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ORIGINAL RESEARCH

An updated survey of attended home delivery and service problems with a focus on applications

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Abstract
The research field of Attended Home Delivery (AHD) and Attended Home Service (AHS) problems has experienced fast growing interest in the last two decades, with the rapid growth of online platforms and e-commerce transactions. The radical changes in consumer lifestyles and habits as well as the COVID-19 pandemic contingency have reinforced that interest, raising further challenges and opportunities that need to be addressed by innovative methodologies and policies. The aim of this work is to provide an extensive literature review on the state of the art for AHD and AHS problems, with a particular focus on real-world applications. A discussion of promising future research directions is also provided.

Keywords Attended home delivery · Attended home service · Demand management · Routing · Integrated demand management and routing

1 Introduction

Attended Home Delivery (AHD) and Attended Home Service (AHS) are last-mile operations where the customer must be present at home for the delivery of goods, the execution of a service or, in some cases, both the delivery of goods and the execution of an additional service (Agatz et al., 2008a; Ehmke, 2012). Examples of AHD and AHS are, among others, the delivery of groceries directly at home, the delivery and installation of large furniture and appliances, or the provision of home healthcare therapies. By definition, they differ from

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Unattended Home Delivery (UHD) and Unattended Home Service (UHS), which are last-mile operations that always involve the delivery of goods or the execution of a service, but can be fulfilled without the customer being present at home. Examples of UHD are the delivery of parcels right in front of the door, inside a nearby parcel locker (Buzzega & Novellani, 2023), or at a different pickup point (Galiullina et al., 2024); an example of AHS is the reading of a meter installed outside a house. In this work, we limit the research area to those operations that are *attended* by the customers and performed by a physical person. For a detailed review on last-mile delivery concepts and recent trends, such as the use of drones and autonomous delivery robots or crowdshipping, we refer the interested reader to Boysen et al. (2021). A particularly recent trend that is worth mentioning is that of crowdkeeping (see, e.g., Wang et al. 2024 for an introductory work), in which the so-called "crowd keepers" voluntarily attend the delivery of parcels on behalf of the customers and then transfer them to the customers on behalf of the delivery company. Another class of problems that shares some similarities with AHD and AHS and which we only briefly mention in this work is that of Same-Day Delivery (SDD). An overview of SDD problems, where the delivery of goods or the execution of a service must be fulfilled within the day of service and the requests arrive dynamically during the operational horizon, is given by Voccia et al. (2019).

AHD problems originated in the context of e-grocery (i.e., the collection of processes lying behind the purchase of groceries online; see, e.g., Punakivi & Saranen 2001 and Lin & Mahmassani 2002 for seminal ideas) and, more generally, e-fulfillment (i.e., the collection of processes lying behind the purchase of physical goods online; see, e.g., Agatz et al. 2008b for an in-depth introductory review). Since the first definition found in the work by Campbell and Savelsbergh (2006), they have seen a continuous increase not only in terms of interest in the research community, but also in terms of importance in many business sectors. The COVID-19 pandemic has just fostered the demand for AHD services, as confirmed by the OECD (2020). In particular, during the first and second quarters of 2020 online retail sales have registered a worldwide increase of 14.8 to 16% in the United States and 30% in the 27 member countries of the European Union, with a similar trend in the Asia-Pacific countries. How long this growth will last and whether we will ever return to the pre-pandemic levels is still matter for debate (Wang et al., 2021). In the meantime AHD has already triggered irreversible changes in the logistics of our cities (Subramanian, 2019), and new trends emerging in large metropolitan areas are posing further challenges (Kushner & Greg, 2021). Among these trends, we mention the delivery of building materials to contractors directly on site and the recent phenomenon of ultra-fast delivery of groceries in as little as 15 min. A further indication that AHD and AHS problems are drawing increasing attention is represented by an analysis that we performed on Scopus and whose results are reported in Fig. 1. We looked at the number of documents per year where the entries "attended home delivery", "attended home service", "attended home deliveries", or "attended home services" appeared between 2006 and 2023. The results show a slightly yet constantly growing trend between 2006 and 2017, followed by a notable increase between 2017 and 2023.

As mentioned before, AHD problems are directly linked to the growth of the *e-grocery* business model, where a fierce competition has arisen around the logistical challenges offered by this particular sector, like the perishability of goods, the unpredictability of demand, the narrow time windows made available to customers for the delivery, and the low profit margins. Even more challenging is the practice of *meal delivery*, which has become increasingly popular in the last years. Another sector that is commonly associated with AHD is the *online retail* of so-called "dry" goods, where the perishability is not an issue, but the parcels may be fragile and require a careful handling, the demand volume can be very high and unpredictable, the goods need to be moved rapidly along the supply chain, and, lastly, the customer might not

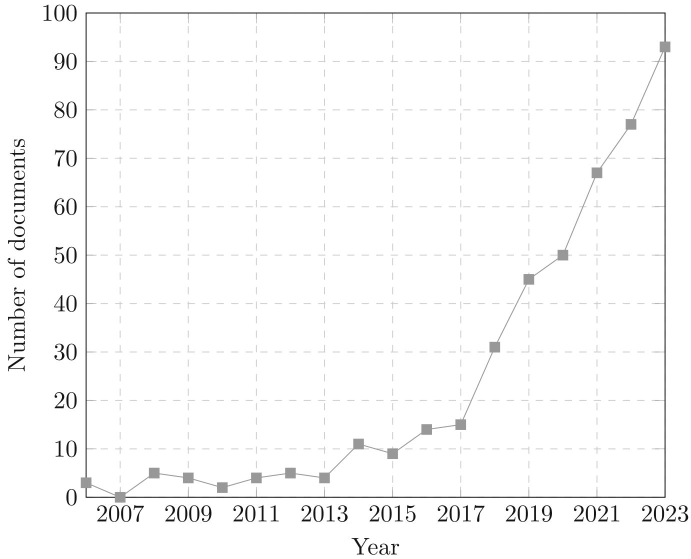

Fig. 1 Documents per year on AHD and AHS published between 2006 and 2023

be at home during the delivery, thus causing additional routing costs and further congestion in city road networks. More traditional sectors are those of *large appliances and furniture*, which usually combine the delivery of goods with the additional installation service. In this sense, we can situate them at the intersection of AHD and AHS problems. Typically, these operations might require a careful handling due to the fragility of some appliances and furniture, but they usually benefit from a larger planning horizon.

The field of AHS itself has received less attention from the research community compared with AHD, but still includes some essential activities like *home healthcare services*, that are important not only to efficiently manage the capability of hospitals but especially to guarantee high-quality therapies to patients who cannot move from home. In this context, we should distinguish between *ordinary* and *extraordinary* care services. The first can be planned over a larger planning horizon, while the latter deal with emergencies and must provide an immediate response. This leads to different problems from an operational research perspective. AHS problems typically arise also in the context of *utilities* (e.g., electricity, gas and water distribution companies, internet and telecommunications service providers, and so forth), where companies might be required by local authorities (see, e.g., Bruck et al. 2018, 2020) to give customers the opportunity to book their installation or maintenance services within publicly available time slots. As for home care services, we should distinguish these ordinary booking activities from extraordinary ones (e.g., a gas leakage) that require an immediate response. So far, we have mentioned only business-to-consumer sectors, but many observations also hold in a business-to-business environment. Indeed, *on-site maintenance and repair services* present similar characteristics to many AHS operations, including the distinction between ordinary and extraordinary services. We refer to the recent survey by Cordeau et al. (2023) for a detailed review on AHD and AHS problems with a focus on applications, and to Vezzali (2024) for the development of Decision Support Systems (DSS) for specific AHS real-world applications.

Addressing real-world AHD and AHS problems is challenging, as it typically implies solving a multi-stage problem: firstly, a *demand management problem*, and consequently, a *routing problem*, where the decisions taken in the previous stage can greatly affect the feasibility as well as the economic profitability of the following decisions.

As described in the recent surveys by Nguyen et al. (2018) and Waßmuth et al. (2023), on the *demand side* companies must be able to find effective ways to efficiently leverage the demand of customers by putting into action Revenue Management (RM) principles. Initially borrowed from the airline industry, the practice of RM has become increasingly popular for AHD and AHS problems. Examples of RM decisions in the context of AHD and AHS problems might regard the basic offering and pricing of time slots, their length, the choice of overlapping versus non-overlapping time slots, or the capacity allocated to each of them. These are typically static decisions. More complex decisions are required in a dynamic environment, where a company might be willing to frequently adjust the offering and pricing of time slots, or increase/decrease the capacity allocated based on the actual demand of customers. The complexity of these decisions is also affected by the immediate responsiveness they typically require.

On the *supply side*, companies seek to limit the operational costs by applying traditional routing techniques, which have been widely studied in the Vehicle Routing Problem (VRP) literature. The degree of complexity of these techniques is affected by the decisions taken at the demand management stage, and by the possible inclusion of stochastic and dynamic routing aspects. In addition, AHD and AHS problems require considerable “back-end” activities in terms of inventory management and order assembly, which are out of scope of this work.

Finally, a meet-in-the-middle approach that is worth considering is to integrate demand management and vehicle routing, as discussed in the recent survey by Fleckenstein et al. (2023). Such an integration requires the anticipation of some routing aspects at the demand management stage, which is complex since the VRP is NP-hard.

AHD and AHS problems can also be classified according to the planning horizon of the decisions that must be taken. Long-term decisions typically dealing with the setup of business (i.e., with lasting effects from months to years), like the opening of new facilities or the creation of demand clusters given an extended geographical area, are taken at a *strategic* level. Medium-term decisions typically dealing with the sizing of business (i.e., with lasting effects from weeks to months), like the design of basic model-weeks for each demand cluster or the allocation of capacity to each single time slot, are taken at a *tactical* level. Finally, short-term decisions typically dealing with the management of business (i.e., with lasting effects of a few hours to a few days), like the dynamic adjustment of the basic time slot offering and pricing or the definition of detailed routing plans for the delivery of goods or the execution of services, are taken at an *operational* level.

Our work makes a number of contributions, namely:

- it extensively reviews the academic literature by distinguishing for the first time between AHD and AHS problems;
- it identifies three classes of problems depending on the extent of the integration between the demand management and the routing stages;
- it looks at these relevant classes of problems through the lens of real-world applications, with the aim of highlighting the main managerial leverages to set up and maintain a profitable business;
- it underlines the most significant future research directions in the AHD and AHS research field.

The remainder of the paper is organized as follows. Mathematical models and solution methods for demand management, routing, and integrated demand management and routing problems in AHD and AHS are reviewed in Sects. 2, 3 and 4, respectively. Then, in Sect. 5 we draw some conclusions on the state of the art of AHD and AHS problems and discuss possible future research directions.

2 Demand management problems in AHD and AHS

The practice of Demand Management (DM) refers to those structural, price and quantity decisions that need to be taken in a business context. Similarly with the previously mentioned RM, DM has its origin in the early 1980 s, when Robert Crandall, then American Airline's vice president of marketing, introduced the first principles of DM in the airline industry (Talluri & Van Ryzin, 2004). Since then, other industries adopted (and adapted) DM techniques, in sectors such as hospitality, transportation, and energy. As explained by Talluri and Van Ryzin (2004), all of these industries share similar conditions that motivate the adoption of DM: *customer heterogeneity*, *demand variability and uncertainty*, *production inflexibility*, *data and information system infrastructure*, and *management culture*. Many of these conditions may well be found in AHD and AHS systems, which probably explains why in recent years the practice of DM has become common in this industry.

