Appl. Math. Mech. -Engl. Ed., 2008, **29**(2):163–170
DOI 10.1007/s10483-008-0204-1

Applied Mathematics and Mechanics (English Edition)

Theoretical prediction of stiffness and strength of three-dimensional and four-directional braided composites *

LI Dian-sen (李典森), LU Zi-xing (卢子兴), LU Wen-shu (卢文书)

(School of Aeronautics Science and Technology, Beijing University of Aeronautics and Astronautics, Beijing 100083, P. R. China)

(Communicated by YUE Zhu-feng)

Abstract Based on unit cell model, the 3D 4-directional braided composites can be simplified as unidirectional composites with different local axial coordinate system and the compliance matrix of unidirectional composites can be defined utilizing the bridge model. The total stiffness matrix of braided composites can be obtained by the volume average stiffness of unidirectional composites with different local axial coordinate system and the engineering elastic constants of braided composites were computed further. Based on the iso-strain assumption and the bridge model, the stress distribution of fiber bundle and matrix of different unidirectional composites can be determined and the tensile strength of 3D 4-directional braided composites was predicted by means of the Hoffman's failure criterion for the fiber bundle and Mises' failure criterion for the matrix.

Key words 3D braiding, strength, stiffness, mechanical properties, composites

Chinese Library Classification TB332
2000 Mathematics Subject Classification 74D10

Introduction

In recent years, 3D braided composites have been developed with an attempt to overcome the poor mechanical properties in the thickness direction and interlaminar delamination of laminated composites. A revival of interest in 3D braided composites has been observed in the aerospace, automobile, marine and construction industries for their better out-of-plane stiffness, strength and impact resistance.

A series of recent analyses[1–3] about the mechanical properties of 3D braided composites focused on the stiffness of such materials, however, there were few literatures on the strength properties of 3D braided composites. Sun[4] proposed a model for the strength analysis of the 3D braided composite which was based on modified classical laminated theory and adopted Tsai-Wu polynomial criterion. Lu[5] introduced a strength prediction method of 3D braided composites by using the model of modulus prediction and the results of FFA. Gu[6] predicted the uniaxial tensile stress-strain curve of 3-dimensional braided preform by describing the yarn trace and the micro-structure of 3-dimensional braided preform in mathematical way. Zuo[7] proposed a theoretic method to calculate the portrait draw strength of 3D braided composite

* Received Aug. 29, 2007 / Revised Dec. 26, 2007
Project supported by the Aeronautics Science Foundation of China (No. 04B51045) and the Common Construction Project of Education Committee of Beijing (No. XK10006052)
Corresponding author LI Dian-sen, Doctor, E-mail: lidiansen@163.com

rectangle section beam, based on the second-rank expression for Tsai-Wu criteria. Dong[8] deduced the mathematical expression of the micro-stress of 3D braided composite based on the homogenization method and the micro-stresses of three-dimensional braided composites were simulated by a FEM method.

Recently, the bridge model developed by Huang[9, 10] has been successfully applied to simulate the nonlinear and strength behaviors of a variety of textile fabric reinforced composites. Motivated by the studies as reported in Refs. [9,10], the aim of this paper is to present such a micromechanics theory to analyze the stiffness and strengths of 3D 4-directional braided composites. The theory is based on a new micromechanical model in conjunction with the use of the bridging model.

1 Unit cell model

3D 4-directional braided composite can be assembled by four interior unit cell models, two surface unit cell models and one corner unit cell model in the interior, surface, corner region, respectively. Figure 1 shows the unit cell models of 3D 4-directional braided composites.

