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DYNAMIC BEHAVIOR OF TWO PARALLEL SYMMETRY CRACKS IN MAGNETO-ELECTRO-ELASTIC COMPOSITES UNDER HARMONIC ANTI-PLANE WAVES *

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Abstract: The dynamic behavior of two parallel symmetry cracks in magneto-electro-elastic composites under harmonic anti-plane shear waves is studied by Schmidt method. By using the Fourier transform, the problem can be solved with a pair of dual integral equations in which the unknown variable is the jumps of the displacements across the crack surfaces. To solve the dual integral equations, the jumps of the displacements across the crack surface were expanded in a series of Jacobi polynomials. The relations among the electric filed, the magnetic flux and the stress field were obtained. From the results, it can be obtained that the singular stresses in piezoelectric/piezomagnetic materials carry the same forms as those in a general elastic material for the dynamic anti-plane shear fracture problem. The shielding effect of two parallel cracks was also discussed.

Key words: magneto-electro-elastic composites; crack; harmonic waves; dual integral equations; intensity factor

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Introduction

Combining two or more distinct piezoelectric and piezomagnetic (magnetostrictive) constituents, piezoelectric/piezoemagnetic composite materials is able to take the advantages of each constituent and consequently have superior coupling magnetoelectric effect as compared to conventional piezoelectric material or piezomagnetic material. The magnetoelectric coupling is a new product property of the composites, since it is absent in each constituent. In some cases, the coupling effect of piezoelectric/piezomagnetic composites can be even obtained a hundred times larger than that in a single-phase magnetoelectric materials. Consequently, they are extensively used as electric packaging, sensors and actuators, *e.g.*, magnetic field probes, acoustic/ultrasonic devices, hydrophones, and transducers with the responsibility of electro-magneto-mechanical energy conversion[1]. When subjected to mechanical, magnetic and electrical loads in service, these magneto-electro-elastic composites can fail prematurely due to some defects, *e.g.* cracks, holes, *etc.* arising during their manufacturing processes. Therefore, it is of great importance to study the magneto-electro-elastic interaction and dynamic fracture behaviors of magneto-electro-elastic composites[2−4]. Liu *et al.*[5] studied the generalized 2D problem of an infinite magnetoelectroelastic plane with an elliptical hole. Gao *et al.*[6] and Wang and Mai[7]

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also studied the fracture problem of the piezoelectric/piezomagnetic composites. The development of piezoelectric-piezomagnetic composites has its roots in the early work of van Suchtelen[8] who proposed that the combination of piezoelectric-piezomagnetic phases may exhibit a new material property-the magnetoelectric coupling effect. Since then, there have not been many researchers studying magnetoelectric coupling effect in BaTiO3-CoFe2O4 composites, and most research results published were obtained in recent years[1−7]. However, relatively litter works have been made for the dynamic fracture analysis duo to the mathematical complexities. To our knowledge, the magneto-electro-elastic dynamic behavior of magneto-electro-elastic composites with two parallel symmetry cracks subjected to harmonic anti-plane shear stress waves has not been studied.

In this paper, the behavior of two parallel symmetry cracks in magneto-electro-elastic composites subjected to harmonic anti-plane shear waves is investigated by use of a somewhat different method, named as the Schmidt method[9]. The Fourier transform is applied and a mixed boundary value problem is reduced to a pair of dual integral equations. To solve the dual integral equations, the jumps of the displacements across the crack surfaces are expanded in a series of Jacobi polynomials. This process is quite different from those adopted in Refs.[2–7] as mentioned above. Numerical solutions are obtained for the stress, the electric displacement and the magnetic flux intensity factors.

1 Formulation of Problem and Solutions of Basic Equations

It is assumed that there are two parallel symmetric cracks of length $2l$ in magneto-electro-elastic composite plane as shown in Fig.1. h is the distance between two parallel cracks. A Cartesian coordinate system(x, y) is positioned as shown in Fig.1. In this paper, the harmonic elastic anti-plane shear stress wave is vertically incident. Let ω be the circular frequency of the incident wave. $-\tau_0$ is a magnitude of the incident wave. In what follows, the time dependence of all field quantities assumed to be of the form $e^{-\mathrm{i}\omega t}$ will be suppressed but understood. The piezoelectric/piezomagnetic boundary-value problem for anti-plane shear is considerably simplified if we consider only the out-of-plane displacement, the in-plane electric and the in-plane magnetic fields. As discussed in Soh's[10] works, since no opening displacement exists for the present anti-plane problem, the crack surfaces can be assumed to be in perfect contact. Accordingly, the electric potential, the magnetic potential, the normal electric displacement and the normal magnetic flux are assumed to be continuous across the crack surfaces. So the boundary conditions of the present problem are (In this paper, we just consider the perturbation fields.)

