



Multifractal characteristics and return predictability in the Chinese stock markets

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Abstract

By employing the Multifractal detrended fluctuation (MFDDFA) analysis methods, the multifractal nature is revealed in the high-frequency data of two typical indexes, the Shanghai Stock Exchange Composite 180 Index (SH180) and the Shenzhen Stock Exchange Composite Index (SZCI). It is found that there is a statistically significant relationship between excess returns and multifractal characteristics, which can be applied to forecast the returns. The in-sample and out-of-sample tests on the return predictability of multifractal characteristics indicate that the multifractal spectral width is a significant return predictor. Additional tests on the S&P 500 index, the exchange rate between Bitcoin and US dollar, four Chinese commodity futures, and the SH180 and SZCI in different sub-periods reveal that the predicting ability of multifractality is robust to the asset type and sample period. The underlying explanation is that multifractal characteristic width contains the information of market volatility and downside tail risk. Our results shed new lights on the application of multifractal nature in asset pricing.

Keywords Multifractal characteristics · Multifractal detrended fluctuation analysis · Return predictability

List of symbols

$p_m(t)$	The minutely price on minute t .
$r_m(t)$	The minutely return on minute t .
$r_d(d)$	The daily return on day d .
$r_f(d)$	The risk-free return on day d .
$r^*(t)$	The daily excess return on day d .

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α	The local singularity exponent.
$f(\alpha)$	The multifractal spectrum.
$\Delta\alpha$	The width of multifractal spectrum.
Δf	The difference between the proportion of the subset with the minimum probability measure and that with the maximum probability measure.
B	The asymmetry of multifractal spectrum.
R_{OS}^2	The percent reduction in mean square forecast error for the predictive regression forecast relative to the historical average benchmark forecast.
MSFE_Adj	The adjusted MSFE statistic, indicating the statistical significance of R_{OS}^2 .
U	The mean-variance utility function.
r^p	The return of the portfolio allocated between a risky asset and a risk-free asset by maximizing the mean-variance utility function U .
ω_t	The optimal portfolio weight for the risky asset.
ΔU	The utility gain between the portfolio based on the multifractal predicting model and the portfolio based on the historical mean model.
RV	The realized volatility.
VaR	The value at risk.
ES	The expected shortfall.

1 Introduction

As we know, stock returns exhibit nonlinear long memory behaviors, which correspond to the multifractal nature (Muzy et al., 2001; Calvet & Fisher, 2002; Jiang & Zhou, 2011). Two stylized facts in financial returns, including fat-tailed distribution and long range dependence, are considered as sources of multifractality (Zhou, 2009; Jiang & Zhou, 2008b; Zhou, 2012; Grahovac & Leonenko, 2014). Multifractal nature in returns makes the price dynamics deviate from the Brownian process, triggering the studies of applying multifractality on uncovering market efficiency (Wang & Wu, 2013; Liu et al., 2010), designing trading strategies (Dewandaru et al., 2015), constructing measures for predicting volatility (Wei & Wang, 2008; Chen & Wu, 2011; Wei et al., 2013; Chen et al., 2014), and to list a few. New theoretical models including multifractal random walk (MRW) (Bacry et al., 2001), the multifractal model of asset returns (MMAR) (Calvet & Fisher, 2002), and Markov-switching multifractal model (MSM) (Calvet & Fisher, 2001) have been proposed to replicate the multi-scaling price behaviors. In comparison of other competitive econometric models, these models can improve the performance of forecasting volatility (Calvet & Fisher, 2001, 2004; Duchon & Robert, 2012; Chuang et al., 2013; Lux et al., 2014; Nasr et al., 2016; Segnon et al., 2017; Wang et al., 2016), predicting financial duration (Chen et al., 2013; Žikeš et al., 2017), and estimating VaR (Batten et al., 2014; Lux & Kaizoji, 2007; Chuang et al., 2013; Lee et al., 2016; Lux et al., 2016; Herrera et al., 2017). The forecasting power of multifractality on volatility indicates that the multifractal characteristics in price dynamic could have a strong connection to the market risk. As high market risk is accompanied with high return, we could infer that strong multifractality in price dynamic will lead to high return. However, such inference is still lack of empirical evidence.

Return predictability has been received considerable research interests, because it can highlight the understanding of asset pricing in academy and improve the performance of stock investments in industry. However, the predictability of stock returns is still under controversy. On the one hand, Welch and Goyal (2008) perform both in-sample and out-of-sample tests

on return predictability with the factors from earlier academic researches and find that those factors are unstable or even spurious. On the other hand, recent researches reveal that the factors, including unexpected changes of oil prices (Casassus & Higuera, 2012), cash flow volatility (Narayan & Westerlund, 2014), aligned technical indicator (Lin, 2018), curvature of the oil futures curve (Chiang & Hughen, 2017), price-to-fundamental ratios (Lawrenz & Zorn, 2017), and daily internet search volume index (SVI) (Chronopoulos et al., 2018), can significantly and economically predict the excess return. However, the predictors lost the forecasting ability when they are published (McLean & Pontiff, 2016), supporting that the predictors usually exhibit time-varying predictability (Devpura et al., 2018).

Multifractality captures the nonlinear scaling behavior for all moments of stock returns and many empirical studies uncover that higher order moments of returns, like skewness and kurtosis (namely *ex ante* volatility), contain the information of future returns (Conrad et al., 2013). Chang et al. (2013) reveal that the market skewness risk premium is statistically and economically significant and cannot be explained by the common risk factors, such as market excess return, the size, book-to-market, momentum, market volatility factors, and firm characteristics. Furthermore, other forms of higher order moments, such as the higher order moments of conditional return distribution and the asymmetry of the cross-sectional distribution, are also good predictors of future returns (Perez-Quiros & Timmermann, 2001; Garcia et al., 2014). In emerging markets, the skewness is mostly positive and idiosyncratic, which can be applied to build an optimal portfolio to have sizeable certainty-equivalent gains (Ghysels et al., 2016). Hwang & Satchell (2001) found that the emerging markets can be better explained by the high-moment CAPMs, which consider the additional risks of skewness and kurtosis.

With the aim of testing whether multifractal characteristics can be used to predict future returns, this paper is organized as follows. Data and methods are given in Sect. 2. Section 3 presents the results of empirical multifractality. The in-sample and out-of-sample tests on return predictability based on multifractality are elaborated in Sect. 4. The robustness tests are given in Sect. 5. Section 6 explains the return predictability of multifractal characteristics. And Sect. 7 concludes.

