Applied Mathematics and Mechanics (English Edition), 2007, **28**(6):769–778

Dynamic model of vertical vehicle-subgrade coupled system under secondary suspension *

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(Communicated by MA Xing-rui)

Abstract As it is known, track transportation can be divided into track system above and track system below. While the train is moving, the parts above and below are interacted and influenced. Therefore, in fact, the problem of track transportation is the match between the vehicle and the railway line system. In this paper, on a basis of dynamic analysis of the vehicle-subgrade model of vertical coupled system under primary suspension, utilizing track maintenance standard and simulating track irregularity excitation, the dynamic interaction of vehicle-track-subgrade system is researched in theory and dynamic model of the vertical vehicle-track-subgrade coupled system under secondary suspension is established by compatibility condition of deformation. Even this model considers the actual structure of a vehicle, also considers vibration characteristic of the substructure of track including subgrade and foundation. All these work want to be benefit for understanding and design about the dynamic characters of subgrade in high speed railway.

Key words secondary suspension, vehicle, subgrade, dynamic model, coupled analysis

Chinese Library Classification U211.5, U260.11
2000 Mathematics Subject Classification 70E55
Digital Object Identifier(DOI) 10.1007/s 10483-007-0607-z

Introduction

For a long time, the research about vehicle-subgrade system dynamics is only limited to solve dynamic equation of simple single-mass system in the irregularity, or subgrade will be simplified as a part of track foundation. And dynamic problem of wheel/track will be summed up as "vehicle dynamics", "track dynamics" and "wheel/rail dynamic interaction relationship", which are some relatively independent research domains. These three problems will not be analyzed as a whole and a system.

In recent years, high speed and fast speed have been the main trend in the world track traffic. Some problems about the dynamic interaction and coupled vibration of vehicle-structure have also become quite remarkable. The research of dynamic interaction relationship between vehicle and bridge system[1−5], and the vehicle-track dynamic model of coupled system[6−11] have all been evolved. And some key problems that obstruct the development of high speed railway

* Received Jul.16, 2006; Revised Dec.19, 2006
Project supported by the Science Foundation of Chongqing Municipal Commission of Education (No.KJ060404), the Natural Science Foundation of Chongqing (No.CSTC, 2006BB6048) and the Startup Foundation of Chongqing Jiaotong University
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have partly been solved, including impact coefficient and lateral stability of bridge, rail wear, joint impact, irregularity of track and so on.

But subgrade dynamic characteristics are not considered as one of the important components of the system as mentioned above. It is known that track traffic can be divided into track system above and track system below. The track system above is the moving part, namely, locomotive and vehicle. Track system below, especial as flexible basement, mainly includes rail, sleeper, ballast and subgrade, endured repeated load of the vehicle. The track will be damaged by the deformation because of the many factors, such as mass of rail, joint, ballast stiffness, fatigue deformation characteristics of foundation and contact effect between sleeper and ballast, and so on. All these factors would influence the normal use of rail and make the rail surface irregular and affect the normal moving function for the wheel on rail. Analysis mentioned above indicates that the track damage is not only the failure in strength of components but also the over-deformation of the system below track. So while analyzing dynamic interaction between vehicle and track, we can not ignore the important effect of the participating vibration of the subgrade in the total system.

Some scholars[12−16] researched the dynamic problem of subgrade from different aspects. But whatever model will not consider that train/track/subgrade is a system. Most of models analyze the stress, formation of foundation under the track based on dynamic load. So it can not reflect fully the dynamic performance of vehicle-subgrade system in running forward, and is not enough for the dynamic analysis of track communication. References [17] and [18] describe the dynamic model of vertical vehicle-subgrade coupled system under the primary suspension system. References [19] and [20] investigate train-bridge coupled system under the primary suspension and secondary suspension. This paper will study train/track/subgrade dynamic interactions in theory according to the actual condition of running train. And a vehicle/train/subgrade dynamic model of vertical coupled system under the secondary suspension will be set up.

