled with fluid are used in many types of engineering structures, such as pressure vessels, oil tankers, aero planes, ships, and marine crafts. In 1952, Junger and Mass[1] first studied the coupled vibration of fluid-filled cylindrical shells. Jain[2] did work to study the free vibration of an orthotropic cylindrical shell that is filled partially or completely with an incompressible non-viscous fluid. Goncalves and Batista[3] analyzed the frequency response of cylindrical shells partially submerged or filled with liquid. Chen and Ding[4] studied the exact solution of the free vibration of a transversely isotropic cylindrical shell filled with fluid. Zhang et al.[5] employed the wave propagation approach to analyze the coupled vibration of fluid-filled cylindrical shells. Zhang et al.[6] developed the wave propagation approach by approximating the eigenvalues of characteristics beam functions and analyzed the vibration characteristics of isotropic cylindrical shells. Li[7] used the wave propagation technique to study the free vibration of circular cylindrical shells for shear diaphragm-shear diaphragm, clamped-clamped,

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and clamped-shear diaphragm boundary conditions. Zhang[8] extended the wave propagation approach to study the coupled frequency of submerged cylindrical shells in fluid and verified the results by comparing the data obtained by the FEM/BEM. Natsuki et al.[9] presented a vibration analysis of fluid-filled double-walled carbon nanotubes using the wave propagation approach. They used the Flugge shell equations governing the vibration of the carbon nanotubes. Haddadpour et al.[10] analyzed the free vibration of the simply supported functionally graded cylindrical shells under thermal effects using four sets of in-plane boundary conditions. Sheng and Wang[11] investigated the vibration of functionally graded cylindrical shells filled with flowing fluid. They used the first-order shear deformation theory to model the dynamic characteristics of functionally graded cylindrical shells filled with flowing fluid.

Functionally graded materials (FGMs) are used to structure cylindrical shells, and their dynamical behaviors are investigated by many researchers. Loy et al.[12] first analyzed the natural frequency spectra for functionally graded cylindrical shells employing the Raleigh-Ritz method. Pradhan et al.[13] studied the vibration characteristics of cylindrical shells made by FGMs, viz., stainless steel and zirconia, under a number of boundary conditions. They used the Rayleigh-Ritz approach with the characteristic beam functions for the axial modal displacement deformations. In 2002, Naeem[14] worked on the vibration analysis of non-rotating and rotating functionally graded cylindrical shells using the Rayleigh-Ritz method and the Galerkin technique, respectively. The Ritz polynomial functions for non-rotating shells and the characteristic beam functions for rotating ones were used to approximate the axial modal dependence. Najafizadeh and Isvandzibaei[15] studied the vibration of thin FGM cylindrical shells with ring supports composed of stainless steel and nickel. Their analysis of the FGM was based on the higher-order shear deformation plate theory. Arshad et al.[16] presented a frequency analysis of the FGM cylindrical shells with various volume fraction laws. They investigated the behavior of shell frequencies for a number of physical parameters. In the present study, the wave propagation approach is applied to the investigation of the vibration characteristics of functionally graded circular cylindrical shells filled with fluid. This approach developed by Zhang et al.[6] has been employed by a number of researchers[7-9] for its simple application. It avoids a large amount of algebraic calculations and manipulations. The analysis is carried out for different boundary conditions, such as the simply supported-simply supported, clamped-simply supported, and clamped-clamped conditions. The validity and the accuracy of the present methodology are confirmed by comparing the present results with those available in the literature.

1 Mathematical equation of motion of shell and fluid

In Fig. 1, a functionally graded circular cylindrical shell with the length L, the mean radius R, and the thickness h is considered for the vibration study. The parameters of the shell material are Young's modulus E, Poisson's ratio ν, and the mass density ρ. An orthogonal coordinate system with the cylindrical coordinates (x, θ, z) is established at the middle surface of the shell. The displacement deformations in the axial, circumferential, and radial directions are denoted by u, v, and w, respectively. The equations for the motion of a cylindrical shell using Love's first-order shell theory[17] are given by

$$\begin{cases} \dfrac{\partial N_x}{\partial x} + \dfrac{1}{R}\dfrac{\partial N_{x\theta}}{\partial \theta} = \rho_{\mathrm{T}}\dfrac{\partial^2 u}{\partial t^2}, \\ \dfrac{\partial N_{x\theta}}{\partial x} + \dfrac{1}{R}\dfrac{\partial N_\theta}{\partial \theta} + \dfrac{1}{R}\dfrac{\partial M_{x\theta}}{\partial x} + \dfrac{1}{R^2}\dfrac{\partial M_\theta}{\partial \theta} = \rho_{\mathrm{T}}\dfrac{\partial^2 v}{\partial t^2}, \\ \dfrac{\partial^2 M_x}{\partial x^2} + \dfrac{2}{R}\dfrac{\partial^2 M_{x\theta}}{\partial x \partial \theta} + \dfrac{1}{R^2}\dfrac{\partial^2 M_\theta}{\partial \theta^2} - \dfrac{N_\theta}{R} = \rho_{\mathrm{T}}\dfrac{\partial^2 w}{\partial t^2}, \end{cases} \tag{1}$$

where N_x, N_θ, and $N_{x\theta}$ are the force resultants, and M_x, M_θ, and $M_{x\theta}$ are the moment