$$\begin{cases} \tau_{yz}^{(1)}(x,h^+) = \tau_{yz}^{(2)}(x,h^-) = -\tau_0, & |x| \le l, \\ w^{(1)}(x,h^+) = w^{(2)}(x,h^-), & |x| > l, \\ \tau_{yz}^{(2)}(x,0^+) = \tau_{yz}^{(3)}(x,0^-) = -\tau_0, & |x| \le l, \\ w^{(2)}(x,0^+) = w^{(3)}(x,0^-), & |x| > l; \end{cases} \tag{1}$$

$$\begin{cases} \phi^{(1)}(x,h^+) = \phi^{(2)}(x,h^-), \quad D_y^{(1)}(x,h^+) = D_y^{(2)}(x,h^-), & |x| < \infty, \\ \psi^{(1)}(x,h^+) = \psi^{(2)}(x,h^-), \quad B_y^{(1)}(x,h^+) = B_y^{(2)}(x,h^-), & |x| < \infty; \end{cases} \tag{2}$$

$$\begin{cases} \phi^{(2)}(x,0^+) = \phi^{(3)}(x,0^-), \quad D_y^{(2)}(x,0^+) = D_y^{(3)}(x,0^-), & |x| < \infty, \\ \psi^{(2)}(x,0^+) = \psi^{(3)}(x,0^-), \quad B_y^{(2)}(x,0^+) = B_y^{(3)}(x,0^-), & |x| < \infty; \end{cases} \tag{3}$$

$$w^{(1)}(x,y) = w^{(2)}(x,y) = w^{(3)}(x,y) = 0 \qquad \text{for } (x^2+y^2)^{1/2} \to \infty, \tag{4}$$

where $\tau_{zk}^{(i)}, D_k^{(i)}$ and$B_k^{(i)}$ $(k = x, y; i = 1, 2, 3)$ are the anti-plane shear stress, the in-plane electric displacement and the in-plane magnetic flux, respectively. $w^{(i)}, \phi^{(i)}$and $\psi^{(i)}$ the mechanical displacement, the electric potential and the magnetic potential, respectively. Note that all quantities with superscript $i (i = 1, 2, 3)$ refer to the upper half plane 1, the layer 2 and the lower half plane 3 as in Fig.1, respectively. In this paper, we only consider that τ_0 is positive.

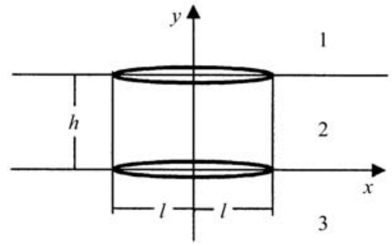

Fig.1 Two parallel symmetry cracks in magneto-electro-elastic composites

It is assumed that the magneto-electro-elastic composite is transversely isotropic. So the constitutive equations for the mode III crack in the magneto-electro-elastic composite can be expressed as

$$\tau_{zk}^{(i)} = c_{44} w_{,k}^{(i)} + e_{15} \phi_{,k}^{(i)} + q_{15} \psi_{,k}^{(i)}, \qquad k = x, y; \quad i = 1, 2, 3, \tag{5}$$

$$D_k^{(i)} = e_{15} w_{,k}^{(i)} - \varepsilon_{11} \phi_{,k}^{(i)} - d_{11} \psi_{,k}^{(i)}, \qquad k = x, y; \quad i = 1, 2, 3, \tag{6}$$

$$B_k^{(i)} = q_{15} w_{,k}^{(i)} - d_{11} \phi_{,k}^{(i)} - \mu_{11} \psi_{,k}^{(i)}, \qquad k = x, y; \quad i = 1, 2, 3, \tag{7}$$

where c_{44} is the shear modulus, e_{15} is the piezoelectric coefficient, ϵ_{11} is the dielectric parameter, q_{15} is the piezomagnetic coefficient, d_{11} is the electromagnetic coefficient, μ_{11} is the magnetic permeability.