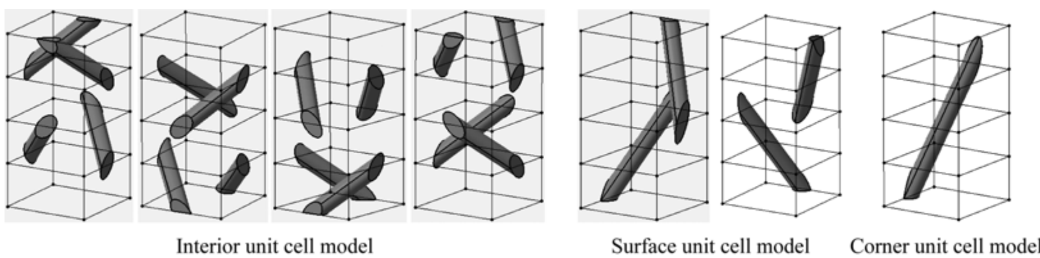

Fig. 1 Unit cell model

The following assumptions have been made for deriving the relationships of braiding parameters of a braided composite:

1. The cross-section for all braiding yarns is elliptical with minor radii d. The deformation coefficient of cross-section is k.

2. The braiding process is quite stable, so that the braided structure is uniform, at least over a certain length of interest. The yarns are in a jamming condition.

3. All yarns in the braided preform have identical yarn packing factor ε.

The relationship between the interior braiding angle γ, surface braiding angle β, corner braiding angle θ, and braiding angle α is presented as the following expressions:

$$\tan\gamma = \frac{1}{\sin\varphi}\tan\alpha = \frac{4}{\sin 2\varphi\sqrt{\sec^2\varphi + 1}}\tan\beta = \frac{3\sqrt{2}}{\sin 2\varphi}\tan\theta, \tag{1}$$

where φ is the angle between the horizontal projection of braiding yarn and thickness direction of the preform, φ is equal to 45° in a perfect state.

The total fiber volume fraction in a braided composite can be written as

$$V_{\mathrm{f}} = C_{\mathrm{i}}V_{\mathrm{i}} + C_{\mathrm{s}}V_{\mathrm{s}} + C_{\mathrm{c}}V_{\mathrm{c}}, \tag{2}$$

where V_{i}, V_{s}, V_{c} is the fiber volume fraction in the preform interior, surface, corner regions respectively and the expressions for them are as follows:

$$V_{\mathrm{i}} = \frac{\pi k \sin 2\varphi}{8\cos\gamma}\varepsilon, \quad V_{\mathrm{s}} = \frac{\pi k \cos\varphi}{4\cos\beta}\varepsilon, \quad V_{\mathrm{c}} = \frac{3\pi k}{16\cos\theta}\varepsilon. \tag{3,4,5}$$

And the volume proportions of the regions to the whole structure are given by the following expressions:

$$C_{\mathrm{i}} = \frac{2(P-1)(Q-1)/\sin 2\varphi}{[(P-1)/\sin\varphi + 2][(Q-1)/\cos\varphi + 2]}, \tag{6}$$

$$C_{\mathrm{s}} = \frac{[2(P-1)+2(Q-1)]/\cos\varphi}{[(P-1)/\sin\varphi + 2][(Q-1)/\cos\varphi + 2]}, \tag{7}$$

$$C_{\mathrm{c}} = \frac{4}{[(P-1)/\sin\varphi + 2][(Q-1)/\cos\varphi + 2]}, \tag{8}$$

where P is the carrier number of rows and Q is the number of columns of the main part. In a jamming condition, k is given as $\sqrt{3}\cos\gamma$. From the analysis above, when given the fiber volume fraction, the yarn packing factor ε can be determined.

2 Elastic properties

The fiber is considered as a transversely isotropic linear elastic solid. The stiffness matrix $[C]_{\mathrm{f}}$ and the compliance matrix $[S]_{\mathrm{f}}$ for the fiber, correlated by $[C]_{\mathrm{f}} = [S]_{\mathrm{f}}^{-1}$, where $[S]_{\mathrm{f}}$ was given by Eq. (5) in Ref. [11]. All the fiber bundles with identical orientation in the braided composite can be modeled as one unidirectional composite. From the unit cell model presented in Section 1, 12 kinds of unidirectional composites in the 3D braided composites can be obtained. Referring to the bridging model in Ref. [10], at any load level, the incremental stresses in the constituent fiber and resin of the unidirectional composite are correlated via a bridging matrix:

$$\{d\sigma_i^{\mathrm{m}}\} = [A_{ij}]\{d\sigma_j^{\mathrm{f}}\}, \tag{9}$$

where $[A_{ij}]$ is the bridging matrix and it can be given by[10]

$$[A_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{bmatrix}. \tag{10}$$

The independent elements of the bridging matrix are given as follows:

$$a_{11} = E^{\mathrm{m}}/E_1^{\mathrm{f}}, \quad a_{12} = a_{13} = (S_{12}^{\mathrm{f}} - S_{12}^{\mathrm{m}})(a_{11} - a_{12})/(S_{11}^{\mathrm{f}} - S_{11}^{\mathrm{m}}),$$
$$a_{21} = a_{31} = 0, \quad a_{23} = a_{32} = 0, \quad a_{22} = a_{33} = \alpha + (1-\alpha)E^{\mathrm{m}}/E_2^{\mathrm{f}} \quad (0 < \alpha < 1),$$
$$a_{44} = \beta + (1-\beta)G^{\mathrm{m}}/G_{23}^{\mathrm{f}} \quad (0 < \beta < 1), \quad a_{55} = a_{66} = \gamma + (1-\gamma)G^{\mathrm{m}}/G_{12}^{\mathrm{f}} \quad (0 < \gamma < 1),$$

where α, β, γ are bridging parameters which can be adjusted by the experimental data of effective transverse moduli and shear moduli of the material.

The unidirectional (UD) composite analysis is generally carried out in the local coordinate system 1-2-3 (Fig. 2), with its local instantaneous compliance matrix given by

$$[S_{ij}]_n = (V_{\mathrm{f}}^n[S_{ij}]_{\mathrm{f}} + V_{\mathrm{m}}^n[S_{ij}]_{\mathrm{m}}[A_{ij}])(V_{\mathrm{f}}^n[I] + V_{\mathrm{m}}^n[A_{ij}])^{-1}, \tag{11}$$

where $[S_{ij}]_{\mathrm{f}}$ and $[S_{ij}]_{\mathrm{m}}$ are the instantaneous compliance matrices of the fiber and matrix materials, and $[I]$ is a unit matrix. Elements of all these matrices can be found in Refs. [10, 11]. V_{f}^n and V_{m}^n refer to the fiber and matrix

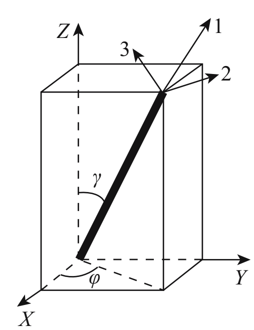

Fig. 2 Schematic of a yarn in local and global coordinate

volume fractions for different unidirectional composites, respectively, and the relation for them is $V_{\mathrm{m}}^{n} = 1 - V_{\mathrm{f}}^{n}$.

The fiber volume fraction for each unidirectional composites are given as follows:

$$V_{\mathrm{f}}^{\mathrm{i1}} = V_{\mathrm{f}}^{\mathrm{i2}} = V_{\mathrm{f}}^{\mathrm{i3}} = V_{\mathrm{f}}^{\mathrm{i4}} = \sqrt{3}\pi\varepsilon/8, \tag{12}$$

$$V_{\mathrm{f}}^{\mathrm{s1}} = V_{\mathrm{f}}^{\mathrm{s2}} = V_{\mathrm{f}}^{\mathrm{s3}} = V_{\mathrm{f}}^{\mathrm{s4}} = \sqrt{6}\pi\varepsilon\cos\gamma/(8\cos\beta), \tag{13}$$

$$V_{\mathrm{f}}^{\mathrm{c1}} = V_{\mathrm{f}}^{\mathrm{c2}} = V_{\mathrm{f}}^{\mathrm{c3}} = V_{\mathrm{f}}^{\mathrm{c4}} = 3\sqrt{3}\pi\varepsilon\cos\gamma/(16\cos\theta). \tag{14}$$