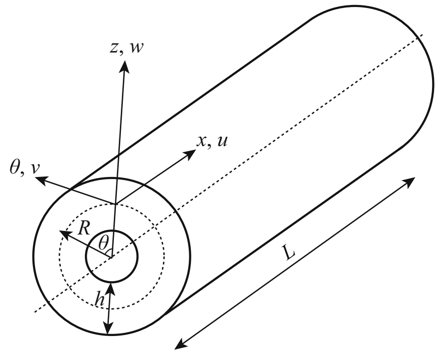

Fig. 1 Geometry of a cylindrical shell

resultants. They are defined as

$$(N_x, N_\theta, N_{x\theta}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_\theta, \sigma_{x\theta}) dz, \quad (M_x, M_\theta, M_{x\theta}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_\theta, \sigma_{x\theta}) z dz, \tag{2}$$

$$\rho_{\mathrm{T}} = \int_{-h/2}^{h/2} \rho dz. \tag{3}$$

For a thin cylindrical shell, the stresses σ_x, σ_θ, and $\sigma_{x\theta}$ involved in Eq. (2) are defined by the two-dimensional Hooke's law,

$$\begin{bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{x\theta} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} e_x \\ e_\theta \\ e_{x\theta} \end{bmatrix}, \tag{4}$$

where the strain components e_x, e_θ, and $e_{x\theta}$ in Eq. (3) are defined as the linear functions of the thickness coordinate z by the following relationships:

$$e_x = e_1 + zk_1, \quad e_\theta = e_2 + zk_2, \quad e_{x\theta} = \gamma + 2z\tau. \tag{5}$$

Here, e_x, e_θ, and $e_{x\theta}$ are, respectively, the strains in the axial and circumferential directions and the shear strain at a distance z from the reference surface. e_1, e_2, and γ are the reference surface strains, and k_1, k_2, and τ are the reference surface curvatures. According to Love's shell theory[17], the expressions for the reference surface strains and curvatures are defined as

$$\begin{cases} [e_1, e_2, \gamma] = \Big[\dfrac{\partial u}{\partial x}, \dfrac{1}{R}\Big(\dfrac{\partial v}{\partial \theta} + w\Big), \dfrac{\partial v}{\partial x} + \dfrac{1}{R}\dfrac{\partial u}{\partial \theta}\Big], \\ [k_1, k_2, \tau] = \Big[-\dfrac{\partial^2 w}{\partial x^2}, -\dfrac{1}{R^2}\Big(\dfrac{\partial^2 w}{\partial \theta^2} - \dfrac{\partial v}{\partial \theta}\Big), -\dfrac{1}{R}\Big(\dfrac{\partial^2 w}{\partial x \partial \theta} - \dfrac{\partial v}{\partial x}\Big)\Big]. \end{cases} \tag{6}$$

With the substitution of Eqs. (5) and (6) into Eq. (4) and the substitution of the resulting equation into Eq. (2), the force and momentum resultants can be obtained as

$$\begin{bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \gamma \\ k_1 \\ k_2 \\ 2\tau \end{bmatrix}, \tag{7}$$

where A_{ij}, B_{ij}, and D_{ij} $(i, j = 1, 2, 6)$ are the extensional, coupling, and bending stiffnesses, respectively. They are defined as

$$[A_{ij}, B_{ij}, D_{ij}] = \int_{-h/2}^{h/2} Q_{ij}[1, z, z^2] dz. \tag{8}$$

Here, Q_{ij} is the reduced stiffness for a thin shell and defined as

$$Q_{11} = \frac{E}{1-\nu^2}, \quad Q_{22} = \frac{E}{1-\nu^2}, \quad Q_{12} = \frac{\nu E}{1-\nu^2}, \quad Q_{66} = \frac{E}{2(1+\nu)}, \tag{9}$$

where E and ν are Young's modulus and Poisson's ratio of the shell material. After the substitution of Eqs. (7)–(9) into Eq. (1), Eq. (1) can be written in a matrix form as

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{10}$$

where L_{ij} $(i, j = 1, 2, 3)$ is the differential operator with respect to x and θ. The fluid filled in the cylindrical shell satisfies the acoustic wave equation, and the equation of the motion of the fluid can be written in the cylindrical coordinate system (x, θ, r) as

$$\frac{1}{r}\frac{\partial}{\partial r}\Big(r\frac{\partial \psi}{\partial r}\Big) + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} + \frac{\partial^2 \psi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2}. \tag{11}$$

Here, t is the time, ψ is the acoustic pressure, and c is the sound speed of the fluid. The x and θ coordinates are the same as those of the shell, and the r coordinate is taken from the axis of the shell.

2 Wave propagation approach

Different numerical methods are employed to solve the shell dynamical equation for studying the free vibration characteristics of circular cylindrical shells associated with fluid effects. Goncalves and Batista[3] used the Galerkin approach to investigate the free vibration of simply supported vertical cylindrical shells filled with or submerged in fluid. In the case of the application of the Galerkin technique for solving the shell equations, the axial modal dependence is approximated by the characteristic beam functions. It involves the integrals of these functions. The evaluation of the integrals requires a very lengthy process of integration. To simplify this approach, Zhang et al.[6] developed the wave propagation approach by taking the approximate eigenvalues of the beam functions. This approach has been employed by a number of authors[7-9]. The modal displacements of the shell equations can be expressed in the form of wave propagation associated with the axial wave number k_m and the circumferential modal parameter n as follows:

$$\begin{cases} u(x, \theta, t) = A_m \sin(n\theta) \exp(\mathrm{i}\omega t - \mathrm{i}k_m x), \\ v(x, \theta, t) = B_m \cos(n\theta) \exp(\mathrm{i}\omega t - \mathrm{i}k_m x), \\ w(x, \theta, t) = C_m \sin(n\theta) \exp(\mathrm{i}\omega t - \mathrm{i}k_m x), \end{cases} \tag{12}$$