The anti-plane governing equations are

$$c_{44} \nabla^2 w^{(i)} + e_{15} \nabla^2 \phi^{(i)} + q_{15} \nabla^2 \psi^{(i)} = \rho \frac{\partial^2 w}{\partial t^2}, \qquad i = 1, 2, 3, \tag{8}$$

$$e_{15} \nabla^2 w^{(i)} - \varepsilon_{11} \nabla^2 \phi^{(i)} - d_{11} \nabla^2 \psi^{(i)} = 0, \qquad i = 1, 2, 3, \tag{9}$$

$$q_{15} \nabla^2 w^{(i)} - d_{11} \nabla^2 \phi^{(i)} - \mu_{11} \nabla^2 \psi^{(i)} = 0, \qquad i = 1, 2, 3, \tag{10}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplace operator. ρ is the density of the piezoelectric/piezomagnetic materials. Because of the assumed symmetry in geometry and loading, it is sufficient to consider only the problem for $0 \le x \le \infty$, $-\infty \le y \le \infty$. A Fourier transform is applied to Eqs.(8)–(10). It is assumed that the solutions are

$$\begin{cases} w^{(1)}(x,y) = \dfrac{2}{\pi} \displaystyle\int_0^\infty A_1(s) \mathrm{e}^{-\gamma y} \cos(sx) ds, \\ \phi^{(1)}(x,y) = \dfrac{a_1}{a_0} w^{(1)}(x,y) + \dfrac{2}{\pi} \displaystyle\int_0^\infty B_1(s) \mathrm{e}^{-sy} \cos(sx) ds, \qquad y \ge h, \\ \psi^{(1)}(x,y) = \dfrac{a_2}{a_0} w^{(1)}(x,y) + \dfrac{2}{\pi} \displaystyle\int_0^\infty C_1(s) \mathrm{e}^{-sy} \cos(sx) ds; \end{cases} \tag{11}$$

$$\begin{cases} w^{(2)}(x,y) = \dfrac{2}{\pi} \displaystyle\int_0^\infty [A_2(s) \mathrm{e}^{-\gamma y} + B_2(s) \mathrm{e}^{\gamma y}] \cos(sx) ds, \\ \phi^{(2)}(x,y) = \dfrac{a_1}{a_0} w^{(2)}(x,y) + \dfrac{2}{\pi} \displaystyle\int_0^\infty [C_2(s) \mathrm{e}^{-sy} + D_2(s) \mathrm{e}^{sy}] \cos(sx) ds, \qquad 0 \le y \le h, \\ \psi^{(3)}(x,y) = \dfrac{a_2}{a_0} w^{(2)}(x,y) + \dfrac{2}{\pi} \displaystyle\int_0^\infty [E_2(s) \mathrm{e}^{-sy} + F_2(s) \mathrm{e}^{sy}] \cos(sx) ds; \end{cases} \tag{12}$$

$$\begin{cases} w^{(3)}(x,y) = \dfrac{2}{\pi}\displaystyle\int_0^\infty A_3(s)\mathrm{e}^{\gamma y}\cos(sx)ds, \\ \phi^{(3)}(x,y) = \dfrac{a_1}{a_0}w^{(3)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty B_3(s)\mathrm{e}^{sy}\cos(sx)ds, \qquad y \le 0, \\ \psi^{(3)}(x,y) = \dfrac{a_2}{a_0}w^{(3)}(x,y) + \dfrac{2}{\pi}\displaystyle\int_0^\infty C_3(s)\mathrm{e}^{sy}\cos(sx)ds, \end{cases} \tag{13}$$

where $A_1(s)$,$B_1(s)$,$C_1(s)$,$A_2(s)$,$B_2(s)$,$C_2(s)$,$D_2(s)$,$E_2(s)$, $F_2(s)$,$A_3(s)$,$B_3(s)$ and $C_3(s)$ are unknown functions. $\gamma^2 = s^2 - \omega^2/c^2$, $c^2 = \mu/\rho$, $\mu = c_{44} + \frac{a_1 e_{15}}{a_0} + \frac{a_2 q_{15}}{a_0}$, $a_0 = \varepsilon_{11}\mu_{11} - d_{11}^2$, $a_1 = \mu_{11}e_{15} - d_{11}q_{15}$, $a_2 = q_{15}\varepsilon_{11} - d_{11}e_{15}$.