2 Data and methods

2.1 Data sets

Our data, including the Shanghai Stock Exchange Composite 180 Index (SH180) and the Shenzhen Stock Exchange Composite Index (SZCI) in the Chinese stock markets, are retrieved from the finance database of Resset (<http://www.resset.cn>). Both indexes cover a period from February 14, 2003 to May 23, 2022 including 4583 trading days in total. By removing the days having recording errors, we left 4569 days for SH180 and 4579 days for SZCI, respectively. There are four trading hours (240 min) on each trading day. For each index, we have the price p_m at each minute on each trading day and thus we define the minutely return $r_m(t)$ as,

$$r_m(t) = \ln p_m(t) - \ln p_m(t - 1). \quad (1)$$

We regard the last price on each trading day as the closing price $p_c(d)$ on that day and the daily return r_d is defined in the following,

$$r_d(d) = \ln p_c(d) - \ln p_c(d - 1). \quad (2)$$

2.2 Multifractal detrended fluctuation analysis (MFDFA)

For a given window of minutely returns $r_m(i)$, $i = 1, \dots, N$, we can define $y(i)$ as follows,

$$y(i) = \sum_{u=1}^i r_m(i), \quad i = 1, 2, \dots, N. \quad (3)$$

The series y is covered by N_s disjoint boxes and each box has the same size s . For our convenience, we label the sub-series in each box as,

$$Y_k(i) = \{y(i) \mid (k-1)s + 1 \leq i \leq ks\}, \quad (4)$$

In some cases, the whole series y cannot be exactly covered by N_s boxes, which means that we have to neglect some data points at the end of the series. In order to avoid this situation, we can utilize $2N_s$ boxes to cover the series, where N_s boxes cover from the beginning and N_s boxes cover from the end. In each box, the sub-series Y_k is regressed by a polynomial $g_l(\cdot)$ of order l (in our work $l = 1$). The overall detrended fluctuation $F_q(s)$ of the sub-series Y_k is defined via the sample variance of the fitting residuals as follows,

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{k=1}^{2N_s} [F_k(s)]^q \right\}^{1/q}. \quad (5)$$

where q can take any real value except for $q = 0$. While $q = 0$, we have

$$F_0(s) = \exp \left\{ \frac{1}{2N_s} \sum_{k=1}^{2N_s} \ln[F_k(s)] \right\}. \quad (6)$$

according to the L'Hôpital's rule. By varying the value of s in the range from $s_{\min} = 20$ to $s_{\max} = N/4$, one can expect that the detrended fluctuation function $F_q(s)$ scales with the size s , which reads

$$F_q(s) \sim s^{h(q)}, \quad (7)$$

where $h(q)$ is the generalized Hurst index. Note that while $q = 2$, $h(2)$ is nothing but Hurst index H . The scaling exponents $\tau(q)$, which is used to reveal the multifractality in the standard multifractal formalism based on the partition function, can be obtained from the following traditional function for each q ,

$$\tau(q) = qh(q) - D_f, \quad (8)$$

where D_f is the fractal dimension of the geometric support of the multifractal measure (in our case $D_f = 1$). The local singularity exponent α of the measure μ and its spectrum $f(\alpha)$ are related to $\tau(q)$ through the Legendre transformation (Halsey et al., 1986),

$$\begin{cases} \alpha = d\tau(q)/dq \\ f(\alpha) = q\alpha - \tau(q) \end{cases}. \quad (9)$$

Taking into account the statistical significance of the estimation of overall fluctuation functions, we focus on $q \in [-4, 8]$.

We further employ three parameters ($\Delta\alpha$, Δf , and B) to capture the characteristics of the multifractal spectrum. Parameter $\Delta\alpha$ stands for the width of multifractal spectrum, defined as $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$. $\Delta\alpha$ quantitatively describes the dispersion of singularity exponents α , and measures the degree of heterogeneity for the probability measure of subsets on the

overall fractal structure (Zhou, 2007). In practice, $\Delta\alpha$ is widely used to gauge the degree of multifractality (Jiang & Zhou, 2008a, b; Zhou, 2009). The larger the value of $\Delta\alpha$ is, the stronger the multifractal nature is. Parameter Δf is estimated via $\Delta f = f(\alpha_{\min}) - f(\alpha_{\max})$, depicting the difference between the proportion of the subset with the minimum probability measure and that with the maximum probability measure. Thus, more measures at the peak leads to $\Delta f < 0$ and more measures at the trough gives $\Delta f > 0$. Parameter B is obtained by fitting the $f(\alpha) \sim \alpha$ curve to the following quadratic function $f(\alpha) = A(\alpha - \alpha_0)^2 + B(\alpha - \alpha_0) + C$ (Shimizu et al., 2002; Munõz-Diosdado & Ríó-Correa, 2006). B captures the asymmetry of the spectrum. The spectrum curve are symmetric when $B = 0$. $B < 0$ means that the spectrum curve is right-hooked, indicating that the data set is dominated by the subsets with large probability measures. In contrast, $B > 0$ means that the spectrum exhibits a left-hooked pattern, suggesting that the subsets with small probability measures take the leading role in the data set.

The shape of the multifractal spectrum and the definition of Δf and B underlie a positive correlation between Δf and B . When $B > 0$ (respectively, $B < 0$), the multifractal spectrum is left-hooked (respectively, right-hooked), which means $f(\alpha_{\min}) > f(\alpha_{\max})$ (respectively, $f(\alpha_{\min}) < f(\alpha_{\max})$), and then we have $\Delta f > 0$ (respectively, $\Delta f < 0$). When B approaches 0, the multifractal spectrum gradually becomes symmetric, which leads to $f(\alpha_{\min}) \approx f(\alpha_{\max})$ and we will obtain $\Delta f \approx 0$. However, Δf and B have very different physical meaning. Δf measures the proportion difference between the minimum measure and the maximum measure, while B indicates which type of measures plays a leading role in the data sets, large measures or small measures.

3 Empirical multifractal characteristics

Using a moving window with a size of 5 days, we perform the multifractal analysis on the returns in each window by means of the MFDFA method. To have an impression that the multifractal spectrum is able to quantitatively capture the market dynamics, we present the results of multifractal analysis on three typical price trajectories, corresponding to the declining, rising, and sideways trend, in three windows for SH 180. Window 1 covers a period from 7 April, 2004 to 13 April, 2004. As shown in Fig. 1 a, b, and c, in window 1, we observe that the price exhibits an decreasing pattern and contains many small returns, which leads to a left-hooked multifractal spectrum. Window 2 spans a period from 16 January, 2004 to 3 February, 2004 and in that window the price has an increasing pattern and the return is dominated by large values, which results in a right-hooked multifractal spectrum, as illustrated in Fig. 1 d, e, and f. Window 3 is from 6 December, 2004 to 10 December, 2004 and the corresponding results are plotted in Fig. 1 g, h, and i. Due to the fact that the price is in a sideways trend and the fraction of large and small returns are almost equal, one can observe a symmetric multifractal spectrum.

We also estimate the three characteristic parameters ($\Delta\alpha$, Δf , B) of multifractal spectrum in the three windows and obtain $\Delta\alpha = 0.33$, $\Delta f = 0.21$, and $B = 0.20$ for window 1, $\Delta\alpha = 0.20$, $\Delta f = -0.13$, and $B = -0.18$ for window 2, and $\Delta\alpha = 0.11$, $\Delta f = 0.01$, and $B = -0.003$ for window 3, respectively. For the width of multifractal spectra, we have $\Delta\alpha^{\text{Window1}} > \Delta\alpha^{\text{Window2}} > \Delta\alpha^{\text{Window3}}$, which is in consistent with the observation that the price in window 1 is the most volatile and the price in window 3 is the least volatile. This indicates that the spectral width $\Delta\alpha$ can be used to quantitatively capture the market volatility. Wei and Wang (2008) have proposed a volatility measure based on the width of