where A_m, B_m, and C_m denote the wave amplitudes in the x, θ, and z directions, respectively. n is the number of the circumferential waves, k_m is the axial wave number that has been specified in Ref. [5] for different boundary conditions, and ω is the natural circular frequency for the cylindrical shell. The associated form of the acoustic pressure field in the contained fluid, which satisfies the acoustic wave equation (11), can be expressed in the cylindrical coordinate system associated with the axial wave number k_m, the radial wave number k_r, and the circumferential modal parameter n as follows:

$$\psi = \psi_m \cos(n\theta) J_n(k_r r) \exp(\mathrm{i}\omega t - \mathrm{i}k_m x), \tag{13}$$

where $J_n(\cdot)$ is the Bessel function of order n. The radial wave number k_r is related to the axial wave number k_m by the usual vector relationship $(k_r R)^2 = \Omega^2 (C_L/C_{\mathrm{f}})^2 - (k_m R)^2$, where Ω is the non-dimensional frequency parameter, and C_L and C_{f} are the sound speeds of the shell and fluid, respectively. To ensure that the fluid remains in contact with the shell wall, the fluid radial displacement and the shell radial displacement must be equal at the interface of the shell inner wall and the fluid. This coupling condition is expressed as

$$-\frac{1}{\mathrm{i}\omega\rho_{\mathrm{f}}} \frac{\partial\psi}{\partial r}\Big|_{r=R} = \frac{\partial w}{\partial t}\Big|_{r=R}, \tag{14}$$

$$\psi_m = (\omega^2 \rho_{\mathrm{f}} / k_r J_n'(k_r R)) W_m, \tag{15}$$

where ρ_{f} is the density of the contained fluid, and the prime on the $J_n(\cdot)$ denotes the differentiation with respect to the argument $k_r R$. Substituting Eq. (12) into Eq. (10) and taking into account the acoustic pressure on the inner wall of the shell and the coupling equation (15), we obtain the equation of the motion of the coupled system in the matrix form as

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} + FL \end{bmatrix} \begin{bmatrix} U_m \\ V_m \\ W_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{16}$$

where C_{ij} $(i, j = 1, 2, 3)$ is the parameter obtained from L_{ij} after it is operated with x and θ. FL is the fluid loading term due to the presence of the fluid acoustic field, and it is given by

$$FL = \Omega^2 (\rho_{\mathrm{f}}/\rho_{\mathrm{s}})(R/h)(k_r R)^{-1} (J_n(k_r R)/J_n'(k_r R)). \tag{17}$$

With Eq. (16), the frequency equation can be obtained in the form of the eigenvalue problem as follows:

$$\begin{bmatrix} T_{11} & T_{12} & T_{13} \\ -T_{12} & T_{22} & T_{23} \\ -T_{13} & T_{23} & T_{33} \end{bmatrix} \begin{bmatrix} A_m \\ B_m \\ C_m \end{bmatrix} = \omega^2 \rho_{\mathrm{t}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_m \\ B_m \\ C_m \end{bmatrix}, \tag{18}$$

where the entries T_{11}, T_{12}, T_{13}, T_{22}, T_{23}, and T_{33} are given in Appendix A.

By keeping the FL term absent, we can obtain the frequency equation for the uncoupled cylindrical shell case. The eigenvalue problem (18) is solved for the natural frequencies and mode shapes with the powerful software package, namely, MATLAB. Only one MATLAB program provides the shell frequencies by changing the values of the axial wave number for a boundary condition. This is the main feature of the wave propagation approach that has been adapted here.

3 Functionally graded materials

FGMs are composite advanced materials and have attracted much attention from material scientists. The Japanese researchers[18-19] studied these materials for their good performance in a very high temperature environment. FGMs have the ability to maintain their integrity at a very high temperature range and offer great promise in applications where the operating conditions are severe. They are microscopically non-homogeneous from the viewpoint of mechanics and thermal mechanics in which mechanical properties vary from one surface to another. They are synthesized by the mixture of two or more different materials. Usually, they are structured from the mixture of ceramic and metals. They are multi-functional materials that combine the desirable high temperature properties and the thermal resistance of a ceramic with the fracture toughness and strength of a metal. The properties of both components can be fully utilized. For example, the toughness of a metal can be mated with the refractoriness of a ceramic. Thus, FGMs are utilized to fabricate cylindrical shells. Their influence on the vibrations of these shells filled with fluid is studied here. In this analysis, a circular cylindrical shell composed of two materials ($M_1 \equiv$ stainless steel and $M_2 \equiv$ nickel) filled with fluid is considered to study its vibration characteristics. The resultant material properties P of an FGM are the function of the material properties and the volume fractions of the constituent materials and described as

$$P = \sum_{k=1}^{l} P_k V_{\mathrm{f}k}, \tag{19}$$

where P_k and $V_{\mathrm{f}k}$ represent the material property and the volume fraction of the constituent material k, respectively. The sum of the volume fractions of all constituent materials equals 1, i.e.,

$$\sum_{k=1}^{l} V_{\mathrm{f}k} = 1. \tag{20}$$

For a cylindrical shell with the uniform thickness h and the middle surface taken as the reference surface, the volume fraction is expressed as

$$V_{\mathrm{f}} = \Big(\frac{z}{h} + 0.5\Big)^{N}, \tag{21}$$

where N is the power law index and $0 \leq N \leq \infty$. For an FGM cylindrical shell structured from two materials M_1 and M_2, the resultant material properties, i.e., Young's modulus E, Poisson's ratio ν, and the mass density ρ, are given by

$$\begin{cases} E = (E_2 - E_1)\Big(\dfrac{z}{h} + 0.5\Big)^{N} + E_1, \\ \nu = (\nu_2 - \nu_1)\Big(\dfrac{z}{h} + 0.5\Big)^{N} + \nu_1, \\ \rho = (\rho_2 - \rho_1)\Big(\dfrac{z}{h} + 0.5\Big)^{N} + \rho_1. \end{cases} \tag{22}$$