So from Eqs.(5)–(7), we have

$$\tau_{yz}^{(1)}(x,y) = -\frac{2}{\pi}\int_0^\infty \{\gamma\mu A_1(s)\mathrm{e}^{-\gamma y} + s[e_{15}B_1(s) + q_{15}C_1(s)]\mathrm{e}^{-sy}\}\cos(sx)ds, \tag{14}$$

$$D_y^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty s[\varepsilon_{11}B_1(s) + d_{11}C_1(s)]\mathrm{e}^{-sy}\cos(sx)ds, \tag{15}$$

$$B_y^{(1)}(x,y) = \frac{2}{\pi}\int_0^\infty s[d_{11}B_1(s) + \mu_{11}C_1(s)]e^{-sy}\cos(sx)ds, \tag{16}$$

$$\begin{aligned} \tau_{yz}^{(2)}(x,y) = -\frac{2}{\pi}\int_0^\infty &\{\gamma\mu A_2(s)\mathrm{e}^{-\gamma y} + s[e_{15}C_2(s) + q_{15}E_2(s)]\mathrm{e}^{-sy} \\ &- \gamma\mu B_2(s)\mathrm{e}^{\gamma y} - s[e_{15}D_2(s) + q_{15}F_2(s)]\mathrm{e}^{sy}\}\cos(sx)ds, \end{aligned} \tag{17}$$

$$D_y^{(2)}(x,y) = \frac{2}{\pi}\int_0^\infty s\{[\varepsilon_{11}C_2(s) + d_{11}E_2(s)]\mathrm{e}^{-sy} - [\varepsilon_{11}D_2(s) + d_{11}F_2(s)]\mathrm{e}^{sy}\}\cos(sx)ds, \tag{18}$$

$$B_y^{(2)}(x,y) = \frac{2}{\pi}\int_0^\infty s\{[d_{11}C_2(s) + \mu_{11}E_2(s)]\mathrm{e}^{-sy} - [d_{11}D_2(s) + \mu_{11}F_2(s)]\mathrm{e}^{sy}\}\cos(sx)ds, \tag{19}$$

$$\tau_{yz}^{(3)}(x,y) = \frac{2}{\pi}\int_0^\infty \{\gamma\mu A_3(s)\mathrm{e}^{\gamma y} + s[e_{15}B_3(s) + q_{15}C_3(s)]\mathrm{e}^{sy}\}\cos(sx)ds, \tag{20}$$

$$D_y^{(3)}(x,y) = -\frac{2}{\pi}\int_0^\infty s[\varepsilon_{11}B_3(s) + d_{11}C_3(s)]\mathrm{e}^{sy}\cos(sx)ds, \tag{21}$$

$$B_y^{(3)}(x,y) = -\frac{2}{\pi}\int_0^\infty s[d_{11}B_3(s) + \mu_{11}C_3(s)]\mathrm{e}^{sy}\cos(sx)ds. \tag{22}$$

To solve the problem, the jumps of the displacements across the crack surfaces are defined as follows:

$$f_1(x) = w^{(1)}(x,h^+) - w^{(2)}(x,h^-), \tag{23}$$

$$f_2(x) = w^{(2)}(x,0^+) - w^{(3)}(x,0^-). \tag{24}$$

Substituting Eqs.(11)–(13) into Eqs.(23)–(24), and applying the Fourier transform and the boundary conditions, it can be obtained

$$A_1(s)\mathrm{e}^{-\gamma h} - A_2(s)\mathrm{e}^{-\gamma h} - B_2(s)\mathrm{e}^{\gamma h} = \bar{f}_1(s), \quad A_2(s) + B_2(s) - A_3(s) = \bar{f}_2(s), \tag{25}$$

$$\frac{a_1}{a_0}[A_1(s)\mathrm{e}^{-\gamma h} - A_2(s)\mathrm{e}^{-\gamma h} - B_2(s)\mathrm{e}^{\gamma h}] + B_1(s)\mathrm{e}^{-sh} - C_2(s)\mathrm{e}^{-sh} - D_2(s)\mathrm{e}^{sh} = 0, \tag{26}$$

$$\frac{a_1}{a_0}[A_2(s) + B_2(s) - A_3(s)] + C_2(s) + D_2(s) - B_3(s) = 0, \tag{27}$$

$$\frac{a_2}{a_0}[A_1(s)\mathrm{e}^{-\gamma h} - A_2(s)\mathrm{e}^{-\gamma h} - B_2(s)\mathrm{e}^{\gamma h}] + C_1(s)\mathrm{e}^{-sh} - E_2(s)\mathrm{e}^{-sh} - F_2(s)\mathrm{e}^{sh} = 0, \tag{28}$$

$$\frac{a_2}{a_0}[A_2(s) + B_2(s) - A_3(s)] + E_2(s) + F_2(s) - C_3(s) = 0. \tag{29}$$