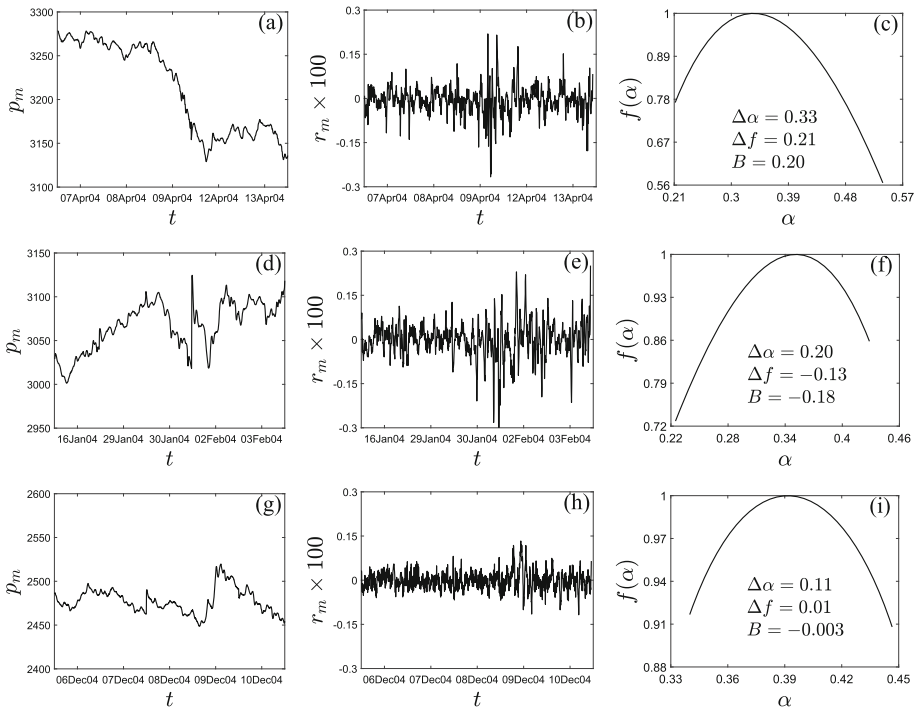


Fig. 1 Results of multifractal analysis on SH180 in three windows, in which the prices exhibit a declining, rising, and sideways trend, respectively. **a–c** Results of window 1, spanning from 7 April 2004 to 13 April 2004. **d–f** Results of window 2, spanning from 16 January 2004 to 3 February 2004. **g–i** Results of window 3, spanning from 6 December 2004 to 10 December 2004. **a, d, g** Plots of the price trajectories. **b, e, h** Plots of returns. **c, f, i** Plots of the multifractal spectrum

multifractal spectrum and achieve better VaR measures comparing with the GARCH-type models (Wei et al., 2013). We also find that the values of Δf and B in these three windows are in consistent with the geometric features of their multifractal spectra.

By performing the multifractal analysis on the returns in each moving window, we will accumulate three series of multifractal characteristics ($\Delta\alpha$, Δf , and B). Table 1 lists the basic information of cumulative return ($\sum r_m$) and multifractal characteristics ($\Delta\alpha$, Δf , and B) for SH180 and SZCI. In Panel A, one can see that the skewness is positive and the kurtosis is much greater than 3 for $\sum r_m$, $\Delta\alpha$, Δf , and B , indicating that the four variables exhibit a right-skewed and fat-tailed distribution. It is also observed that for $\Delta\alpha$ and Δf the gap between the mean and the median are very small and for B the mean is less than the median, indicating that the extreme values in $\Delta\alpha$, Δf , and B have negligible effects on their mean values.

In Panel B of Table 1, one can find that there is no correlation between the daily returns r_d on the day exactly after the moving windows and three multifractal parameters ($\Delta\alpha$, Δf , and B), as their correlation coefficients are very close to 0 and none of them is significant. The cumulative returns in moving windows $\sum r_m$ are found to be negatively and weakly correlated with $\Delta\alpha$ for SH180 and significantly correlated with Δf for both indexes. The correlation of $\Delta\alpha$ and Δf is weak, positive, and significant for both indexes, because both coefficients are around 0.05. We also see that Δf and B are significantly positive correlated

and the correlation coefficients are 0.14 and 0.13 for SH180 and SZCI, agreeing with the underlying correlation between Δf and B inferring from their definitions and the geometric features of multifractal spectrum.

In Panel C of Table 1, we find that $\Delta\alpha$ and Δf exhibit very strong autocorrelating behaviors, since their autocorrelation coefficients of lags 1 and 5 are positive, large, and significant. The autocorrelation coefficient of cumulative returns $\sum r_m$ in moving windows is around 0.78 at lag 1 for both indexes and quickly fall to 0 with the increasing of lag, as the autocorrelation coefficient of lag 5 approaches 0. The autocorrelation in B is weak, because only the coefficient at lag 1 is significantly positive and the values are less than 0.1 for both indexes. In addition, the Ljung and Box Q statistics of lags 30 and 50 are statistically significant for the four variables, showing that the null hypothesis of no autocorrelation is rejected at the 1% level. Such results imply the existence of autocorrelation in $\sum r_m$, $\Delta\alpha$, Δf , and B , which can be attributed to the fact that the four variables are estimated from overlapping moving windows.

We report the results of augmented Dickey-Fuller (ADF) unit root tests and ARCH tests in Panels D and E of Table 1. For the ADF unit root test, the optimal lag length is determined according to the Schwarz information criterion. For both indexes, the ADF statistics suggest that the null hypothesis of having a unit root in the series of $\sum r_m$, $\Delta\alpha$, Δf , and B is rejected at the 1% level, indicating the stationarity of the four series. Panel F lists the F -statistics of the ARCH tests at lags 1, 5, 10 and 15, suggesting that the null hypothesis of no ARCH effects is rejected at any level of significance. The results clearly speak for the presence of heteroscedasticity in the series of $\sum r_m$, $\Delta\alpha$, Δf , and B for both indexes.

4 Predictive power of multifractal characteristics

Practitioners and researchers have put great efforts on predicting future returns. One natural thought is to use the information in the option market, since it reflects the expectation of the market participants, to predict the future price movements. Bu et al. (2019) employ the VIX volatility curve to forecast future stock returns in developed markets. And the variance risk premium is also regarded as a potential predictor for returns (Pyun, 2019). Factors such as time-varying consumption-aggregate wealth ratio (Chang et al., 2019), (time-varying) tail risk dynamics (Chevapatrakul et al., 2019) are found to exhibit certain predictive ability on the excess returns. In emerging markets, Chang et al. (2009) found that foreign capital flows in the option market can be used to predict the underlying asset returns in Taiwan. It is found that simple technical trading rules can yield positive excess returns even when the trading costs is considered, showing the evidence of predictability (Chang et al., 2004; Gunasekarage & Power, 2001).

4.1 In-sample tests

The daily excess return r^* on day d is defined as the difference between the index return r_d and the market risk-free return r_f ,

$$r^*(d) = r_d(d) - r_f(d). \quad (10)$$

And the corresponding multifractal characteristics on day d , denoted as $\Delta\alpha_{d-4:d}$, $\Delta f_{d-4:d}$, and $B_{d-4:d}$, are estimated from day $(d - 4)$ to day d . We first separate the multifractal characteristic $\Delta\alpha_{d-4:d}$ into six groups based on the following bins $(-\infty, 0.05)$, $[0.05, 0.1]$,

Table 1 Basic information of the cumulative return ($\sum r_m$) and multifractal characteristics ($\Delta\alpha$, Δf , and B) in moving windows for SH180 and SZCI.