In the local coordinate system, the stiffness matrix and the compliance matrix for each UD composite, correlated by $[C_{ij}]_n = [S_{ij}]_n^{-1}$, the average stiffness matrix in the local coordinate system can be transformed to a representation in the global coordinate system (X-Y-Z), by a rotation about the common Z axis, through a fiber orientation angle. The fiber orientation angle for different UD composite is (γ,φ), $(\gamma,-\varphi)$, $(-\gamma,-\varphi)$, $(-\gamma,\varphi)$, (β,φ), $(\beta,-\varphi)$, $(-\beta,-\varphi)$, $(-\beta,\varphi)$, (θ,φ), $(\theta,-\varphi)$, $(-\theta,-\varphi)$, $(-\theta,\varphi)$, respectively. Then the stiffness matrix for each UD composite at the global coordinate system ($OXYZ$) can be derived as

$$[\overline{C_{ij}}]_n = [T_\sigma]_n[C_{ij}]_n[T_\sigma]_n^{\mathrm{T}}, \tag{15}$$

in which $[T_\sigma]$ is the transformation matrix of stress vector, and superscript "T" refers to transpose of the matrix. The expression for $[T_\sigma]$ can be found in Ref. [12]

Stiffness matrix from each UD composite is summed up for the stiffness of the 3D braided composite by the volume average. Namely,

$$[\overline{C}] = \sum_{n=\mathrm{i1}}^{\mathrm{c4}} V_n[\overline{C_{ij}}]_n, \tag{16}$$

where the volume proportions of each UD composite to the whole composite are given by the following expressions:

$$V_{\mathrm{i1}} = V_{\mathrm{i2}} = V_{\mathrm{i3}} = V_{\mathrm{i4}} = \frac{(P-1)(Q-1)}{2[\sqrt{2}(P-1)+2][\sqrt{2}(Q-1)+2]}, \tag{17}$$

$$V_{\mathrm{s1}} = V_{\mathrm{s2}} = V_{\mathrm{s3}} = V_{\mathrm{s4}} = \frac{\sqrt{2}[(P-1)+(Q-1)]}{2[\sqrt{2}(P-1)+2][\sqrt{2}(Q-1)+2]}, \tag{18}$$

$$V_{\mathrm{c1}} = V_{\mathrm{c2}} = V_{\mathrm{c3}} = V_{\mathrm{c4}} = \frac{1}{[\sqrt{2}(P-1)+2][\sqrt{2}(Q-1)+2]}. \tag{19}$$

Correspondingly, compliance matrix for 3D 4-directional braided composites can be calculated by $[S'] = [\overline{C}]^{-1}$, and finally the engineering elastic constant of 3D 4-directional braided composite can be received by the elements of the compliance matrix.

3 Strength analysis

When the tensile loading $\{\sigma_i\} = \{\sigma_x, 0, 0, 0, 0, 0\}^{\mathrm{T}}$ is applied, the stress-strain relationship for the whole composite is given by $\{\sigma_i\} = [\overline{C}]\{\varepsilon_j\}$, where $\{\varepsilon_j\}$ is the total strain of the whole composite. Suppose that uniform strain exists for each UD composite and the whole composite, the total stress of composite is written to the following expressions in the global coordinate system:

$$\begin{aligned}\{\sigma_i\} &= \Big(\sum_{n=\mathrm{i1}}^{\mathrm{c4}} V_n[\overline{C_{ij}}]_n\Big)\{\varepsilon_j\}\\ &= (V_{\mathrm{i1}}[\overline{C_{ij}}]_{\mathrm{i1}})\{\varepsilon_j\} + (V_{\mathrm{i2}}[\overline{C_{ij}}]_{\mathrm{i2}})\{\varepsilon_j\} + \cdots (V_{\mathrm{c4}}[\overline{C_{ij}}]_{\mathrm{c3}})\{\varepsilon_j\} + (V_{\mathrm{c4}}[\overline{C_{ij}}]_{\mathrm{c4}})\{\varepsilon_j\}\\ &= \{\sigma_i\}_{\mathrm{i1}} + \{\sigma_i\}_{\mathrm{i2}} + \cdots + \{\sigma_i\}_{\mathrm{c3}} + \{\sigma_i\}_{\mathrm{c4}}.\end{aligned} \tag{20}$$