A superposed bar indicates the Fourier transform throughout the paper. Substituting Eqs.(14)–(22) into Eqs.(1)–(3), it can be obtained

$$\gamma\mu A_1(s)\mathrm{e}^{-\gamma h} + s[e_{15}B_1(s) + q_{15}C_1(s)]\mathrm{e}^{-sh} - \gamma\mu A_2(s)\mathrm{e}^{-\gamma h} - s[e_{15}C_2(s) + q_{15}E_2(s)]\mathrm{e}^{-sh} + \gamma\mu B_2(s)\mathrm{e}^{\gamma y} + s[e_{15}D_2(s) + q_{15}F_2(s)]\mathrm{e}^{sh} = 0, \tag{30}$$

$$\gamma\mu A_2(s) + s[e_{15}C_2(s) + q_{15}E_2(s)] - \gamma\mu B_2(s) - s[e_{15}D_2(s) + q_{15}F_2(s)] + \gamma\mu A_3(s) + s[e_{15}B_3(s) + q_{15}C_3(s)] = 0, \tag{31}$$

$$-[\varepsilon_{11}B_1(s) + d_{11}C_1(s)]\mathrm{e}^{-sh} + [\varepsilon_{11}C_2(s) + d_{11}E_2(s)]\mathrm{e}^{-sh} - [\varepsilon_{11}D_2(s) + d_{11}F_2(s)]\mathrm{e}^{sh} = 0, \tag{32}$$

$$-\varepsilon_{11}C_2(s) - d_{11}E_2(s) + \varepsilon_{11}D_2(s) + d_{11}F_2(s) - \varepsilon_{11}B_3(s) - d_{11}C_3(s) = 0, \tag{33}$$

$$-[d_{11}B_1(s) + \mu_{11}C_1(s)]\mathrm{e}^{-sh} + [d_{11}C_2(s) + \mu_{11}E_2(s)]\mathrm{e}^{-sh} - [d_{11}D_2(s) + \mu_{11}F_2(s)]\mathrm{e}^{sh} = 0, \tag{34}$$

$$-d_{11}C_2(s) - \mu_{11}E_2(s) + d_{11}D_2(s) + \mu_{11}F_2(s) - d_{11}B_3(s) - \mu_{11}C_3(s) = 0. \tag{35}$$

By solving twelve Eqs.(25)–(35) with twelve unknown functions $A_1(s)$, $B_1(s)$, $C_1(s)$, $A_2(s)$, $B_2(s)$, $C_2(s)$, $D_2(s)$, $E_2(s)$, $F_2(s)$, $A_3(s)$, $B_3(s)$ and $C_3(s)$ and applying the boundary condition (1) to the results, it can be obtained

$$\frac{2}{\pi}\int_0^\infty \bar{f}_1(s)\cos(sx)ds = 0, \quad \frac{2}{\pi}\int_0^\infty \bar{f}_2(s)\cos(sx)ds = 0, \quad |x| > l, \tag{36}$$

$$\frac{1}{\pi}\int_0^\infty [g_1(s)\bar{f}_1(s) + g_2(s)\bar{f}_2(s)]\cos(sx)ds = -\tau_0, \quad |x| \le l, \tag{37}$$

$$\frac{1}{\pi}\int_0^\infty s[g_2(s)\bar{f}_1(s) + g_1(s)\bar{f}_2(s)]\cos(sx)ds = -\tau_0, \quad |x| \le l. \tag{38}$$