	SH180			SZCI				
	$\sum r_m$	$\Delta\alpha$	Δf	B	$\sum r_m$	$\Delta\alpha$	Δf	B
<i>Panel A: Basic statistics</i>								
mean $\times 100$	-0.25	17.09	4.31	5.20	-0.29	15.33	2.17	3.08
median $\times 100$	-0.24	16.39	4.08	7.51	-0.32	14.79	2.37	4.60
max	0.24	0.53	0.73	7.32	0.26	0.68	0.78	7.13
min	-0.19	0.01	-0.40	-4.61	-0.17	0.01	-0.42	-4.72
stdev	0.04	0.07	0.11	0.46	0.04	0.07	0.10	0.45
skew	0.27	0.53	0.24	4.25	0.37	1.15	0.48	5.51
kurt	5.09	3.28	5.90	114.61	5.15	8.21	8.44	117.43
<i>Panel B: Correlations</i>								
r_d		0.00	0.00	0.01		0.01	0.01	0.02
$\sum r_m$		-0.07***	0.04**	0.01		-0.02	0.06***	0.01
$\Delta\alpha$			0.07***	-0.01*			0.05**	-0.02
Δf				0.14***				0.13***
<i>Panel C: Autocorrelations</i>								
rho(1)	0.78***	0.83***	0.70***	0.07***	0.79***	0.78***	0.69***	0.09***
rho(5)	0.02	0.54***	0.27***	-0.02	0.05**	0.34***	0.26***	0.03

Table 1 continued

SH180		SZCI						
	$\sum r_m$	$\Delta\alpha$	Δf	B	$\sum r_m$	$\Delta\alpha$	Δf	B
Q(30)	3613.82***	25486.88***	6554.78***	149.94***	3703.75***	11416.35***	6347.66***	209.23***
Q(50)	3844.04***	37193.02***	8025.57***	216.63***	4018.37***	15567.39***	7442.48***	233.54***
<i>Panel D: Unit root tests</i>								
Optimal lag	5.	8.	6.	1.	8.	7.	7.	0.
ADF Statistics	-15.56***	-2.73***	-10.89***	-39.77***	-16.03***	-3.39***	-11.05***	-50.04***
<i>Panel E: ARCH tests</i>								
ARCH(1)	1177.57***	1291.75***	985.26***	119.44***	1302.25***	1821.52***	1262.24***	185.25***
ARCH(5)	1187.25***	1308.76***	997.97***	143.07***	1308.93***	1851.50***	1323.53***	200.29***
ARCH(10)	1189.58***	1334.95***	1007.71***	143.90***	1312.23***	1883.95***	1334.45***	290.55***
ARCH(15)	1191.27***	1343.38***	1009.96***	145.21***	1326.41***	1887.04***	1339.97***	299.99***

Panel A reports mean, median, maximum (max), minimum (min), standard deviation (stdev), skewness (skew), and kurtosis (kurt) of four variables. Panel B lists the correlation coefficients between pairs of variables, r_{ij} (daily returns on the day exactly after the moving windows), $\sum r_m$, $\Delta\alpha$, Δf , and B in moving windows. The autocorrelation coefficients of lag 1 and 5 and the Ljung-Box Q tests of lag 30 and 50 are presented in Panel C. The ADF unit root tests and ARCH tests are listed in Panels D and E. Note that *, **, and *** indicate the significant level of 5%, 1%, and 0.1%, respectively

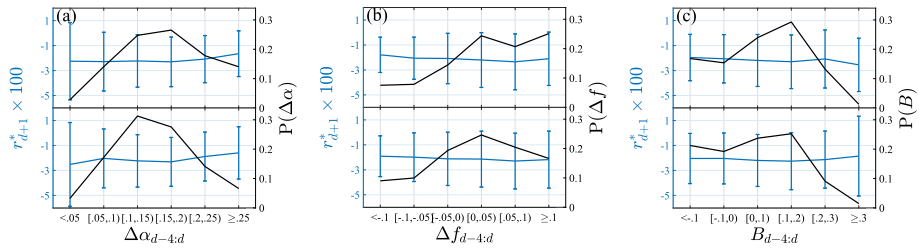


Fig. 2 Illustration of the predictive power of multifractal characteristics ($\Delta\alpha$, Δf , and B) on the future excess returns for SH180 (top panels) and SZCI (bottom panels). **a** Plots of the average excess returns with respect to the average $\Delta\alpha$ on the left y-axis and the fraction of $\Delta\alpha$ in each bin on the right y-axis. **b** The same as **a**, but for Δf . **c** The same as **a**, but for B

Table 2 Results of Granger causality tests between excess returns r^* and multifractal characteristics

Index	$r^* \rightarrow \Delta\alpha$	$\Delta\alpha \rightarrow r^*$	$r^* \rightarrow \Delta f$	$\Delta f \rightarrow r^*$	$r^* \rightarrow B$	$B \rightarrow r^*$
SH180	4.53*	17.42***	0.13	0.39	6.04*	0.10
SZCI	1.12	5.41*	0.50	1.24	8.00**	1.06

$\Delta\alpha$, Δf , and B . $x \rightarrow y$ indicates the null hypothesis that x does not Granger cause y . Note that *, **, and *** indicate the significant level of 5%, 1%, and 0.1%, respectively

[0.1, 0.15), [0.15, 0.2), [0.2, 0.25), and [0.25, ∞). In each group, we calculate the average value of $\Delta\alpha_{d-4:d}$ and the average value of excess returns r_{d+1}^* on day $d + 1$. In Fig. 2 a, we illustrate the errorbar plots of the average excess return $\langle r_{d+1}^* \rangle$ with respect to the average spectral width $\Delta\alpha_{d-4:d}$ on the left y-axis and the fraction of the spectral width $\Delta\alpha_{d-4:d}$ in each bin on the right y-axis. The top panel is the results of SH180 and the bottom panel is the results of SZCI. One can see that there is a slightly increasing trend for $\langle r_{d+1}^* \rangle$ with the increasing of $\langle \Delta\alpha_{d-4:d} \rangle$ for both indexes, indicating the predicting ability of $\Delta\alpha$ on future excess returns. For Δf and B , we perform the same analysis. The corresponding results are shown in Fig. 2 b, c. One can observe a slightly decreasing trend for $\langle r_{d+1}^* \rangle$ with the increasing of $\langle \Delta f_{d-4:d} \rangle$ for both indexes, implying that there exists to be a negative correlation between Δf and future excess returns. However, we cannot find any dependence between $\langle B \rangle$ and $\langle r_{d+1}^* \rangle$, suggesting no information of future excess returns in B .

We further conduct the Granger causality tests between the excess returns r^* and the multifractal characteristics $\Delta\alpha$, Δf , and B . The corresponding null hypothesis of $x \rightarrow y$ is that x does not Granger cause y . The results are listed in Table 2. One can see that the excess return r^* is the Granger causality of $\Delta\alpha$ and B for SH180 at the level of 5% and of B for SZCI at the level of 1%. The null hypothesis that $\Delta\alpha$ does not Granger cause r^* is rejected for SH180 (respectively, SZCI) at the significant level of 0.1% (5%), indicating that the lagged $\Delta\alpha$ may contain the information of future excess returns r^* . For r^* and Δf , the Granger tests cannot reject the null hypothesis, indicating no Granger causality between them.