From these expressions, we know that when $z = -h/2$, $E = E_1$, $\nu = \nu_1$, and $\rho = \rho_1$; when $z = h/2$, $E = E_2$, $\nu = \nu_2$, and $\rho = \rho_2$. This shows that, at the inner surface of the shell, the material properties correspond to the constituent material M_1, and at the outer surface of the shell, they are associated with the constituent material M_2. The material property in an FGM varies in the thickness direction. Thus, for the present cylindrical shell, the material properties change gradually from the material M_1 at the inner surface to the material M_2 at the outer surface of the shell. The classical shell theories are applicable for the FGM shell if the thickness-to-radius ratio is less than 1/20.

4 Numerical results and discussions

To check the validity, efficiency, and accuracy of the present method, some comparisons of the results are made for various boundary conditions. In Table 1, the uncoupled and coupled non-dimensional frequency parameters $\Omega = \omega R\sqrt{(1-\nu^2)\rho/E}$ are given for an isotropic cylindrical shell for clamped-clamped edge conditions and compared with the values determined by Zhang et al.[5]. They solved the polynomial frequency equation numerically whereas in the present study, the MATLAB command was used to solve the eigenfrequency equation. The values of the frequency parameters are obtained for the axial wave number $m = 1$, 2, 3, and $L/R = 20$ and $h/R = 0.01$ are taken as the geometrical parameters. It is observed that the present results are very close and some a bit lower than those obtained in Ref. [5]. In Table 2, the natural frequencies (Hz) for a simply supported functionally graded circular cylindrical shell without fluid are listed for the half-axial wave number $m = 1$ with the geometrical parameters $L/R = 20$ and $h/R = 0.002$. They are compared with those evaluated by Loy et al.[12]. The values of the volume fraction exponent N taken into consideration are equal to 0.5, 1, and 15. The shell is composed of stainless steel at the inner surface and nickel at the outer surface of the shell. The two sets of natural frequencies are very close. From all these comparisons, it is evident that the present wave propagation approach is accurate and efficient and can be easily employed to carry out the vibration analysis of cylindrical shells.

Table 1 Comparison of frequency parameters $\Omega = R\omega\sqrt{(1-\nu^2)\rho/E}$ for clamped-clamped isotropic cylindrical shell ($m = 1$, $L/R = 20$, and $h/R = 0.01$)

Order	(m, n)	Ref. [5]		Present	
		Uncoupled frequency	Coupled frequency	Uncoupled frequency	Coupled frequency
1	(1, 2)	12.17	4.93	12.120 7	4.908 3
2	(1, 3)	19.61	8.94	19.606 1	8.924 07
3	(1, 4)	36.47	18.26	36.474 3	18.235 88
4	(2, 2)	28.06	11.48	28.057 2	11.351 2
5	(2, 3)	23.28	10.64	23.270 7	10.581 2
6	(2, 4)	37.37	18.73	37.368 8	18.669 5
7	(3, 3)	31.98	14.66	31.956 8	14.506 4
8	(3, 4)	39.78	19.96	39.761 1	19.842 5

Table 2 Comparison of the natural frequencies (Hz) for an FGM cylindrical shell with simply supported end conditions ($h/R = 0.002$ and $L/R = 20$)

No.	$N = 0.5$		$N = 1$		$N = 15$	
	Ref. [6]	Present	Ref. [6]	Present	Ref. [6]	Present
1	13.321	13.321	13.211	13.211	12.933	12.933
2	4.516	4.516 2	4.480	4.479	4.383	4.383
3	4.191	4.190 3	4.156 9	4.156	4.065	4.065
4	7.097 2	7.096 7	7.038	7.037 9	6.885	6.885
5	11.336	11.335	11.241	11.240 7	10.999	10.998
6	16.594	16.593 5	16.455	16.454 9	16.101	16.101
7	22.826	22.825 8	22.635	22.634 9	22.148	22.148
8	30.023	30.022 5	29.771	29.771	29.132	29.132
9	38.181	38.181 1	37.862	37.861 5	37.048	37.048
10	47.301	47.300 5	46.905	46.904 6	45.897	45.897