Therefore, the stress for each UD composite in the global coordinate system can be formally expressed as

$$\{\sigma_i\}_n = V_n[\overline{C_{ij}}]_n\{\varepsilon_j\}. \tag{21}$$

By making use of the bridging model[11], when given the constituents properties and fiber volume fraction of each UD composite, the stresses generated in the fiber and resin materials at the local coordinate system are determined as

$$\{\sigma_i^{\mathrm{f}}\}_n = (V_{\mathrm{f}}^n[I] + V_{\mathrm{m}}^n[A_{ij}])^{-1}\{\sigma_j\}_n = [B_{ij}]\{\sigma_j\}_n, \tag{22}$$

$$\{\sigma_i^{\mathrm{m}}\}_n = [A_{ij}](V_{\mathrm{f}}^n[I] + V_{\mathrm{m}}^n[A_{ij}])^{-1}\{\sigma_j\}_n = [A_{ij}][B_{ij}]\{\sigma_j\}_n, \tag{23}$$

where $\{\sigma_j\}_n$ is the stress for each UD composite in the local coordinate system.

Substituting the stress in Eq. (21) into Eqs. (22) and (23), and solving for $\{\sigma_i^{\mathrm{f}}\}_n$, $\{\sigma_i^{\mathrm{m}}\}_n$ with respect to $\{\sigma_i\}_n$ by coordinate-system transformation, we obtain the total stresses in the constituents (the fiber and matrix) at the local coordinate system:

$$\{\sigma_i^{\mathrm{f}}\}_n = [B_{ij}](([T_\sigma]_n^{-1})^{\mathrm{T}}\{\sigma_i\}_n, \tag{24}$$

$$\{\sigma_i^{\mathrm{m}}\}_n = [A_{ij}][B_{ij}](([T_\sigma]_n^{-1})^{\mathrm{T}}\{\sigma_i\}_n. \tag{25}$$

The fiber can be considered as a transversely isotropic linear elastic composite. Hence, the Hoffman criterion to govern the tensile failure of the fiber is adopted and the expression is simply given by

$$\begin{aligned} &C_1(\sigma_1-\sigma_2)^2 + C_2(\sigma_2-\sigma_3)^2 + C_3(\sigma_3-\sigma_1)^2 \\ &+ C_4\sigma_1 + C_5\sigma_2 + C_6\sigma_3 + C_7\tau_{23}^2 + C_8\tau_{31}^2 + C_9\tau_{12}^2 = 1, \end{aligned} \tag{26}$$

where $C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9$ can be computed as described in Ref. [5].

The matrix is considered as an isotropic material and a generalized Mises criterion to detect tensile failure of the matrix is expressed as

$$(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 + 6(\tau_{12}^2 + \tau_{23}^2 + \tau_{31}^2) = 2\sigma_{\mathrm{m}}^2. \tag{27}$$

When the stresses in the matrix meet Eq. (27), the yield failure occurs in some local regions of the matrix, but the whole composite still has the capacity of load bearing. In this paper, when the stresses of the fiber bundles in the surface region of the 3D 4-directional braided composite meet Eq. (26), the whole composite is considered as losing the capacity of bear loading and the applied load is taken as the strength of the composite. The calculated results showed that the fiber bundles in the interior region failed first, followed by the fiber bundles in the surface region, and finally the fiber bundles in the corner region failed with increasing the applied load. When the fiber bundles in the interior region fail, we believe that the damage takes place in the whole composite and the bear loading capacity decreases, but the composite can still bear loading. The experimental results[13] also showed that the accumulated damage process existed during the loading.