1 Vehicle dynamic model and balance equations

1.1 Vehicle-subgrade dynamic simplified model

According to the different vehicle structures (mainly because of suspension style), the unified vertical coupled model of vehicle-subgrade may be three different styles. One is the secondary suspension vehicle-track model that mainly reflects the interaction between passenger coach and track. A modern new freight train, like the British LTF, is also regarded as this model. Based on this model, the vehicle-subgrade dynamic simplified model in this paper is as shown in Fig.1. In this model, the coach-body and the wheel of the vehicle are both considered rigid body; The train, consisted of many same or different locomotives or vehicles (they will here in after be called vehicles), passes line at even speed; two degrees of freedom of sink-emerge and nod will be considered for each coach-body and each bogie and one degree of freedom of the wheel will be considered. Furthermore, it is assumed that the wheels of the vehicle keep close touch with the track surface. In this paper we mainly study the vehicle-track-subgrade system's vibration caused by vehicle running at high speed. Especially, we do research about the influence of the change of the part below track on the vehicle running quality, including comfortable degree, safety and irregularity. In order to simplify the analysis process, it is also simplified to the primary spring suspension as shown in Refs.[17,18].

1.2 Dynamic equation of vehicle system under the secondary suspension

Based on the assumption as mentioned above, dynamic equation of vehicle system, such as sixth-axis vehicle, is set up.

The dynamic balance equations of the i coach-body are

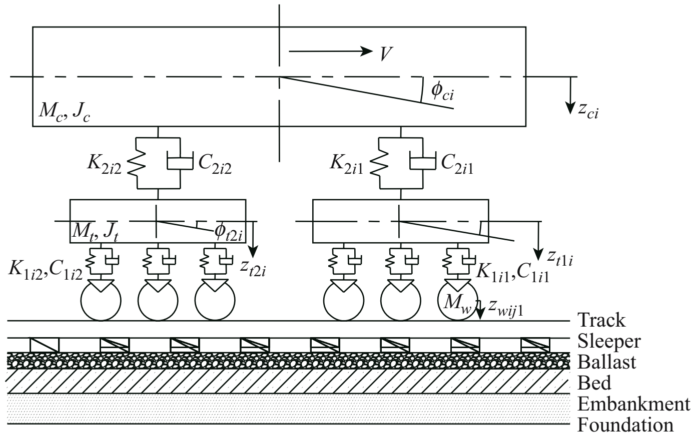

Fig.1 Vertical vehicle-subgrade dynamic simplified model

$$\begin{cases} M_{ci}\ddot{z}_{ci} + \sum_{j=1}^{2} C_{2ij}(\dot{z}_{ci} - \dot{z}_{tij}) + \sum_{j=2}^{2} K_{2ij}(z_{ci} - z_{tij}) = 0, \\ J_{ci}\ddot{\varphi}_{ci} + \sum_{j=1}^{2} C_{2ij}(l_{ci}^2\dot{\varphi}_{ci} - \eta_{ij}l_{ci}\dot{z}_{tij}) + \sum_{j=1}^{2} K_{2ij}(l_{ci}^2\varphi_{ci} - \eta_{ij}l_{ci}\dot{z}_{tij}) = 0. \end{cases} \tag{1}$$

Formula (1) represents the sink-emerge and nod dynamic balance equation of coach-body. Its matrix form is

$$\begin{aligned} &\begin{bmatrix} M_{ci} & 0 \\ 0 & J_{ci} \end{bmatrix} \begin{Bmatrix} \ddot{z}_{ci} \\ \ddot{\varphi}_{ci} \end{Bmatrix} + \begin{bmatrix} C_{czi} & 0 \\ 0 & C_{c\varphi i} \end{bmatrix} \begin{Bmatrix} \dot{z}_{ci} \\ \dot{\varphi}_{ci} \end{Bmatrix} + \begin{bmatrix} K_{czi} & 0 \\ 0 & K_{c\varphi i} \end{bmatrix} \begin{Bmatrix} z_{ci} \\ \varphi_{ci} \end{Bmatrix} \\ &- \begin{bmatrix} C_{2i} & C_{2i} \\ C_{2i}l_{ci} & -C_{2i}l_{ci} \end{bmatrix} \begin{Bmatrix} \dot{z}_{til} \\ \dot{z}_{ti2} \end{Bmatrix} - \begin{bmatrix} K_{2i} & K_{2i} \\ K_{2i}l_{ci} & -K_{2i}l_{ci} \end{bmatrix} \begin{Bmatrix} z_{til} \\ z_{ti2} \end{Bmatrix} = 0, \end{aligned} \tag{2}$$