A widely accepted classification of demand management decisions in AHD and AHS is the one proposed by Agatz et al. (2013). On one dimension, the authors distinguish between *slotting* and *pricing* decisions, that deal with the proposal of time slots to customers and the definition of prices for each time slot, respectively. On the other dimension, they distinguish between *differentiated* (or *static*) and *dynamic* decisions, where the former are taken off-line and are usually based on forecasts, while the latter are taken in real time.

The main difference between DM in traditional industries, where costs are generally supposed to be fixed, and DM in AHD and AHS, is that decisions taken at this level greatly affect the resulting routing costs. Therefore, even at an early stage, it is necessary to seek a trade-off between revenue maximization and cost balance, which is not trivial.

In this section, we review several demand management models proposed in the literature on AHD and AHS problems, where a routing part may be considered but is not the core of the research. An overview of the main characteristics of the reviewed articles is provided in Table 1. A particular emphasis is put on real-world applications. In addition, we highlight that column "Cost Estimation" includes both rather simple methods, used to compute the additional routing cost while accepting an incoming request, and more sophisticated methods, used to estimate the opportunity cost of accepting an incoming request and foregoing a potentially more profitable future request.

For a more detailed study on DM/RM, we refer the interested reader to the reviews by Strauss et al. (2018) and Klein et al. (2020), where in the latter a specific section is dedicated to innovative applications of RM in AHD.

2.1 Slotting problems

Following the research avenue opened by Asdemir et al. (2009), Yang et al. (2016), Yang and Strauss (2017), and Klein et al. (2018), which is discussed in the subsequent sections on pricing and integrated demand management and routing problems, Mackert (2019) proposed a new approach for the dynamic Time Slot Management Problem (TMSP), a tactical problem

Table 1 Overview of the main characteristics of demand management problems in AHD and AHS

Sector	Real-world application	Degree of dynamism	Problem	Planning horizon	Objective	Main framework	Choice model	Cost estimation	References
E-grocery	No	Dynamic	Slotting	Operational	Max PR	LP	GAM	SB, MILP	Mackert (2019)
E-grocery	Yes	Dynamic	Slotting	Operational	Max AR	SIM	SP	IH	Köhler et al. (2020)
E-grocery	Yes	Dynamic	Slotting	Operational	Max RV	SIM	MNL	ADP	Lang et al. (2021a)
E-grocery	No	Dynamic	Slotting	Tact./Oper.	Multiple	SIM	MNL	IH	Lang et al. (2021b)
E-grocery	Yes	Dynamic	Slotting	Operational	Max AR	SIM	SP	IH	Burian et al. (2024)
E-grocery	No	Dynamic	Pricing	Operational	Max PR	DP	MNL	–	Asdemir et al. (2009)
E-grocery	No	Dynamic	Pricing	Operational	Max PR	ADP	MNL	IH, SB	Klein et al. (2018)
E-grocery	Yes	Static	Pricing	Tactical	Max PR	MILP	GNR	SB	Klein et al. (2019)
E-grocery	No	Dynamic	Pricing	Operational	Max PR	QP	SP	ADP	Vinsensius et al. (2020)
Large appl.	No	Static	Pricing	Operational	Min TC	DP	AP	–	Yıldız and Savelsbergh (2020)
E-grocery	No	Dynamic	Pricing	Operational	Max PR	LP	MNL	CR	Strauss et al. (2021)
E-grocery	Yes	Stat./Dyn.	Pricing	Operational	Multiple	SIM	NL	IH	Köhler et al. (2023)

ADP approximate dynamic programming, *AP* acceptance probabilities, *AR* number of accepted requests, *CR* cluster-first, route-second, *DP* dynamic programming, *GAM* general attraction model, *GNR* general nonparametric rank-based, *IH* insertion heuristics, *LP* linear programming, *MILP* mixed integer linear programming, *MNL* multinomial logit, *NL* nested logit, *PR* profit, *QP* quadratic program, *RV* revenue, *SB* seed-based, *SIM* simulation, *SP* selection probabilities, *TC* total cost in AHD aimed at determining an efficient set of time slots for each region within a delivery area with the objective of minimizing the delivery costs while satisfying given service requirements. In particular, the author was the first to introduce a customer-choice model in the context of slotting problems; namely, he used a General Attraction Model (GAM) (see, e.g., Gallego et al. 2015), of which the Multinomial Logit (MNL), largely found in the stream of literature on pricing problems, is a special case. The advantage of using the GAM, instead of the MNL, is to avoid a potential overestimation of the choice probabilities in particular settings. Another noteworthy contribution of this work is the definition of a novel Mixed Integer Linear Programming (MILP) model to approximate the value function, hence the opportunity costs, of the Dynamic Programming (DP) framework underlying the slotting problem. In doing so, the author built upon the work of Klein et al. (2018), combining insertion heuristics, for the computation of the routing costs associated to already accepted orders, and a dynamic seed-based scheme, to estimate the delivery costs of expected future orders. The resulting online slotting problem is solved through a Linear Programming (LP) formulation derived from a Non-Linear Binary Program. In the computational experiments performed using relaxed versions of the proposed MILP model to favor real-time decisions, the results show a potential increase of 4 to 7% in terms of average profit compared to benchmark policies.

The idea of adding flexibility to the slotting problem was introduced in the work of Köhler et al. (2020), where the authors presented four alternative algorithmic approaches to derive the time slot offering for each incoming customer request. Their main contribution was to investigate the effect of proposing both long time windows (i.e., of 4 h), to preserve a certain flexibility in building the tentative routing plan during the booking horizon (especially in the early phases), and short time windows (i.e., of 30 min), which are commonly used in the e-grocery business sector. The results obtained on different demand scenarios (one derived from a German e-grocer) were greatly affected by the customers' willingness to accept long time windows, but they showed a clear potential in terms of increased number of accepted orders compared to the benchmark approach in which only short time windows are offered.

In the first of a series of papers on dynamic slotting, Lang et al. (2021a) studied incremental modular approaches that rely on the idea of anticipating, through simulation during an offline phase preceding the booking horizon, the information on delivery schedules and opportunity cost. In particular, the authors solve a Team Orienteering Problem with Multiple Time Windows to build anticipatory schedule patterns, while they apply an Approximate Dynamic Programming (ADP) to estimate the opportunity cost (taking inspiration from the work of Yang and Strauss (2017) on dynamic pricing that is reviewed in the following section). During the online booking phase, an Assortment Optimization Problem is solved to derive the set of time slots proposed for each incoming request, adding a Theft-based mechanism to dynamically adjust delivery capacity by "stealing" extra capacity from neighboring areas of the previously determined schedule patterns.

In their following work, Lang et al. (2021b) were the first to introduce the Multi-Criteria Dynamic Slotting Problem, where they seek to (i) maximize revenue, (ii) maximize the visibility of branded trucks, and (iii) maximize the social influence produced by the most influencing groups of customers, using a scalarized objective function. The last two objectives are in line with marketing principles, but the proposed approach is flexible and adaptable to other sets of criteria.

Building upon the work of Köhler et al. (2020), Burian et al. (2024) further investigated the idea of proposing flexible time windows for attended home deliveries in urban as well as rural areas. Considering two different delivery areas (i.e., the densely populated city of Vienna and the sparsely populated Upper Austria), the authors compared six algorithmic approaches (three of which were proposed in the original work, while the others are new) to derive the time slot offering for customer requests under two alternative demand scenarios (i.e., in case of equal preference between time slots and in case of strong preference for the most popular time slots). The results confirm the increase in the number of accepted orders, both in urban and rural areas, when flexible time windows are used, also providing meaningful insights for e-grocery retailers (and others) interested in setting up/sizing their business in rural areas.

2.2 Pricing problems

Asdemir et al. (2009) developed a dynamic pricing model that dynamically adjusts the delivery prices of multiple delivery options over a discrete booking horizon according to the remaining time, the residual capacity, and the affinity of customers with a particular class (which characterizes their arrival probability, expected profit, predictable utility for each delivery option and price sensitivity). The authors adopt a Logit-based model to reproduce the customer-choice behavior and a discrete-time, discrete-state Markov Decision Process (MDP) to set the pricing decisions of the e-grocer. Using simple examples, they demonstrate how optimal prices may change over time and how an increase or decrease in terms of capacity can influence them, even in the case when more than one class of customers is considered.

Klein et al. (2018) presented a novel MILP formulation to approximate the opportunity costs in dynamic pricing problems. In the proposed approach, which is repeated in an iterative way for each customer request received within a discrete booking horizon, the authors combine insertion heuristics (to compute the delivery cost for already accepted orders), an MNL model (to anticipate expected customers' reactions to future pricing decisions and, consequently, estimate future revenues), a dynamic seed-based approximation (to estimate the delivery costs of expected future orders), and the MILP formulation (to approximate the value function of a customer request in a DP framework). The results show an average increase in terms of total profits compared to common policies (e.g., fixed price and order value-based), as well as the "Foresight Policy" by Yang et al. (2016), which is considered as a benchmark policy. The so-obtained total profit is on average 5.5% higher in the first case, and 2.3% higher in the latter case. In addition, they find that a regular recalculation of the opportunity costs is preferable rather than a periodic, less frequent recalculation.

Klein et al. (2019) were the first to address the problem of pricing from a tactical perspective, proposing different variants of an exact MILP formulation for the Differentiated Time Slot Pricing Problem (DTSPP). In their work, motivated by an industrial partnership with a German e-grocer, the customer-choice behavior is modeled using a general nonparametric rank-based approach where the preferences of customers (assuming that all customers in a particular segment share the same preferences) are expressed through simple preference lists of slot-price tuples. The restrictions imposed by the DM problem are embedded into the MILP formulation in a first group of constraints, while the restrictions imposed by the routing problem (namely, route construction, demand and capacity, and time windows) are embedded into a second group of constraints. Given the NP-hardness of the DTSPP, the authors proposed two alternative model-based approximations for the routing constraints, one seed-based (Fisher & Jaikumar, 1981) while the other adapting and extending the approach found in Agatz et al. (2011). After an extensive computational study, the authors show that at a tactical level it is preferable to adopt model-based approaches that embed routing constraints. In fact, an early approximation of the delivery costs results in higher profits compared to diffused practical pricing approaches. In this sense, a trade-off between more accurate formulations, where the delivery cost approximation is more elaborate at the expense of an increase in the integrality gap, and less accurate formulations, where the delivery cost approximation is particularly rough but optimality can be reached, needs to be found.

Vinsensius et al. (2020) developed an Incentive-Routing Optimization framework for solving the dynamic pricing problem in AHD, where the pricing problem itself is formulated as a Quadratic Programming (QP) model with the objective of maximizing the total expected profits. As in Campbell and Savelsbergh (2006), the authors adopt a simple model to shape the customer-choice behavior, based on selection probabilities and a linear response to incentives. The QP formulation receives as an input the marginal fulfillment cost of each incoming order, which is computed through an ADP mechanism. The boundary condition for the ADP is obtained by solving an independent VRP with Service Choice for each time slot; to reduce the computational time, this particular sub-problem is solved using a minimum-regret construction heuristic (Pisinger & Ropke, 2007). Compared to a "Free Choice" policy, where the customers are free to choose their preferred time slot, and a "Myopic Incentive" policy, where the incentives are set based only on the QP model (with a myopic marginal cost anticipation), the "ADP Incentive" approach proposed by the authors shows better results in terms of total costs and fulfilled orders. The results are confirmed by a sensitivity analysis on some parameters (e.g., order density, arrival probability, and number of vehicles).