4 Results and discussion

The mechanical properties for the T300 carbon fiber constitute is $E_{\mathrm{f1}} = 220$ GPa, $E_{\mathrm{f2}} = 13.8$ GPa, $G_{\mathrm{f12}} = 9$ GPa, $\mu_{\mathrm{f12}} = 0.2$, $\mu_{\mathrm{f23}} = 0.25$, $\sigma_{\mathrm{f}} = 3$ GPa, $\sigma_{\mathrm{f}}^- = 2.07$ GPa, $\tau_{\mathrm{f}} = 943$ MPa. The mechanical properties for the epoxy resin is $E_{\mathrm{m}} = 4.5$ GPa, $\mu_{\mathrm{m}} = 0.34$, $\sigma_{\mathrm{m}}^* = 70.6$ MPa, $\sigma_{\mathrm{m}} = 80$ MPa, $\sigma_{\mathrm{m}}^- = 79$ MPa, $\varepsilon_{\mathrm{mu}} = 1.7\%$, $\tau_{\mathrm{m}} = 46$ MPa. The bridge parameters are chosen as $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 0.5$. The meaning for the above symbols are identical with that used in Ref. [5].

Figure 3 shows the variation of the elastic constant with the braiding angle and the fiber volume fraction and the comparison between the results obtained from the present analytical and the stiffness averaged method[11]. It is observed that the elastic constants in variation trend predicted by these two models are quite close. Close results are also reported for E_z, G_{xy}, G_{yz}, and μ_{zx}, but transverse tensile stiffness E_x predicted by the present analytical model are a little bit smaller than that from the stiffness averaged method and Poisson ratio μ_{xy} from the present analytical model are a little bit higher. In addition, compared with the predictive results in Ref. [12], similar results can be obtained, which shows the validity of the present analytical