In this study, the vibration characteristics of functionally graded circular cylindrical shells filled with fluid are investigated under a number of boundary conditions. The shells are assumed to be fabricated from stainless steel and nickel as the functionally graded constituent materials. With the change of the configuration of the shell materials, they are classified into two types. The Type I functionally graded shell consists of stainless steel at the inner surface and nickel at the outer surface whereas, in the Type II shell, the order of the constituent materials is reversed. The functionally graded circular cylindrical shell is filled with water of the sound speed $c = 1\,500$ m/s and the mass density $\rho_3 = 1\,000$ kg/m^3. The material properties for stainless steel and nickel are calculated at $T = 300$ K and presented in Table 3. The properties of the FGM in Table 3 are temperature dependent to reinforce the strength of the shells. The coupled frequencies of the cylindrical shells are evaluated under a normal environment for simply supported-simply supported (SS-SS), clamped-simply supported (C-SS), and clamped-clamped (C-C) end conditions. Table 4 shows the comparison and variation of the coupled and uncoupled natural frequencies (Hz) of the Type I cylindrical shell for the SS-SS, C-SS, and C-C boundary conditions. It is observed that the frequency increases with N for these boundary conditions. It also increases with the circumferential wave number for the coupled case whereas it varies in the uncoupled case like the isotropic shell without fluid. However, for the axial wave number, it increases with m. The frequency increases as the boundary constraints are added more. It is highest for the C-C and lowest for the SS-SS boundary conditions. In Table 5, the variation of the natural frequencies (Hz) for the Type II cylindrical shell is listed for the SS-SS, C-SS, and C-C conditions. The values of the shell frequency decrease with N. It shows that the behavior of the variation of the frequency is opposite to that for the Type I shell. In Fig. 2, the coupled natural frequency (Hz) for the Type I FGM circular cylindrical shell is drawn against the circumferential wave number (n) for the SS-SS, C-SS, and C-C boundary conditions. It is observed that the same behavior is exhibited by the Type II FGM cylindrical shell. Figure 3 shows the variation of the coupled frequency of the Type I FGM circular cylindrical shell with circumferential wave number n and the fixed value of N for three boundary conditions. It is observed that the natural frequency is large for the C-C condition and low for the SS-SS condition. The same trends for the Type II shell are exhibited. Figure 4 shows that the coupled frequency of the FGM shell increases as the values of N increase, and for the Type II shell, there is a decrease in the frequency with the increase of the values of N. It is also observed that the frequency decreases with the fluid. For the Type I shell, there is an increase in the ratio of the coupled frequency to the uncoupled frequency with the increase of the values of N. However, the ratio decreases as the circumferential wave number increases for the fixed value of N. The ratio of the coupled frequence to the uncoupled frequency decreases with the increase of the values of N and also decreases with the circumferential wave number for the fixed value of N.

Table 3 Properties of constituent materials

Materials	Coefficients	P_0	P_{-1}	P_1	P_2	P_3	P
	$E/(\mathrm{N{\cdot}m^{-2}})$	201.04E+09	0	3.079E−04	−6.534E−07	0	2.077 88E+11
Stainless steel	ν	0.326 2	0	−2.002E−04	3.797E−07	0	0.317 756
	$\rho/(\mathrm{kg\cdot m^{-3}})$	8 166	0	0	0	0	8 166
	$E/(\mathrm{N{\cdot}m^{-2}})$	223.95E+9	0	−2.794E−4	−3.998E−9	0	2.050 98E+11
Nickel	ν	0.310 0	0	0	0	0	0.310 0
	$\rho/(\mathrm{kg\cdot m^{-3}})$	8 900	0	0	0	0	8 900

5 Concluding remarks

In the analysis, the vibration frequencies of the FGM circular cylindrical shells filled with fluid are studied with regard to the volume fraction laws. The shell frequencies vary with the power law exponent for three boundary conditions, i.e., the simply supported-simply supported, clamped-simply supported, and clamped-clamped conditions. The coupled frequency (Hz) of the FGM shell is affected by the fluid. There is a remarkable decrease in the coupled frequency

Table 4 Coupled and uncoupled frequencies of the Type I FGM circular cylindrical shell for SS-SS, C-C, and C-SS boundary conditions

Boundary conditions	m	n	$N = 0.3$ Uncoupled	$N = 0.3$ Coupled	$N = 0.3$ Ratio	$N = 1$ Uncoupled	$N = 1$ Coupled	$N = 1$ Ratio
SS-SS	1	2	4.416 0	0.903 1	4.889 8	4.473 7	0.904 9	4.943 9
		3	4.094 0	0.959 9	4.265 0	4.147 8	0.962 0	4.311 6
		4	6.939 1	1.823 4	3.805 6	7.032 5	1.828 4	3.846 3
		5	11.086 8	3.198 2	3.466 6	11.237 2	3.207 7	3.503 2
	2	2	16.628 2	3.398 8	4.892 4	16.848 0	3.406 2	4.946 3
		3	8.716 6	2.041 9	4.268 9	8.829 9	2.046 2	4.3153
		4	8.262 3	2.169 7	3.808 0	8.370 4	2.174 8	3.848 8
		5	11.483 1	3.310 9	3.468 3	11.636 6	3.320 1	3.504 9
C-C	1	2	9.523 3	1.947 3	4.890 5	9.648 7	1.951 4	4.944 5
		3	5.742 2	1.345 8	4.266 8	5.816 6	1.348 6	4.313 1
		4	7.326 4	1.924 7	3.806 5	7.423 7	1.929 6	3.847 3
		5	11.200 0	3.230 2	3.467 3	11.351 0	3.239 5	3.503 9
	2	2	25.549 7	5.218 9	4.895 6	25.887 8	5.230 3	4.949 6
		3	12.846 7	3.007 3	4.271 8	13.014 5	3.013 8	4.318 3
		4	9.928 4	2.605 9	3.810 0	10.057 3	2.611 7	3.850 9
		5	12.042 7	3.471 0	3.469 5	12.202 2	3.480 2	3.506 2
C-SS	1	2	6.706 3	1.371 4	4.890 1	6.794 3	1.374 2	4.944 2
		3	4.747 5	1.112 9	4.265 9	4.809 3	1.115 3	4.312 1
		4	7.080 2	1.860 3	3.805 9	7.174 9	1.865 2	3.846 7
		5	11.128 3	3.209 9	3.466 9	11.278 8	3.219 3	3.503 5
	2	2	20.872 5	4.265 1	4.893 8	21.148 5	4.274 4	4.947 7
		3	10.647 9	2.493 5	4.270 3	10.786 7	2.498 8	4.316 8
		4	8.998 5	2.362 5	3.808 9	9.115 7	2.367 9	3.849 7
		5	11.721 5	3.379 1	3.468 8	11.877 5	3.388 2	3.505 5