Yıldız and Savelsbergh (2020) studied the Pricing for Delivery Flexibility Problem where, unlike in other reviewed articles, they seek to minimize the total expected cost (which comprises both the delivery costs and the discounts offered to customers for changing the delivery day). The idea is to increase the delivery flexibility by proposing a discount to those customers that accept a different delivery day than the preferred one, with the objective to reduce the delivery costs. To solve the problem, the authors implemented an exact DP algorithm where the customer-choice behavior is modeled through acceptance probabilities. Several computational experiments were performed to evaluate the potential of cost reduction in the presence of different properties. The results show an expected cost reduction of more than 30% in the best cases, albeit a similar approach may be applicable only to those cases where the level of detail is the delivery day and the demand volume is not too high (e.g., large appliances).

The opportunity of proposing flexible time slots (either adjacent or non-adjacent) compared to single standard time slots is investigated in the work by Strauss et al. (2021), where a dynamic pricing approach based on an LP formulation is developed. The authors show how the offering of flexible time slots to customers may be beneficial for companies in reducing delivery costs, as it gives them more flexibility to build their routes. An additional and interesting insight regards the composition of the proposed flexible time slots. Indeed, a combination of more popular and less popular non-adjacent time slots is able to generate higher total profits compared to adjacent time slots, especially when the capacity is tight relative to the demand.

A promising work that is worth mentioning and might open new directions for dynamic pricing implementations is the one by Lebedev et al. (2021), where the authors studied several mathematical properties of the pricing problem, in the context of AHD, that can be used to find closer approximations of the value function in DP algorithms.

Motivated by the real-world case of a German online supermarket operating in Berlin and following up on the work by Köhler et al. (2020) on slotting, Köhler et al. (2023) presented an interesting study on pricing strategies for AHD. As a novel contribution to the literature, the authors introduced a Nested Logit model to reproduce the customer-choice behavior and compared four different pricing strategies (i.e., static one-price strategy, static multi-price strategy, dynamic one-price strategy, and dynamic multi-price strategy) to gain useful managerial insights. The results show that dynamic pricing strategies outperform commonly used static pricing strategies, although they are more difficult to implement in practice.

3 Routing problems in AHD and AHS

In the broad sense, the VRP consists in determining a set of minimum-cost routes to serve a set of customer requests, given a starting depot, a fleet of vehicles, and specific constraints depending on the application at hand. A rich body of literature on the family of VRPs is available, as these problems have been widely studied for more than 60 years and represent one of the main application areas in combinatorial optimization. We refer to Toth and Vigo (2014) for an extensive review on the VRP and its main variants, and to Wang and Wasil (2021) and Mor and Speranza (2022) for recent surveys.

Given that they are associated with last-mile delivery operations, AHD and AHS problems are strongly related to city logistics, as the majority of deliveries is naturally condensed in populated urban areas. A detailed overview of VRPs arising in city logistics is provided by Cattaruzza et al. (2017). In recent years, we have also seen the emergence of new VRP variants in line with the increasing complexity and variety of real-world applications; a brief overview of this topic can be found in the survey of Vidal et al. (2020), where the authors focus on emerging metrics to evaluate VRP solutions (which may give several hints for novel multi-criteria formulations), integrated approaches where the VRP is linked to upstream decisions and sometimes conceived as an evaluation tool for these decisions (which, to some extent, can be the case of AHD and AHS applications), and refinements of existing models.

When we consider the routing stage of AHD and AHS problems, we are interested in solving a Vehicle Routing Problem with Time Windows (VRPTW), in which capacity constraints are typically not binding if compared to time window constraints. For state-of-the-art works on the VRPTW we refer to Bräysy and Gendreau (2005a) for route construction methods and local search algorithmic techniques, Bräysy and Gendreau (2005b) for metaheuristic algorithms, Kallehauge (2008) and Baldacci et al. (2012) for exact solution approaches, Vidal et al. (2013) for an efficient hybrid genetic algorithm, and Desaulniers et al. (2014) for mathematical formulations, as well as exact and heuristic methods. Recently, new VRPTW extensions have emerged, by considering stochastic service times (Errico et al., 2018), multiple trips per vehicle and time-dependent travel times (Pan et al., 2021), as well as synchronized visits (see, e.g., Liu et al. 2019 and Polnik et al. 2021). In addition, the Electric VRPTW has received much attention for its practical implications (see, e.g., Schneider et al. 2014; Desaulniers et al. 2016; Hiermann et al. 2016; Keskin & Çatay 2016, 2018; Keskin et al. 2019, 2021; Duman et al. 2022; Lam et al. 2022).

In multi-stage AHD and AHS problems, the VRPTW may be used as a boundary condition in a DP framework, where the selected customer-choice model most of the times is an MNL model and a VRPTW must be solved for each state to update such boundary condition. However, this makes the AHD/AHS problem intractable due to the NP-hardness of the VRPTW (see, e.g., Savelsbergh 1985). This drawback can be partially overcome, at the expense of optimality, by applying approximate techniques.

The anticipation of the routing costs during the demand management stage is another critical aspect in AHD and AHS problems. As described in more detail in Sect. 4.1, an early approximation of the routing cost leads to higher profits compared to pure revenue management approaches that are still diffused in practice. This idea was also investigated by Bühler et al. (2016), who proposed four MILP models, all based on the Set Covering formulation for the VRP. The four models are conceived to be integrated into more developed DM models as "plug-in" modules to anticipate the estimation of the routing costs. The results show that the proposed models, decremental in terms of decision variables and constraints, approximate well the routing costs (i.e., the overestimation is no more than 10% compared to benchmark exact models, and slightly less than 3% compared to benchmark heuristics) in an acceptable computational time, thus being promising for real-world applications and suitable for decision support at a tactical level. In the aforementioned work by Klein et al. (2019), the authors built on these preparatory findings by introducing a routing module into their MILP formulation for the DTSPP.

Since a detailed review of routing problems would be too ambitious, we limit the scope of this section to the main routing models developed to solve specific AHD and AHS problems. An overview of the main characteristics of the reviewed articles is provided in Table 2. We remark that a particular emphasis is put on real-world applications.

3.1 Routing problems in AHD

In the first work of a series of articles on VRPTW variants for AHD problems, Azi et al. (2007) defined the Single-Vehicle Routing Problem with Time Windows and Multiple Routes (S-VRPMTW), where during a typical workday a single vehicle performs multiple routes of short duration for the delivery of perishable goods. Given the impossibility of serving all customers within the required time window, the multiple objectives are to maximize the number of customers served and to minimize the total distance (for the same number of customers served). The problem is solved using a two-phase solution approach based on the exact algorithm for the Elementary Shortest Path Problem (ESPP) proposed by Feillet et al. (2004).

In their second paper, Azi et al. (2010) defined a multiple-vehicle generalization of the S-VRPMTW, named the Vehicle Routing Problem with Time Windows and Multiple Routes (VRPMTW). Here, the multiple objectives are to maximize the total revenue and to minimize the total distance, and the problem is solved via Branch-and-Price (BP). In particular, the primary problem is a Set Partitioning Problem (SPP) formulation solved through column generation, while the pricing subproblem is an ESPP solved using the aforementioned algorithm by Feillet et al. (2004).

A few years later, Azi et al. (2014) solved the VRPMTW by means of an Adaptive Large Neighborhood Search (ALNS) algorithm. Interestingly, the authors demonstrate the advantage of applying destruction and insertion operators at different levels (customer, route, and workday) instead of using only customer-based operators.

Building upon their previous results, Azi et al. (2012) solved the dynamic VRPMTW, where the source of dynamicity is given by the arrival of new customer requests during the operational horizon. Note that such requests are inserted in future routes, as the current ones are fixed. Compared to the previously mentioned ALNS, a dynamic environment (in which the acceptance rule is slightly modified to take care of dynamicity) and an event management mechanism (to handle different types of events) were added. The results show that the proposed non-myopic approach (where future requests are considered) outperforms the myopic approach (where future requests are not considered) in terms of profit (computed as the total revenue associated with the served customers minus the total distance), percentage of served customers, number of routes per day, and number of customers per route, at the expense of considerably higher computational times (however acceptable and compatible with the response time required by an offline real-world application).

Table 2 Overview of the main characteristics of routing problems in AHD and AHS

Sector	Real-world application	Planning horizon	Objective	Model structure	Constraints	Solution method	References
E-grocery	No	Operational	Multiple	MILP	CP, TW	2-SA	Azi et al. (2007)
E-grocery	No	Operational	Multiple	MILP	CP, TW	BP	Azi et al. (2010)
E-grocery	No	Operational	Max PR	MILP	CP, TW	ALNS	Azi et al. (2012)
E-grocery	No	Operational	Multiple	MILP	CP, TW	ALNS	Azi et al. (2014)
Multiple	No	Operational	Multiple	–	CP, TW	TS, LP	Jabali et al. (2015)
Online retail	No	Operational	Multiple	MILP	TW	ALNS	Özarık et al. (2021)
Online retail	No	Operational	Multiple	MILP	TW, PC, MV	ALNS	Özarık et al. (2023)
Meal delivery	Yes	Operational	Min CCo	MILP	S-CS, T-CS, CtD	CRG	Yıldız and Savelsbergh (2019)
Meal delivery	Yes	Operational	Min CtD	MILP	S-CS, T-CS	CRG	Yıldız and Savelsbergh (2019)
Meal delivery	Yes	Operational	Min RtD	MILP	S-CS, T-CS, CtD	CRG	Yıldız and Savelsbergh (2019)
Meal delivery	Yes	Operational	Min CtDO	MILP	S-CS, T-CS	CRG	Yıldız and Savelsbergh (2019)
Meal delivery	Yes	Operational	Min RtP	MILP	S-CS, T-CS, CtD	CRG	Yıldız and Savelsbergh (2019)
Meal delivery	Yes	Operational	Min ESD	MDP	DD	ACA	Ulmer et al. (2021)
Large appl.	Yes	Operational	Min TC	MILP	CP, TW, PC, SYN	ALNS	Ali et al. (2021)
Multiple	Yes	Operational	Min TC	MILP	CP, TW, IT, E, PE, CS	HGS	Biswas et al. (2024)
Home health.	No	Operational	Multiple	MILP	TW, PC, SYN	LBH	Bredström and Rönnqvist (2008)
Home health.	Yes	Tact./Oper.	Maxmin	MILP	SK, CCa, WL	PGP	Cappanera and Scutellà (2015)
Home health.	Yes	Tact./Oper.	Minmax	MILP	SK, CCa, WL	PGP	Cappanera and Scutellà (2015)
Home health.	Yes	Tact./Oper.	Minmax	MILP	SK, CCa, WL	PGP, RO	Cappanera et al. (2018)
Home health.	No	Operational	Min TC	MILP	TW, WT, OT	TS	Zhan and Wan (2018)
Home health.	Yes	Operational	Min TC	MILP	SK, TW, OT, IT	LNS, SPP	Grenouilleau et al. (2019)
Home health.	No	Operational	Min TC	MILP	TW, WT, OT	LM	Zhan et al. (2021)