A standard univariate predictive regression framework is employed to test the predictive power of multifractal characteristics $M_{d-4:d}$ from day $d - 4$ to day d on the excess return on day $d + 1$,

$$r_{d+1}^* = \alpha_r + \beta_{r,i} M_{i,d-4:d} + \varepsilon_d, d = 5, \dots, T \quad (11)$$

where $M_1 = \Delta\alpha$, $M_2 = \Delta f$, $M_3 = B$, $M_4 = \Delta\alpha\Delta f$, $M_5 = \Delta\alpha B$, and $M_6 = \Delta f B$. The corresponding results of in-sample tests are shown in Table 3 for both indexes, in which

the regression slopes, intercepts, and the adjusted R^2 statistics are reported. The p -values, which are obtained from the NW t -tests (Newey & West, 1987), are also listed in the parentheses under the regression parameters. The null hypothesis of the NW t -tests is that the regressing slopes ($\beta_{r,i}$) are vanishing. If the null hypothesis is rejected for $\beta_{r,i}$, we can conclude that multifractal characteristic M_i contains information of predicting future excess return r_{d+1}^* . And accepting null hypothesis means that multifractal characteristic M_i is not a return predictor.

In panel A of Table 3, we list the results of in-sample tests for SH180. One can find that except $\Delta\alpha$, Δf , and ΔfB the other multifractal characteristics give insignificant regression coefficients. The regression coefficient of $\Delta\alpha$ is significant at the level of 1% and the adjusted R^2 statistics is 0.19%, meaning that $\Delta\alpha$ can explain 0.19% of the excess returns. In Panel B, we report the regressing results of in-sample tests for SZCI. One can observe that the regressing coefficients of $\Delta\alpha$, Δf , $\Delta\alpha \Delta f$, and ΔfB are statistically significant. It is also observed that the regression coefficients of other multifractal characteristics are not statistically significant and the adjusted R^2 are vanishing. Usually, $\Delta\alpha$ can be regarded as a measurement of volatility. The predicting ability of $\Delta\alpha$ can be linked to the market risk. Higher market risk, associating with high market volatility, results in higher risk premium (Merton, 1980; French et al., 1987).

We further run the in-sample regression by incorporating the market volatility, such that

$$r_{d+1}^* = \alpha_r + \beta_{r,i} M_{i,d-4:d} + \psi_r v_{d-4:d} + \varepsilon_d, d = 5, \dots, T \quad (12)$$

where $v_{d-4:d}$ is the realized market volatility from day $d-4$ to d , given by summing the square of minutely returns. The regression results of Eq. (12) are listed in Table 4. It is observed that the results in Table 4 are in accordance with those in Table 3. We find that the coefficients of $\Delta\alpha$ are statistically significant for both indexes when adding the market volatility into the regression, revealing that the multifractal characteristic $\Delta\alpha$ do have additional information beyond the market volatility.

4.2 Out-of-sample tests

Out-of-sample tests are usually encouraged to evaluate the return predictability by excluding the using of future information and over-fitting in-sample tests. Two statistics, the R_{OS}^2 statistic (Campbell & Thompson, 2008) and adjusted MSFE statistic (Clark & West, 2007), are employed to assess the out-of-sample forecasting performance. The R_{OS}^2 statistic is defined as follows,

$$R_{OS}^2 = 1 - \frac{\sum_{d=n}^{T-1} (r_{d+1}^* - \hat{r}_{d+1}^*)^2}{\sum_{d=n}^{T-1} (r_{d+1}^* - \bar{r}_{d+1}^*)^2} \quad (13)$$

where r_{d+1}^* is the actual excess return, \hat{r}_{d+1}^* is the predicting excess return, and \bar{r}_{d+1}^* is the benchmark of historical average returns. R_{OS}^2 measures the percent reduction in mean square forecast error for the predictive regression forecast relative to the historical average benchmark forecast. According to the definition, we can infer that R_{OS}^2 locates in the range of $(\infty, 1]$. The predictive regression forecast \hat{r}_{d+1}^* is better than the historical average \bar{r}_{d+1}^* from the perspective of mean squared forecasting errors (MSFE) when $R_{OS}^2 > 0$. The adjusted MSFE statistic, proposed by Clark and West (2007), is used to indicate the statistical significance of $R_{OS}^2 > 0$ (Neely et al., 2014; Phan et al., 2015; Guo & Tao, 2017; Chen et

Table 3 In-sample tests of predicting the excess return r_{d+1}^* on day $d + 1$ with multifractal characteristics $M_{d-4:d}$ from day $d - 4$ to day d

$M_{t,d-4:d}$	$\beta_{r,1}$	$\beta_{r,2}$	$\beta_{r,3}$	$\beta_{r,4}$	$\beta_{r,5}$	$\beta_{r,6}$	α_r	R^2
<i>Panel A: SH180</i>								
$\Delta\alpha$	0.0041** (0.0014)						0.0001 (0.0002)	0.0019
Δf		-0.0011* (0.0004)					-0.0005*** (0.0001)	0.0003
B			0.0001 (0.0001)				-0.0005*** (0.0001)	0.0001
$\Delta\alpha\Delta f$				0.0003 (0.00035)			-0.0005*** (0.0001)	0.0000
$\Delta\alpha B$					0.0014 (0.0012)		-0.0005*** (0.0001)	0.0003
$\Delta f B$						-0.0014* (0.0006)	-0.0005*** (0.0001)	0.0013
<i>Panel B: SZCI</i>								
$\Delta\alpha$	0.0023** (0.0007)						-0.0009*** (0.0002)	0.0005
Δf		-0.0016* (0.0008)					-0.0005*** (0.0001)	0.0006

Table 3 continued

$M_{i,d-4;d}$	$\beta_{r,1}$	$\beta_{r,2}$	$\beta_{r,3}$	$\beta_{r,4}$	$\beta_{r,5}$	$\beta_{r,6}$	α_r	R^2
B			0.0001 (0.0001)				-0.0005*** (0.0001)	0.0002
$\Delta\alpha\Delta f$				0.0149*** (0.0035)			-0.0006*** (0.0001)	0.0041
$\Delta\alpha B$					-0.0021 (0.0012)		-0.0005*** (0.0001)	0.0006
$\Delta f B$						-0.0039*** (0.0010)	-0.0006*** (0.0001)	0.0032

The regression model is $r_{d+1}^* = \alpha_r + \beta_{r,i} M_{i,d-4;d} + \varepsilon_d$, where $M_1 = \Delta\alpha$, $M_2 = \Delta f$, $M_3 = B$, $M_4 = \Delta\alpha\Delta f$, $M_5 = \Delta\alpha B$, and $M_6 = \Delta f B$. In each panel, we list the regression coefficient β , the intercept α , and the adjusted R^2 statistic. Note that *, **, and *** indicate the significant level of 5%, 1%, and 0.1%, respectively. Numbers in parentheses are standard deviations

Table 4 Comparison of the predictive ability between multifractal characteristic and market volatility