From Eqs.(36)–(38), it can be obtained

$$\bar{f}_1(s) = \bar{f}_2(s) \Rightarrow f_1(x) = f_2(x), \quad \tau_{yz}^{(1)}(x,h) = \tau_{yz}^{(2)}(x,h) = \tau_{yz}^{(2)}(x,0) = \tau_{yz}^{(3)}(x,0), \tag{39}$$

$$D_y^{(1)}(x,h) = D_y^{(2)}(x,h) = D_y^{(2)}(x,0) = D_y^{(3)}(x,0), \tag{40}$$

$$B_y^{(1)}(x,h) = B_y^{(2)}(x,h) = B_y^{(2)}(x,0) = B_y^{(3)}(x,0), \tag{41}$$

where $g_1(s) = -c_{44}\gamma - (\gamma - s)\frac{e_{15}a_1 + q_{15}a_2}{a_0}$ and $g_2(s) = -c_{44}\gamma\mathrm{e}^{-\gamma h} - (\gamma\mathrm{e}^{-\gamma h} - s\mathrm{e}^{-sh})\frac{e_{15}a_1 + q_{15}a_2}{a_0}$. $\lim_{s\to\infty} g_1(s)/s = -c_{44}$ and $\lim_{s\to\infty} g_2(s)/s = 0$. To determine the unknown functions $\bar{f}_1(s)$ and $\bar{f}_2(s)$, the above two pairs of dual integral equations (36)–(38) must be solved.

2 Solution of Dual Integral Equations

From the natural property of the displacement along the crack line, it can be obtained that the jump of the displacements across the crack surface is a finite, continuous and differentiable function. Hence, the jump of the displacements across the crack surfaces can be represented by the following series:

$$f_1(x) = f_2(x) = \sum_{n=1}^{\infty} b_n P_{2n-2}^{(\frac{1}{2},\frac{1}{2})}(\frac{x}{l})(1 - \frac{x^2}{l^2})^{\frac{1}{2}}, \quad 0 \le x \le l,\ y = 0, \tag{42}$$

$$f_1(x) = f_2(x) = w^{(1)}(x,h^+) - w^{(2)}(x,h^-) = 0, \quad 0 \le x \le l,\ y = 0, \tag{43}$$

where b_n are unknown coefficients to be determined and $P_n^{(1/2,1/2)}$ is a Jacobi polynomial[11]. The Fourier transform of Eqs.(42)–(43) is[12]

$$\bar{f}_1(s)=\sum_{n=1}^{\infty} b_n G_n \frac{1}{s}\mathrm{J}_{2n-1}(sl), \quad G_n=2\sqrt{\pi}(-1)^{n-1}\frac{\Gamma(2n-\frac{1}{2})}{(2n-2)!}, \tag{44}$$

where $\Gamma(x)$ and $\mathrm{J}_n(x)$ are the Gamma and the Bessel functions, respectively.

Substituting Eq.(44) into Eqs.(36)–(38), respectively. It can be shown that Eqs.(36) are automatically satisfied. After integration with respect to x in $[0,x]$, Eqs.(37)–(38) reduce to

$$\frac{1}{\pi}\sum_{n=1}^{\infty} b_n G_n \int_0^{\infty} \frac{1}{s}[\frac{g_1(s)+g_2(s)}{s}]\mathrm{J}_{2n-1}(sl)\sin(sx)ds=-\tau_0 x. \tag{45}$$

Equations (45) can now be solved for the coefficients b_n by the Schmidt method[9]. It can be seen in Refs.[13,14]. Here, it is omitted.

3 Intensity Factors

The coefficients b_n are known, so that the entire perturbation stress field, the perturbation electric displacement and the magnetic flux can be obtained. However, in fracture mechanics, it is of importance to determine the perturbation stress τ_{yz} and the perturbation electric displacement D_y in the vicinity of the crack tips. In the case of the present study, $\tau_{yz}^{(1)}, \tau_{yz}^{(2)}, \tau_{yz}^{(3)}$, $D_y^{(1)}, D_y^{(2)}, D_y^{(3)}, B_y^{(1)}, B_y^{(2)}$ and $B_y^{(3)}$ along the crack line can be expressed, respectively, as

$$\begin{aligned}\tau_{yz}^{(1)}(x,h)&=\tau_{yz}^{(2)}(x,h)=\tau_{yz}^{(2)}(x,0)=\tau_{yz}^{(3)}(x,0)=\tau_{yz}\\&=\frac{1}{\pi}\sum_{n=1}^{\infty} b_n G_n \int_0^{\infty}[\frac{g_1(s)+g_2(s)}{s}]\mathrm{J}_{2n-1}(sl)\cos(xs)ds,\end{aligned} \tag{46}$$

$$\begin{aligned}D_y^{(1)}(x,h)&=D_y^{(2)}(x,h)=D_y^{(2)}(x,0)=D_y^{(3)}(x,0)=D_y\\&=-\frac{e_{15}}{\pi}\sum_{n=1}^{\infty} b_n G_n \int_0^{\infty}[1+\mathrm{e}^{-sh}]\mathrm{J}_{2n-1}(sl)\cos(xs)ds,\end{aligned} \tag{47}$$

$$\begin{aligned}B_y^{(1)}(x,h)&=B_y^{(2)}(x,h)=B_y^{(2)}(x,0)=B_y^{(3)}(x,0)=B_y\\&=-\frac{q_{15}}{\pi}\sum_{n=1}^{\infty} b_n G_n \int_0^{\infty}[1+\mathrm{e}^{-sh}]\mathrm{J}_{2n-1}(sl)\cos(xs)ds.\end{aligned} \tag{48}$$