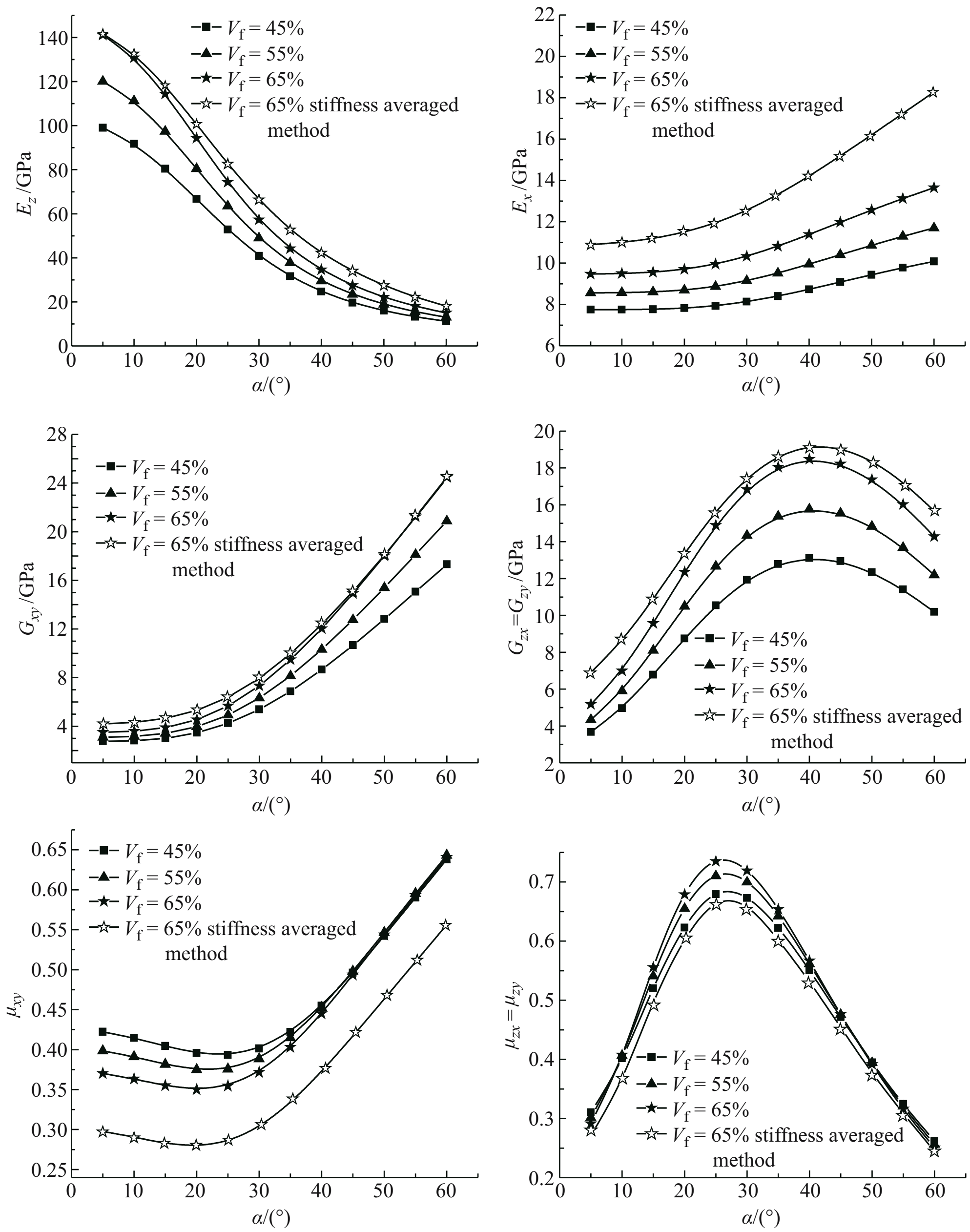

Fig. 3 Elastic constants variation with braiding angle and fiber volume fraction

model. Here, the variation tendency for elastic constant with the braiding angle and the fiber volume fraction in detail is neglected.

The variation of the predicted strength of 3D braided composites with the braid angle and the fiber volume fraction is plotted in Fig. 4. It can be seen that the strength decreases monotonically with increasing the braid angle .When the braiding angle α is less than 30°, the strength decreases linearly and the velocity of declining is great. When the braiding angle α is more than 30°, the strength decreases nonlinearly and the velocity of declining reduces obviously. When α is more than 40°, the variation of the strength is very small and the strength tends to be constant when braiding angle is big enough. We notice that the strength increases with increasing the fiber volume fraction and the smaller the braiding angle is, the more sensitive the variation is. The variation magnitude of the strength with the fiber volume fraction is decreased with increasing the braiding angle. It is also seen from Fig. 4 that the predicted strength has a discrepancy between the present result and the one given by the FEA method[5]. It is mainly attributed to the fact that the model used in Ref. [5] is too simple and that the predictions must depend on the experiment results, which is difficult to give effective predictions in a large braiding angle range.

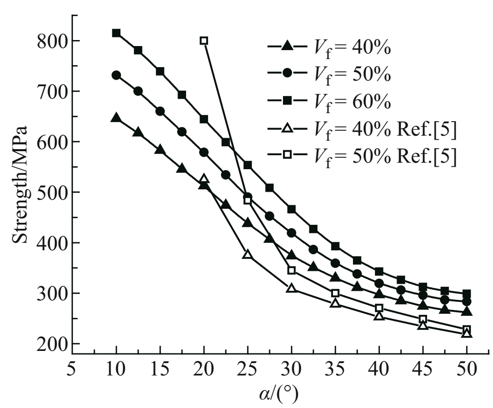

Fig. 4 Strength variation with braiding angle and fiber volume fraction

In order to verify the validation of the present model further, Table 1 lists a comparison of the predictive values and experimental data[13] for the mechanical properties of 3D 4-directional braided composite. It can be seen that the predictions of the axial elastic modulus agree well with the experimental results. We notice that there is a little difference between the predictions and the experimental results for the strength, but the error is in the engineering limit range. It is concluded that the present model is an effective way of predicting the stiffness and strength properties of 3D 4-directional braided composite from the discussion in Fig. 2 and Fig. 3 and the data in Table 1.