$M_{i,d-4,d}$	ψ_r	$\beta_{r,1}$	$\beta_{r,2}$	$\beta_{r,3}$	$\beta_{r,4}$	$\beta_{r,5}$	$\beta_{r,6}$	α_r	R^2
<i>Panel A: SH180</i>									
$\Delta\alpha$	0.0358*** (0.0099)	0.0042** (0.0014)						-0.0001 (0.0002)	0.0046
Δf	0.0376*** (0.0100)		-0.0016 (0.0009)					-0.0007*** (0.0001)	0.0034
B	0.0342*** (0.0100)			0.0001 (0.0009)				-0.0007*** (0.0001)	0.0034
$\Delta\alpha * \Delta f$	0.0349*** (0.0100)				-0.0015 (0.0036)			-0.0007*** (0.0001)	0.0027
$\Delta\alpha * B$	0.0339*** (0.0099)					0.0012 (0.0012)		-0.0007*** (0.0001)	0.0028
$\Delta f * B$	0.0347*** (0.0099)						-0.0015* (0.0006)	-0.0008*** (0.0001)	0.0040
<i>Panel B: SZCI</i>									
$\Delta\alpha$	0.0767*** (0.0092)	0.0007** (0.0002)						-0.0012*** (0.0002)	0.0157
Δf	0.0823***		-0.0032**					-0.0012***	0.0177

Table 4 continued

$M_{i,d-4;d}$	ψ_r	$\beta_{r,1}$	$\beta_{r,2}$	$\beta_{r,3}$	$\beta_{r,4}$	$\beta_{r,5}$	$\beta_{r,6}$	α_r	R^2
B	(0.0092) 0.0771*** (0.0091)		(0.0010)	0.0001 (0.0001)				(0.0001) -0.0011*** (0.0001)	0.0157
$\Delta\alpha * \Delta f$	0.0719*** (0.0094)				0.0076* (0.0036)			-0.0011*** (0.0001)	0.0166
$\Delta\alpha * B$	0.0777*** (0.0091)					-0.0023 (0.0012)		-0.0011*** (0.0001)	0.0164
$\Delta f * B$	0.0778*** (0.0091)						-0.0040*** (0.0010)	-0.0012*** (0.0001)	0.0190

The regression model are $r_{d+1}^* = \alpha_r + \beta_{r,i} M_{i,d-4;d} + \psi_r v_{d-4;d} + \varepsilon_d$, where $M_1 = \Delta\alpha$, $M_2 = \Delta f$, $M_3 = B$, $M_4 = \Delta\alpha\Delta f$, $M_5 = \Delta\alpha B$, and $M_6 = \Delta f B$. In each panel, we list regression the coefficients of β and ψ , the intercept α , and the adjusted R^2 statistics. Note that *, **, and *** indicate the significant level of 5%, 1%, and 0.1%, respectively. Numbers in parentheses are standard deviations

al., 2017). The corresponding null hypothesis is $R_{OS}^2 \leq 0$, meaning that the MSFE of the historical average benchmark is less than or equal to that of predictive regression forecast.

Our out-of-sample tests are conducted in both moving windows and expanding windows. Here is the detailed procedure. The predictive regression (Eq. (11)) is estimated in each window and the obtained parameters are then used to generate the out-of-sample prediction for the day following that window. These steps are repeated by expanding or sliding the window till one reaches the end of sample period. The initial window includes the first 600 data points, about 2.5 years of data. The results are reported in Table 5. Panel A lists the results of the out-of-sample tests for SH180, one can find that $\Delta\alpha$, Δf , B and $\Delta\alpha B$ have positive R_{OS}^2 in both moving and expanding windows, indicating that the predictive regression forecast is superior to the historical average, and the other predictors have $R_{OS}^2 < 0$. According to the adjusted MSFE statistics, we find that the R_{OS}^2 not greater than 0 is rejected for $\Delta\alpha$ and Δf at the significant level of 0.1% and 5% in the moving window and the expanding window. The results of the out-of-sample tests for SZCI are reported in Panel B of Table 5. One can see that R_{OS}^2 corresponding to the predictors B and $\Delta\alpha B$ are negative and the other predictors all have positive R_{OS}^2 in the moving window and expanding window. However, only $\Delta\alpha$ and Δf are statistically significant according to the adjusted MSFE statistics both in the moving window tests and Δf in the expanding window tests. We also find that including more data points into the predictive regression forecast will shrink the predictive ability of predictors, evidenced by lower R_{OS}^2 in expanding window tests comparing with that in moving window tests. The out-of-sample tests reveal that the multifractal characteristics $\Delta\alpha$ and Δf are good return predictors.

Following Campbell and Thompson (2008); Rapach & Zhou (2013); Wang et al. (2018), we employ the mean-variance utility function to test the economic significance of the return predictability based on multifractality. Such utility-based method treats the predictions as inputs of a specific trading rule or an asset allocation decision derived from maximizing the expected utility. Supposing that a mean-variance investor with relative risk aversion γ allocates his portfolio between a risky asset and a risk-free asset with the purpose of maximizing the following utility function,

$$U = \max \left[E_t (r_{t+1}^p) - \frac{\gamma}{2} \text{Var}_t (r_{t+1}^p) \right] \quad (14)$$

where r_{t+1}^p represents the return of the portfolio and γ measures the risk aversion. The portfolio returns r_{t+1}^p can be estimated as follows,

$$r_{t+1}^p = r_{t+1}^f + \omega_t r_{t+1}, \quad (15)$$

where ω_t corresponding to the optimal portfolio weight for the risky asset by solving Eq. (14),

$$\omega_t = \frac{1}{\gamma} \left(\frac{\hat{r}_{t+1}}{\hat{\sigma}_{r,t+1}^2} \right) \quad (16)$$

where \hat{r}_{t+1} is the expected excess returns, giving by the forecasting model and $\hat{\sigma}_{t+1}^2$ is the variance of excess returns. Once we have the portfolio r_t^p at different time t , we are able to calculate the utility U through Eq. (14).

By setting $\gamma = 5$ (for $\gamma = 3$ very similar results are obtained), we estimate the utility \hat{U} of the portfolio constructed based on the excess returns given by our return forecasting model, Eq. (11). For comparison, we also calculate the utility of the portfolio \bar{U} obtained from a benchmark model, in which the excess returns are given by the historical mean returns. The utility gains ΔU is defined as the difference between the utility of two portfolios, that is

Table 5 Out-of-sample tests of predicting the excess index return r_{d+1}^* on day $d + 1$ with multifractal characteristics $M_{d-4,d}$ from day $d - 4$ to day d

	Moving window			Expanding window		
	R_{OS}^2 (%)	MSFE_adj	ΔU (%)	R_{OS}^2 (%)	MSFE_adj	ΔU (%)
<i>Panel A: SH180</i>						
$\Delta\alpha$	0.63	3.01***	0.0052	0.05	1.50***	0.0002
Δf	0.26	-0.12*	0.0014	0.11	0.97*	0.0001
B	0.63	0.98	0.0018	0.25	-0.54	0.0001
$\Delta\alpha\Delta f$	-0.87	-0.54	0.0014	-0.13	0.06	0.0000
$\Delta\alpha B$	0.48	0.81	0.0024	0.34	-0.41	0.0001
$\Delta f B$	-1.66	0.28	0.1096	-0.15	0.04	0.0036
<i>Panel B: SZCI</i>						
$\Delta\alpha$	0.24	2.41*	0.0030	0.16	0.37*	0.0002
Δf	0.01	1.82**	0.0037	0.11	2.62**	0.0004
B	0.93	-0.71	0.0019	0.27	-0.15	0.0002
$\Delta\alpha\Delta f$	0.68	0.27	0.0036	0.09	1.08	0.0005
$\Delta\alpha B$	0.47	-0.36	0.0021	0.60	0.38	0.0002
$\Delta f B$	-0.62	0.60*	0.0325	-0.07	0.22	0.0089