Table 2 continued

Sector	Real-world application	Planning horizon	Objective	Model structure	Constraints	Solution method	References
Home health.	No	Tact./Oper.	Min TC	MILP	WL	LBBD	Naderi et al. (2023)
Home health.	No	Operational	Min TWT	MILP	TW, WL, ST	LNS, LP	Arda et al. (2024)
Home health.	Yes	Tact./Oper.	Min TWT	MILP	SK, CCa, WL	BB	Parreño-Torres et al. (2024)
Home health.	Yes	Tact./Oper.	Maxmin	MILP	SK, CCa, WL	BB	Parreño-Torres et al. (2024)
Home health.	Yes	Tact./Oper.	Minmax	MILP	SK, CCa, WL	BB	Parreño-Torres et al. (2024)
Home health.	Yes	Tact./Oper.	Multiple	MILP	SK, CCa, WL	BB	Parreño-Torres et al. (2024)
Maintenance	Yes	Operational	Min TC	MILP	TW, TB, SK	ALNS	Kovacs et al. (2012)
Maintenance	No	Operational	Min Makespan	MDP	EL, PC	RTR	Chen et al. (2016)
Maintenance	Yes	Operational	Min TC	MILP	TW, TB, SK, WT, OT	BP	Zamorano and Stolletz (2017)
Maintenance	Yes	Operational	Multiple	MILP	SK, PC, IN, DT, TW, BR	BB	Mathlouthi et al. (2018)
Maintenance	Yes	Operational	Multiple	MILP	SK, PC, IN, DT, TW, BR	BP	Mathlouthi et al. (2021a)
Maintenance	Yes	Operational	Multiple	MILP	SK, PC, IN, DT, TW, BR	TS	Mathlouthi et al. (2021b)
Maintenance	Yes	Operational	Min TRD	MILP	RD, PC, MV	ILS	Atefi et al. (2023)
Maintenance	Yes	Tactical	Multiple	2-SP	TW	ALNS	Nielsen and Pisinger (2023)

2-SA two-phase solution approach, *2-SP* two-stage stochastic programming, *ACA* anticipatory customer assignment, *ALNS* adaptive large neighborhood search, *BB* Branch-and-bound, *BP* branch-and-price, *BR* breaks, *CCa* continuity of care, *CCo* courier compensation, *CP* Capacity, *CRG* column- and row-generation, *CS* customer satisfaction, *CtD* click-to-door time, *CtDO* click-to-door overage, *DD* delivery deadline, *DT* distance traveled, *E* emissions, *EL* experience level, *ESD* expected sum of the delay, *HGS* hybrid genetic search, *ILS* iterated local search, *IN* inventory, *IT* idle time, *LBBD* logic-based benders decomposition, *LBH* local branching heuristic, *LM* L-shaped method, *LNS* large neighborhood search, *LP* linear programming, *MDP* Markov decision process, *MILP* mixed integer linear programming, *MV* multiple visits, *OT* overtime, *PC* precedence, *PE* professional earnings, *PGP* pattern generation policy, *PR* profit, *RD* route duration, *RO* robust optimization, *RtD* ready-to-door time, *RtP* ready-to-pickup time, *RTR* record-to-record travel algorithm, *S-CS* spatial consistency, *SK* skill, *SPP* set packing/partitioning problem, *ST* stability, *SYN* synchronization, *T-CS* time consistency, *TB* team building, *TC* total cost, *TRD* total route duration, *TS* tabusearch, *TW* time windows, *TWT* total working time, *WL* workday length, *WT* waiting time

An interesting characteristic introduced by Jabali et al. (2015) is the use of self-imposed endogenous time windows rather than the exogenous ones typically considered in the VRPTW literature. Those self-imposed time windows are assigned to the customers by the company which, in turn, is committed to respecting them. A similar approach may be applicable to sectors like online retail, large appliances and furniture, as well as utilities. Another important feature included in this work is the presence of stochastic travel times that are dependent on a random variable representing a non-negative delay. Such delay is added to the base travel time. To solve the problem, the authors proposed a collaborative two-stage hybrid algorithm. First, the routing part is solved via Tabu Search (TS) using three alternative criteria for choosing a move. Second, the scheduling part, which takes as an input the solution found at the previous stage, is solved through an LP formulation that includes buffer times to handle the uncertainty given by the adoption of stochastic travel times. From a practical perspective, the use of self-imposed time windows may represent an unconventional policy (compared to the common practice of letting customers select their favorite time windows) to lighten the time window constraints, thus reducing the operating costs while keeping a certain service level.

Resuming the idea originally proposed by Pan et al. (2017) of using customer-related data to improve the effectiveness of AHD systems, Özarık et al. (2021) defined the Vehicle Routing and Scheduling Problem with Time-Dependent Costs (VRSPTDC). The problem is a variant of the VRPTW, as it adds a time-dependent penalty cost to the objective function. Such penalty cost is directly linked to the so-called "customer availability profiles" (introduced for the first time by Florio et al. 2018) that identify, for each customer, the probability of being present at home when the delivery is performed. In case the customer is absent during the first attempt of delivery, the authors assume that the next attempt is outsourced to an external courier, thus causing additional costs. From a practical perspective, the issue of low hit rates (i.e., frequent unsuccessful deliveries due to the absence of customers) is still one of the most significant problems in last-mile delivery. The VRSPTDC is solved using an ALNS-based metaheuristic algorithm with several removal and insertion operators. The results indicate the existence of a trade-off between the minimization of travel costs and the increase of hit rates. However, by taking advantage of customer-related data, it is possible to reach relevant cost savings. In particular, introducing the information on customer availability, in combination with the practice of waiting before serving a customer, may generate up to 40% in cost savings. Last but not least, the ALNS-based algorithm produced good results in comparison with a state-of-the-art MILP solver, and showed short computational times, which is desirable for a potential real-world application. In their following work, Özarık et al. (2023) focused on the impact of possible second visits to absent customers on the same delivery day by defining the Vehicle Routing and Scheduling Problem with Time-Dependent Costs and Multiple Customer Visits and solving it using a parallelized version of an ALNS algorithm. Interestingly, the results show an average cost reduction of 8% (which in some cases can be as high as 32%) if second visits on the same delivery day are allowed.

3.1.1 A focus on the meal delivery routing problem

Given the outstanding expansion of the food delivery sector in the last few years, a necessary exception from the main scope of our work is required for the Meal Delivery Routing Problem (MDRP). This problem is part of AHD (in the sense that the customer must be present at home for the delivery of food), but it also comprises typical elements of SDD (with new requests coming during the operational horizon) as well as the use of innovative practices arising in last-mile logistics, like crowdshipping and bundle generation. For an overview on last-mile delivery challenges and, in particular, routing problems with crowdshipping we refer to Archetti and Bertazzi (2021), while for a recent work on routing with bundle generation and occasional drivers we refer to Mancini and Gansterer (2022).

Among the first to study the MDRP, Yıldız and Savelsbergh (2019) introduced a mathematical formulation which is adaptable to different objectives that may be worth considering for an online food ordering and delivery platform (e.g., courier compensation, click-to-door time, ready-to-door time, click-to-door overage, namely the difference between the drop-off time of an order and its placement time plus the target click-to-door time, and ready-to-pickup time). Interestingly, their work is based on the concept of work package, which is a possible way to serve a bundle of orders. To solve the problem, the authors implemented a column- and row-generation algorithm, enhanced by a selective column inclusion scheme, that proved to be effective on the MDRPLIB instance set publicly made available by Grubhub (an American online ordering and delivery platform and a subsidiary of Just Eat Takeaway). In addition, a noteworthy analysis reported by the authors demonstrates that guaranteeing a minimum-pay to couriers does not cause a dramatic increase in terms of total cost (i.e., 9% in the worst case); to the contrary, it ensures a large availability of couriers. In our opinion, such an analysis may well contribute to the wide debate on policies for platform workers.

The Restaurant Meal Delivery Problem (RMDP) was addressed by Ulmer et al. (2021). Inspired by the previous work of Ulmer et al. (2020), the authors defined the RMDP as a route-based MDP, solving it by means of an Anticipatory Customer Assignment (ACA) heuristic algorithm. Such an approach was strengthened by the use of time buffering and postponement to soften the effects of stochasticity and dynamicity. The proposed policy was tested in an extensive computational study on real-world data from Iowa City. In comparison with the common-sense benchmark policy of assigning an incoming order to the driver that is able to deliver it as fast as possible, which is typically used in current practice, the results show that the ACA, relying on both time buffering and postponement, achieves strong improvements in terms of total delay. In particular, the use of time buffering itself produces significant improvements, as it decreases the effects of uncertain events. With the addition of postponement, it is also possible to take advantage of newly collected information which favor the assignment, as well as the bundling, of orders. From a practical perspective, the proposed algorithm proved to be robust in the presence of variability and suitable to solve real-world problems.

3.2 Routing problems in AHS

In this section, we are interested in reviewing some recent articles on routing problems for AHS.

A particularly interesting problem at the intersection between AHD and AHS is the Delivery Installation and Routing Problem (DIRP) investigated by Ali et al. (2021). The DIRP is inspired by a real-world application encountered in the sector of large appliances and furniture, where the deliveries and the installations are performed by two heterogeneous fleet of deliverymen and installers, respectively. This particular application requires the synchronization of worker skills and is characterized by the presence of temporal precedence constraints (i.e., an installer must wait for a deliveryman to complete the delivery service before reaching the location of a customer and starting the installation service). In some cases, the installation may be directly performed by the deliveryman (with a lower efficiency as such figure is less specialized than an installer). The authors defined the DIRP using a flexible MILP formulation, from which specific variants of the VRP can be easily derived (i.e., in case all the installations are performed only by deliverymen we refer to the VRP with time windows and driver-specific times, while in case all the installations are performed only by installers we refer to the VRP with multiple synchronization constraints). In addition, a variant of the DIRP was discussed in which the deliveryman and the installer can perform an installation together (instead of assuming that only one worker can perform the installation, as in the previous case). To solve the problem, the authors implemented an ALNS algorithm and compared its performance with that of the MILP formulation solved by a commercial solver. The results show that the ALNS algorithm is able to find good-quality solutions in short computing times both for test instances, as well as for real-world instances obtained from an industrial partner. Two noticeable insights emerged from the sensitivity analysis performed by the authors. The first is that using two heterogeneous fleets of deliverymen and installers has a positive impact in terms of total routing cost reduction. The second demonstrates the existence of a correlation between the efficiency of the deliverymen and the percentage of installations performed by the installers.

In a very recent work, Biswas et al. (2024) addressed home services from a broader perspective by defining the Home Services Assignment and Routing Problem with the Triple Bottom Line (HSARP-TBL). In particular, the authors focused on on-demand home services (e.g., cleaning, plumbing, electrical work, furniture repair) provided to customers by independent providers found through online service platforms, and proposed an MILP formulation for the HSARP-TBL. The problem is solved using a Hybrid Genetic Search algorithm. What is worth mentioning here is the integration of the three pillars of the triple bottom line (i.e., economic, social, and environmental) in the form of additional constraints of the MILP formulation.