Table 1 Comparison of predictive and experimental data

Braiding structure	Braiding angle/(°)	Fiber volume fraction/%	Axial elastic modulus		Error /%	Tensile strength		Error /%
			Experimental data/GPa	Predictive data/GPa		Experimental data/MPa	Predictive data/MPa	
4D	21	45	67.2	63.87	−4.96	665	543	−18.35
4D	42	58	28.7	28.17	−1.84	283	321	13.43
4D	48	45	22.4	22.98	2.59	254	267	5.12

5 Conclusions

Based on the unit cell model, the stiffness and strength of 3D 4-directional braided composites are predicted by a micromechanical method in conjunction with the bridge model. It is believed that the braiding angle and the fiber volume fraction are the important factor. The predictions by the present analytical approach show a good agreement with the experiment results, which proves the practicability of the bridging model in the prediction of the mechanical properties of 3D braiding composites.

Acknowledgements The authors gratefully acknowledge the support by the Aeronautics Science Foundation of China under the grant No. 04B51045, the Common Construction Project of Education Committee of Beijing under the grant No. XK10006052 and the Doctor Innovation Foundation of Beijing University of Aeronautics and Astronautics.

References

[1] Ma C L, Yang J M, Chou T W. Elastic stiffness of three-dimensional braided textile structural composites[M]. ASTM, STP893. In: *Composite Materials: Testing and Design.* Philadelphia: American Society for Testing Material, 1986, 404–421.

[2] Chen L, Tao X M, Choy C L. Mechanical analysis of 3-D braided composites by the finite multiphase element method[J]. *Composites Science and Technology*, 1999, **59**:2383–2391.

[3] Sun Xuekun, Sun Changjie. Mechanical properties of three-dimensional braided composites[J]. *Composite Structures*, 2004, **65**(3–4):485–492.

[4] Sun Huiyu, Wu Changchun, Bian Enrong. A study on in-plane stiffness and strength of three-dimensionally braided composites[J]. *Acta Materiae Compositae Sinica*, 1998, **15**(4):102–106 (in Chinese).

[5] Lu Zixing, Liu Zhenguo, Mai Hanchao, et al. Numerical prediction of strength for 3D braided composites[J]. *Journal of Beijing University of Aeronautics and Astronautics*, 2002, **28**(5):563–565 (in Chinese).

[6] Gu Bohong. Prediction of the uniaxial tensile curve of 4-step 3-dimensional braided perform[J]. *Composite Structures*, 2004, **64**(2):235–241.

[7] Zuo Weiwei, Xiao Laiyuan, Liao Daoxun. Calculating the strength of 3D-braided composite rectangle beam by using Tsai-Wu criteria[J]. *Journal of Huazhong University of Science & Technology*, 2006, **34**(12):74–76 (in Chinese).

[8] Dong Jiwei, Sun Liangxin, Hong Ping. Homogenization-based method for simulating micro-stress of 3-D braided composites[J]. *Acta Materiae Compositae Sinica*, 2005, **22**(6):139–143 (in Chinese).

[9] Huang Zhengming. A bridging model prediction of the ultimate strength of composite laminates subjected to biaxial loads[J]. *Composites Science and Technology*, 2004, **64**(3–4):395–448.

[10] Huang Zhengming. The micromechanics of composite materials[M]. Beijing: Science Publishing Company, 2004 (in Chinese).

[11] Li Diansen, Lu Zixing, Chen Li, Li Jialu. Theoretical prediction of the elastic properties of three-dimensional and six-directional braided composites[J]. *Acta Materiae Compositae Sinica*, 2006, **4**(23):112–118 (in Chinese).

[12] Yang Zhenyu, Lu Zixing. Theoretical prediction of the elastic properties of three-dimensional and four-directional braided composites[J]. *Acta Materiae Compositae Sinica*, 2004, **21**(2):134–141 (in Chinese).

[13] Lu Zixing, Feng Zhihai, Kou Changhe, et al. Studies on tensile properties of braided structural composite materials[J]. *Acta Materiae Compositae Sinica*, 1999, **16**(3):129–134 (in Chinese).