Boundary conditions	m	n	$N = 5$ Uncoupled	$N = 5$ Coupled	$N = 5$ Ratio	$N = 10$ Uncoupled	$N = 10$ Coupled	$N = 10$ Ratio
SS-SS	1	2	4.549 8	0.907 6	5.013 0	4.568 3	0.908 3	5.029 5
		3	4.218 3	0.965 0	4.371 3	4.236 8	0.966 1	4.385 5
		4	7.150 5	1.834 0	3.898 9	7.181 2	1.836 1	3.9111
		5	11.424 9	3.217 9	3.550 4	11.473 4	3.221 6	3.5614
	2	2	17.132 6	3.415 8	5.015 7	17.199 5	3.418 1	5.031 9
		3	8.980 7	2.052 7	4.375 1	9.017 9	2.054 6	4.389 1
		4	8.513 0	2.182 0	3.901 5	8.550 3	2.184 7	3.913 7
		5	11.832 7	3.331 2	3.552 1	11.883 9	3.335 2	3.563 2
C-C	1	2	9.812 1	1.956 9	5.014 1	9.850 8	1.958 3	5.0303
		3	5.916 1	1.352 9	4.372 9	5.941 5	1.354 4	4.386 8
		4	7.549 2	1.935 7	3.900 0	7.582 2	1.938 1	3.912 2
		5	11.541 3	3.250 1	3.551 1	11.590 7	3.253 9	3.562 1
	2	2	26.324 5	5.244 8	5.019 2	26.426 9	5.248 3	5.035 3
		3	13.236 0	3.023 2	4.378 1	13.289 6	3.025 8	4.392 1
		4	10.229 3	2.620 6	3.903 4	10.273 6	2.623 7	3.915 7
		5	12.408 8	3.492 1	3.553 4	12.463 0	3.496 5	3.564 4
C-SS	1	2	6.909 6	1.378 2	5.013 5	6.937 2	1.379 2	5.029 9
		3	4.891 4	1.118 8	4.372 0	4.912 7	1.120 1	4.385 9
		4	7.295 7	1.871 0	3.899 4	7.327 3	1.873 2	3.911 6
		5	11.467 5	3.229 6	3.550 7	11.516 4	3.233 4	3.561 7
	2	2	21.505 5	4.286 4	5.017 1	21.589 3	4.289 2	5.033 4
		3	10.970 5	2.506 7	4.376 5	11.015 5	2.508 9	4.390 6
		4	9.271 4	2.375 8	3.902 4	9.311 8	2.378 7	3.914 7
		5	12.078 1	3.399 7	3.552 7	12.130 7	3.403 9	3.563 8

Table 5 Coupled and uncoupled frequencies of the Type II FGM circular cylindrical shell for SS-SS, C-C, and C-SS boundary conditions

Boundary conditions	m	n	$N=0.3$ Uncoupled	$N=0.3$ Coupled	$N=0.3$ Ratio	$N=1$ Uncoupled	$N=1$ Coupled	$N=1$ Ratio
SS-SS	1	2	4.539 1	0.907 9	4.999 6	4.479 5	0.906 1	4.943 7
		3	4.212 0	0.966 1	4.359 8	4.156 2	0.964 0	4.311 4
		4	7.134 6	1.834 8	3.888 5	7.038 0	1.829 8	3.846 3
		5	11.396 3	3.218 3	3.541 1	11.240 7	3.208 7	3.503 2
	2	2	17.081 2	3.414 8	5.002 1	16.854 1	3.407 5	4.946 2
		3	8.962 9	2.054 1	4.363 4	8.845 7	2.049 9	4.315 2
		4	8.501 0	2.184 7	3.891 2	8.389 0	2.179 6	3.848 9
		5	11.809 6	3.333 4	3.542 8	11.650 7	3.324 1	3.504 9
C-C	1	2	9.784 4	1.956 7	5.000 5	9.654 7	1.952 6	4.944 5
		3	5.907 2	1.354 5	4.361 2	5.830 1	1.351 7	4.313 2
		4	7.536 1	1.937 5	3.889 6	7.435 4	1.932 6	3.847 4
		5	11.515 2	3.251 2	3.541 8	11.359 0	3.241 8	3.503 9
	2	2	26.243 5	5.242 9	5.005 5	25.894 1	5.231 5	4.949 7
		3	13.204 8	3.024 2	4.366 4	13.031 2	3.017 7	4.318 3
		4	10.214 9	2.623 8	3.893 2	10.081 4	2.618 0	3.850 8
		5	12.388 3	3.495 4	3.544 2	12.223 1	3.486 2	3.506 1
C-SS	1	2	6.891 3	1.378 3	4.999 9	6.800 3	1.375 5	4.943 9
		3	4.884 6	1.120 2	4.360 5	4.820 6	1.117 9	4.312 2
		4	7.281 3	1.872 3	3.889 0	7.183 3	1.867 4	3.846 7
		5	11.440 1	3.230 3	3.541 5	11.284 4	3.220 9	3.503 5
	2	2	21.44	4.284 9	5.003 6	21.154 7	4.275 7	4.947 7
		3	10.946 6	2.507 9	4.364 8	10.803 1	2.502 6	4.316 8
		4	9.258 6	2.378 8	3.892 1	9.137 2	2.373 5	3.849 7
		5	12.056 4	3.402 4	3.543 5	11.894 9	3.393 2	3.505 5