where M_{ci} and J_{ci} represent the mass and the inertial moment of the i coach-body, respectively; z_{ci} and φ_{ci} represent the vertical displacement and the angle of mass center of the i coach-body, respectively; K_{czi} and C_{czi} represent the general stiffness and general damping of the sink-emerge of the coach-body, respectively; $K_{c\varphi i}$ and $C_{c\varphi i}$ represent the general stiffness and general damping of the nod, respectively; K_{2i} and C_{2i} represent the secondary vertical spring stiffness and the damping of the bogie, respectively. If $K_{2i} = K_{2i1} = K_{2i2}$, $C_{2i} = C_{2i1} = C_{2i2}$, and the spring are parallel connection, then $K_{czi} = 2K_{2i}$, $K_{c\varphi i} = 2K_{2i}l_{ci}^2$, $C_{czi} = 2C_{2i}$, $C_{c\varphi i} = 2C_{2i}l_{ci}^2$. l_{ci} is the half distance of the coach-body. z_{ti1} and z_{ti2} represent the vertical displacements of center mass of the front and back bogie, respectively. The parameter meanings could be seen in Fig.1.

The matrix form of the dynamic balance equations of the j bogie of the i coach-body is

$$\begin{aligned} &\begin{bmatrix} M_{tij} & 0 \\ 0 & J_{tij} \end{bmatrix} \begin{Bmatrix} \ddot{z}_{tij} \\ \ddot{\varphi}_{tij} \end{Bmatrix} + \begin{bmatrix} C_{tzij} & 0 \\ 0 & C_{t\varphi ij} \end{bmatrix} \begin{Bmatrix} \dot{z}_{tij} \\ \dot{\varphi}_{tij} \end{Bmatrix} + \begin{bmatrix} K_{tzij} & 0 \\ 0 & K_{t\varphi ij} \end{bmatrix} \begin{Bmatrix} z_{tij} \\ \varphi_{tij} \end{Bmatrix} \\ &- \begin{bmatrix} C_{2i} & \eta_j C_{2i}l_{ci} \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{z}_{ci} \\ \dot{\varphi}_{ci} \end{Bmatrix} - \begin{bmatrix} K_{2i} & \eta_j K_{2i}l_{ci} \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} z_{ci} \\ \varphi_{ci} \end{Bmatrix} = \begin{Bmatrix} \sum_{l=1}^{N_k}[(K_{1i}z_{wijl} + C_{1i}\dot{z}_{wijl})] \\ \sum_{l=1}^{N_k} 2\eta_l l_w(K_{1i}z_{wijl} + C_{1i}\dot{z}_{wijl}) \end{Bmatrix}, \end{aligned} \tag{3}$$

where M_{tij} and J_{tij} represent the mass and the inertial moment of the j bogie of the i coach-body, respectively; z_{tij} and φ_{tij} represent the vertical displacement and the angle of the j bogie of center mass of the i coach-body, respectively; K_{tzij} and C_{tzij} represent the general stiffness and general damping of the sink-emerge of the j bogie of the i coach-body, respectively; $K_{t\varphi ij}$ and $C_{t\varphi ij}$ represent the general stiffness and general damping of the nod, respectively. The spring and damper below are parallel connection, then $C_{tzij} = C_{2i}+N_kC_{li}$, $K_{tzij} = K_{2i}+N_kK_{1i}$, $C_{t\varphi ij} = N_kC_{1i}l_{wi}^2$, $K_{t\varphi ij} = N_kK_{li}l_{wi}^2$. K_{1i} and C_{1i} represent the primary vertical spring stiffness and the damping of the bogie, respectively. N_k is the number of the wheel of each bogie, N_w is the number of the wheel of each coach-body, and $N_k = 0.5N_w$. z_{wijl} represents the vertical displacement of the l wheel of the j bogie of the i coach-body. l_w is the half distance of the axle of a bogie. η_l is the symbol function of the wheel. When wheel l is the front of bogie, $\eta_l = 1$, the middle of bogie, $\eta_l = 0$, and the back of bogie, $\eta_l = -1$. η_j is the symbol function of the bogie. When bogie j is in front vehicle $\eta_j = 1$, and in back vehicle $\eta_j = -1$. Other parameter's value and meanings are the same as mentioned above.

2 Track and subgrade model

Track and subgrade models in the previous research[6−8,16] are all considered to be the supporting model of multi-layer bases track below. Track is considered to be Euler's spring sustaining beam. The mass of sleeper concentrates on the node of the track element. There is a dispersed sustaining system below each sleeper. The ballast will be considered to be loose medium without considering the effect of the vertical vibration among them. So the model of lumped parameters shown in Fig.2 is adopted.