By examining Eqs.(46)–(48), the singular parts of the stress field, the electric displacement and the magnetic flux can be expressed respectively as follows ($l<x$):

$$\tau=-\frac{c_{44}}{\pi}\sum_{n=1}^{\infty} b_n G_n H_n(x),\ D=-\frac{e_{15}}{\pi}\sum_{n=1}^{\infty} b_n G_n H_n(x),\ B=-\frac{q_{15}}{\pi}\sum_{n=1}^{\infty} b_n G_n H_n(x), \tag{49}$$

where $H_n(x)=\frac{(-1)^{n-1}l^{2n-1}}{\sqrt{x^2-l^2}[x+\sqrt{x^2-l^2}]^{2n-1}}$.

We obtain the stress intensity factor K as

$$K=\lim_{x\to l^+}\sqrt{2(x-l)}\cdot\tau=-\frac{2c_{44}}{\sqrt{\pi l}}\sum_{n=1}^{\infty} b_n \frac{\Gamma(2n-\frac{1}{2})}{(2n-2)!}. \tag{50}$$

We obtain the electric displacement intensity factor K^{D} as

$$K^{\mathrm{D}} = \lim_{x\to l^+} \sqrt{2(x-l)} \cdot D = -\frac{2e_{15}}{\sqrt{\pi l}} \sum_{n=1}^{\infty} b_n \frac{\Gamma(2n-\frac{1}{2})}{(2n-2)!} = \frac{e_{15}}{c_{44}} K. \tag{51}$$

We obtain the magnetic flux intensity factor K^{B} as

$$K^{\mathrm{B}} = \lim_{x\to l^+} \sqrt{2(x-l)} \cdot B = -\frac{2q_{15}}{\sqrt{\pi l}} \sum_{n=1}^{\infty} b_n \frac{\Gamma(2n-\frac{1}{2})}{(2n-2)!} = \frac{q_{15}}{c_{44}} K. \tag{52}$$

4 Numerical Calculations and Discussion

From Refs.[13–14], it can be seen that the Schmidt method performs satisfactorily if the first ten terms of the infinite series (45) are retained. The precision of present solution can satisfy the demands of the practical problem. At $-l \le x \le l, y = 0$, it can be obtained that $\tau_{yz}^{(1)}/\tau_0$ is very close to negative unity. Hence, the solution of present paper can also be proved to satisfactory the boundary condition (1). The materials are assumed to be that $c_{44} = 44.0$ GPa, $e_{15} = 5.8$ C/m^2, $\varepsilon_{11} = 5.64 \times 10^{-9}$ C^2/(N $\cdot$ m^2), $q_{15} = 275.0$ N/(A $\cdot$ m), $d_{11} = 0.005 \times 10^{-9}$ Ns/(V $\cdot$ C), $\mu_{11} = -297.0 \times 10^{-6}$ Ns^2/C^2, $\rho = 1500$ kg/m^3. The numerical results of the present paper are shown in Figs.2–6.

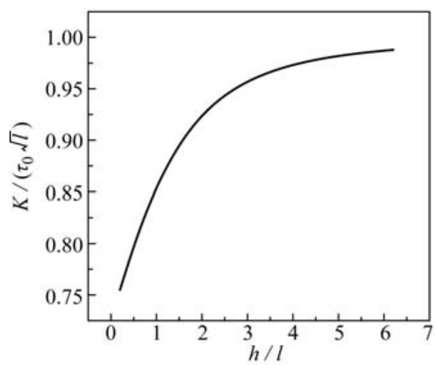

Fig.2 The stress intensity factor *versus* h/l for $\omega l/c = 0$

From the results, the following observations are very significant:

(i) The dynamic stress, the electric displacement and the magnetic flux intensity factors not only depend on the crack length, the distance between two parallel cracks, the wave velocity, the circular frequency of the incident waves, but also on the properties of the materials. From the results, it can be shown that the singular dynamic stress in piezoelectric/piezomagnetic materials carries the same forms as those in the general elastic materials. The electro-magneto-elastic coupling effects can be obtained as shown in Eqs.(50)–(52).