3.2.1 A focus on the home healthcare routing and scheduling problem

Given their practical implications, we cannot forget to mention relevant works, in the context of home care services, on service planning and patient-to-nurse assignment. Among these, we refer to Eveborn et al. (2006, 2009), where the authors described a DSS developed for the Swedish healthcare system, which is based on an SPP formulation and a repeated matching algorithm for optimizing the generation of attended home visiting schedules. Another noticeable work is that of Duque et al. (2015), where the case of *Landelijke Thuiszorg*, a Belgian home care service provider, is described. For what concerns the assignment of patients to traveling nurses, Hertz and Lahrichi (2009) developed an assignment algorithm to solve a real-world problem arising in a small area of Montréal (Québec), while Carello and Lanzarone (2014) and Lanzarone and Matta (2014) addressed the robust nurse-to-patient assignment problem by focusing on structural policies to guarantee the continuity of care (which means that a patient must be visited by a restricted group of caregivers). For more references on routing and scheduling problems in home healthcare we refer the interested reader to the surveys by Fikar and Hirsch (2017) and by Euchi et al. (2022).

Starting from the real-world application described by Eveborn et al. (2006, 2009), Bredström and Rönnqvist (2008) defined a novel MILP formulation for the Vehicle Routing and Scheduling Problem with Time Windows (VRSPTW). The peculiarity of the VRSPTW is given by the presence of pairwise temporal precedence constraints and pairwise synchronization constraints. As discussed by the authors, similar constraints may be found in homecare staffing and scheduling problems, where different staff members are required to visit a patient one after the other or simultaneously. The problem was solved using a local branching heuristic inspired by Fischetti et al. (2004). This solution method was tested by considering alternative objective functions (i.e., minimization of preferences, minimization of traveling time, minimization of maximal difference in workload among staff members, or minimization of a weighted sum of multiple objectives).

Cappanera and Scutellà (2015) addressed the Palliative Home Care Problem (PHCP), an important problem arising in home healthcare that refers to the provision of palliative therapies to terminal patients. The authors modeled the PHCP through an MILP formulation where assignment, scheduling and routing decisions are taken in an integrated fashion. Two alternative objective functions, *maxmin* (which balances the operator workload by maximizing the minimum utilization factor) and *minmax* (which balances the operator workload by minimizing the maximum utilization factor), were defined and used to guide the solution process. The MILP formulation was strengthened with the addition of symmetry breaking constraints and valid inequalities. To solve the PHCP, the authors implemented three alternative pattern generation policies (a greedy heuristic procedure, a realistic procedure based on current practice, and a flow-based model), where patterns are alternative schedules of visits that are generated a priori for each patient. The generated patterns are given as input to the MILP formulation that solves the original PHCP. This approach proved to be effective on different sets of realistic instances. From a practical perspective, it is worth highlighting that the selection of *maxmin* as the objective function of the MILP formulation produces more balanced solutions in terms of workload among operators. On the contrary, the selection of *minmax* produces less costly solutions, as the total travel time for the operators is minimized.

Extending the previous work by Cappanera and Scutellà (2015), Cappanera et al. (2018) generalized the Home Care Problem (HCP) by taking into account demand uncertainty. In particular, the authors adopted the cardinality-constrained framework proposed by Bertsimas and Sim (2004) to define the sequence-preserving Γ-Robust Home Care Problem ($sRHC_\Gamma$). In this robust version of the HCP, uncertainty is handled by considering additional uncertain requests; among these, at most Γ requests must be inserted into each solution tour (where Γ is a given parameter). The decisions of the $sRHC_\Gamma$ are guided by the aforementioned *minmax* objective function. The proposed approach turned out to produce more robust solutions compared to the nominal formulation, showing a high degree of fairness in terms of caregiver utilization factor and a low approximation error. The authors also experimented with a decomposition approach by fixing the scheduling decisions. This approach proved to be suitable for solving larger instances.

Zhan and Wan (2018) defined the Routing and Appointment Scheduling with Team Assignment (RASTA) problem, which arises in the context of home healthcare and integrates decisions on team assignment, routing and scheduling. The authors formulated the RASTA as an MILP model and solved it by implementing a TS algorithm, where the initial feasible routing schedule is built using a modified parallel savings algorithm. This initial solution is then improved by invoking classical local search operators (e.g., 2-opt, relocate, and Or-opt) until a termination criterion is reached, while the customers' appointment times are determined by solving a scenario-based LP formulation (which considers the routing schedule as an input). The stochastic information on service times was estimated based on the results found by Lanzarone et al. (2010). The proposed methodology proved to be effective on small sets of randomly generated instances, leaving room for potential extensions. In their following work, Zhan et al. (2021) focused on the Routing and Appointment Scheduling problem by defining a novel MILP formulation and solving it via the L-shaped method. Additionally, a heuristic algorithm to handle large-size instances was also developed.

Motivated by a collaboration with Alayacare, a Canadian start-up based in Montréal (Québec), Grenouilleau et al. (2019) studied the Home Health Care Routing and Scheduling Problem (HHCRSP). In particular, the authors defined the problem as an SPP with the objective of selecting the best daily routes for each caregiver. Such routes are built by taking into account the patients' mandatory requirements, the caregivers' skills, and the required time windows. Several objectives, such as the number of missing optional requirements, the travel time, the continuity of care, and a penalty for non-compliance with minimum and maximum working hours, are inserted into the weighted sum cost function that is computed for each route. The weekly overtime and idle time for each caregiver, and the number of unscheduled visits are then added in the overall objective function of the SPP formulation as additional objectives. A Large Neighborhood Search (LNS) algorithm is used to find the set of feasible routes that are given as input to a relaxed version of the SPP, after which a constructive heuristic algorithm is called to rebuild the integrality of solutions. Interestingly, the proposed approach outperformed Alayacare's current solution by 37% in terms of total travel time and 16% in terms of continuity of care, thus proving to be effective in solving real-world instances. The HHCRSP with temporal dependencies under uncertainty was later addressed in the work of Shahnejat-Bushehri et al. (2021), where the authors defined the problem using a robust optimization model and solved it by implementing three alternative metaheuristic algorithms.

Naderi et al. (2023) studied the multi-period Home Healthcare Planning and Scheduling Problem (HHCPS), in which the decisions on the assignment, scheduling and routing of caregivers are taken in an integrated fashion. For tractability reasons, a priori generated patterns are given as input to the proposed MILP formulation. To solve the problem, the authors implemented an exact algorithm relying on a logic-based Benders decomposition. A robust version of the HHCPS that accounts for uncertainty in travel and service times was also developed.

Another important problem arising in home healthcare that refers to the administration of chemotherapy to cancer patients, named Home Chemotherapy Delivery Problem (HCDP), was addressed in the work of Arda et al. (2024). The HCDP consists of two interdependent subproblems, (i) drug production scheduling and (ii) drug administration routing and scheduling, and is characterized by hard stability and time window constraints. The authors solved it by implementing an LNS algorithm, which relies on an LP model to reoptimize the schedules and guarantee the feasibility of solutions. The performance of the proposed method was compared with a compact MILP formulation for the HCDP also proposed by the authors.

Building upon the work of Cappanera and Scutellà (2015), Parreño-Torres et al. (2024) modeled the Palliative Home Health Care Routing and Scheduling Problem through an MILP formulation with five alternative objective functions (i.e., minimization of total workload, *maxmin*, *minmax*, minimization of total workload and *maxmin*, and minimization of total workload and *minmax*). Interestingly, the results show that the formulation using a scalarized objective function that minimizes the total workload and maximizes the minimum percentage workload (i.e., *maxmin*) is able to find a better trade-off between total workload and workload distribution among caregivers (which is important in the context of palliative home care).

3.2.2 A Focus on the technician routing and scheduling problem

Starting from the problem formulation given by Cordeau et al. (2010) and motivated by a collaboration with an infrastructure service provider, Kovacs et al. (2012) were among the first to address the Service Technician Routing and Scheduling Problem (STRSP). In particular, the authors presented an MILP formulation for the STRSP and implemented two alternative versions of an ALNS algorithm, one without team building and the other with team building. Both ALNS algorithms rely on several destroy and repair operators from the literature. The proposed algorithms were tested on benchmark instances as well as on real-world instances, showing a significant average cost reduction of almost 11% compared to the manual plans adopted by the company. Other pioneering works on the Technician Routing and Scheduling Problem (TRSP) and the Technician Dispatching Problem (TDP) that are worth mentioning are those by Pillac et al. (2013) and Cortés et al. (2014), respectively.

Later, Chen et al. (2016) studied a novel problem variant, named Technician Routing and Scheduling Problem with Experienced-based Service Times (TRSP-EST). Here, the authors formally described the problem as an MDP, and developed a myopic solution approach based on a daily routing problem solved with a metaheuristic algorithm. The noteworthy contribution of this work is to consider, for the first time in the routing literature, different learning curves and heterogeneity of technicians and to derive some "rules of thumb" that can be used from a managerial perspective. In particular, the results demonstrate the advantage of considering learning curves and heterogeneity of technicians instead of static productivity. In addition, the authors emphasize the idea that the routing aspect should be favored in the presence of fast-learning and experienced technicians, while the scheduling aspect should be favored in the presence of slow-learning and inexperienced technicians. In their following works, Chen et al. (2017, 2018) addressed the multi-period Technician Routing and Scheduling Problem with Experienced-based Service Times and Stochastic Customers by focusing on the problem of assigning tasks to technicians and omitting the routing component. In particular, the authors proposed an ADP-based solution approach, in which the so-called "cost-to-go" is computed by looking ahead both one period and over the entire planning horizon. Recently, Chen et al. (2024) solved the multi-period TRSP-EST by considering both learning curves of technicians and future information over the entire planning horizon. In particular, the authors introduced an implicit cross-training mechanism that assigns unfamiliar tasks and customer types to technicians, with the aim of increasing their experience level and, consequently, the overall workforce flexibility.

Motivated by the real-world case of an external maintenance provider specialized in electric forklifts, Zamorano and Stolletz (2017) defined the Multi-period Technician Routing and Scheduling Problem (MPTRSP) and solved it using two alternative BP algorithms based on different decomposition schemes (i.e., a day decomposition and a team-day decomposition). Compared to the literature on Workforce Scheduling and Routing, of which the MPTRSP is a generalization, the novel contribution of this work is to consider multiple periods and team building simultaneously. The numerical experiments conducted on test instances show that the BP algorithm based on the team-day decomposition scheme, which results in more but easier-to-solve subproblems, performs better in terms of computing times and gap to optimality. The same experiments are repeated on real-world as well as larger instances, confirming the effectiveness of the proposed solution approach. Additional experiments conducted on other test instances indicate a negative correlation between time window length and overall costs, which is noticeable from a practical perspective, and a positive correlation between time window length and computing times.

In the first of a series of papers on technician routing and scheduling, Mathlouthi et al. (2018) presented a novel MILP formulation for a Multi-attribute Technician Routing and Scheduling Problem (MATRSP) solving it using a commercial solver. This work is motivated by a real-world application arising at a company providing maintenance and repair services for electronic transaction equipment. The noteworthy contribution of this work is to accurately define a complex problem by combining a number of heterogeneous characteristics (required skills, precedence constraints for special parts, inventory levels for spare parts, maximum traveled distance, breaks, and time windows). Several computational experiments are performed to assess the effect of certain parameter variations, such as the percentage of special parts, technician skills, the impact of service times, and the number of technicians.