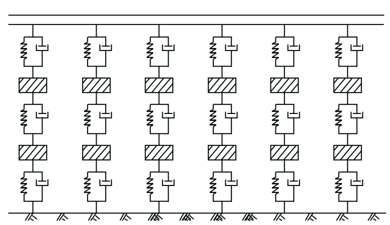

Fig.2 Three layers of bases supporting model below track

The biggest advantage of this model is that it simplifies the computational work of the numerical analysis.

Germany route's model[21] is also the longitudinal dispersed model that is mono-layer spring bearing model of sleeper-track, but the longitudinal plane of the track in dispersed dots is simulated by the method of the imitate approaching fit.

Although ballast, embankment and ground are a kind of loose, uncontinuous medium, the simple longitudinal dispersed model without considering the longitudinal connective effect among them does not conform to the character of their vibration. Besides, failure to consider the subgrade and ground vibration is also one of the main shortcomings of the models mentioned above.

The model of track-subgrade analyzed in this paper is still divided in finite element model by conventional method shown in Fig.3. The track is divided in beam element or quadrangle element. The sleeper, subgrade and foundation are separated in quadrangle element. The effect of working property, the interaction and the coordination and the parameter on the track and subgrade model in vibration condition will be fully considered. The dynamic equation is

$$\boldsymbol{m}\ddot{\boldsymbol{\delta}} + \boldsymbol{c}\dot{\boldsymbol{\delta}} + \boldsymbol{k}\boldsymbol{\delta} = \boldsymbol{f}, \tag{4}$$

where $\boldsymbol{f}$ is the node load or force vector; $\boldsymbol{m}, \boldsymbol{k}, \boldsymbol{c}$ are the mass, stiffness and damping matrixes, respectively; $\boldsymbol{\delta}$ is the node displacement vector.

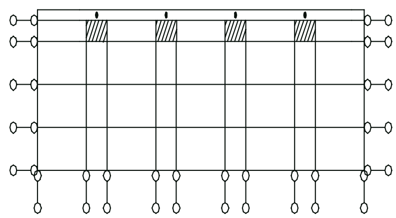

Fig.3 Finite element model

As the finite element model, if the dynamic equations of the system of vehicle and subgrade are directly calculated, the calculation work will be very great large. So the track and subgrade model is built by sub-structure principle. Firstly, the free frequency and vibration mode of every step is solved. Secondly, by using the normalization of vibration mode, thousands of coupled equations will be changed into special mode equations. Because the vibration reaction of structure is mainly controlled by the first several low stages of vibration mode, the calculation work will be greatly reduced by using the first several stages of vibration mode. In practical calculation, the vehicle load acts on subgrade by track and all the structural vibrations affect the vehicle by track too. So every stages of vibration mode can be taken from the joint of the track surface or the position where the wheel is on the track.

The vertical displacement $z_r(x)$ of any cross-section of track-subgrade can be summed up by several vibration mode functions. If we adopt the N stages of vibration mode to calculate, we have

$$z_r(x) = \sum_{n=1}^{N} A_n \Phi_n(x), \tag{5}$$

where $\Phi_n(x)$ is the n-th stage of vibration mode in some position, A_n is the corresponding generalized coordinate (i.e., the generalized displacement in some time steps), x is the horizontal position of some cross-section.

The corresponding n-th stage modal equation can be gained by mode analysis

$$M_n \ddot{A}_n + C_n \dot{A}_n + K_n A_n = F_n, \tag{6}$$

or can be written into

$$\ddot{A}_n + 2\xi_n \omega_n \dot{A}_n + \omega_n^2 A_n = F_n, \tag{7}$$

where F_n is the generalized force according to the n-th stage of vibration mode; M_n, K_n and C_n are the generalized mass, stiffness and damping, respectively; ω_n is the circular frequency of the n-th stage; ξ_n is the normalization damping coefficient.

The method of the generalized force F_n is defined as follows.