Boundary conditions	m	n	$N=5$ Uncoupled	$N=5$ Coupled	$N=5$ Ratio	$N=10$ Uncoupled	$N=10$ Coupled	$N=10$ Ratio
SS-SS	1	2	4.406 3	0.903 4	4.877 5	4.389 3	0.902 7	4.862 4
		3	4.088 5	0.961 0	4.254 4	4.071 5	0.959 9	4.241 6
		4	6.924 7	1.824 2	3.796 0	6.896 6	1.822 2	3.784 8
		5	11.060 7	3.198 5	3.458 1	11.016 3	3.194 9	3.448 1
	2	2	16.580 8	3.397 9	4.879 7	16.519 5	3.395 6	4.865 0
		3	8.700 7	2.043 4	4.258 0	8.666 5	2.041 4	4.245 4
		4	8.251 9	2.172 4	3.798 5	8.217 6	2.169 7	3.787 4
		5	11.462 5	3.313 1	3.459 8	11.415 4	3.309 1	3.449 7
C-C	1	2	9.497 8	1.947 1	4.877 9	9.462 3	1.945 7	4.863 2
		3	5.734 4	1.347 4	4.255 9	5.711 0	1.345 9	4.243 3
		4	7.314 8	1.926 5	3.796 9	7.284 6	1.924 1	3.786 0
		5	11.176 4	3.231 3	3.458 8	11.131 0	3.227 5	3.448 8
	2	2	25.474 7	5.216 9	4.883 1	25.381 1	5.213 5	4.868 3
		3	12.818 3	3.008 3	4.261 0	12.769 1	3.005 7	4.248 3
		4	9.916 0	2.609 2	3.800 4	9.875 2	2.606 1	3.789 3
		5	12.024 6	3.474 3	3.461 0	11.974 8	3.469 9	3.451 1
C-SS	1	2	6.689 5	1.371 5	4.877 5	6.664 3	1.370 5	4.862 7
		3	4.741 6	1.114 4	4.254 8	4.72 2	1.113 1	4.242 2
		4	7.067 2	1.861 6	3.796 3	7.038 2	1.859 4	3.785 2
		5	11.103 3	3.210 6	3.458 3	11.058 5	3.206 9	3.448 3
	2	2	20.811 9	4.263 7	4.881 2	20.735 2	4.260 9	4.866 4
		3	10.626 3	2.494 8	4.259 4	10.585 0	2.492 5	4.246 7
		4	8.987 5	2.365 5	3.799 4	8.950 3	2.362 7	3.788 2
		5	11.702 3	3.381 8	3.460 4	11.654 0	3.377 6	3.450 4

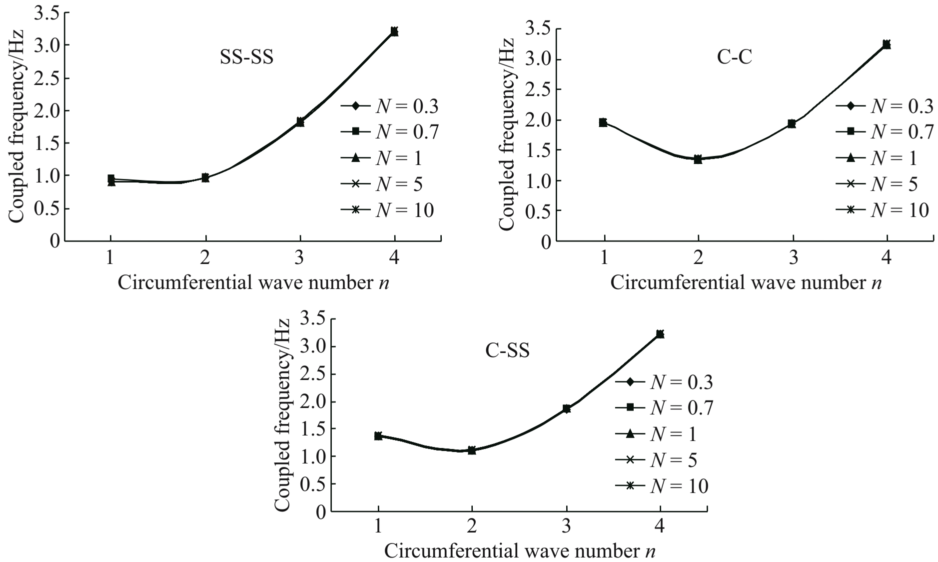

Fig. 2 Variation of coupled frequency (Hz) with circumferential wave number n of the Type I FGM circular cylindrical shell for SS-SS, C-C, and C-SS boundary conditions

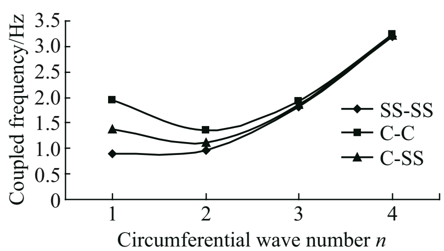

Fig. 3 Variation of coupled frequency (Hz) with circumferential number n of the Type I FGM circular cylindrical shell for three boundary conditions

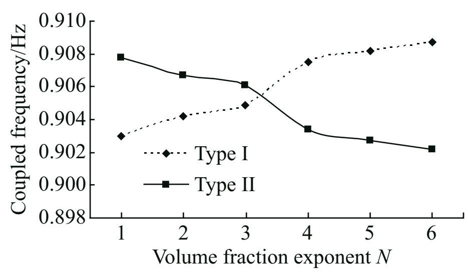

Fig. 4 Variation of coupled frequency (Hz) with volume fraction exponents N of the Type I and Type II FGM circular cylindrical shells for SS-SS boundary condition

(Hz) of the FGM shell as compared with the uncoupled frequency of the FGM circular cylindrical shell. Based on the approximate eigenvalues of characteristic beam functions, the wave propagation approach is employed to solve the shell governing equation. The approach is simple and fast and presents accurate results. The present analysis can be extended to study other shell problems like the vibrations of rotating shells and laminated shells.