(ii) The interaction of two parallel symmetry cracks increase when the distance between two parallel symmetry cracks increases as shown in Figs.2–5. This phenomenon is called crack shielding effect as discussed in Ratwani's paper[15]. For the electric displacement and the magnetic flux intensity factors, they have the same changing rule as the stress intensity factor as shown in Figs.3–5. However, the amplitude values of $K/(\tau_0\sqrt{l}), K^{\mathrm{D}}/(\tau_0\sqrt{l})$ and $K^{\mathrm{B}}/(\tau_0\sqrt{l})$ are different from each others. The amplitude values of $K^{\mathrm{D}}/(\tau_0\sqrt{l})$ and $K^{\mathrm{B}}/(\tau_0\sqrt{l})$ are very small as shown in Figs.4–5.

(iii) The stress intensity factors tend to increase with increase in the circular frequency of the incident waves, until reaching a maximum at $\omega l/c = 0.6$, then it decreases in magnitude as shown in Fig.6. However, the stress intensity factors tend to increase again with increase in the circular frequency of the incident waves for $\omega l/c > 2.0$. This may be cased by electro-magneto-elastic coupling effects and the high circular frequency of the incident waves.

(iv) The solution of this problem can be returned to the static solution for $\omega l/c = 0$. From the results, it can be shown that the stress intensity factor tends to a unit for $h/l > 5.5$ when $\omega l/c = 0$ as shown in Fig.2. This is consistent with the fracture problem in the general elastic materials for the anti-plane shear fracture problem.

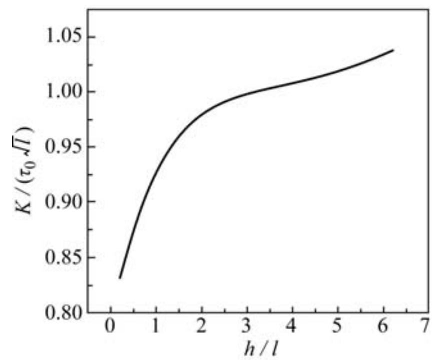

Fig.3 The stress intensity factor *versus* h/l for $\omega l/c = 0.4$

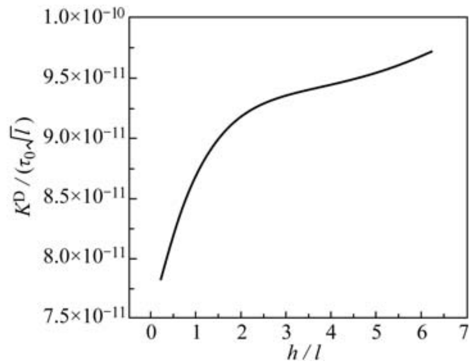

Fig.4 The electric displacement intensity factor *versus* h/l for $\omega l/c = 0.4$

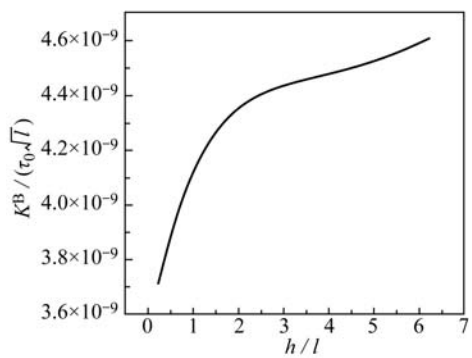

Fig.5 The magnetic flux intensity factor *versus* h/l for $\omega l/c = 0.4$

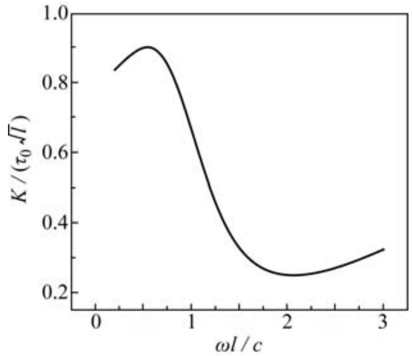

Fig.6 The stress intensity factor *versus* $\omega l/c$ for $h/l = 0.5$

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