When the l wheel of the j bogie of the i coach-body is running through the length of the analyzed track, the force from the wheel acting on the track includes the inertia force of the wheel and the vertical force from the spring and damper of the bogie. Then the vertical force is

$$P_{jlw} = P_{sl} + K_{1i}(z_{tij} + \eta_{jl} l_{wi} \varphi_{tij} - z_{wijl}) + C_{1i}(\dot{z}_{tij} + \eta_{jl} l_{wi} \dot{\varphi}_{tij} - \dot{z}_{wijl}) - m_{wijl} \ddot{z}_{wijl}, \tag{8}$$

where P_{sl} is the average wheel's static load, $P_{sl} = [(0.5M_{ci} + M_{ti})/N_k + m_{ijl}]g$; m_{wijl} is the mass of the l wheel of the j bogie of the i coach-body; z_{wijl}, $\dot{z}_{wijl}$ and $\ddot{z}_{wijl}$ are the vertical displacement, velocity and acceleration of the l wheel of the j bogie of the i coachbody, respectively. The other symbols are the same as above.

If there are N_v coach-bodies, every coach-body including N_l bogies and every bogie including N_k wheels running on the length of the analyzed track, the generalized force F_n corresponding the n-th vibration mode is

$$F_n = \sum_{i=1}^{N_v}\sum_{j=1}^{N_l}\sum_{l=1}^{N_k} P_{jlw}\Phi_n(x_{ijl}). \tag{9}$$

Considering Eqs.(8) and (9), we have

$$\begin{aligned} F_n = & \sum_{i=1}^{N_v}\sum_{j=1}^{N_l}\sum_{l=1}^{N_k} P_{jlw}\Phi_n(x_{ijl})[P_{sl} + K_{1i}(z_{tij} + \eta_{jl}l_{wi}\varphi_{tij} - z_{wijl}) \\ & + C_{1i}(\ddot{z}_{tij} + \eta_{jl}l_{wi}\dot{\varphi}_{tij} - \dot{z}_{wijl}) - m_{wijl}\ddot{z}_{wijl})], \end{aligned} \tag{10}$$

where x_{ijl} is the position of the track surface of the l wheel of the j bogie of the i coachbody.

3 Solution of coupled dynamic equation

As mentioned above, though the model of track and subgrade can be transformed into special mode equation by sub-structure principle and the dynamic analysis of the vehicle-subgrade model can be calculated by several low stages of vibration mode. It will be an important problem how to connect the model of track-subgrade and the vehicle model, and how to solve them.

In the past, the dynamics coupled model of vehicle-track was solved by Hertz (non-linear elastic contact) theory[6−8] according to wheel/rail contact force. However, in this paper the problem is solved by displacement compatibility condition of vehicle-subgrade dynamic model.

3.1 Displacement equation of the wheel (connection equation)

According to the assumption mentioned above, while the train is running, the harmonious displacement equation of the wheel's vertical displacement and the dynamic displacement of the track is

$$z_{wij} = z_r(x_{ijl}) + z_s(x_{ijl}), \tag{11}$$

where $z_s(x_{ijl})$ is the track vertical profile irregularity in x_i position of track surface, and is a simulated value.

Combining Eqs.(5) and (11) gives

$$z_{wijl} = \sum_{n=1}^{N} A_n\Phi_n(x_{ijl}) + z_s(x_{ijl}). \tag{12}$$

Combining the right of Eqs.(2), (3) and (12), then the matrix form of the dynamic balance equations of the coach-body and the bogie will be given as

$$\begin{aligned} & \sum_{j=1}^{N_l}\begin{Bmatrix} \sum_{l=1}^{N_k}(K_{1i}z_{wijl} + C_{1i}\dot{z}_{wijl}) \\ \sum_{l=1}^{N_k}\eta_{jl}l_{wi}(K_{1i}z_{wijl} + C_{1i}\dot{z}_{wijl}) \end{Bmatrix} \\ = & \sum_{j=1}^{N_l}\begin{Bmatrix} \sum_{l=1}^{N_k}\left[\sum_{n=1}^{N}\Phi_n(x_{ijl})(K_{1i}A_n + C_{1i}\dot{A}_n) + K_{1i}z_s(x_{ijl}) + C_{1i}\dot{z}_s(x_{ijl})\right] \\ \sum_{l=1}^{N_k}\left[\sum_{n=1}^{N}\Phi_n(x_{ijl})\eta_{jl}l_{wi}(K_{1i}A_n + C_{1i}\dot{A}_n) + \eta_{jl}l_w(K_{1i}z_s(x_{ijl}) + C_{1i}\dot{z}_s(x_{ijl}))\right] \end{Bmatrix}. \end{aligned} \tag{13}$$