In their following work, Mathlouthi et al. (2021a) implemented a BP algorithm to solve the MATRSP. As in Azi et al. (2010), the primary problem is formulated as an SPP, while the pricing subproblem is an Elementary Shortest Path Problem with Resource Constraints (ESPPRC) which is solved using both the algorithm by Feillet et al. (2004) and the Decremental State-Space Relaxation (DSSR) algorithm by Righini and Salani (2008). Also, two alternative branching strategies are presented here. The results demonstrate that the DSSR implementation with the ternary branching strategy obtains the best results. In addition, the BP algorithm proved to solve to optimality larger instances (with up to 45 tasks) as compared to the MILP formulation presented in Mathlouthi et al. (2018) and solved with a commercial solver.

Mathlouthi et al. (2021b) developed a TS metaheuristic algorithm with adaptive memory for the MATRSP. Interestingly, the algorithm found the same optimal values as the exact method by Mathlouthi et al. (2021a) for instances with up to 45 tasks and solved instances with up to 200 tasks within 2 h, which is compatible with practical implementations.

A specific variant of the TRSP for monitoring water distribution networks was studied in the work of Atefi et al. (2023). The problem is characterized by maximum route duration, precedence constraints, and multiple visits to so-called key centers due to the presence of some special nodes that require a key (to be picked up at a particular key center) to perform the service. To solve the problem, the authors proposed an MILP model and an Iterated Local Search (ILS) algorithm. These methods were tested on randomly created instances as well as on the benchmark instances for the Asymmetric Distance-Constrained VRP proposed by Almoustafa et al. (2013). The ILS was also used to solve realistic instances derived from the water distribution network of the city of Mashhad (Iran). Interestingly, the ILS proved to find good-quality solutions in short computing times for instances with up to 200 nodes.

Nielsen and Pisinger (2023) approached the TRSP from a tactical perspective by defining the problem as a Two-stage Stochastic Programming (2-SP) model. The first stage aims at partitioning the plane (which is identified by the depots and the locations of customers to be serviced) into slices and assigning the slices to working days, and is solved with a balanced sweep algorithm; the second stage is a TRSP, which is modeled through an MILP formulation and solved with an ALNS algorithm. The problem originated from a real TRSP at *TDC-NET*, the Danish wired telecommunication infrastructure owner, in which we have both static (i.e., installations) and dynamic (i.e., maintenance services) tasks; the former are known in advance and have long service windows, while the latter arrive dynamically and have short service windows. The results of the computational experiments show an average 10% reduction in driving distance when the proposed tactical planning is used. A sensitivity analysis on the percentage of technicians to be dedicated to dynamic tasks is also performed.

The considerable interest in the research field of TRSP is confirmed by the number of articles recently published. In particular, Delavernhe et al. (2024) addressed a maintenance and routing optimization problem, of which the TRSP is only one part. It is worth noting that the authors integrated in the problem formulation several aspects typically considered independently, like probabilistic models for assessing the machine states, decisions on maintenance operations based on machine states, and experience levels of technicians. Gamst and Pisinger (2024) solved the same problem found in Nielsen and Pisinger (2023) by considering investment decisions (i.e., minimization of capital expenditures, CAPEX, and minimization of operational costs, OPEX). Nowak and Szufel (2024) focused on the specific TRSP for the sharing economy, where a heterogeneous team of independent technicians with different experience levels is managed by a centralized service provider.

4 Integrated demand management and routing problems in AHD and AHS

Many authors have been approaching the field of integrated demand management and routing from their methodological backgrounds since the mid-2000 s. In Sect. 4.1, we review the most relevant articles in the literature on AHD and AHS and give an overview of their main characteristics in Table 3. In Sect. 4.2, we then focus on the Time Window Assignment Vehicle Routing Problem (TWAVRP).

4.1 Integrated problems

Although the authors do not refer directly to the problem of integrating demand management and routing, the paper of Bent and Van Hentenryck (2004) may be considered a pioneering work in this area, as it anticipates the idea of using stochastic information in the decision to accept or reject a request. Indeed, the Scenario Based Planning Approach (SBPA) to dynamic stochastic VRPTW they proposed fits well with the ordering phase of AHD problems that precedes the cutoff time, when the order requests arrive and must be accepted or rejected. Also, the SBPA may be applied for practical implementations of maintenance and repair services, where it is not known a priori when the next call will arrive. The basic principle of SBPA is to keep in memory a set of routing plans that are updated at each execution step. These routing plans are generated by considering information on already known requests as well as possible future requests. The plan to be implemented is then selected by means of a so-called consensus function. The experimental results show that the SBPA performs well compared to less sophisticated methodologies in terms of number of customers served and number of vehicles used.

Among the first to see a potential in the integration between order promise and order delivery phases, Campbell and Savelsbergh (2005) proposed several insertion-based heuristics for AHD problems. In particular, the authors developed a number of probability-based heuristics where the information on potential future orders is considered in the decision to either accept or reject an order. Compared to the common practice of accepting a fixed number of orders per time slot and using simple dynamic insertion heuristics, the proposed probability-based heuristics are constantly more efficient in capturing the economic profitability of incoming requests. The authors extensively tested such heuristics by varying some experimental characteristics. In many cases, the probability-based heuristics were able to come very close to the results obtained in the presence of perfect information and, except in one case, they showed computational times that are compatible with practical implementations.

Building upon their previous work (i.e., Campbell and Savelsbergh 2005), Campbell and Savelsbergh (2006) addressed the use of incentive schemes to steer customer behavior in AHD services. In particular, the authors propose two alternative LP formulations to solve the Home Delivery Problem with Time Slot Incentives and the Home Delivery Problem with Wider Slot Incentives, respectively, that do not incorporate a proper customer-choice model but use, instead, simple selection probabilities. In both formulations, an estimation of the delivery costs of accepted orders, performed using a combination of insertion heuristics and randomization, is inserted in the objective function. In addition, the feasibility of the routes under construction is checked. Interestingly, the results show that companies could take advantage from the use of incentive schemes to reduce delivery costs and, consequently, increase profits even in the early stages of the decision process. The authors also demonstrate that developing incentives schemes for wider time slots is easier and has the potential to

Table 3 Overview of the main characteristics of integrated demand management and routing problems in AHD and AHS

Sector	Real-world application	Planning horizon	Objective	Modeling approach	Constraints	Choice model	Solution method	References
Maintenance	No	Operational	Max AR	–	TW	–	SBPA	Bent and Van Hentenryck (2004)
E-grocery	No	Operational	Max PR	–	TW	RP	IH	Campbell and Savelsbergh (2005)
E-grocery	No	Operational	Max PR	LP	TW, IL	SP	LP, IH	Campbell and Savelsbergh (2006)
E-grocery	Yes	Tactical	Min RC	CA	CP, TW	–	GH, CR	Agatz et al. (2011)
E-grocery	Yes	Tactical	Min RC	ILP	CP, TW	–	ILP, SB	Agatz et al. (2011)
Online retail	Yes	Operational	Max AR	–	TW	–	SIM, IH	Ehmke and Campbell (2014)
E-grocery	Yes	Operational	Max PR	DP	–	MNL	GH, IH	Yang et al. (2016)
Large appl.	No	Tactical	Min RC	MILP	CP, TW	–	TS	Hernandez et al. (2017)
Online retail	No	Operational	Multiple	MILP, SDP	CP, TW	–	TS, ADP	Han et al. (2017)
E-grocery	Yes	Operational	Max PR	DP	–	MNL	ADP, CR	Yang and Strauss (2017)
Utilities	Yes	Tactical	Min RC	ILP	CP, TW	SS	LNS, ILP	Bruck et al. (2018)
Online retail	Yes	Tact./Oper.	Min TC	2-SP	CP, TW, DT, NB	–	MLM	Restrepo et al. (2019)

Table 3 continued

Sector	Real-world application	Planning horizon	Objective	Modeling approach	Constraints	Choice model	Solution method	References
Utilities	Yes	Strat./Tact.	Min RC	MILP	CP, TW	–	LNS, ILP	Bruck et al. (2020)
E-grocery	No	Operational	Max PR	DP	–	MNL	ADP, IH, GH	Koch and Klein (2020)
E-grocery	Yes	Operational	Max PR	DP	–	MNL	ADP, IH, SA	Abdollahi et al. (2023)
Utilities	Yes	Operational	Min ERC	DP	CP	–	RH, LNS, TH	Keskin et al. (2023)
Retail	Yes	Strat./Tact.	Min ETC	MILP	CP, TW	–	BPC	Spliet and Gabor (2015)
Retail	Yes	Strat./Tact.	Min ETC	MILP	CP, TW	–	BPC	Spliet and Desaulniers (2015)
Retail	Yes	Strat./Tact.	Min ETC	MILP	CP, TW	–	BPC	Spliet et al. (2018)
Retail	Yes	Strat./Tact.	Min ETC	MILP	CP, TW	–	BC	Dalmeijer and Spliet (2018)
Retail	Yes	Strat./Tact.	Min ETC	MILP	CP, TW, MP, SD	–	3-SA	Neves-Moreira et al. (2018)
Retail	Yes	Strat./Tact.	Min ETC	2-SP	CP, TW	–	SDA	Subramanyam et al. (2018)
Maintenance	No	Tact./Oper.	Multiple	2-SP	CP, TW	–	ALNS	Vareias et al. (2019)
Retail	Yes	Strat./Tact.	Min ETC	MILP	CP, TW	–	BPC	Dalmeijer and Desaulniers (2021)
Multiple	No	Tact./Oper.	Min TWVI	RO	CP, TW	–	BC	Hoogeboom et al. (2021)
Maintenance	Yes	Tact./Oper.	Multiple	2-SP	CP, TW	–	ARSH	Yu et al. (2023)
Large appl.	Yes	Tact./Oper.	Min ETC	2-SP	CP, TW	–	SAAM, ALNS	Côté et al. (2024)
Multiple	No	Operational	Min ETWS	NL-SP	SL	–	TWH	Ulmer et al. (2024)

2-SP two-stage stochastic programming, *3-SA* three-phase solution approach, *ARSH* assignment-routing-scheduling heuristics, *ADP* approximate dynamic programming, *ALNS* adaptive large neighborhood search, *AR* number of accepted requests, *BC* branch-and-cut, *BPC* branch-price-and-cut, *CA* continuous approximation, *CP* capacity, *CR* cluster-first, route-second, *DP* dynamic programming, *DT* distance traveled, *ERC* expected routing costs, *ETWS* expected time window size, *ETC* expected total cost, *GH* greedy heuristic, *IH* insertion heuristics, *IL* incentive limit, *ILP* integer linear programming, *LNS* large neighborhood search, *LP* linear programming, *MILP* mixed integer linear programming, *MLM* multicut L-shaped method, *MNL* multinomial logit, *MP* multiple product, *NB* neighboring, *NL-SP* non-linear stochastic programming, *PR* profit, *RC* routing costs, *RH* rolling horizon, *RO* robust optimization, *RP* realization probabilities, *SAAM* sample average approximation method, *SA* simulated annealing, *SB* seed-based, *SBPA* scenario based planning approach, *SD* split deliveries, *SDA* scenario decomposition algorithm, *SDP* stochastic dynamic programming, *SIM* simulation, *SL* service level, *SP* selection probabilities, *SS* simulation strategies, *TC* total cost, *TH* touting heuristic, *TS* tabu search, *TW* time windows, *TWH* time window heuristic, *TWVI* time window violation index produce an increase in profits as well (additionally determining a benefit in terms of flexibility in building efficient routes).