References

[1] Junger, M. C. and Mass, C. Vibration of elastic shells in a fluid medium and the associated radiation of sound. *Journal of Applied Mechanics* **74**, 439–445 (1952)

[2] Jain, R. K. Vibration of fluid-filled, orthotropic cylindrical shells. *Journal of Sound and Vibration* **37**(3), 379–388 (1974)

[3] Goncalves, P. B. and Batista, R. C. Frequency response of cylindrical shells partially submerged or filled with liquid. *Journal of Sound and Vibration* **113**(1), 59–70 (1987)

[4] Chen, W. Q. and Ding, H. J. Natural frequency of fluid-filled transversely isotropic cylindrical shells. *International Journal of Mechanical Sciences* **41**(6), 677–684 (1999)

[5] Zhang, X. M., Liu, G. R., and Lam, K. Y. Coupled vibration analysis of fluid-filled cylindrical shells using the wave propagation approach. *Journal of Applied Acoustics* **62**(3), 229–243 (2001)

[6] Zhang, X. M., Liu, G. R., and Lam, K. Y. Vibration analysis of thin cylindrical shells using wave propagation approach. *Journal of Sound and Vibration* **239**(3), 397-403 (2001)

[7] Li, X. B. Study of free vibration analysis of circular cylindrical shells using wave propagation. *Journal of Sound and Vibration* **311**(3-5), 667–682 (2008)

[8] Zhang, X. M. Frequency analysis of submerged cylindrical shells with the wave propagation approach. *International Journal of Mechanical Sciences* **44**(7), 1259–1273 (2002)

[9] Natsuki, T., Ni, Q.-Q., and Endo, M. Vibration analysis of fluid-filled carbon nanotubes using the wave propagation approach. *Applied Physics A-Material Science and Processing* **90**(3), 441–445 (2008)

[10] Haddadpour, H., Mahmoudkhani, S., and Navazi, H. M. Free vibration analysis of functionally graded cylindrical shells including thermal effects. *Journal of Thin-Walled Structures* **45**(6), 591–599 (2007)

[11] Sheng, G. G. and Wang, X. Thermomechanical vibration analysis of a functionally graded shell with flowing fluid. *European Journal of Mechanics-A/Solids* **27**(6), 1075–1087 (2008)

[12] Loy, C. T., Lam, K. Y., and Reddy, J. N. Vibration of functionally graded cylindrical shells. *International Journal of Mechanical Sciences* **41**(3), 309–324 (1999)

[13] Pradhan, S. C., Loy, C. T., Lam, K. Y., and Reddy, J. N. Vibration characteristics of functionally graded cylindrical shells under various boundary conditions. *Applied Acoustics* **61**(1), 111–129 (2000)

[14] Naeem, M. N. *Vibrational Frequency Analysis of Nonrotating and Rotating FGM Circular Cylindrical Shells*, Ph. D. dissertation, University of Manchester Institute of Science and Technology, Manchester, UK (2002)

[15] Najafizadeh, M. M. and Isvandzibaei, M. R. Vibration of functionally graded cylindrical shells based on higher order shear deformation plate theory with ring support. *Acta Mechanica* **191**(1-2), 75–91 (2007)

[16] Arshad, S. H., Naeem, M. N., and Sultana, N. Frequency analysis of functionally graded material cylindrical shells with various volume fraction law. *Journal of Mechanical Engineering Science* **221**(12), 1483–1495 (2007)

[17] Love, A. E. H. *A Treatise on the Mathematical Theory of Elasticity*, 4th Ed., Cambridge University Press, Cambridge (1952)

[18] Hirano, T., Teraki, J., and Yamada, T. On the design of functionally gradient materials. *Proceedings of the First International Symposium on Functionally Gradient Materials* (eds. Yamanouchi, M., Koizumi, M., Hiral, T., and Shiota, I.), Sendai, Japan, 5–10 (1990)

[19] Koizumi, M. Functionally gradient materials, the concept of FGM. *Ceramic Transactions* **34**, 3–10 (1993)

Appendix A

$$
\begin{aligned}
T_{11} &= (K_m)^2 A_{11} + \frac{n^2}{R^2} A_{66} - \omega^2 h \rho_{\mathrm{T}}, \\
T_{12} &= -\frac{\mathrm{i} K_m n}{R}\Big((A_{12}+A_{66}) + \frac{B_{12}}{R} + \frac{2B_{66}}{R}\Big), \\
T_{13} &= \mathrm{i} K_m\Big(\frac{A_{12}}{R} + (K_m)^2 B_{11} + \frac{n^2}{R^2}(2B_{66}+B_{12})\Big), \\
T_{22} &= (K_m)^2\Big(A_{66} + \frac{4B_{66}}{R} + \frac{4D_{66}}{R^2}\Big) + \frac{n^2}{R^2}\Big(A_{22} + \frac{2B_{22}}{R} + \frac{D_{22}}{R^2}\Big) - \omega^2 h \rho_{\mathrm{T}}, \\
T_{23} &= n(K_m)^2\Big(\frac{B_{12}+B_{66}}{R} + \frac{D_{12}+4D_{66}}{R^2}\Big) + \frac{n}{R^2}\Big(A_{22} + \frac{B_{22}}{R}\Big) + \frac{n^3}{R^3}\Big(B_{22} + \frac{D_{22}}{R}\Big), \\
T_{33} &= \frac{(K_m)^2 n^2}{R^2}(D_{12}+2D_{66}) + \frac{n^2}{R^3}\Big(2B_{22} + \frac{n^2 D_{22}}{R}\Big) + (K_m)^2\Big(\frac{2B_{12}}{R} + (K_m)^2 D_{11}\Big) + \frac{A_{22}}{R^2} - \omega^2 h \rho_{\mathrm{T}} + FL.
\end{aligned}
$$