3.2 Dynamic equilibrium equation of vehicle-subgrade coupled system under secondary suspension

The dynamic equation of the vehicle and the mode equation of track-subgrade have been mentioned above respectively. As the dynamic property of the vehicle-subgrade coupled system, they must be combined. If we substitute into the displacement connection equation, the dynamic equations of the vertical vehicle-subgrade coupled system under secondary suspension will be obtained:

$$\begin{bmatrix} M_{ci} & 0 \\ 0 & J_{ci} \end{bmatrix} \begin{Bmatrix} \ddot{z}_{ci} \\ \ddot{\varphi}_{ci} \end{Bmatrix} + \begin{bmatrix} C_{czi} & 0 \\ 0 & C_{c\varphi i} \end{bmatrix} \begin{Bmatrix} \dot{z}_{ci} \\ \dot{\varphi}_{ci} \end{Bmatrix} + \begin{bmatrix} K_{czi} & 0 \\ 0 & K_{c\varphi i} \end{bmatrix} \begin{Bmatrix} z_{ci} \\ \varphi_{ci} \end{Bmatrix}$$
$$- \begin{bmatrix} C_{2i} & C_{2i} \\ C_{2i}l_{ci} & -C_{2i}l_{ci} \end{bmatrix} \begin{Bmatrix} \dot{z}_{til} \\ \dot{z}_{ti2} \end{Bmatrix} - \begin{bmatrix} K_{2i} & K_{2i} \\ K_{2i}l_{ci} & -K_{2i}l_{ci} \end{bmatrix} \begin{Bmatrix} z_{til} \\ z_{ti2} \end{Bmatrix} = 0, \tag{14a}$$

$$\begin{bmatrix} M_{tij} & 0 \\ 0 & J_{tij} \end{bmatrix} \begin{Bmatrix} \ddot{z}_{tij} \\ \ddot{\varphi}_{tij} \end{Bmatrix} + \begin{bmatrix} C_{tzij} & 0 \\ 0 & C_{t\varphi ij} \end{bmatrix} \begin{Bmatrix} \dot{z}_{tij} \\ \dot{\varphi}_{tij} \end{Bmatrix} + \begin{bmatrix} K_{tzij} & 0 \\ 0 & K_{t\varphi ij} \end{bmatrix} \begin{Bmatrix} z_{tij} \\ \varphi_{tij} \end{Bmatrix}$$
$$- \begin{bmatrix} C_{2i} & \eta_j C_{2i} l_{ci} \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{z}_{ci} \\ \dot{\varphi}_{ci} \end{Bmatrix} - \begin{bmatrix} K_{2i} & \eta_j K_{2i} l_{ci} \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} z_{ci} \\ \varphi_{ci} \end{Bmatrix}$$
$$= \sum_{l=1}^{N_k} \begin{Bmatrix} \left[\sum_{n=1}^{N} \Phi_n(x_{ijl})(K_{1i}A_n + C_{1i}\dot{A}_n) + K_{1i}z_s(x_{ijl}) + C_{1i}\dot{z}_s(x_{ijl})\right] \\ \left[\sum_{n=1}^{N} \Phi_n(x_{ijl})\eta_{jl}l_{wi}(K_{1i}A_n + C_{1i}\dot{A}_n) + \eta_{jl}l_{wi}(K_{1i}z_s(x_{ijl}) + C_{1i}\dot{z}_s(x_{ijl}))\right] \end{Bmatrix}, \tag{14b}$$

$$M_n\ddot{A}_n + C_n\dot{A}_n + K_nA_n$$
$$= \sum_{i=1}^{N_v}\sum_{j=1}^{N_l}\sum_{l=1}^{N_k} P_{jlw}\Phi_n(x_{ijl}) \cdot \big\{P_{sl} + K_{1i}(z_{tij} + \eta_{jl}l_{wi}\varphi_{tij} - z_{wijl})$$
$$+ C_{1i}(\ddot{z}_{tij} + \eta_{jl}l_{wi}\dot{\varphi}_{tij} - \dot{z}_{wijl}) - m_{wijl}\big[\sum_{n=1}^{N}\ddot{A}_n\Phi_n(x_{ijl}) + \ddot{z}_s(x_{ijl})\big]\big\}. \tag{14c}$$

The total number of equations is $[2 \cdot N_v(1+N_l)+N]$ where N is the number of track-subgrade mode equations. Several representative low vibration modes will be considered, for example, N is 10. The dynamic equations will be solved by Newmark-β method.