A milestone in the field of AHD is the work of Agatz et al. (2011), where the TMSP in AHD was defined for the first time. The authors studied the particular TMSP arising at Albert Heijn, the leading Dutch e-grocer at the time, and proposed two alternative formulations for the problem, in which the expected delivery costs are minimized. The first extends the Continuous Approximation (CA) approach found in Daganzo (1987); in particular, the authors start from a base schedule (e.g., the one adopted by the company) and iteratively improve it until the expected routing costs do not decrease anymore or a maximum number of iterations is reached. In this formulation, a "cluster-first, route-second" strategy is used to approximate the delivery costs. The second formulation is an Integer Linear Programming (ILP) model that relies on the seed-based scheme originally proposed by Fisher and Jaikumar (1981) to approximate the routing costs. As shown by the computational experiments both formulations produce high-quality schedules, resulting in a slight reduction of delivery costs compared to the schedule used by the company. But the greatest potential generated by the two formulations is that of automating the schedule design process; in this sense, the CA approach is better than the ILP model as it requires shorter computational times. Further remarkable findings are presented in the what-if analyses conducted by the authors, where the effects of potential changes (increase of demand, increase or decrease of vehicle capacity, increase or reduction of service level, and use of alternative time slot templates) are investigated. Among them, they remark the existence of a trade-off between the time slot length and the routing efficiency (with an increase of up to 25% in delivery costs going from an entire shift length to a two-hour length). Also, they highlight the idea that introducing a demand clustering may have a beneficial effect of approximately 10% reduction in terms of delivery costs.

Building upon the work of Campbell and Savelsbergh (2005) as well as the results previously found by Ehmke et al. (2012a, b), Ehmke and Campbell (2014) developed and compared novel customer acceptance mechanisms for AHD applications in metropolitan areas. The innovative idea behind their work is represented by the introduction of time-dependent and stochastic travel time information in the decision-making process of accepting or rejecting an incoming order request. In particular, to take care of possible lateness, due to variable travel times in rush hours, and the so-called "lateness propagation" effect, which depends on accumulated travel time variations during the execution of delivery routes, the authors included a thorough computation of individual buffer times. Such computation was integrated in a time-dependent variant of the I1 insertion heuristic algorithm originally developed by Solomon (1987). The results obtained from several rounds of simulation show that the proposed acceptance mechanism generally outperforms alternative approaches, both static and dynamic, in terms of the number of accepted requests and potential to avoid lateness. The authors also investigated the effect of changes in some input parameters (e.g., distribution of customer locations between downtown and suburban areas, service times, time window length, lateness avoidance, and confluence of requests in popular time slots) and provided meaningful practical insights.

Yang et al. (2016) defined a DP framework for the dynamic pricing of delivery time slots based on a thorough demand model, where the arrival of customers for a single delivery day is estimated using a time-dependent Poisson process, while the selection of time slots within a given delivery day is modeled through an MNL model. The dynamic program is defined to gain insights for the development of good pricing policies, as it is not solvable in short computing times due to the curse of dimensionality and the VRPTW that must be solved at each stage. To overcome this problem, during the online booking phase an approximation of the routing costs is computed based on the insertion heuristics by Campbell and Savelsbergh (2006) and an online pricing problem is solved. As a valuable result, the authors show that a dynamic pricing policy that includes an estimation of the delivery costs for expected future orders, instead of focusing only on already accepted orders, is preferable. Moreover, they show how a similar policy produces a remarkable increase in terms of total profits (i.e., 3.8% on average) compared to the common industrial practices of using static prices or order-based prices for time slots. This effect is even more evident when capacity is scarce. The work was motivated by an industrial partnership with a major e-grocer in the United Kingdom that provided anonymized booking data that were used to train the models and perform different runs of simulation. Building upon their previous work and using the same sample data provided by a major e-grocer operating in the Greater London area, Yang and Strauss (2017) developed an APD procedure. In particular, the proposed approach adopts a dynamic pricing policy that incorporates both approximated delivery costs (obtained by applying the "cluster-first, route-second" approach originally proposed by Daganzo 1987) and estimated revenues to compute the opportunity costs from expected future orders. Remarkably, the results show an average total profit increase of more than 2% compared to base policies where no opportunity cost is considered, and a computational time compatible with real-world applications.

A different interpretation of the Tactical Time Slot Management Problem (TTSMP) was given in the work of Hernandez et al. (2017), where the authors defined the TTSMP through an MILP formulation and solved it heuristically. In particular, two alternative heuristics were proposed. The first heuristic relies on a three-phase decomposition, that initially solves a Periodic Vehicle Routing Problem (PVRP), in which the time slots in the TTSMP correspond to the periods in the PVRP, subsequently merges the routes obtained from Phase 1 over each day, and, finally, solves a VRPTW for each day in the planning horizon (i.e., optimizes the routes merged during Phase 2). The second heuristic interprets the TTSMP as a Periodic Vehicle Routing Problem with Time Windows (PVRPTW), in which the days in the TTSMP correspond to the periods in the PVRPTW while the time slots correspond to the time windows. Both problems were solved using a TS algorithm that has proven to be efficient for these problems (see, e.g., Cordeau et al. 1997, 2001). Although the first heuristic is competitive for being more generic and tractable with state-of-the-art techniques and available software, it is generally outperformed by the second heuristic both in terms of computational times and solution quality.

Inspired by the work of Schmid and Doerner (2014), Han et al. (2017) developed an integrative approach for solving the appointment scheduling and routing problem in the context of AHD. What characterizes this work is the inclusion of random customer behavior in the proposed model by considering no-show probabilities and random response times during the delivery phase. Such randomness typically represents a remarkable issue in real-world applications, frequently causing inefficient re-routing, potential disruptions, and extra costs. To solve the problem, the authors implemented a hybrid heuristic algorithm, which iteratively combines a TS metaheuristic, for solving the routing part, and an approximate DP algorithm, for solving the scheduling part. The results show how the proposed integrative approach outperforms a traditional hierarchical approach. However, the computational times obtained on large instances warn against a potentially low compatibility with real-world cases, as the developed algorithm took almost 20 h to solve instances with up to 5 vehicles and 50 customers.

In their work at the border between AHD and SDD, Restrepo et al. (2019) introduced for the first time the Integrated Shift Scheduling and Load Assignment Problem. The problem, originating from a real-world start-up company offering last-mile delivery services in many cities of France, is formulated as a 2-SP model. In particular, the first stage aims at designing tactical schedules for couriers, which are allocated to a restricted number of geographic areas, while the second stage defines the assignment of customer orders to couriers. In this work, we have a co-presence of stochasticity (given a portion of stochastic orders generated using a Poisson distribution) and dynamicity (given a portion of orders that must be fulfilled according to a same-day delivery policy). To solve the problem, the authors implemented a multicut L-shaped method with some additional algorithmic refinements to generate initial cuts and derive valid inequalities. The main idea underlying this work is represented by the opportunity of using the tactical model to compare alternative policy offerings and to evaluate their impact on total cost and solution quality. In addition, the results show the advantage of including uncertainty when generating tactical solutions.

A very interesting real-world application of differentiated slotting in the context of utilities was studied in the work of Bruck et al. (2018). Here, the authors addressed a particular problem arising from an Italian gas distribution company, named *IRETI*, in which the required Quality of Service (QoS) level is exogenously fixed by the public authority that regulates the market, so there is no opportunity to influence the demand of customers using RM principles. As a consequence, the design of good quality time slot tables is fundamental to limit the routing costs generated after the actual demand is revealed. For doing so, the authors developed a three-step approach having at its core an LNS algorithm that iteratively improves an initial set of time slot tables by means of destroy and repair methods. Interestingly, the customer-choice behavior in the process of booking the preferred time slot for the execution of a service was reproduced using four alternative simulation strategies. The cost of the solutions computed by the LNS algorithm is evaluated through a Multidepot multiple Traveling Salesman Problem (MmTSP), which relies on a time-extended network. Note that a different MmTSP is solved for each day in the booking horizon. The results obtained on real-case instances showed an expected reduction of routing costs in the order of 5% to 15% compared to the company's solution.

Addressing the same real-world application described by Bruck et al. (2018), Bruck et al. (2020) developed a DSS to solve the practical problem of defining the organizational model for a so-called "minimum territorial area" (ATEM), given the QoS levels imposed by the public authority regulating the gas distribution market. The DSS is intended to support *IRETI* in solving a three-stage problem, in which the decisions are sequential. In the first stage, a number of municipalities are clustered by solving a p-Median Facility Location Problem; in the second stage, an initial model-week is generated for each cluster by using an improved ILP formulation compared to the one in Bruck et al. (2018) and an LNS algorithm; in the third stage detailed technician routing plans are created by solving an MmTSP for each day in the simulating horizon and several key performance indicators are provided in output to the decision makers. Interestingly, dynamic changes are made to the model-weeks during the simulation, thus reproducing a common practice to address demand fluctuations. Also, it is worth noting that the DSS has integrated a machine learning submodule that gives the opportunity to design solutions in the presence of missing information (i.e., by predicting the demand of partially known or totally unknown ATEMs).

Extending previous works and combining them with ideas from recent streams of literature on the VRP, Koch and Klein (2020) proposed a route-based ADP approach for dynamic pricing, where the opportunity cost due to the displacement of potential future orders is carefully estimated through a route-based formulation borrowed from the Stochastic Dynamic VRP literature (see, e.g., Ulmer et al. 2020). In particular, the authors used artificial routes to improve the estimation of future routing costs and introduced a time window budget approach to better evaluate the idle time of vehicles within the time windows. These features serve as an input for the online pricing problem, which is solved using an efficient heuristic algorithm. Computational experiments show that the performance of the route-based ADP approach with time window budget is superior compared not only to another ADP approach with waiting time (proposed by the same authors), but also to other policies adapted from the literature (among which the one by Yang and Strauss 2017). Such superiority is expressed both in terms of average profit and number of served customers. Another valuable change that the authors introduced in this work, compared to the previous literature, is represented by the use of a finite-mixture MNL model as the customer-choice model.

Following up on the works by Yang et al. (2016) and Yang and Strauss (2017), Abdollahi et al. (2023) presented a new dynamic pricing approach in which the opportunity cost estimation is based on a combination of actual orders with time windows and forecast orders without time windows. Interestingly, each time an incoming requests is accepted and inserted in a route, a forecast order is removed from that route and the underlying dynamic VRPTW is re-optimized to adjust the pricing offer for future requests. Compared to commonly used static pricing policies, the proposed approach performed better in terms of total profits, with an increase between 13.57% and 21.43%.

Motivated by the real-world application of a waste collection company operating in the United Kingdom, Keskin et al. (2023) presented a very interesting work on the dynamic multi-period VRP, in which the demand of customers is leveraged by the practice of touting (i.e., contacting customers who are expected to place an order soon and offering to anticipate it). The problem is defined as a dynamic program and solved with a rolling horizon algorithm, where touting decisions are taken based on specifically developed touting heuristics, while routing decisions are taken based on an LNS algorithm. The results obtained on real instances provided by the waste collection company show how the practice of touting (accompanied by efficient routing algorithms) can reduce the total distance traveled and the overall number of routes, while improving the utilization of available capacity.

4.2 The time window assignment vehicle routing problem

In this section, we survey a particular class of integrated demand management and routing problems, the TWAVRP, in which time windows must be assigned to customers before demand is known, followed by the creation of routing schedules that minimize the expected routing costs.