This table lists the statistics of R_{OS}^2 , MSFE_{adj}, and ΔU in the moving and expanding window analysis. R_{OS}^2 measures the percent reduction in mean square forecast error for the predictive regression forecast based on the multifractal characteristic given in the first column relative to the historical average benchmark forecast. The adjusted MSFE statistic evaluates the significant level of $R_{OS}^2 > 0$. ΔU is the utility gain between the portfolio based on the multifractal predicting model and the portfolio based on the historical mean model. Note that *, **, and *** indicate the significant level of 5%, 1%, and 0.1%, respectively

$\Delta U = \hat{U} - \bar{U}$. This utility gain can be interpreted as the portfolio management fee that an investor would be willing to pay to have access to the information in the predictive regression forecast relative to the information in the historical average forecast alone (Rapach & Zhou, 2013). Because of the trading constraint, the optimal ω_t is limited in the range of [0, 1.5]. The utility gain ΔU is reported in Table 5 for different multifractal characteristics. We must note that significant R_{OS}^2 doesn't ensure positive utility gains, according to the results of the south Asia stock market (Rahman et al., 2019). Comparing with the expanding window, the utility gains in the moving windows are much more volatile, which is in line with the intuition that the latest information are more valuable for investments. And the positive utility gains of the multifractal characteristic $\Delta\alpha$ and Δf for SH180 and SZCI in the moving window tests and the expanding window tests indicate that a mean-variance investor is willing to pay a portfolio management fee to follow the forecasting returns giving by our multifractal predictive models, as opposed to the historical means.

5 Robustness tests

To check the predicting ability of multifractal characteristics on the financial returns in different markets, we perform the same analysis on the S&P 500 index, the exchange rate between Bitcoin and US dollar, and the Chinese commodity futures of bean pulp, corn, copper, and natural rubber. The S&P 500 index covers from January 2, 1989 and the Bitcoin

covers from February 10, 2014 to January 18, 2018. The prices of four commodity futures are recorded from March 10, 2005 to May 23, 2022 for bean pulp, from April 11, 2005 to May 23, 2022 for corn, from April 25, 2006 to May 23, 2022 for cooper, and from February 28, 2008 to May 23, 2022 for natural rubber. The corresponding results are reported in Table 6. One can find that all R_{OS}^2 are significantly positive and all ΔU are not less than zero in the moving window and expanding window analysis, supporting that the multifractal characteristic is a robust excess return predictor in different markets.

We further separate the sample period of SH180 and SZCI into different sub-periods, including the pre-COVID period (4 June, 2003–9 January, 2020), the COVID period (10 January, 2020 - 23 May, 2022), the period before the market crash in 2015 (4 June, 2003–15 June, 2015), the period after the market crash in 2015 (16 June, 2015–23 May, 2022), the period of bull market (30 June, 2014–June 15, 2015), and the period of bear market (June 16, 2015–June 30, 2016). The predicting performance of multifractal characteristics is investigated in these sub-periods. The statistics of performance are listed in Table 7. Again, one can see that the regressing results in sub-periods are very similar to those in the entire periods. $\Delta\alpha$ and Δf exhibit significant predictability as R_{OS}^2 is significantly positive and ΔU is not less than zero. This result indicates that the return predicting power of multifractal characteristic is robust in different sub-periods. Furthermore, it is also observed that multifractal characteristics exhibit higher return predictability in the period after the market crash in 2015 and the period of bear market. The possible reason is that volatile market increases the strength of multifractality, and in turn improves the return predictability, which motivates us to test the economic explanation on the return predictability of multifractal characteristics in the following section.

6 Economic explanations

In this section, we try to explore the economic explanations on why the multifractal characteristics can predict the future returns. As the multifractal characteristics are used to capture the market volatility, one possible explanation is that the multifractal measurements reflect the market volatility risks. As the multifractality is originated from the fat-tailed distribution and more extreme events can lead to stronger multifractality, thus another possible explanation is that multifractal characteristics reflect the the market tail risk. We thus employ the realized volatility (RV) (respectively, the value at risk (VaR) and expected shortfall (ES)) to measure the market risk (respectively, the tail risk) and run the following regression,

$$RF_{i,d+1} = \alpha_v + \sum_i \beta_i M_{i,d-4,d} + \epsilon_{d+1} \quad (17)$$

where $RF_1 = RV$, $RF_2 = VaR$, $RF_3 = ES$, $M_1 = \Delta\alpha$, $M_2 = \Delta f$, $M_3 = B$.

Table 8 reports the regressing results. For both indexes, the coefficients of $\Delta\alpha$ are significant at the level of 0.1%, indicating that the spectral width $\Delta\alpha$ is able to explain the three risk measures and agrees with the idiosyncratic volatility puzzle and the idiosyncratic tail risk puzzle in the Chinese stock markets (Nartea et al., 2013; Gu et al., 2018; Wan, 2018; Long et al., 2018). The predictor Δf is also statistically significant for the RV , VaR and ES of both indexes, indicating that the spectral height Δf has explanatory power for the volatility risk and the downside tail risk. Therefore, both $\Delta\alpha$ and Δf can be regarded as a risk measure including the information of market volatility and downside tail risk.

Table 6 Robustness tests of out-of-sample predicting excess returns from different markets with multifractal characteristics

Excess return	$M_{d-4;d}$	Moving window			Expanding window		
		R^2_{OS} (%)	MSFE_adj	ΔU (%)	R^2_{OS} (%)	MSFE_adj	ΔU (%)
S&P 500	$\Delta\alpha$	0.13	0.66*	0.0620	0.32	1.97*	0.0000
S&P 500	Δf	0.99	3.98**	0.0064	0.23	5.01**	0.0000
Bitcoin	$\Delta\alpha$	0.47	-1.30*	2.2031	0.85	1.26*	0.0240
Bitcoin	Δf	0.32	-2.00*	6.5270	0.32	-0.98**	0.0000
Bean pulp	$\Delta\alpha$	0.25	2.80**	0.1203	0.41	3.68***	0.0045
Bean pulp	Δf	0.23	0.94*	0.7795	0.00	2.82**	0.0231
Corn	$\Delta\alpha$	0.41	-1.58*	0.0881	0.04	2.32*	0.0012
Corn	Δf	0.05	-1.36*	0.5512	0.12	-0.38**	0.0055
Copper	$\Delta\alpha$	0.52	2.58**	0.5276	0.08	3.21*	0.0000
Copper	Δf	0.26	1.36*	1.2387	0.04	-0.29*	0.0000
Natural rubber	$\Delta\alpha$	0.43	3.28**	0.1416	0.31	2.92**	0.1052
Natural rubber	Δf	0.56	1.66*	0.5647	0.08	3.70***	0.0061

This table lists the statistics of R^2_{OS} , MSFE_adj, and ΔU in the moving expanding window analysis. R^2_{OS} measures the percent reduction in mean square forecast error for the predictive regression forecast based on the multifractal characteristic given in the first column relative to the historical average benchmark forecast. The adjusted MSFE statistic evaluates the significant level of $R^2_{OS} > 0$. ΔU is the utility gain between the portfolio based on the multifractal predicting model and the portfolio based on the historical mean model. Note that *, **, and *** indicates the significant level of 5%, 1%, and 0.1%, respectively