3.3 Simulation of vertical irregularity of the track surface

While the vehicles are running along the track, the vibration of vehicle and track will be induced because of track irregularity. Different kinds of track irregularity are produced because of such factors as the deformations of rail structure, joint, weld and subgrade base[21]. They all could be described by random irregularity.

The usual method to measure the vertical irregularity of track is chord measurement[22]. They are often described by SIN or COS function. In this paper, corresponding to different frequencies, the irregularity value is

$$z_t(x) = z_0 \sin(2\pi f_s x), \tag{15}$$

where $z_t(x)$ is the irregularity value at certain positions, z_0 is the amplitude of vibration, f_s is the corresponding frequency.

From this concept and by the maintenance and management goal of irregularity[23], the track surface irregularity can be stimulated as

$$z_s(x) = A_s \sin\left(\frac{2\pi V}{L_s}t + \eta\right), \tag{16}$$

where V is the speed of train, L_s is the wave length corresponding to the management aim, A_s is the management aim of irregularity, η is the random number corresponding to different positions (0∼1.0). Then

$$\dot{z}_s(x_{ijl}) = \frac{2\pi V}{L_s} A_s \cos\left(\frac{2\pi V}{L_s} t + \eta\right), \tag{17}$$

$$\ddot{z}_s(x_{ijl}) = -\frac{4\pi^2 V^2}{L_s^2} \cdot A_s \cdot \sin\left(\frac{2\pi V}{L_s} t + \eta\right). \tag{18}$$

4 Theoretical analysis of vehicle-subgrade

According to the dynamic interaction model of vehicle-subgrade and the dynamic balance equations mentioned above, the following parameters of the vehicle running quality and the subgrade design can be obtained by program using Newmark-β method.

(i) The acceleration of the i-th coach-body,

$$a_i = \left(\ddot{z}_{ci} + \frac{l_p}{l_u} \cdot \ddot{\varphi}_{ci}\right), \tag{19}$$

where l_p is the length apart from the center of coachbody, l_u is the distance between the center of wheel and the center of coachbody, φ_{ci} is the same meaning above.

(ii) The j-th wheels' reduced load of the i-th coach-body,

$$D_j = \frac{p_{jlw} - p_{sl}}{p_{sl}} = \frac{p_{jw}}{p_{sl}} - 1, \tag{20}$$

where p_{jlw} is the wheel's vibrating load while the train running, p_{js} is the average wheel's static load.

(iii) The dynamic displacement or dynamic deflection of track surface in the position $x = x_p$,

$$z_r(x) = \sum_n^N A_n \Phi_n(x)|_{x=x_p}, \tag{21}$$

where A_n is the generalized coordinate (displacement) corresponding to the n-th vibration mode, $\Phi_n(x)$ is the n-th vibration mode function in some track position ($x = x_p$).

5 Conclusions

The meanings and conclusion about establishment of the dynamic model of the vertical vehicle-subgrade coupled system includes:

(i) If track-subgrade is regarded as a part of vibration structure of vehicle-subgrade in dynamic analysis, it will perfectly reflect the dynamic property of vehicle-subgrade.

(ii) For the low speed railway, the problems of vehicle-subgrade system can solved by dispersing them respectively into static, semi-static and dynamic problem of each subsystem. Of course, the dynamic model of vehicle-subgrade system in this paper is a model suitable for high speed railway.

(iii) If track-subgrade is regarded as a part of vibration structure of the vehicle model, it can fully reflect the interactions between the vehicle and the substructure, and provide a basis of theoretical analysis for the determination of the part design parameters (especially for subgrade design parameter).

(iv) Vehicle-subgrade dynamic model considering secondary suspension will give the basis for more exactly analyzing interaction between vehicle and subgrade.

(v) Subgrade design parameter, such as track foundation below (including subgrade stiffness, sleeper spacing, foundation condition, track surface maintenance standard) can be considered in this model to solve the problem of vehicle running quality (including vehicle comfortable degree and safety). It provides an analytical basis to better solve the match between substructure and vehicle system above in track transportation.

Of course, this model also need further research and improvement through more complicated vehicle system, comprehensive coupled system and accurate disperse of structure element of substructure.

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