The TWAVRP was introduced for the first time in the paper of Spliet and Gabor (2015), where the authors presented a compact MILP formulation which considers multiple scenarios corresponding to different realizations of demand. In particular, they distinguished between exogenous and endogenous time windows to identify, respectively, time windows imposed by an external stakeholder and time windows agreed upon by the customer and supplier. To solve the problem, the authors proposed a Branch-Price-and-Cut (BPC) algorithm, in which the restricted primary problem is solved via column generation while the secondary pricing problem, an ESPPRC in which vehicle capacity and time windows are the resource constraints, is decomposed by scenario and solved using basic route relaxation techniques (i.e., allowing all cyclic routes but eliminating 2-cycle routes). An acceleration strategy and some valid inequalities were also proposed. The computational experiments proved that the proposed BPC algorithm can solve to optimality instances with up to 25 customers and 3 demand scenarios. Interestingly, the authors compared the results found by the BPC algorithm for the TWAVRP with those obtained by a heuristic procedure to solve the VRPTW with average demand (i.e., which corresponds to a one-scenario TWAVRP), showing that the routing costs of VRPTW solutions with average demand are on average 1.85% higher.

In a follow-up work, Spliet and Desaulniers (2015) defined the Discrete Time Window Assignment Vehicle Routing Problem (DTWAVRP), which differs from the TWAVRP in that a finite set of candidate time windows is given for each customer. Building upon their previous approach, the authors proposed an exact BPC algorithm, in which the secondary pricing problem is solved using the *ng*-route relaxation technique by Baldacci et al. (2011), with $\Delta_{ng} \in \{1, 5, n\}$. Also, five column generation heuristics (i.e., one restricted master heuristic, two diving heuristics, and two rounding heuristics) were developed. When solving the DTWAVRP using the exact BPC algorithm, the authors demonstrate how the configuration with $\Delta_{ng} = 5$ represents a good compromise between short computing times and solution quality if compared to the configurations allowing all cyclic routes (i.e., when $\Delta_{ng} = 1$) and elementary paths only (i.e., when $\Delta_{ng} = n$). The five column generation heuristics, in turn, proved to find solutions with relatively small gap to optimality (i.e., between 0.29% and 4.30%) for instances with up to 25 customers and 5 demand scenarios, while they were able to solve instances with up to 60 customers, although without proving optimality. Among them, the so-called TWDiving-Tabu heuristic produced the best results. Additional experiments were performed to compare a multiple-scenario TWDiving-Tabu heuristic with a single-scenario average demand based TWAVRP. These experiments confirmed the potential of the TWDiving-Tabu heuristic in generating solutions with lower expected routing costs as well as the advantage of considering multiple scenarios.

A novel formulation of the TWAVRP with time-dependent travel times was presented in the work of Spliet et al. (2018), where the authors developed an innovative labeling algorithm to solve the secondary pricing problem based on the contributions of Ioachim et al. (1998) and Feillet et al. (2004), and built upon the TS column generator originally proposed by Spliet and Desaulniers (2015). Also, new arc-synchronization inequalities were formulated to strengthen the BPC algorithm used to solve the problem.

In their paper, Dalmeijer and Spliet (2018) defined an alternative MILP formulation for the TWAVRP based on the two-commodity network flow approach for the Capacitated VRP by Baldacci et al. (2004) and the well-known MTZ-inequalities. The authors solved the problem via Branch-and-Cut (BC) with the addition of a tailored class of valid inequalities for the TWAVRP (i.e., the precedence inequalities) and the introduction of a new branching rule. The results show that the proposed BC algorithm clearly outperforms the BPC algorithm of Spliet and Gabor (2015) in terms of computing times and gap to optimality. More interestingly, the BC algorithm is able to solve to optimality larger instances with up to 35 customers and 3 scenarios, while showing small optimality gap for instances with up to 40 customers.

Starting from a real-world application and data provided by a large European food retailer, an extended version of the TWAVRP with product dependent time windows was studied by Neves-Moreira et al. (2018). The impact of realistic features like multi-product deliveries and fleet requirements (e.g., temperature at which products are kept during transportation and compatibility between vehicle and retail site capacities) were also investigated by the authors. To solve the problem, a three-phase approach consisting of (i) route generation, (ii) initial solution construction, and (iii) improvement matheuristic (see, e.g., Boschetti & Maniezzo 2022 for an overview of this topic) was developed. The benefit from considering multi-product deliveries, instead of single-product deliveries only, was confirmed by the computational experiments in which an average saving of 6.44% in terms of total routing costs was achieved thanks to the additional flexibility of multi-product deliveries. Furthermore, in line with the results obtained by Spliet and Gabor (2015), the authors demonstrate that a stochastic multiple-scenario approach is preferable to a deterministic single-scenario approach with average demand (with the former that outperforms the latter by 5.3% on average). Some useful managerial insights were also derived from a sensitivity analysis. In particular, the authors proved that further savings can be achieved by increasing the time window length and product flexibility (in terms of the minimum quantity of the main product that must be delivered in multi-product deliveries).

In their work, Subramanyam et al. (2018) took advantage from the similarities between the TWAVRP and the Consistent VRP (see, e.g., Kovacs et al. 2014 for an overview of this problem) to adapt the decomposition algorithm previously proposed by Subramanyam and Gounaris (2018) for the Consistent TSP. Such an algorithm turned out to outperform state-of-the-art solution methods both for the TWAVRP and the DTWAVRP, thus demonstrating a good efficiency and versatility in solving problem of this class.

In Vareias et al. (2019), a TWAVRP with stochastic travel times is solved with the goal of designing routes having minimum traveling distance and minimum earliness and lateness penalty costs due to time windows violation. The problem is solved by means of two mathematical models and an ALNS.

Building upon the work of Dalmeijer and Spliet (2018), Dalmeijer and Desaulniers (2021) introduced an edge-based branching method to eliminate orientation symmetry from the search tree of a BPC, and they presented enhancements to make this method efficient in practice. They consistently reduced the number of explored nodes and solved 25 TWAVRP benchmark instances to proven optimality for the first time.

A robust formulation of the TWAVRP for solving problems in which the probability distribution of travel and service times is partially unknown was presented in Hoogeboom et al. (2021). Their formulation is based on a time window violation index that measures the risk associated with the violation of the time windows assigned to destination nodes. This index is inspired by the Requirements Violation Index originally proposed by Jaillet et al. (2016). The problem was solved via BC and the results were compared with those obtained by a stochastic variant of the TWAVRP in which the probability distribution of travel times is known.

Starting from a real-world application and data provided by Ford Motor Company and comparing with the work of Vareias et al. (2019), Yu et al. (2023) addressed a particular TWAVRP under multiple sources of uncertainty (travel and service times, continuous time windows of variable length, and possible service cancellations by the customers). The problem is formulated as a 2-SP model and solved using "Assignment-Routing-Scheduling" heuristics.

Multiple sources of uncertainty (number of customers, customer location, customer demand, and service times) are also considered in the recent work of Côté et al. (2024), where the authors address a multi-period stochastic variant of the TWAVRP. The work is motivated by a real-world application arising at a Canadian retailer that sells and delivers large appliances and furniture in Edmonton and Calgary, and their respective surrounding areas. The problem is modeled through a 2-SP formulation and solved using a heuristic approach. In particular, the first-stage solution is obtained by applying the "sample average approximation method" found in Kleywegt et al. (2002), while the second-stage solution is obtained by means of an ALNS algorithm. Some speed-up techniques for the ALNS are also proposed. Interestingly, the proposed method was able to improve the company's solution, still manually computed, and provide meaningful insights from a practical perspective.

Extending the previous work by Ulmer and Thomas (2019), in which the focus was on predicting mean arrival times to customers, Ulmer et al. (2024) proposed a heuristic algorithm for determining narrow and reliable time windows to be communicated to customers when they request a service. Unlike many other works on the TWAVRP, the authors assume that routing and scheduling decisions are exogenous to the algorithm and only induce the distributions of possible arrival times to customers. With this information, they prove that it is possible to infer the expected time window size of an incoming request. The problem is formulated as a Non-Linear Stochastic Program, and a “Time Window” heuristic is developed to solve it. Particularly interesting managerial insights are derived from an extensive computational study.

5 Conclusions and future research directions

This work has provided a detailed literature review on the state of the art for Attended Home Delivery (AHD) and Attended Home Service (AHS) problems, a research field that is experiencing increasing attention, as confirmed by the fast-growing number of documents published each year on this class of problems. Given its strong practical relevance, a particular focus has been put on real-world applications with the purpose of gaining useful managerial insights. Indeed, AHD and AHS problems owe their popularity to the rapid diffusion of online platforms, where a particularly high demand is registered for e-grocery and online retail transactions.

Since the seminal works in this topic, an increased awareness of the multi-stage nature of AHD and AHS problems, where the decisions taken at the first level greatly affect the feasibility as well as the economic profitability of the decisions taken at the second level, has emerged. Demand management and routing are well-established research fields per se, but the integration of demand management and routing decisions represents the complex part of solving real-world AHD and AHS problems, as these decisions are affected by uncertainty.

Many authors have proposed several sophisticated methods to solve alternately demand management problems (where the information related to the routing subproblem is estimated or forecast) or routing problems (where the information related to the demand management subproblem is oversimplified and used as an input or, once again, forecast), but the search for a more effective integration of these two stages may represent one of the most significant future research directions in AHD and AHS.

In this sense, a promising approach may be that of using Dynamic Programming as the main framework, but great efforts are needed to overcome the issues of dimensionality and complexity of solving a Vehicle Routing Problem with Time Windows as the boundary condition for each state. An alternative approach may be that of borrowing some ideas from the Stochastic Dynamic Vehicle Routing Problem literature to roughly solve the online demand management problem by anticipating some routing aspects that must be fine-tuned offline.

The sustainability of AHD and AHS systems is another relevant topic having received little attention as compared to the wide literature on AHD and AHS problems. The recent work of Agatz et al. (2021) presents an interesting discussion on the effectiveness of using “green” incentives to steer customer choices, along with traditional price incentives, while Zhang et al. (2023) investigate the opportunity of integrating “green labels” with the practice of order consolidation (i.e., postponing some deliveries and consolidating the deliveries for the same customer). As sustainability may represent for AHD and AHS problems an additional objective, which may be conflicting with profit maximization or cost minimization, the benefit from introducing multi-criteria problem formulations is worth exploring. Also, further objectives may emerge and be considered in the future. For this reason, the introduction of Multi-Criteria Decision Analysis for solving AHD and AHS problems may represent another future research directions in this field.

Other interesting research directions may include the use of machine learning techniques to support online time slot decisions (see, e.g., van der Hagen et al. 2024), the extensive adoption of data science approaches to analyze large amounts of historical order data and, consequently, better understand the preferences of customers (see, e.g., Köhler et al. 2024), and the exploitation of opportunity sales to generate additional profits (see, e.g., Ötken et al. 2023).

Finally, we have seen that real-world AHD and AHS applications may be encountered in heterogeneous business sectors, although the problem at its core maintains a similar structure (with some exceptions). In upcoming years, we expect a denser transfer of ideas and technologies among different sectors as well as the emergence of innovative areas of application.

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Declarations

Conflict of interest The authors declare that they have no Conflict of interest.

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