Table 7 Robustness tests of out-of-sample predicting excess returns in different periods with multifractal characteristics

Excess return	M_d-4d	Moving window			Expanding window		
		R^2_{OS} (%)	MSFE_adj	ΔU (%)	R^2_{OS} (%)	MSFE_adj	ΔU (%)
<i>Panel A: Pre-COVID period (4 June, 2003 - 9 January, 2020)</i>							
SH180	$\Delta\alpha$	0.70	2.77**	0.0061	0.06	1.31***	0.0002
SH180	Δf	0.30	0.15*	0.0016	0.08	0.88*	0.0001
SZCI	$\Delta\alpha$	0.20	2.36*	0.0035	0.17	0.49*	0.0002
SZCI	Δf	0.04	1.94*	0.0043	0.06	2.51*	0.0005
<i>Panel B: COVID period (10 January, 2020 - 23 May, 2022)</i>							
SH180	$\Delta\alpha$	0.11	-0.95*	0.0001	0.05	0.38*	0.0000
SH180	Δf	0.23	-0.68*	0.0004	0.37	-0.69	0.0000
SZCI	$\Delta\alpha$	0.56	0.24*	0.0003	0.02	0.84***	0.0000
SZCI	Δf	0.29	-0.13	0.0004	0.61	-2.16*	0.0000
<i>Panel C: Before the market crash in 2015 (4 June, 2003 - 15 June, 2015)</i>							
SH180	$\Delta\alpha$	0.66	3.20**	0.0076	0.03	1.70**	0.0002
SH180	Δf	0.21	0.13*	0.0019	0.11	0.31*	0.0001
SZCI	$\Delta\alpha$	0.37	2.87**	0.0048	0.13	0.88*	0.0003
SZCI	Δf	0.24	2.06*	0.0063	0.09	2.69**	0.0006

Table 7 continued

Excess return	$M_{d-4,d}$	Moving window			Expanding window		
		R^2_{OS} (%)	MSFE_adj	ΔU (%)	R^2_{OS} (%)	MSFE_adj	ΔU (%)
<i>Panel D: After the market crash in 2015 (16 June, 2015 - 23 May, 2022)</i>							
SH180	$\Delta\alpha$	3.96	1.17*	0.0020	0.28	2.96**	0.0001
SH180	Δf	0.38	-1.41*	0.0007	0.65	-3.00**	0.0000
SZCI	$\Delta\alpha$	1.37	-0.63*	0.0006	0.21	-0.24*	0.0000
SZCI	Δf	0.47	0.69*	0.0003	0.49	-0.05*	0.0001
<i>Panel E: Bull market (30 June, 2014 - 15 June, 2015)</i>							
SH180	$\Delta\alpha$	0.20	-0.31*	0.0000	0.93	0.26	0.0000
SH180	Δf	0.43	1.24**	0.0029	0.62	2.55*	0.0000
SZCI	$\Delta\alpha$	0.04	-0.68*	0.0002	0.41	-0.74**	0.0000
SZCI	Δf	0.84	0.32*	0.0107	1.13	-0.28*	0.0001
<i>Panel F: Bear market (16 June, 2015 - 30 June, 2016)</i>							
SH180	$\Delta\alpha$	8.29	2.01*	0.0132	1.04	3.93*	0.0004
SH180	Δf	0.43	-0.02	0.0001	0.88	-3.64***	0.0000
SZCI	$\Delta\alpha$	2.42	0.15*	0.0012	0.43	0.96*	0.0000
SZCI	Δf	-0.90	0.04*	0.0000	0.21	-1.32*	0.0002

This table lists the statistics of R^2_{OS} , MSFE_adj, and ΔU in the moving expanding window analysis. R^2_{OS} measures the percent reduction in mean square forecast error for the predictive regression forecast based on the multifractal characteristic given in the first column relative to the historical average benchmark forecast. The adjusted MSFE statistic evaluates the significant level of $R^2_{OS} > 0$. ΔU is the utility gain between the portfolio based on the multifractal predicting model and the portfolio based on the historical mean model. Note that *, **, and *** indicates the significant level of 5%, 1%, and 0.1%, respectively

Table 8 Linking multifractal characteristics $M_{d-4:d}$ from day $d - 4$ to day d with risk measures on day $d + 1$

	<i>RV</i>	<i>VaR</i>	<i>ES</i>	<i>RV</i>	<i>VaR</i>	<i>ES</i>
	Panel A: SH180			Panel B: SZCI		
$\Delta\alpha$	0.0027*** (0.0006)	0.0222*** (0.0050)	0.0302*** (0.0056)	0.0144*** (0.0024)	0.0040 (0.0053)	0.0185** (0.0067)
Δf	0.0160*** (0.0013)	0.0382*** (0.0032)	0.0428*** (0.0037)	0.0180*** (0.0016)	0.0297*** (0.0036)	0.0326*** (0.0042)
<i>B</i>	0.0004** (0.0002)	0.0012** (0.0004)	0.0013** (0.0005)	0.0012*** (0.0002)	0.0012* (0.0005)	0.0010 (0.0006)
R^2	3.58%	3.81%	4.64%	3.81%	1.53%	1.78%

This table reports the regression coefficients β of realized volatility (*RV*), value at risk (*VaR*), and expected shortfall (*ES*), and the adjusted R^2 statistics. Note that *, **, and *** indicate the significant level of 5%, 1%, and 0.1%, respectively. Numbers in parentheses are standard deviations

7 Conclusion

In this paper, we apply MF DFA to detect the multifractal characteristics in high-frequency data of SH180 and SZCI. We find that both indexes exhibit strong multifractality. We extract three measures of multifractal spectra ($\Delta\alpha$, Δf , and *B*) from returns in moving windows with a size of five days and found that there is a positive correlation between $\Delta\alpha$ and future excess returns, and a negative correlation between Δf and future excess returns. With the factor model, the return predictability is observed for the multifractal spectral width $\Delta\alpha$ in both in-sample and out-of-sample tests, which is further corroborated by the robustness tests on the S&P 500 index, the exchange rate between Bitcoin and US dollar, four Chinese commodity futures, and the SH180 and SZCI in different sub-periods.

One possible explanation of such predicting ability is that multifractal characteristics can be linked to market volatility and the return predictability may be economically explained by the theory of risk premium. Another possible explanation is that the multifractality in stock returns originates from the fat-tailed distribution, which may contain the information of downside tail risks. Our work suggests that the multifractal characteristics $\Delta\alpha$ and Δf jointly include the information of market volatility and downside tail risk, which indicates that multifractality may have potential applications on the risk management. On the one hand, one can design better risk measures based on multifractal characteristics. On the other hand, one can develop hybrid models by combining multifractality and machine learning techniques to improve the accuracy of volatility prediction (Pradeepkumar & Ravi, 2016; Pradeepkumar & Ravi, 2017; Pradeepkumar & Ravi, 2017b; Pradeepkumar & Ravi, 2020; Ravi et al., 2017). Furthermore, the future work can also be focused on testing the return predictability of multifractal characteristics on various industry sectors and regions, and developing trading strategies based on multifractal characteristics.

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